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# BLOW UP AT INFINITY OF WEAK SOLUTIONS FOR A HIGHER-ORDER PARABOLIC EQUATION WITH LOGARITHMIC NONLINEARITY

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Abstract. The main goal of this work is to study the inital boundary value problem for a higher-order parabolic equation with logarithmic source term

$$u_t + (-\Delta)^m u = u \ln |u| .$$

We obtain blow-up at $+\infty$ of weak solutions, by employing potential well technique. This improves and extends some previous studies.

## 1. Introduction

In this paper, we cosider the following higher-order parabolic problem with logarithmic nonlinearity

$$\begin{cases} u_t + Au = u \ln |u| , & x \in \Omega, \quad t > 0, \\ D^\gamma u(x,t) = 0, \quad |\gamma| \le m-1, & x \in \partial\Omega, \quad t > 0, \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{1.1}$$

where $A = (-\Delta)^m$, $m \geq 1$ a positive integer, $\Omega$ is a bound domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$, $\gamma = (\gamma_1, \gamma_2, ..., \gamma_n)$ is multi-index, $\gamma_i$ $(i = 1, 2, ..., n)$ are non-negative integers, $|\gamma| = \gamma_1 + \gamma_2 + ... + \gamma_n$, $D^\gamma = \frac{\partial^{|\gamma|}}{\partial x_1^{\gamma_1} \partial x_2^{\gamma_2} ... \partial x_n^{\gamma_n}}$ are multi-index derivative operator, $\Delta = \sum\limits_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$ is the Laplace operator.

When $m = 1$, the equation (1.1) becomes a heat equation as follows

$$u_t - \Delta u = u \ln |u| . \tag{1.2}$$

In the equation (1.2), Chen et al. [2] obtained under some suitable conditions for the global existence, decay estimate and blow-up at $+\infty$ of weak solutions, via the logarithmic Sobolev inequality and potential well technique. Also, Han [5] obtained the blow-up at infinity of solutions, via the logarithmic Sobolev inequality. Additionally, Chen and Tian [3] obtained the global existence of solution, blow-up at $+\infty$ of solution, by adding strong damping term to the equation (1.2).

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Peng and Zhou [10] studied the following semilinear heat equation with logarithmic nonlinearity

$$u_t - \Delta u = u^{p-2} u \ln |u| ,$$

where $2 < p$. They studied the existence of the unique global weak solutions and blow-up in the finite time of weak solutions, via potential well technique and energy technique.

Li and Liu [8] established a class of fourth-order parabolic equation with logarithmic source term as follows

$$u_t + \Delta^2 u = u^{p-2} u \ln |u| ,$$

where $2 < p$. They studied the existence of global solutions, by using potential well technique. In addition, they also studied result of decay and finite time blow-up for weak solutions.

Nhan and Truong [9] studied the following nonlinear pseudo-parabolic equation

$$u_t - \Delta u_t - div\left(|\nabla u|^{p-2} \nabla u\right) = |u|^{p-2} u \log |u| .$$

They obtained results as regard the existence or non-existence of global solutions. Also, He et al. [6] proved the decay and the finite time blow-up for weak solutions of the equation.

Resently many other authors investigated higher-order hyperbolic and parabolic type equation [4, 7, 11, 12, 13, 14, 15]. Ishige et al. [7] studied the Cauchy problem for nonlinear higher-order heat equation as follows

$$u_t + (-\Delta)^m u = |u|^p .$$

They obtained existence of solutions of the Cauchy problem by introducing a new majorizing kernel. In addition, they studied the local existence of solutions under the different conditions.

Xiao and Li [13] considered initial boundary value problem for nonlinear higher-order heat equations of

$$u_t + (-\Delta)^m u_t + (-\Delta)^m u = f(u).$$

They established the existence of weak solution to the static problem, by using the potential well technique.

The remainder of our work is organized as follows. In Section2, some important Lemmas are given. In Section 3, the main result is proved.

## 2. Preliminaries

Let $\|u\|_{H^m(\Omega)} = \left(\sum_{|\gamma|\le m} \|D^\gamma u\|^2_{L^2(\Omega)}\right)^{\frac{1}{2}}$ denote $H^m(\Omega)$ norm, let $H_0^m(\Omega)$ denote the closure in $H^m(\Omega)$ of $C_0^\infty(\Omega)$. Let $\|.\|_r$ and $\|.\|$ denote the usual $L^r(\Omega)$ norm and $L^2(\Omega)$ norm.

For $u \in H_0^m(\Omega)\backslash\{0\}$, we define the energy functional

$$J(u) = \frac{1}{2}\left\|A^{\frac{1}{2}} u\right\|^2 - \frac{1}{2}\int_\Omega |u|^2 \ln |u|\, dx + \frac{1}{4}\|u\|^2 , \tag{2.1}$$

and Nehari functional

$$I(u) = \left\|A^{\frac{1}{2}} u\right\|^2 - \int_\Omega |u|^2 \ln |u|\, dx. \tag{2.2}$$

By (2.1) and (2.2), we obtain

$$J(u)=\frac{1}{2}I(u)+\frac{1}{4}\|u\|^{2}. \tag{2.3}$$

Further, let

$$d=\inf_{u\in\mathcal{N}}J(u), \tag{2.4}$$

denote the potential depth, where $\mathcal{N}$ is the Nehari manifold, which is defined by

$$\mathcal{N}=\{u\in H_0^m(\Omega)\backslash\{0\}:I(u)=0\}.$$

**Lemma 2.1.** [1]. *Let $k$ be a number with $2\le k<+\infty$, $n\le 2m$ and $2\le k\le\frac{2n}{n-2m}$, $n>2m$. Then there is a constant $C$ depending*

$$\|u\|_k\le C\left\|A^{\frac{1}{2}}u\right\|,\quad \forall u\in H_0^m(\Omega).$$

**Lemma 2.2.** *$J(t)$ is a nonincreasing function for $t\ge 0$ and*

$$J'(u)=-\int_\Omega u_t^2dx\le 0. \tag{2.5}$$

*Proof.* Multiplying the equation (1.1) by $u_t$ and integrating on $\Omega$, we get

$$\int_\Omega u_t^2dx+\int_\Omega Auu_tdx=\int_\Omega uu_t\ln|u|\,dx.$$

By straightforward calculation, we obtain

$$\int_\Omega u_t^2dx+\frac{1}{2}\frac{d}{dt}\left\|A^{\frac{1}{2}}u\right\|^2=\frac{1}{2}\frac{d}{dt}\int_\Omega|u|^2\ln|u|\,dx-\frac{1}{4}\frac{d}{dt}\|u\|^2,$$

which yields that

$$\frac{1}{2}\frac{d}{dt}\left\|A^{\frac{1}{2}}u\right\|^2-\frac{1}{2}\frac{d}{dt}\int_\Omega|u|^2\ln|u|\,dx+\frac{1}{4}\frac{d}{dt}\|u\|_2^2=-\int_\Omega u_t^2dx.$$

Thus, we get

$$\frac{d}{dt}\left(\frac{1}{2}\left\|A^{\frac{1}{2}}u\right\|^2-\frac{1}{2}\int_\Omega|u|^2\ln|u|\,dx+\frac{1}{4}\|u\|_2^2\right)=-\int_\Omega u_t^2dx. \tag{2.6}$$

By 2.1 and 2.6, we obtain

$$\frac{d}{dt}J(u)=-\int_\Omega u_t^2dx. \tag{2.7}$$

Moreover, Integrating (2.7) with respect to $t$ on $[0,t]$, we arrive at

$$\int_0^t\|u_s(s)\|^2ds+J(u(t)=J(u_0). \tag{2.8}$$

□

**Lemma 2.3.** *Let $u\in H_0^m(\Omega)\backslash\{0\}$ and $j(\lambda)=J(\lambda u)$. Then we get*

(i) $\lim_{\lambda\to 0^+}j(\lambda)=0$ *and* $\lim_{\lambda\to+\infty}j(\lambda)=-\infty$,
(ii) *there is a unique $\lambda^*>0$ such that $j'(\lambda^*)=0$,*

(iii) *$j(\lambda)$ is increasing on $(0,\lambda^*)$, decreasing on $(\lambda^*,+\infty)$ and taking the maximum at $\lambda^*$,*

(iv) *$I(\lambda u)>0$ for $\lambda\in(0,\lambda^*)$, $I(\lambda u)<0$ for $\lambda\in(\lambda^*,+\infty)$ and $I(\lambda^* u)=0$.*

*Proof.* By the definition of $j$, for $u\in H_0^1(\Omega)\backslash\{0\}$, we get

$$j(\lambda)=\frac{\lambda^2}{2}\left\|A^{\frac{1}{2}}u\right\|^2-\frac{\lambda^2}{2}\int_\Omega |u|^2\ln|u|\,dx-\frac{\lambda^2}{2}\ln\lambda\,\|u\|_2^2+\frac{\lambda^2}{4}\|u\|^2. \tag{2.9}$$

By (2.9), we have

$$\begin{aligned}\frac{d}{d\lambda}j(\lambda) &= \lambda\left\|A^{\frac{1}{2}}u\right\|^2-\lambda\int_\Omega |u|^2\ln|u|\,dx\\ &\quad -\lambda\ln\lambda\,\|u\|^2-\frac{\lambda}{2}\|u\|^2+\frac{\lambda}{2}\|u\|^2\\ &= \lambda\left(\left\|A^{\frac{1}{2}}u\right\|^2-\int_\Omega |u|^2\ln|u|\,dx-\ln\lambda\,\|u\|^2\right).\end{aligned}$$

Moreover, by taking

$$\lambda^*=\lambda^*(u)=\exp\left(\frac{\left\|A^{\frac{1}{2}}u\right\|^2-\int_\Omega |u|^2\ln|u|\,dx}{\|u\|^2}\right)$$

By (2.2), we get

$$\begin{aligned}I(\lambda u) &= \left\|A^{\frac{1}{2}}(\lambda u)\right\|^2-\int_\Omega |\lambda u|^2\ln|\lambda u|\,dx\\ &= \lambda^2\left\|A^{\frac{1}{2}}u\right\|^2-\lambda^2\int_\Omega |u|^2\ln|u|\,dx-\lambda^2\ln\lambda\,\|u\|^2\\ &= \lambda j'(\lambda).\end{aligned}$$

So, $I(\lambda u)>0$ for $\lambda\in(0,\lambda^*)$, $I(\lambda u)<0$ for $\lambda\in(\lambda^*,+\infty)$ and $I(\lambda^* u)=0$. Therefore, the proof is completed. □

**Lemma 2.4.** *$d$ defined by (2.4) is positive and there exists a positive function $u\in\mathcal{N}$ such that $J(u)=d$.*

*Proof.* Let $\{u_r\}_r^\infty\subset\mathcal{N}$ be a minimizing sequence for $J$, which means that

$$\lim_{r\to\infty}J(u_r)=d. \tag{2.10}$$

We can easy show that $\{|u_r|\}_r\subset\mathcal{N}$ is also a minimizing sequence for $J$ due to $|u_r|\in\mathcal{N}$ and $J(|u_r|)=J(u_r)$. Therefore, we can suppose that $u_r>0$ a.e. $\Omega$ for all $r\in N$.

Moreover, we have already observed that $J$ is coercive on $\mathcal{N}$ which satisfies that $\{u_r\}_r^\infty$ is bounded in $H_0^m(\Omega)$. Let $\mu>0$ be small enough such that $2+\mu<\frac{2n}{n-2}$. Since $H_0^m(\Omega)\hookrightarrow L^{2+\mu}(\Omega)$ is compact, so there exists a function $u$ and a subsequence of $\{u_r\}_r^\infty$, still denote by $\{u_r\}_r^\infty$, such that

$$u_r\to u \text{ weakly in } H_0^m(\Omega),$$

$$u_r\to u \text{ strongly in } L^{2+\mu}(\Omega),$$

$$u_r(x)\to u(x) \text{ a.e. in } \Omega.$$

Also, $u \geq 0$ a.e. in $\Omega$. First, we prove $u \neq 0$. From the dominated convergence theorem, we have

$$\int_\Omega |u|^2 \ln |u| \, dx = \lim_{r\to\infty} \int_\Omega |u_r|^2 \ln |u_r| \, dx, \tag{2.11}$$

and

$$\int_\Omega |u|^2 \, dx = \lim_{r\to\infty} \int_\Omega |u_r|^2 \, dx. \tag{2.12}$$

From the weak lower semicontinuity of $H_0^m(\Omega)$, we get

$$\left\| A^{\frac{1}{2}} u \right\|^2 \leq \liminf_{r\to\infty} \left\| A^{\frac{1}{2}} u_r \right\|^2. \tag{2.13}$$

Then it follows from (2.1), (2.10), (2.11), (2.12) and (2.13) that

$$\begin{aligned} J(u) &= \frac{1}{2} \left\| A^{\frac{1}{2}} u \right\|^2 - \frac{1}{2} \int_\Omega |u|^2 \ln |u| \, dx + \frac{1}{4} \|u\|^2 \\ &\leq \liminf_{r\to\infty} \frac{1}{2} \left\| A^{\frac{1}{2}} u_r \right\|^2 - \lim_{r\to\infty} \frac{1}{2} \int_\Omega |u_r|^2 \ln |u_r| \, dx + \lim_{r\to\infty} \frac{1}{4} \|u_r\|^2 \\ &= \liminf_{r\to\infty} \left( \frac{1}{2} \left\| A^{\frac{1}{2}} u_r \right\|^2 - \frac{1}{2} \int_\Omega |u_r|^2 \ln |u_r| \, dx + \frac{1}{4} \|u_r\|^2 \right) \\ &= \liminf_{r\to\infty} J(u_r) = d. \end{aligned} \tag{2.14}$$

Using (2.2), (2.11) and (2.13), we have

$$\begin{aligned} I(u) &= \left\| A^{\frac{1}{2}} u \right\|^2 - \int_\Omega |u|^2 \ln |u| \, dx \\ &\leq \liminf_{r\to\infty} \left\| A^{\frac{1}{2}} u_r \right\|^2 - \lim_{r\to\infty} \int_\Omega |u_r|^2 \ln |u_r| \, dx \\ &= \liminf_{r\to\infty} \left( \left\| A^{\frac{1}{2}} u_r \right\|^2 - \int_\Omega |u_r|^2 \ln |u_r| \, dx \right) \\ &= \liminf_{r\to\infty} I(u_r) = 0. \end{aligned} \tag{2.15}$$

Since $u_r \in \mathcal{N}$, we have $I(u_r) = 0$. So, by Lemma 1 and the fact $x^{-\mu} \ln x \leq (e\mu)^{-1}$ for $x \geq 1$, we get

$$\begin{aligned} \left\| A^{\frac{1}{2}} u_r \right\|^2 &= \int_\Omega |u_r|^2 \ln |u_r| \, dx \\ &\leq (e\mu)^{-1} \int_\Omega |u_r|^{2+\mu} \, dx \\ &= (e\mu)^{-1} \|u_r\|_{2+\mu}^{2+\mu} \\ &\leq C \left\| A^{\frac{1}{2}} u_r \right\|_2^{2+\mu}, \end{aligned}$$

where $C$ is Sobolev emdedding constant. This satisfies that

$$\int_\Omega |u_r|^2 \ln |u_r| \, dx = \left\| A^{\frac{1}{2}} u_r \right\|^2 \geq C. \tag{2.16}$$

By (2.11) and (2.16), we have

$$\int_\Omega |u|^2 \ln |u| \, dx \geq C.$$

Thus, we have $u \in H_0^m(\Omega)\backslash\{0\}$.

If $I(u_r) < 0$, from Lemma 3, there exists a $\lambda^*$ such that $I(\lambda^* u) = 0$ and $0 < \lambda^* < 1$. Thus, $\lambda^* u \in \mathcal{N}$. It follows from (2.3), (2.4), (2.12) and (2.13) that

$$\begin{aligned} d &\leq J(\lambda^* u) \\ &= \frac{1}{2} I(\lambda^* u) + \frac{1}{4} \|\lambda^* u\|^2 \\ &= \frac{(\lambda^*)^2}{4} \|u\|^2 \\ &\leq (\lambda^*)^2 \liminf_{r\to\infty} \frac{1}{4} \|u_r\|^2 \\ &= (\lambda^*)^2 \liminf_{r\to\infty} J(u_r) \\ &= (\lambda^*)^2 d, \end{aligned}$$

which indicates $\lambda^* \geq 1$ by $d > 0$. It contradicts $0 < \lambda^* < 1$. By (2.15), we have $I(u) = 0$. For this reason, $u \in \mathcal{N}$. From (2.10), we have $J(u) \geq d$. From (2.14), we have $J(u) \leq d$. So, $J(u) = d$. □

## 3. Main results

**Definition 3.1.** (Maximal Existence Time). Assume that $u(t)$ be weak solutions of problem (1.1). We define the maximal existence time $T_{\max}$ as follows

(i) If $u(t)$ exists for all $0 \leq t < \infty$, then $T_{\max} = +\infty$ ;
(ii) If there exists a $t_0 \in (0, \infty)$ such that $u(t)$ exists for $0 \leq t < t_0$, but doesn't exists at $t = t_0$, then $T_{\max} = t_0$.

**Definition 3.2.** (Blow-up at $+\infty$). Let $u(t)$ be a weak solution of (1.1). We call $u(t)$ blow-up at $+\infty$ if the maximal existence time $T_{\max} = +\infty$ and

$$\lim_{t\to+\infty} \|u(t)\|^2 = +\infty.$$

**Theorem 3.3.** *Assume that $u_0 \in H_0^m(\Omega)\backslash\{0\}$, $J(u_0) < d$ and $I(u_0) < 0$. Let $u(t)$ be a weak solution to the problem (1.1). Then $u(t)$ blows up at $+\infty$ such that*

$$\lim_{t\to+\infty} \|u(t)\|^2 = \infty.$$

*Proof.* Let $u(t)$ be weak solution of (1.1) with $J(u_0) < d$ and $I(u_0) < 0$. Let $F : [0, \infty) \to \mathbb{R}^+$, and

$$F(t) = \int_0^t \|u(s)\|^2 \, ds. \tag{3.1}$$

Then, a direct calculation gives

$$F'(t) = \|u(t)\|^2 . \tag{3.2}$$

From (2.2) and (3.2), we get

$$\begin{aligned} F''(t) &= 2 \int_\Omega u u_t dx \\ &= 2 \int_\Omega u^2 \ln|u| \, dx - 2 \int_\Omega A u^2 dx \\ &= -2I(u). \end{aligned} \tag{3.3}$$

By (3.2) and (3.3), we get

$$\begin{aligned} F'(t)\ln F'(t) - F''(t) &= \|u(t)\|^2 \ln \|u(t)\|^2 + 2I(u) \\ &= 2\|u(t)\|^2 \ln \|u(t)\| + 2\left\|A^{\frac{1}{2}}u\right\|^2 - 2\int_\Omega |u|^2 \ln |u|\, dx \\ &\geq 0, \end{aligned}$$

which, in turn, yields that

$$(\ln F'(t))' \leq \ln F'(t).$$

This means

$$\ln F'(t) \leq e^t \ln F'(0) = e^t \ln \|u_0\|^2.$$

Then

$$\|u(t)\|^2 \leq \|u_0\|^{e^t}, \quad \forall t \geq 0,$$

which yields that $u(t)$ does not blow up in finite time.

On the other hand, using the Hölder inequality and combining (3.3), we have

$$\begin{aligned} \frac{1}{4}(F'(t))^2 &= \frac{1}{4}\left(\int_0^t F''(s)ds\right)^2 \\ &= \left(\int_0^t \int_\Omega u u_s dx ds\right)^2 \\ &\leq \int_0^t \|u(s)\|^2 ds \int_0^t \|u_s\|^2 ds. \end{aligned} \tag{3.4}$$

From (2.3) and (3.3), it follows

$$\begin{aligned} F''(t) &= -2I(u) \\ &= -4J(u) + \|u\|^2 \\ &\geq -4J(u_0) + 4\int_0^t \|u_s(s)\|^2 ds + \|u\|^2. \end{aligned} \tag{3.5}$$

By Lemma 3, there exists a $\lambda^* \in (0,1)$ such that $I(\lambda^* u(t)) = 0$. Thus, by the definition of $d$, it follows that

$$\begin{aligned} d &= \inf_{u\in\mathcal{N}} J(u) \leq J(\lambda^* u(t)) \\ &= \frac{1}{2}I(\lambda^* u(t)) + \frac{1}{4}\|\lambda^* u(t)\|^2 \\ &= \frac{1}{2}I(\lambda^* u(t)) + \frac{(\lambda^*)^2}{4}\|u(t)\|^2 \\ &\leq \frac{1}{4}\|u(t)\|^2. \end{aligned} \tag{3.6}$$

Combining (3.5) and (3.6), we have

$$\begin{aligned} F''(t) &\geq -4J(u_0) + 4\int_0^t \|u_s(s)\|^2 ds + \|u\|^2 \\ &\geq 4(d - J(u_0)) + 4\int_0^t \|u_s(s)\|^2 ds. \end{aligned} \tag{3.7}$$

Using (3.1), (3.4) and (3.7), we get

$$\begin{aligned} F(t)F''(t) &\geq 4\left(d - J(u_0)\right)F(t) + 4\int_0^t \|u(s)\|^2 \|u_s(s)\|^2\,ds \\ &\geq 4\left(d - J(u_0)\right)F(t) + \left(F'(t)\right)^2. \end{aligned} \tag{3.8}$$

Then, we see that

$$F(t)F''(t) - \left(F'(t)\right)^2 \geq 4\left(d - J(u_0)\right)F(t).$$

By $J(u_0) < d$ and $I(u) < 0$, then we know

$$F(t)F''(t) - \left(F'(t)\right)^2 > 0.$$

On the other hand, by straightforward calculation, it is clear that

$$\left(\ln F(t)\right)' = \frac{F'(t)}{F(t)}, \tag{3.9}$$

and

$$\left(\ln F(t)\right)'' = \frac{F(t)F''(t) - \left(F'(t)\right)^2}{\left(F(t)\right)^2} > 0. \tag{3.10}$$

From (3.10), we know that $\left(\ln F(t)\right)'$ is increasing with respect to $t$, using this fact, integrating (3.9) from $t_0$ to $t$, we get

$$\left(\ln F(t)\right)' = \left(\ln F(t_0)\right)' + \int_{t_0}^t \frac{F(s)F''(s) - \left(F'(s)\right)^2}{\left(F(s)\right)^2}ds,$$

and

$$\begin{aligned} \ln F(t) - \ln F(t_0) &= \int_{t_0}^t \left(\ln F(s)\right)'\,ds \\ &= \int_{t_0}^t \frac{F'(s)}{F(s)}ds \\ &\geq \frac{F'(t_0)}{F(t_0)}\left(t - t_0\right), \end{aligned}$$

where $0 \leq t_0 \leq t$. Then

$$F(t) \geq F(t_0)\exp\left(\frac{F'(t_0)}{F(t_0)}\left(t - t_0\right)\right).$$

Since $F(0) = 0$ and $F'(0) > 0$, we can take $t_0$ small enough such that $F'(t_0) > 0$ and $F(t_0) > 0$. Then for sufficiently large $t$,

$$\begin{aligned} \|u(t)\|^2 &= F'(t) \\ &\geq \frac{F'(t_0)}{F(t_0)}F(t) \\ &\geq F'(t_0)\exp\left(\frac{F'(t_0)}{F(t_0)}\left(t - t_0\right)\right) \\ &= \|u(t_0)\|^2\exp\left(\frac{F'(t_0)}{F(t_0)}\left(t - t_0\right)\right) \\ &\geq \|u_0\|^2\exp\left(\frac{F'(t_0)}{F(t_0)}\left(t - t_0\right)\right), \quad t \geq t_0, \end{aligned}$$

i.e.,

$$\lim_{t\to+\infty} \|u(t)\|^2 = +\infty.$$

This shows that weak solution $u(t)$ of the problem (1.1) blows up at $+\infty$. □

## 4. Conclusion

In this paper, we examined the initial boundary value problem for a higher-order parabolic equation with logarithmic nonlinearity. We obtained blow-up at infinity of weak solution, by using the potential well method and logarithmic convexity method.

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