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CONNECTEDNESS IN TEMPORAL INTUITIONISTIC FUZZY TOPOLOGY IN CHANG'S SENSE

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Abstract. In this paper, connectedness in temporal intuitionistic fuzzy topology in Chang's sense is introduced and investigated. In the content of the paper, basic definitions, theorems and propositions about connectedness in temporal intuitionistic fuzzy topology in Chang's sense are given.

1. Introduction

Fuzzy logic was firstly defined by Zadeh in 1965 [21]. Then,Intuitionistic fuzzy set (shortly IFS) was defined by K.Atanassov [1, 2]. Intuitionistic fuzzy logic comes into play in situations where fuzzy logic cannot respond or is insufficient. The intuitionistic fuzzy set theory is useful in various application areas such as; medicine, medical diagnosis, medical application, career determination, real life situations, education, decision making, multi criteria decision making, artificial intellligence, networking, computer,smart systems, economy and various fields. The concept of fuzzy topology was defined by Chang in 1968 [4]. Çoker generalized the concept of fuzzy topology in the sense of intuitionistic fuzzy set theory in 1997. The fuzzifying of the concept of topology was made by Šostak in 1985 [19]. Coker and Demirci [6] defined the concept of intuitionistic fuzzy set in Šostak's sense in 1996. Temporal intuitionistic fuzzy set, another approach in which temporal variables also participated in calculating the membership and non-membership degrees, was defined by Atanassov in 1991 [3]. This is one of the most important extensions of IFS. In recent years, Šostak's mean temporal intuitionistic fuzzy topology was defined by Kutlu and Bilgin [9]. Also, the other fundamental concepts of Šostak's mean temporal intuitionistic fuzzy topology defined by the author in [10, 9, 11]. The concepts of temporal and overall intuitionistic fuzzy topology in Chang's sense firstly defined by Kutlu in 2019 [13]. In this study, Kutlu gave basic definitions and theorems and explained them in detail. The concept of temporal intuitionistic fuzzy has recently started to attract the attention of researchers [13, 12, 14]. The concept of connectedness in intuitionistic fuzzy topological spaces in Šostak's sense is investigated by

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El-Latif and Khalaf [7]. Connectedness in intuitionistic fuzzy special topological spaces is researched by Özçağ and Çoker [17]. Connectedness in intuitionistic fuzzy topological spaces is investigated by Kim and Abbas [8]. In this paper, connectedness in temporal intuitionistic fuzzy topology in Chang's sense is introduced and investigated. Basic definition, theorem and propositions about connectedness in temporal intuitionistic fuzzy topology in Chang's sense are given.

2. Preliminaries

Definition 2.1. [1] An intuitionistic fuzzy set in a non-empty set X given by a set of ordered triples $A = \{(x, \mu_A(x), \eta_A(x)) : x \in X\}$ where $\mu_A(x) : X \to I$,$\eta_A(x) : X \to I$ and $I = [0, 1]$, are functions such that $0 \leq \mu(x) + \eta(x) \leq 1$for all $x \in X$. For $x \in X$, $\mu_A(x)$ and $\eta_A(x)$ represent the degree of membership and degree of non-membership of x to A respectively. For each $x \in X$; intuitionistic fuzzy index of x in A can be defined as follows $\pi_A(x) = 1 - \mu_A(x) - \eta_A(x)$. π_A is the called degree of hesitation or indeterminacy.

By $IFS(X)$, we denote to the set of all intuitionistic fuzzy sets.

Definition 2.2. [1] Let$A, B \in IFS(X)$. Then,
(i) $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x)$ and $\eta_A(x) \geq \eta_B(x)$for$\forall x \in X$,
(ii) $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$,
(iii) $A^c = \{(x, \eta_A(x), \mu_A(x)) : x \in X\}$,
(iv) $\bigcap A_i = \{(x, \wedge\mu_{A_i}(x), \vee\eta_{A_i}(x)) : x \in X\}$,
(v) $\bigcup A_i = \{(x, \vee\mu_{A_i}(x), \wedge\eta_{A_i}(x)) : x \in X\}$,
(vi) $\tilde{0} = \{(x, 0, 1) : x \in X\}$ and $\tilde{1} = \{(x, 1, 0) : x \in X\}$.

Definition 2.3. [5, 1] Let a and b be two real numbers in $[0, 1]$ satisfying the inequality$a + b \leq 1$. Then, the pair $\langle a, b\rangle$ is called an intuitionistic fuzzy pair. Let $\langle a_1, b_1\rangle$ and $\langle a_2, b_2\rangle$be two intuitionistic fuzzy pair (briefly IF-pair). Then define
(i) $\langle a_1, b_1\rangle \leq \langle a_2, b_2\rangle \Leftrightarrow a_1 \leq a_2$ and $b_1 \geq b_2$,
(ii) $\langle a_1, b_1\rangle = \langle a_2, b_2\rangle \Leftrightarrow a_1 = a_2$ and $b_1 = b_2$,
(iii) If $\{\langle a_i, b_i\rangle ; i \in J\}$ is a family of intuitionistic fuzzy pairs, then
$\vee\langle a_i, b_i\rangle = \langle\vee a_i, \wedge b_i\rangle$ and $\wedge\langle a_i, b_i\rangle = \langle\wedge a_i, \vee b_i\rangle$,
(iv) The complement of $\langle a, b\rangle$is defined by $\overline{\langle a, b\rangle} = \langle b, a\rangle$,
(v) $1^{\sim} = \langle 1, 0\rangle$ and $0^{\sim} = \langle 0, 1\rangle$. .

Definition 2.4. [6] An intuitionistic fuzzy topology in Chang's sense (briefly, CT-IFS) on a non-empty set X is a family τ_t of TIFSs satisfying the following axioms:
I. $\tilde{0} \in \tau$ and $\tilde{1} \in \tau$,
II. $A_1 \cap A_2 \in \tau$ for each A_1, $A_2 \in \tau$,
III. $\bigcup\limits_{i \in I} A_i \in \tau$ For any arbitrary family $\{A_i ; i \in I\} \in \tau$,

Definition 2.5. [3]. Let E be an universe and T be a non-empty time-moment set. We call the elements of T "time moments". Based on the definition of IFS, a temporal intuitionistic fuzzy set (breifly TIFS) A is defined as the following: $A(T) = \{(x, \mu_A(x, t), \eta_A(x, t)) : (x, t) \in E \times T\}$ where:
(a) $A \subseteq E$ is a fixed set
(b) $\mu_A(x, t) + \eta_A(x, t) \leq 1$ for every $(x, t) \in E \times T$
(c) $\mu_A(x, t)$ and $\eta_A(x, t)$ are the degrees of membership and non-membership, respectively, of the element $x \in E$ at the time moment $t \in T$

By $TIFS^{(X,T)}$, we denote to the set of all TIFSs over nonempty set X and time-moment set T. For brevity, we write A instead of$A\left(T\right)$. The hesitation degree of a TIFS is defined as$\pi_A\left(x,t\right)=1-\mu_A\left(x,t\right)-\ \eta_A\left(x,t\right)$. Obviously, every ordinary IFS can be regarded as TIFS for which T is a singleton set. All operations and operators on IFS can be defined for TIFSs.

Definition 2.6. [13] Let $A\left(T'\right)=\left\{\left(x,\mu_A\left(x,t\right),\eta_A\left(x,t\right)\right):\left(x,t\right)\in X\times T'\right\}$ and $B\left(T''\right)=\left\{\left(x,\mu_B\left(x,t\right),\eta_B\left(x,t\right)\right):\left(x,t\right)\in X\times T''\right\}$ where$T'$and T''have finite number of distinct time-elements or they are time intervals. Then,

$$A\left(T'\right)\cap B\left(T''\right)=\left\{\left(x,\ \min\left(\bar{\mu}_A\left(x,t\right),\bar{\mu}_B\left(x,t\right)\right),\max\left(\bar{\eta}_A\left(x,t\right),\bar{\eta}_B\left(x,t\right)\right):\left(x,t\right)\in X\times\left(T'\cup T''\right)\right)\right.$$

$$A\left(T'\right)\cup B\left(T''\right)=\left\{\left(x,\ \max\left(\bar{\mu}_A\left(x,t\right),\bar{\mu}_B\left(x,t\right)\right),\min\left(\bar{\eta}_A\left(x,t\right),\bar{\eta}_B\left(x,t\right)\right):\left(x,t\right)\in X\times\left(T'\cup T''\right)\right)\right.$$

Also from definition of subset in IFS theory, Subsets of TIFS can be defined as the follow: $A\left(T'\right)\subseteq B\left(T''\right)\Leftrightarrow\bar{\mu}_A\left(x,t\right)\leq\bar{\mu}_B\left(x,t\right)$ and $\bar{\eta}_A\left(x,t\right)\geq\bar{\eta}_B\left(x,t\right)$ for every $\left(x,t\right)\in X\times\left(T'\cup T''\right)$ where

$$\bar{\mu}_A\left(x,t\right)=\begin{cases}\mu_A\left(x,t\right), & if\ t\in T'\\ 0, & if\,t\in T''-T'\end{cases}$$

$$\bar{\mu}_B\left(x,t\right)=\begin{cases}\mu_B\left(x,t\right) & if\,t\in T''\\ 0, & if\,t\in T'-T''\end{cases}$$

$$\bar{\eta}_A\left(x,t\right)=\begin{cases}\eta_A\left(x,t\right), & if\,t\in T'\\ 1, & if\,t\in T''-T'\end{cases}$$

$$\bar{\eta}_B\left(x,t\right)=\begin{cases}\eta_B\left(x,t\right), & if\,t\in T''\\ 1, & if\,t\in T''-\ T'\end{cases}$$

It is obviously seen that $\bar{\mu}_A\left(x,t\right)=\mu_A\left(x,t\right)$, $\bar{\mu}_B\left(x,t\right)=\mu_B\left(x,t\right)$, $\bar{\eta}_A\left(x,t\right)=\eta_A\left(x,t\right)$, $\bar{\eta}_B\left(x,t\right)=\eta_B\left(x,t\right)$ when $T'=T''$.

Let Jbe an arbitrary index set. Then we define that $T=\bigcup_{i\in J}T_i$where T_i is a time set for each $i\in J$. Thus, we can extend the definition of union and intersection of TIFSs family $F=\left\{A_i\left(T_i\right)=\left(x,\mu_{A_i}\left(x,t\right),\eta_{A_i}\left(x,t\right)\right):x\in X\times T_i,\,i\in J\right\}$ as follows:

$$\bigcup_{i\in J}A\left(T_i\right)=\left\{\left(x,\ \max_{i\in J}\left(\bar{\mu}_{A_i}\left(x,t\right)\right),\min_{i\in J}\left(\bar{\eta}_{A_i}\left(x,t\right)\right):\ \left(x,t\right)\ \in X\times T\right)\right.,$$

$$\bigcap_{i\in J}A\left(T_i\right)=\left\{\left(x,\ \min_{i\in J}\left(\bar{\mu}_{A_i}\left(x,t\right)\right),\max_{i\in J}\left(\bar{\eta}_{A_i}\left(x,t\right)\right):\ \left(x,t\right)\ \in X\times T\right)\right.,$$

where

$$\bar{\mu}_{A_j}\left(x,t\right)=\begin{cases}\mu_{A_j}\left(x,t\right), & if\,t\in T_j\\ 0, & if\,t\in T-T_j\end{cases}$$

and

$$\bar{\eta}_{A_j}\left(x,t\right)=\begin{cases}\eta_{A_j}\left(x,t\right), & if\,t\in T_j\\ 1, & if\,t\in T-T_j\end{cases}.$$

The operations defined above are defined over all of the time moments. In the following definition, these operations will be defined for an individual time moment.

Definition 2.7. [13] Let

$$A(T') = \{(x, \mu_A(x,t), \eta_A(x,t)) : (x,t) \in X \times T'\}$$
$$B(T'') = \{(x, \mu_B(x,t), \eta_B(x,t)) : (x,t) \in X \times T''\}$$

where T' and T''have finite number of distinct time-elements or they are time intervals. Then, the definitions of instant intersection and instant union of TIFSs are defined as follows:

$$\begin{aligned} A(T') \cap_{t_0} B(T'') &= \{(x, \min(\bar{\mu}_A(x,t_0), \bar{\mu}_B(x,t_0)), \max(\bar{\eta}_A(x,t_0), \bar{\eta}_B(x,t_0)) : \\ &\quad (x,t_0) \in X \times (T' \cup T'')\}, \\ A(T') \cup_{t_0} B(T'') &= \{(x, \max(\bar{\mu}_A(x,t_0), \bar{\mu}_B(x,t_0)), \min(\bar{\eta}_A(x,t_0), \bar{\eta}_B(x,t_0)) : \\ &\quad (x,t_0) \in X \times (T' \cup T'')\}. \end{aligned}$$

Also from definition of subset in IFS theory, instant subsets of TIFS can be defined as the following: $A(T') \subseteq_{t_0} B(T'') \Leftrightarrow \bar{\mu}_A(x,t_0) \leq \bar{\mu}_B(x,t_0)$ and $\bar{\eta}_A(x,t) \geq \bar{\eta}_B(x,t)$ for every $(x,t_0) \in X \times (T' \cup T'')$ where

$$\bar{\mu}_A(x,t_0) = \begin{cases} \mu_A(x,t_0), & if\, t_0 \in T' \\ 0, & if\, t_0 \in T'' - T' \end{cases}$$

$$\bar{\mu}_B(x,t_0) = \begin{cases} \mu_B(x,t_0), & if\, t_0 \in T'' \\ 0, & if\, t_0 \in T' - T'' \end{cases}$$

$$\bar{\eta}_A(x,t_0) = \begin{cases} \eta_A(x,t_0), & if\, t_0 \in T' \\ 1, & if\, t_0 \in T'' - T' \end{cases}$$

$$\bar{\eta}_B(x,t_0) = \begin{cases} \eta_B(x,t_0), & if\, t_0 \in T'' \\ 1, & if\, t_0 \in T'' - T' \end{cases}$$

[13] Let J be an arbitrary index set. Then we define that $T = \bigcup_{i \in J} T_i$where T_i is a time set for each $i \in J$. Thus, we can extend the definition of union and intersection of TIFSs family $F_{t_0} = \{A_i(T_i) = (x, \mu_{A_i}(x,t_0), \eta_{A_i}(x,t_0)) : (x,t_0) \in X \times T_i,\, i \in J\}$ as follows:

$$\bigcup_{i \in J}{}^{t_0} A(T_i) = \left\{ \left(x,\ \max_{i \in J} (\bar{\mu}_{A_i}(x,t_0)), \min_{i \in J} (\bar{\eta}_{A_i}(x,t_0)) : (x,t_0) \in X \times T \right) \right.$$

$$\bigcap_{i \in J}{}^{t_0} A(T_i) = \left\{ \left(x,\ \min_{i \in J} (\bar{\mu}_{A_i}(x,t_0)), \max_{i \in J} (\bar{\eta}_{A_i}(x,t_0)) : (x,t_0) \in X \times T \right) \right.$$

where

$$\bar{\mu}_{A_j}(x,t_0) = \begin{cases} \mu_{A_j}(x,t_0), & if\, t_0 \in T_j \\ 0, & if\, t_0 \in T - T_j \end{cases}$$

$$\bar{\eta}_{A_j}(x,t_0) = \begin{cases} \eta_{A_j}(x,t_0), & if\, t_0 \in T_j \\ 1, & if\, t_0 \in T - T_j \end{cases} .$$

In fact, these TIFS operators can be seen as IFS operators over TIFSs, since they are defined for a single time moment.

Definition 2.8. [13] $\underset{\sim}{0}^t$ and $\underset{\sim}{1}^t \in TIFS^{(X,T)}$ are defined as:

$$\underset{\sim}{0}^t = \{(x,0,1) : (x,t) \in X \times T\}$$

and

$$\underset{\sim}{1}^t = \{(x,1,0) : (x,t) \in X \times T\}$$

for each time moment t, i.e. $\mu_{\underset{\sim}{0}^t}(x,t) = 0$, $\eta_{\underset{\sim}{0}^t}(x,t) = 1$ and $\mu_{\underset{\sim}{1}^t}(x,t) = 1$, $\eta_{\underset{\sim}{1}^t}(x,t) = 0$ for each $(x,t) \in X \times T$.

Definition 2.9. $\underset{\sim}{0}^{t_0}$ and $\underset{\sim}{1}^{t_0} \in TIFS^{(X,T)}$ are defined as:

$$\underset{\sim}{0}^{t_0} = \left\{ \left(x, \mu_{\underset{\sim}{0}^{t_0}}(x,t), \eta_{\underset{\sim}{0}^{t_0}}(x,t) \right) : \ (x,t) \in X \times T \right\}$$

and

$$\underset{\sim}{1}^{t_0} = \left\{ \left(x, \mu_{\underset{\sim}{1}^{t_0}}(x,t), \eta_{\underset{\sim}{1}^{t_0}}(x,t) \right) : \ (x,t) \in X \times T \right\}$$

for individual time moment $t_0 \in T$, i.e. $\mu_{\underset{\sim}{0}^t}(x,t_0) = 0$, $\eta_{\underset{\sim}{0}^t}(x,t_0) = 1$ and $\mu_{\underset{\sim}{1}^{t_0}}(x,t_0) = 1$, $\eta_{\underset{\sim}{1}^{t_0}}(x,t_0) = 0$ for each $(x,t_0) \in X \times \{t_0\}$.

3. MAIN RESULTS

Definition 3.1. [13] An temporal intuitionistic fuzzy topology in Chang's sense (briefly, CT-TIFS) on a non-empty set X is a family τ_{t_0} of TIFSs satisfying the following axioms for fixed time moment t_0

I. $\underset{\sim}{0}^{t_0} \in \tau_{t_0}$ and $\underset{\sim}{1}^{t_0} \in \tau_{t_0}$,

II. For each A_1, $A_2 \in \tau_{t_0}$, there exist a $F \in \tau_{t_0}$ such that $\mu_F(x,t_0) = \mu_{A_1 \cap_{t_0} A_2}(x,t_0)$, $\eta_F(x,t_0) = \eta_{A_1 \cap_{t_0} A_2}(x,t_0)$ for each $(x,t_0) \in X \times \{t_0\}$.

III. For any arbitrary family $\{A_i;\, i \in I\} \in \tau_{t_0}$, there exist a $D \in \tau_{t_0}$ such that $\mu_D(x,t_0) = \mu_{\underset{i\in I}{\bigcup}{}^{t_0} A_i}(x,t_0)$ and $\eta_D(x,t_0) = \eta_{\underset{i\in I}{\bigcup}{}^{t_0} A_i}(x,t_0)$ for each $(x,t_0) \in X \times \{t_0\}$.

The pair $((X,T), \tau_{t_0})$ is called temporal intuitionistic fuzzy topological space in Chang's sense. Any member of τ_{t_0} is called temporal intuitionistic fuzzy open set (TIFOS). On the other hand, the complement of any member of τ_{t_0} is called intuitionistic fuzzy closed set (TIFCS). It is obtained intuitionistic fuzzy topological space in Chang's sense from every temporal intuitionistic fuzzy topological space in Chang's sense by the following method.

Proposition 1. [13] Let τ_{t_0} is an temporal intuitionistic fuzzy topological space in Chang's sense on non-empty set X and time moment set T, Then we define IFS's from every $A \in \tau_{t_0}$TIFSs by following way: $\mu_{\hat{A}}(x) = \mu_{A_i}(x,t_0)$ and $\eta_{\hat{A}}(x) = \eta_A(x,t_0)$. So that the new family $\tau^{t_0} = \left\{ \hat{A} : A \in \tau_{t_0} \right\}$ obtained from τ_{t_0} is a intuitionistic fuzzy topology in Chang's sense.

Definition 3.2. [13] Let τ_{t_0} is an temporal intuitionistic fuzzy topological space in Chang's sense on non-empty set X and time moment set $T$and $A \in \tau_{t_0}$. Then temporal intuitionistic fuzzy interior and temporal intuitionistic fuzzy closure of A defined as follows: $\mathrm{int}_{t_0}(A) = \cup \{G\,;\, G \in \tau_{t_0},\ G \subseteq A\}$, $cl_{t_0}(A) = \cap \left\{C\,;\, \bar{C} \in \tau_{t_0},\ A \subseteq C\right\}$. Following propositions are valid for both of fuzzy and intuitionistic fuzzy case [19, 4, 5, 6, 18, 16, 15], it can be proved as in the above-mentioned articles.

Definition 3.3. Let τ_{t_0} be a temporal intuitionistic fuzzy topological space in Chang's sense on non-empty set X and time moment set. Then,

i. A is a TIFCS in $\tau_{t_0} \Leftrightarrow cl_{t_0}(A) = A$,

ii. A is a TIFOS in $\tau_{t_0} \Leftrightarrow \mathrm{int}_{t_0}(A) = A$,

iii. $cl_{t_0}(A) = \overline{\mathrm{int}_{t_0}(A)}$ for any $A \in TIFS^{(X,T)}$,

iv. $\mathrm{int}_{t_0}(A) = \overline{cl_{t_0}(A)}$ for any $A \in TIFS^{(X,T)}$,

v. $\operatorname{int}_{t_0}(A) \subseteq A$ for any $A \in TIFS^{(X,T)}$,
vi. $A \subseteq cl_{t_0}(A)$ for any $A \in TIFS^{(X,T)}$,
vii. $A \subseteq B \Rightarrow \operatorname{int}_{t_0}(A) \subseteq \operatorname{int}_{t_0}(B)$ for any $A, B \in TIFS^{(X,T)}$,
viii. $A \subseteq B \Rightarrow cl_{t_0}(A) \subseteq cl_{t_0}(B)$ for any $A, B \in TIFS^{(X,T)}$,
ix. $cl_{t_0}(cl_{t_0}(A)) = cl_{t_0}(A)$ for any $A \in TIFS^{(X,T)}$,
x. $\operatorname{int}_{t_0}(\operatorname{int}_{t_0}(A)) = \operatorname{int}_{t_0}(A)$ for any $A \in TIFS^{(X,T)}$,
xi. $\operatorname{int}_{t_0}(A \cap B) = \operatorname{int}_{t_0}(A) \cap \operatorname{int}_{t_0}(B)$ for any $A, B \in TIFS^{(X,T)}$,
xii. $cl_{t_0}(A \cup B) = cl_{t_0}(A) \cup cl_{t_0}(B)$ for any $A, B \in TIFS^{(X,T)}$,
xiii. $\operatorname{int}_{t_0}\left(\underset{\sim}{1}^{t_0}\right) = \underset{\sim}{1}^{t_0}$,
xiv. $cl_{t_0}\left(\underset{\sim}{0}^{t_0}\right) = \underset{\sim}{0}^{t_0}$. We will give definitions of temporal intuitionistic fuzzy continuous functions and open function definitions, which are defined for fuzzy and intuitionistic fuzzy sets in [19, 4, 5, 6, 18, 16, 15], respectively.

Theorem 3.4. $((X,T), \tau_{t_0})$ *is an temporal intuitionistic fuzzy topology in Chang's sense on nonempty set x an time moment T and $X_1, X_2 \neq \emptyset$, $X = X_1 \cup_{t_0} X_2$ and $X_1 \cap_{t_0} X_2 = \emptyset_{t_0}$ are subsets; the following expressions are equivalent:*
(i) $((X,T), \tau_{t_0})$ temporal intuitionistic fuzzy topology in Chang's sense is the topological sum of X_1 and X_2 spaces.
(ii) X_1 and X_2 sets are both temporal intuitionistic fuzzy open set (TIFOS) and temporal intuitionistic fuzzy closed set (TIFCS) in X.
(iii) X_1 (or X_2) is both TIFOS and TIFCS.
(iv) $cl_{t_0}(X_1) \cap_{t_0} X_2 = \emptyset_{t_0}$ and $X_1 \cap_{t_0} cl_{t_0}(X_2) = \emptyset_{t_0}$

Definition 3.5. Let τ_{t_0} is a temporal intuitionistic fuzzy topology in Chang's sense on nonempty set X and time moment set $t_0 \in T$ and $A, B \in \tau_{t_0}$. $((X,T), \tau_{t_0})$ is called to be temporal disconnected at time moment t_0 if there are sets of A and B nonempty set with;

$$\begin{aligned} A \cup_{t_0} B &= X \\ A \cap_{t_0} cl_{t_0}(B) &= \emptyset_{t_0} \\ cl_{t_0}(A) \cap_{t_0} B &= \emptyset_{t_0} \end{aligned}$$

otherwise $((X,T), \tau_{t_0})$ is called to be temporal connected.

Proposition 2. Let $((X,T), \tau_{t_0})$ is a temporal intuitionistic fuzzy topology in Chang's sense, the following expressions are equivalent:
(i) $((X,T), \tau_{t_0})$ temporal intuitionistic fuzzy topological space is temporal disconnected.
(ii) $((X,T), \tau_{t_0})$ has at least both open and closed subset that is nonempty and distinct from itself.

Proposition 3. A temporal intuitionistic fuzzy topological space $((X,T), \tau_{t_0})$ is temporal connected at time moment t_0 if and only if it has no subset, both open and closed, other than empty and itself.

Proof. (i)$\Rightarrow$ (ii) If the temporal intuitionistic fuzzy topological space $((X,T), \tau_{t_0})$ is temporal disconnected, there are sets A and B different from the empty set as $A \cup_{t_0} B = X$, $A \cap_{t_0} cl_{t_0}(B) = \emptyset_{t_0}$ and $cl_{t_0}(A) \cap_{t_0} B = \emptyset_{t_0}$. Since $B = (cl_{t_0}(A))^c$ and $A = (cl_{t_0}(B))^c$; A and B sets are open. But since $A = B^c$; A is both open and closed and $A \neq \emptyset_{t_0}, A \neq X$
(ii)$\Rightarrow$ (i) If $A \in X$ is subset of both open and closed, which is different from the

empty and itself, $A \cup_{t_0} B = X$ and $A \cap_{t_0} B = \emptyset_{t_0}$ as $B = A^c$. Since $A \neq X$; $B \neq \emptyset_{t_0}$. Since A is closed; $cl_{t_0}(A) = A$ and $cl_{t_0}(A) \cap_{t_0} B = \emptyset_{t_0}$. Since A is open, B is closed and $A \cap_{t_0} cl_{t_0}(B) = \emptyset_{t_0}$. Then $((X,T), \tau_{t_0})$ temporal intuitionistic fuzzy topological space is temporal disconnected. □

Proposition 4. $((X,T), \tau_{t_0})$ temporal intuitionistic fuzzy topological space in Chang's sense is temporal connected at time moment t_0, if and only if it has no subset, both open and closed, other than empty and itself.

Theorem 3.6. $((X,T), \tau_{t_0})$ *temporal intuitionistic fuzzy topological space in Chang's sense, the following expressions are equivalent:*
(i) $((X,T), \tau_{t_0})$ *temporal intuitionistic fuzzy topological space in Chang's sense is temporal disconnected at time moment* t_0.
(ii) $((X,T), \tau_{t_0})$ *temporal intuitionistic fuzzy topological space in Chang's sense is the topological sum.*
(iii) X *is the union of two distinct open sets other than empty.*
(iv) X *is the union of two distinct closed sets other than empty.*
(v) X *has at least one subset, both open and closed that is distinct from empty and itself.*

Theorem 3.7. $((X,T), \tau_{t_0})$ *temporal intuitionistic fuzzy topological space in Chang's sense, the following expressions are equivalent:*
(i) $((X,T), \tau_{t_0})$ *temporal intuitionistic fuzzy topological space in Chang's sense is temporal connected at time moment* t_0.
(ii) $((X,T), \tau_{t_0})$ *temporal intuitionistic fuzzy topological space in Chang's sense cannot be any topological sum.*
(iii) X *cannot be written as the union of two distinct open sets other than empty.*
(iv) X *cannot be written as the union of two distinct closed sets other than empty.*
(v) Both open and closed subsets of X *are only* X *and the empty set.*

4. Conclusion

In this paper, connectedness in temporal intuitionistic fuzzy topology in Chang's sense is introduced and investigated. Basic definition, theorem and propositions about connectedness in temporal intuitionistic fuzzy topology in Chang's sense are given.

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The Declaration of Conflict of Interest/ Common Interest

The author(s) declared that no conflict of interest or common interest

The Declaration of Ethics Committee Approval

This study does not be necessary ethical committee permission or any special permission.

The Declaration of Research and Publication Ethics
The author(s) declared that they comply with the scientific, ethical, and citation rules of Journal of Universal Mathematics in all processes of the study and that they do not make any falsification on the data collected. Besides, the author(s) declared that Journal of Universal Mathematics and its editorial board have no responsibility for any ethical violations that may be encountered and this study has not been evaluated in any academic publication environment other than Journal of Universal Mathematics.

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