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Reflective Thinking in The Problem-Solving Process: A Model Proposal

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Abstract

Problem solving is one of the basic skills that individuals need in a lot of fields throughout their lives. Problem solving is among the process standards in mathematics education; mathematics education aims to develop students' problem-solving skills in teaching all content domains. Reflective thinking is an important skill that directly affects problem solving and ensures its successful outcome. This research aims to determine the reflective thinking skills used in problem-solving and to model this process. The research was conducted as a case study, and the participants were selected using criterion sampling. Twenty pre-service primary teachers with different mathematics achievements participated in the study. Data collection tools are worksheets, think-aloud protocol, and semi-structured interview forms. In order to determine the reflective thinking skills used in problem-solving, a non-routine problem was solved by pre-service teachers in the worksheets through the think-aloud protocol and interviews were conducted with pre-service teachers. The data were analyzed considering the components and indicators of reflective thinking. The research results showed that the components of reflective thinking were used in all the problem-solving stages. Based on this result, a model proposal was developed regarding the problem solving process in which reflective thinking is used, and the results were discussed in light of the relevant literature.

Keywords

Problem-solving, reflective problem-solving, reflective thinking, components of reflective thinking.

Ethics Committee Approval: Ethics committee permission for this study was obtained from Ondokuz Mayıs University Social and Human Sciences Research Ethics Committee with the decision dated 26.11.2021 and numbered 11/2021-930.

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INTRODUCTION

Problem-solving is one of the critical skills that children need throughout their lives (Haylock & Cockburn, 2014). Developing problem-solving skills is one of the objectives of the mathematics curriculum in formal education (Ministry of National Education [MNE], 2018). Because, there is a general acceptance that students should be trained as skilled problem solvers through mathematics education (National Council of Teachers of Mathematics [NCTM], 1980; Schoenfeld, 1992; Van de Walle, Karp & Bay-Williams, 2014). Therefore, in mathematics education, mathematical concepts should be taught to students through problem situations based on real-life contexts so that they can transfer their mathematical knowledge and concepts to real life and solve these problems (Schoenfeld, 1992; Van de Walle et al., 2014). Like Gagne, many psychologists and educators see problem-solving as the most important learning achievement in life (Jonassen, 2000).

While solving a problem, students perform different cognitive actions; such as understanding the problem statement, selecting the necessary data for the solution, applying concepts and operations to the solution, solving the problem, and deciding whether the solution is correct. These actions contribute to the student's cognitive development (Bernardo, 1999; Charles, 1985). Because with problem-solving, children are taught not only the rules and strategies specific to the subject but also ways of thinking and approaches that can be used to develop a rule, formula, and self-learning strategies (Donnelly & Fitzmaurice, 2005; NCTM, 2000; Olkun & Toluk Uçar, 2012). Besides, problem-solving includes high-level mental skills such as critical thinking, reflective thinking, decision-making, and questioning (Demirel, 2002). In this process, students gain ways of thinking, curiosity, and confidence that they can use in situations they encounter outside of school and overcome difficulties. Therefore, problem-solving should become a lifestyle for students (Altun, 2014; NCTM, 2000).

The Stages of Problem Solving

Different models have been proposed in the literature on the stages of problem-solving. The first of these belongs to Dewey (1910). Dewey (1910, p. 72) explained five stages of problem-solving: "a felt difficulty, its location and definition, suggestion of possible solution, development by the reasoning of the bearings of the suggestion, and further observation and experiment leading to its acceptance or rejection." The famous mathematician George Polya (1945) developed the most known problem-solving model. He listed problem-solving stages as understanding the problem, devising a plan, carrying out the plan, and looking back. Polya's problem-solving model is dynamic. While determining a strategy, the student can reread it, thinking he did not understand the problem. He can go back to devising a plan or implementing the plan while evaluating the solution; can make a new plan, or try a different method for the solution. These stages are as follows (Altun, 2014; Baykul, 2011; Polya, 1945; Van de Walle et al., 2014):

1. Understanding the problem: In this stage, it is essential to understand the information given in the problem. This stage includes determining what is given and requested in the problem and summarizing it. The student reads the problem and explains it in his own words.

2. Devising a plan: The student thinks about how to solve the problem. The relationships between what is given and what is requested are investigated. He determines how to solve the problem. It includes actions such as making a plan for the solution, benefiting from the solution of a similar problem that has been solved before, determining the relations, and dividing the problem into sub-stages.

3. Carrying out the plan: At this stage, the student solves the problem by using the methods determined before. He carries out the plan, constructs mathematical sentences, performs the operations in order, and tries to reach the result.

4. Looking back: The student checks the result to determine whether it is correct. Looking back is a critical step in the problem-solving process, but many students overlook it when they reach the answer in the third stage. According to Polya (1945), at this stage, the student examines the problem-solving stages from beginning to end and explains why the strategy was successful or unsuccessful. He identifies the situations that work and those that do not in the problem-solving process. Therefore, this stage provides the student with essential experiences for future problem-solving situations.

The problem-solving models proposed by Dewey (1910) and Polya (1945) formed the basis of many models developed after them (Rott, Specht & Knipping, 2021). Schoenfeld (1982) developed a different model with the stages of analysis, design, exploration, implementation, and verification. Mason, Burton, and Stacey (1982) proposed a problem-solving model of three stages. This model consists of entry, attack, and review stages. Unlike other researchers, Wilson, Fernandez, and Hadaway (1993) emphasized the importance of the managerial process in problem-solving. They developed a problem-solving model consisting of understanding the problem, devising a plan, carrying out the plan, looking back, problem-posing, and managerial process. A different approach belongs to Haylock and Cockburn (2014). They defined three components of problem-solving as givens-goal-gap. They described problem-solving as closing the gap between the given and the goal. This gap starts with understanding what is given, and sometimes by working forward from what is given, sometimes by going backward from the goal, and sometimes by using both, working back and forth; this gap ends, and the problem is solved (Haylock & Cockburn, 2014).

What is Reflective Thinking?

For Dewey (1933, p. 3), the creator of the concept of reflective thinking, "reflective thinking is a thought process that involves turning a subject over in mind and giving it serious and consecutive consideration." Rodgers (2002) summarized Dewey's thoughts on reflective thinking in four main points:

1. Reflection is a meaning-making process that enables the learner to more deeply understand the relationships and connections of their experiences with other experiences and ideas. It makes the continuity of learning possible and enables the individual's and society's progress.

2. Reflection is a systematic, rigorous, disciplined way of thinking rooted in scientific research.

3. Reflection must take place in society, in interaction with others.

4. Reflection requires attitudes that value the personal and intellectual development of oneself and others.

Reflective thinking includes rethinking and evaluating past events and experiences to obtain better and more efficient solutions in the future (Buzdar & Akhtar, 2013). Reflection is a way of correcting mistakes in people's actions and decisions to fulfill a task, but it also allows people to examine the assumptions they use to make sense of the world (Mezirow, 1990, 1991). This way of thinking does not arise spontaneously in daily life because it is a conscious mental activity. It occurs due to active effort; it is epigenetic (hereditary but not genetic) and needs to be learned and encouraged (Gelter, 2003).

Problem Solving and Reflective Thinking

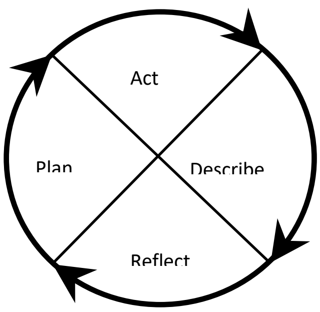
Reflective thinking is an important factor in problem-solving and mathematics achievement (Baş & Kıvılcım, 2013; Hong & Choi, 2011; Lai & Land, 2009) because it is one of the essential elements of problem-solving (Mason, 2009). Reflective thinking can be best observed in problem-solving because it allows the evaluation of the solutions put forward for the problem and selecting the best solution (Bingham, 2004; Shermis, 1992). According to Schön (1987), reflective thinking occurs when unexpected or unusual results in problem-solving conflict with one's knowledge. In such an imbalance, the person focuses on the situation and applies reflection to address the problem. Therefore, Schön (1987) considers reflection as a conversation between the problem solver and the problematic situation.

Hong and Choi (2011, p. 689) defined reflective thinking in the context of design problems as "conscious mental activities that examine designers' courses of action, decisions, and their inner selves in given situations throughout a design process." For them, associating the task in the problem with previous knowledge, skills and experiences is possible by reflecting on problem-solving. It also lets to determine the mistakes made in this process (Hong & Choi, 2011; Mason, 2009). This reflection increases students' learning experiences rather than memorizing theoretical formulas (Schön, 1987). It also supports the development of students' thinking and reasoning skills (Epstein, 2003). There is a strong and significant positive relationship between mathematics achievement, reflective thinking toward problem-solving, and metacognitive awareness (Toraman, Orakcı, & Aktan, 2020). In addition, reflective thinking toward problem-solving significantly predicts students' academic success in mathematics and geometry courses (Baş & Kıvılcım, 2013). Therefore, it is crucial to develop the reflective thinking skills of students while solving problems in mathematics lessons.

In the literature, studies are carried out on reflective thinking types, stages, and realization times. Schön (1987) expressed two types of reflection: reflection in action and reflection on action. "Reflection in action" is the reflection that emerges during the realization of the action, focuses on solving the problem, and rearranges the action. "Reflecting on action" is evaluating the action after the action has taken place, looking back, and thinking about the action deliberately and systematically (Schön, 1987). According to Tripp (2003, p. 10), reflection is a "conscious attempt to evaluate the process and outcomes of the action as experienced by the actor." It consists of plan, act, describe, and reflect stages. Tripp (2003) defined the stages of reflective thinking with the reflective practice cycle. Figure 1 shows Tripp's (2003)'s reflective practice cycle.

Figure 1

Reflective Practice Cycle (Tripp, 2003)



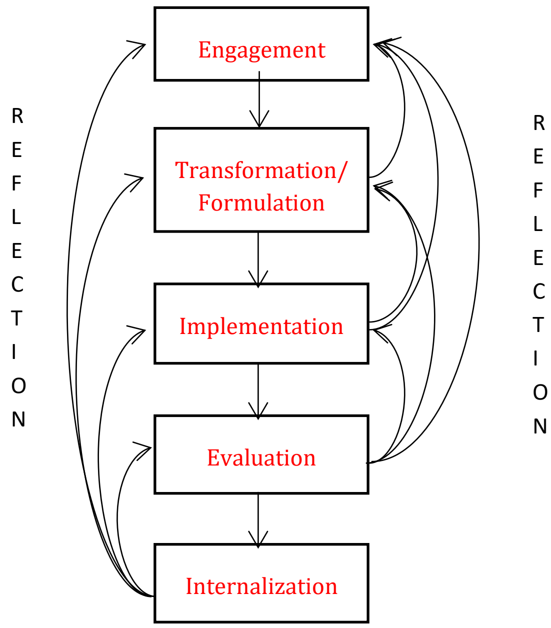
Tripp (2003) also determined the realization time of reflective thinking in problem-solving. Reflective thinking is planned simultaneously with searching for a solution to the problem in the "plan" stage. Reflective thinking occurs while the plan is implemented for the solution. Then the results are evaluated and reflected. This process continues in a loop. Developing reflective thinking skills is possible by being more conscious in the planning and monitoring stages of the action, reflecting on the action in the evaluation stage, and reviewing the reflective thinking processes (Tripp, 2003).

Kızılkaya and Aşkar (2009) mention three types of actions in the reflective problem-solving process. The first is questioning which seeks answers to questions produced by the person or directed to him from the outside. The other is evaluation; the person is looking back at his action and determining what is right and wrong. The last one is reasoning which involves investigating the cause of one's actions and examining cause-effect relationships according to the result.

Yimer and Ellerton (2010), on the other hand, proposed a five-stage model to describe the cognitive and metacognitive skills that pre-service teachers use in non-routine mathematical problem-solving. Researchers also examined pre-service teachers' reflections and modeled in Figure 2.

Figure 2

The problem-solving model of Yimer and Ellerton (2010)



On the other hand, Hong and Choi (2011) explained the reflective thinking used in design problems with a three-dimensional model. According to this model, three critical dimensions are level, timing, and objects of reflection. Timing is the model's first dimension and has two types: problem-driven and solution-driven approaches. The second dimension consists of the objects of reflective thinking, and the third dimension is the level of reflection. In a different study, Kholid, Sa'dijah, Hidayanto, and Permadi (2020) examined the reflective thinking skills of pre-service mathematics teachers in the problem-solving process. The research results revealed three types of reflective thinking components in terms of coping with the difficulty in the problem: assumptive, virtual, and connective. Another result of this study is that the participants used all reflective thinking components in problem-solving, even if not at the maximum level.

Components of reflective thinking have also been investigated in other studies. According to Zehavi and Mann (2005), reflective thinking comprises techniques, monitoring, insight, and conceptualization. Techniques is concerned with how an individual uses techniques and strategies to solve a mathematical problem using effective principles. Monitoring is an activity used to monitor the solution of math problems. It refers to the individual checking whether the solution to a mathematical problem is correct. Insight is how an individual uses his intellectual and emotions to solve problems when he fails. Conceptualization is the component related to how an individual uses his ability to develop and understand concepts, analyze information and apply the solution to a problem (Zehavi & Mann, 2005). Besides, in different studies, it has been determined that reflective thinking has indicators such as doing something based on a plan, improving the level of knowledge, self-monitoring, and providing reasonable evidence to decide on a problem (Dervent, 2015; Ghanizadeh, 2017; Gencel & Saracaloğlu, 2018; Hsieh & Chen, 2012; Rieger, Radcliffe & Doepker, 2013; Sivaci, 2017; Satjatam, Sarintip & Teerachai, 2016). However, it was observed that there is a gap in the literature about modeling the reflective problem-solving process.

Aim of The Study

This research examines the reflective thinking skills used in the problem-solving and aims to model this process. The sub-problems of the research can be expressed as follows:

1. What are the reflective thinking components used in the problem-solving process?

2. How can the reflective problem-solving process be modeled?

METHOD

A case study, one of the qualitative research designs, was used in the study. The case study is a type of research in which a single individual, group, or important subject is comprehensively examined and studied (Fraenkel, Wallen, & Hyun, 2012). In the study, the reflective thinking skills used by pre-service teachers while solving a non-routine problem were examined in detail.

Participants

The research was conducted in the Faculty of Education of a university in North Anatolia in the fall semester of the 2021-2022 academic year. Twenty pre-service teachers from different grade levels studying in the Department of Primary Education participated in the research. Participants were selected among volunteer pre-service teachers using the criterion sampling method. The pre-service teachers from high, medium, and low success levels participated in the study, taking into account the achievement scores of the Basic Mathematics course. Thus, it was aimed to examine the reflective thinking skills used by pre-service teachers of all achievement levels. Seventeen participants are female, and three are male.

Data Collection Tools

The data collection tools are worksheets, think-aloud protocol, and semi-structured interview forms.

Worksheets: In order to determine the reflective thinking skills used by pre-service teachers in the problem-solving process, a non-routine mathematical problem was asked to them in the worksheets.

According to Kitchener and Fischer (1990), well-structured problems can be solved with a "high degree of certainty"; the problem's parameters are known, and there is only one correct answer that can be verified by the appropriate use of an algorithm, formula, or procedure. However, non-routine problems cannot be defined precisely. Real-life problems of this type have unknown parameters. The reasoning required to propose reasonable solutions for such problems is similar to what Dewey describes as reflective thinking (Kitchener & Fischer, 1990). Although different researchers have developed many models for problem-solving stages, the most widely known was developed by Polya (1945). Therefore, the reflective thinking components used in the problem-solving stages of Polya were examined in the research and the worksheets were organized according to Polya's problem-solving stages. Two lecturers from mathematics education and primary education examined the problem. Afterward, a pilot study was conducted with a randomly selected group of pre-service teachers, and the worksheet was revised based on feedbacks.

Think Aloud Protocol: A thinking-aloud protocol was applied to determine the reflective thinking skills used in the problem-solving process. The think-aloud protocol is a verbal performance-based assessment technique in which participants state aloud everything they thought and did while performing a given task (such as reading a verbal text or solving a math problem) (Özkubat & Özmen, 2018; Rosenzweig et al., 2011). Before applying the think-aloud protocol, participants were trained about thinking aloud. The think-aloud protocol directive at the worksheet was also explained to them (Özkubat & Özmen, 2018):

“You have to solve this problem by thinking aloud. You should audibly explain everything that comes to your mind, your thoughts, and the reasons for all your procedures. How you solve the problem is more important than the outcome. That is why you need to explain how you think about the solution. This worksheet was organized according to Polya's problem-solving stages. It would be best if you expressed aloud what comes to your mind at each stage. In order to analyze this whole process, I need to record your voice with this voice recorder."

Semi-Structured Interview Form: A semi-structured interview form was also applied to pre-service teachers, and interview questions were asked to better reveal their reflective thinking skills. For example, at the stage of understanding the problem, some pre-service teachers read the problem and started to solve it. In order to determine their reflective thinking skills, the interview question of "what other methods do you use to understand the problem?" was asked to them. Some pre-service teachers skipped looking back stage after solving the problem. At this stage, the following questions were also asked to them:

“How would you evaluate the results after solving the problem? How do you check the correctness of the solution? What methods do you use? What will you do if the result is wrong?”

Applications

In the applications, the purpose of the study was explained first. Then the implementation of the think-aloud protocol was expressed to participants. It was requested that they solve the problem according to Polya's problem-solving stages and they explain their thoughts, procedures, and reasons at each stage aloud. The applications were conducted with each pre-service teacher in the researcher's office. The applications were started by saying, "I want you to solve the problem by thinking aloud," and when they were silent, they were asked, “What do you think now?” or “Can you please think aloud?” In each stage of the solution, interview questions were also asked. Each implementation lasted an average of 25-30 minutes and was recorded with a voice recorder.

Data Analysis

The data obtained from the audio recordings of the think-aloud protocols, interviews, and worksheets were analyzed and interpreted together for in-depth analysis. The data analysis steps suggested by Özkubat and Özmen (2018) were followed in the analysis of the think-aloud protocol and interviews. These steps are analyzing the qualitative data and transforming it into quantitative by recording them in the Think Aloud Protocol Coding Form. The audio recordings were transcribed on the computer. Reflective thinking components and indicators suggested by Kholid et al. (2020) were taken into account in the analysis of the transcripts. Kholid et al. (2020) identified indicators and components of reflective thinking suggested by Zehavi and Mann (2005) in their study. These components are presented in Table 1.

Table 1

Reflective Thinking Components and Indicators (adapted from Kholid et al., 2020)

Components	Indicators	Codes
Techniques	1. Finding how to understand what the given question means	T1
	2. Finding how to understand what the question is	T2
	3. Inferring the question’s meaning	T3
Monitoring	1. Monitoring the steps of the solution to mathematics questions	M1
	2. Monitoring whether the answers are correct or not	M2
	3. Using strategies for solving the questions	M3
Insight	1. Being ready to correct the wrong questions	I1
	2. Understanding how to prevent any difficulty	I2
Conceptualization	1. Thinking about other ways of solving the questions	C1
	2. Relating relevant concepts to solving the questions	C2

The reflective thinking components used in the problem-solving stages were coded considering the indicators in Table 1. The codes were recorded in the coding form, and total frequencies were calculated. Besides, the findings included examples from the worksheets, and direct quotations from the pre-service teachers' views. Pre-service teachers were coded according to their gender and mathematics achievement. For example, PT1FM refers to the first female pre-service teacher with moderate mathematics achievement. PT2MH refers to the number 2 male pre-service teacher with a high mathematics achievement level.

Validity and Reliability of the Research

In qualitative research, validity is divided into internal and external (Guba & Lincoln, 1982). To ensure the credibility of the study within the scope of internal validity, data were collected by triangulation (worksheets, think-aloud protocol, and interviews). For external validity (transferability), the participants were selected according to the criterion sampling method, one of the purposive sampling methods. The inclusion criterion in the sampling was also specified. In addition, the applications and participants within the scope of the research were described in detail. For the reliability of the data analysis, the codings were presented to a lecturer in mathematics education. The lecturer examined the analysis of the transcripts according to the indicators in Table 1. Then, the differences were determined, and the codings were corrected by exchanging ideas between the lecturer and the researcher. This way, the data analysis ended after all the codings were examined and arranged in consensus.

Ethical Principles

Ethical principles were complied with during the implementation of the research. Ethics committee permission for this study was obtained from Ondokuz Mayıs University Social and Human Sciences Research Ethics Committee with the decision dated 26.11.2021 and numbered 11/2021-930.

FINDINGS

Reflective thinking components used in the problem-solving process

The components of reflective thinking used in the problem-solving process were analyzed and the results were presented in Table 2.

Table 2

Reflective thinking components used in the problem-solving process

	Reflective thinking components and indicators	f	%
Understanding the problem	Techniques T1, T2, T3	20	17,54
	Insight I2	4	3,51
	Conceptualization C2	5	4,39
Devising a plan	Monitoring M1	6	5,26
	Insight I2	4	3,51
	Conceptualization C2	9	7,89
Carrying out the plan	Monitoring M1, M3	14	12,28
	Insight I2	6	5,26
	Conceptualization C2	5	4,39
Looking back	Monitoring M1, M2	12	10,53
	Insight I1, I2	14	12,28
	Conceptualization C1	15	13,16
Total		114	100

Table 2 shows the components of reflective thinking used in problem-solving, and it is seen that the components are used in all problem-solving stages. An example of the T1 indicator of the Techniques component is so: *"I try to use a few variables to understand and solve the problem. For example, instead of x and y, I just use only x. Instead of x + y = 15, I would say y=15-x. So much unknown confuses me"* (PT19MM). Another indicator of the Techniques component is: *"If the question is long, I write what is given. If it is short, it is not necessary. However, if it is long, I forget what is given. I read the question one by one to understand better. For example, I reviewed it again since this question has a lot of data. I concretize the question in my head. I think about why it is given and what is asked for by writing it down on paper. On the one hand, I am thinking about how to solve this. I think how to understand, follow a solution, and perceive it more easily"* (PT4FH). A different view of this component is, *"After reading and understanding for the first time, I read and start solving for the second time. Because I know what is coming, I will try to find the price for 1 GB. I need to find the fixed fee. Let's call* *the fixed fee x. I understood the problem. However, I need to set a fixed fee. I doubted there. Should I say x to fixed fee? Should I say x to 1 GB? I could not decide. That is why I am rereading it."* (PT13FM).

Table 2 shows that the Monitoring, Insight, and Conceptualization components are used in the problem-solving stages of devising a plan, carrying out the plan, and looking back, although their indicators differ. A statement containing both the Monitoring and Conceptualization components says: *"To solve the question, I first read the order of operations; I think about why I am doing it. I do not do anything without thinking about the question. I think about the relationships between the givens. I do not do random procedures. There is a sequence of actions; I predict the next action."* (PT3FL). Here is an example of the M2 indicator of the Monitoring component: *"I am evaluating the result. Here I will find the person using 30 GB per month. I said x to that. I found that y is three and x is 30. He paid 75 liras in total. It confirmed the result; it is true."* (PT8FH). Some of the views on M3 were expressed as follows: *"Now I am reading aloud. However, when I read it out loud, I do not understand. I understand when I read it silently. I usually understand all the questions the first time I read them. I read with attention to numbers. For example, when I first read this question, I put 15 GB in my mind"* (PT2FL). *"When I first read, I read numerical data by writing. If the question is easy, I understand it in the first reading. Sometimes I have to reread the hard questions"* (PT8FH).

As a result of data analysis, it was determined that the Insight and Conceptualization components of reflective thinking are used in all problem-solving stages. An example of the I1 indicator of the Insight component is, *"I will now check the result I found. I made a mistake; now I am doing it again. Yes, I made a mistake here; I am correcting it. This time, I am verifying the result. I have been checking from the beginning. Was there a transaction error? I am looking at the numbers, division, and proportions. I got 400. I will recheck this by dividing it. The answer has changed. I had found 35 before. When I checked, I found 33 this time"* (PT5FM). A view of both the Insight and the Conceptualization components is: *"When I read the problem first, I think about how to solve it. What is already given is in the form of a continuation of each other. I make a connection between what was given. Why was it given? How do I find the ones that are not given? I try to pick the ones that work for me"* (PT15FH).

An example of the C1 indicator of the Conceptualization component was expressed as: *"After solving the problem, I do not try many different ways. If the answer is correct, I continue. However, if the solution is too long, I think about a different way. I think there must be a shorter solution."* (PT19MM). Another example of the C2 indicator is *"While reading the question, at the same time I think about how to do it by relating the givens, a solution is formed in my head."* (PT2FL).

The model of the reflective problem-solving process

Table 2 shows that reflective thinking emerged in all stages of problem-solving. This process was modeled in Figure 3.

Figure 3

Model of the problem-solving process, including reflective thinking

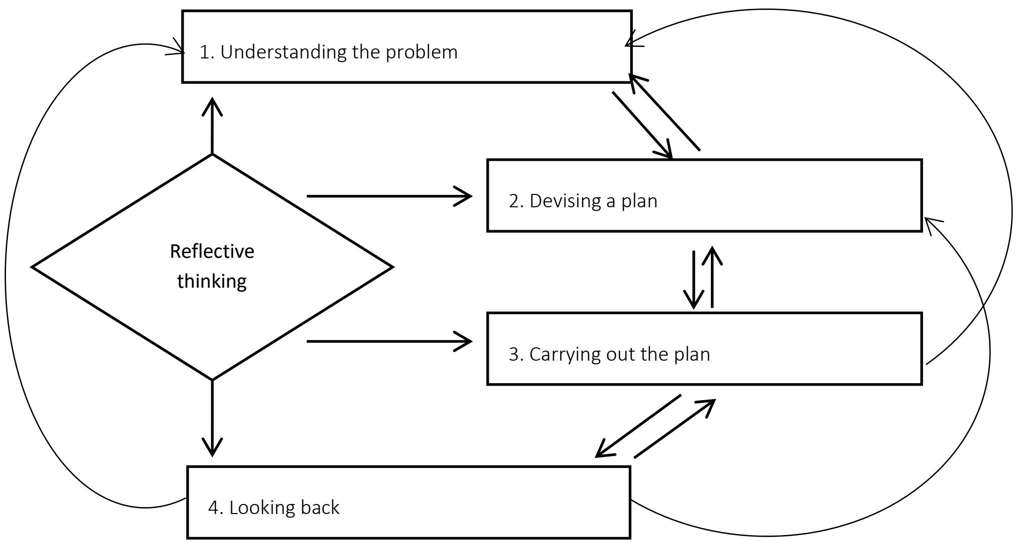
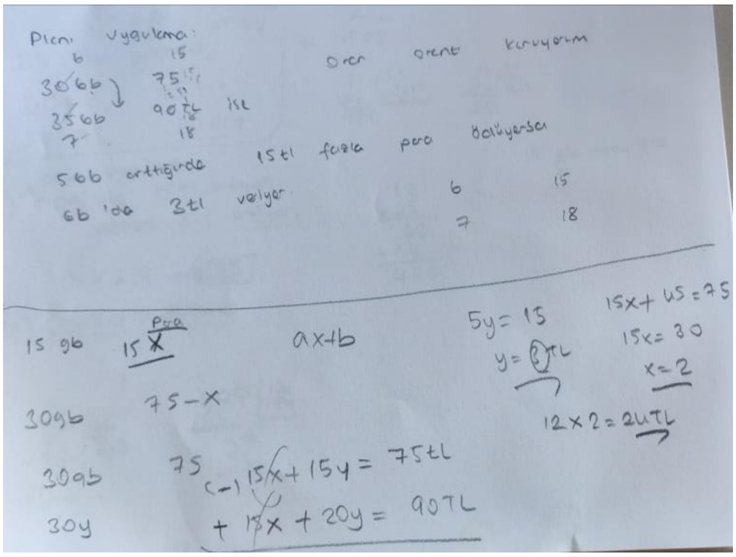


Figure 3 shows the modeling of the problem-solving process that includes reflective thinking. During the applications, the transitions of the pre-service teachers between the stages of problem-solving were represented by arrows in the model. The pre-service teachers started problem-solving by reading the problem and continued by making a plan for the solution. Then, the planned solution was implemented, and the correctness of the solution was tested. Besides, some pre-service teachers reread it, thinking that they did not understand the problem while planning the solution: *"Now I am rereading the question, I am trying to understand how many GB was given in the beginning, I am trying to understand it again, I realized that I did not understand it"* (PT19MM). Some pre-service teachers returned to the previous stage, devised a plan again, and changed the solution path when they could not reach the result: *"There is no result; I think I need to try a different way"* (PT11FL). In addition, when the pre-service teachers understood that the result was incorrect in the looking back stage, they tried to reach the right solution by checking all the stages respectively: *"Sometimes I make many transaction errors. If the result is wrong, I delete all the transactions and do it again. Sometimes I misunderstand the question. I am trying to read and understand the question again. I need to check it"* (PT12MH). In this problem-solving process, reflective thinking emerged as a skill that guides all stages. It was observed that the pre-service teachers, who could not reach the correct result, clearly used reflective thinking skills. When the result was wrong, they stated that they returned to the previous stages in order, sought the cause of the error, and tried to reach the correct result; *"I put the numbers to try if the answer is correct. It should give a total of 90. It is true. However, I would go back to the beginning if it was wrong. I would check transactions first. If there were no transaction errors, I would reread it. I probably thought I did not understand the question"* (PT19MM). In Figure 4, a worksheet belonging to a pre-service teacher is presented.

Figure 4

An example of the worksheets (PT7FM)



RESULTS, DISCUSSIONS, AND SUGGESTIONS

Reflective thinking in problem-solving is needed in uncertain and problematic situations (Kitchener & Fischer, 1990). This research was conducted to determine the reflective thinking skills used in problem-solving and to model this process. The research results showed that reflective thinking emerges at all stages of problem-solving and affects the whole process. The research results are align with the results of the research conducted by Kholid et al. (2020). Kholid et al. (2020) also determined that pre-service mathematics teachers use all reflective thinking components in the problem-solving. The research results revealed that reflective thinking was used while understanding the problem, planning for the solution, choosing a strategy, applying the plan, and evaluating the solution. Therefore, it can be thought that reflective thinking is practical at every stage of problem-solving and makes it possible to be successful. Reflective thinking is a problem-solving approach that starts due to mental complexity, requiring identifying the problem and looking at this problem from different perspectives (Dewey, 1933; Kember et al., 2000; Öztürk, 2003). Other researchers also stated that it is essential for students to think reflectively in identifying and solving problems and adapting them to different situations (Dewey, 1933; Schön, 1983, 1987). Van de Walle et al. (2014) also emphasize the importance of students reflecting on their experiences in the problem-solving process. They state that students should have time to think and discuss their solutions. Reflection during the problem-solving process can increase students' awareness of their mental activities, including cognitive and metacognitive thinking, and enable them to approach their learning consciously. Thus, it can provide an opportunity for them to review and internalize not only the procedures and algorithms for the problem’s solution but also the thinking methods and strategies.

Another point to be emphasized is that students should encounter non-routine problems to enable them to use their reflective thinking skills. Non-routine problems require individuals to reflect on problems using their existing knowledge, experiences, and beliefs (Kember et al., 2000). In schools, students often encounter routine problems with only one solution. However, as reflection skills develop, their ability to cope with non-routine problems will also improve (Schön, 1987). Therefore, developing students' reflective thinking skills is necessary by including non-routine problems in mathematics lessons.

The role of the teacher in this process is also critical because it is very important to support and guide students in reflective thinking skills. The teacher can help students with written diaries, peer discussion, and evaluation (Lai & Land, 2009). They can also apply various strategies to encourage reflective thinking like asking open-ended questions, accepting contradictory comments, and writing down what children say (Epstein, 2003).

The model of reflective problem-solving confirmed its cyclical and dynamic nature. It has been revealed that the problem-solving process includes up and down transitions according to the success of solving the problem and the accuracy of the result. In other words, the student can return to the previous step between successive stages. While trying to determine a solution, he can reread it, thinking he does not understand the problem. However, the false result, especially in the evaluation phase, causes a return to the previous stages of problem-solving. When the false result emerges, the student thinks about where the error originates and returns to the stages of understanding the problem, planning, and implementing it to identify the error. Reflective thinking is applied more actively and mainly after this stage. The problem-solver reread the problem, questions whether he has understood it correctly, and addresses it differently. Returning to the planning stage, he reviews the strategy and considers whether there is a need for a new plan/strategy/solution. Then he checks the stage of carrying out the plan and examines whether there are errors. He tries to find the correct result. In this process, it was understood that reflective thinking facilitates reaching the correct result by giving direction to all stages. It is seen that the model developed for reflective thinking skills in the problem-solving process is compatible with the model developed by Yimer and Ellerton (2010) in terms of the fact that reflective thinking is effective in the whole problem-solving process.

Reflection, embedded in the whole problem-solving, can increase the students’ problem-solving success and improve their mathematical competencies. Therefore, it should be ensured that students use reflective thinking more consciously in problem-solving activities in classroom practices. In this regard, it should be ensured that both pre-service primary teachers and primary teachers should be trained in reflective thinking in the problem-solving process.

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