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On Fuzzy Hypersoft Topological Spaces

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Makalenin Alanı: Matematik

Makale Bilgileri	Öz
Geliş Tarihi 13.07.2021 **Kabul Tarihi** 12.04.2022 **Anahtar Kelimeler** Fuzzy hypersoft küme Fuzzy hypersoft topoloji Fuzzy hypersoft taban Fuzzy hypersoft alt uzay	Bu makale fuzzy hypersoft kümeler üzerinde bir topoloji kurmayı amaçlamaktadır. Bu çalışmada fuzzy hypersoft topoloji üzerinde fuzzy hypersoft açık(kapalı) kümeler, fuzzy hypersoft iç, fuzzy hypersoft kapanış, fuzzy hypersoft taban ve fuzzy hypersoft alt uzay topoloji kavramları tanıtılmış ve bazı önemli teoremler ispatlanmıştır.

Article Info	Abstract
Received 13.07.2021 **Accepted** 12.04.2022 **Keywords** Fuzzy hypersoft set Fuzzy hypersoft topology Fuzzy hypersoft base Fuzzy hypersoft subspace	This paper is aimed at constructing a topology on fuzzy hypersoft sets. The concepts of fuzzy hypersoft topology, fuzzy hypersoft open(closed) sets, fuzzy hypersoft interior, fuzzy hypersoft closure, fuzzy hypersoft base and fuzzy hypersoft subspace topology are also introduced here and some important theorems have been established.

1. INTRODUCTION

In sociology, economics, climate, engineering, etc., we can not use classical mathematical methods to solve any kind of problems because these sort of problems have their own difficulties. The Fuzzy set theory, first proposed in 1965 by researcher Zadeh (Zadeh, 1965), has become a very useful technique for solving these types of problems and provides an effective structure for representing ambiguous concepts by enabling partial membership. Both mathematicians and computer scientists have studied Fuzzy Set Theory and, over the

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years, fuzzy control systems, fuzzy logic, fuzzy topology, etc. Many implementations of fuzzy set theory have appeared as such. There is also a probability theory besides this theory, a rough set theory (Pawlak, 1982) that deals with solving these problems. Molodtsov (Molodtsov, 1999) introduced the idea of soft set theory, which is a completely new approach to modeling uncertainty, each of these theories has its inherent difficulties. The fundamental results of this new theory were established by Molodtsov and the soft set theory was successfully applied in many directions, such as smoothness of functions, operations analysis, Riemann integration, game theory, probability theory, etc. by Molodtsov. Maji (Maji et al., 2003) identified and studied several fundamental notions of soft set theory. The study of Maji et al. (Maji et al., 2003) was improved by Pei and Miao (Pei & Miao, 2005), Feng et al. (Feng et al., 2008), Chen et al. (Chen et al., 2005), Aktaş and Çağman (Aktaş & Çağman, 2007), Irfan Ali et al. (Ali et al., 2009) and Ozturk and Bayramov (Ozturk & Bayramov, 2014).

The research involving both fuzzy sets and soft sets was initiated by Maji et al. (Maji et al., 2001). The fuzzy soft set structure, which is a combination of soft set structure and fuzzy set structure, has been actively used by researchers and many studies have been added to the literature (Ahmad & Kharal, 2009; Roy & Maji, 2007; Yang et al., 2009). It is pointed out in (Pei & Miao, 2005) that several scientists from many fields have studied information systems and that certain compact ties exist between soft sets and information systems. It is also shown in (Pei & Miao, 2005) that soft sets are a class of special information systems , called fuzzy information systems, and that it is possible to unify research on soft sets and information systems. In addition , it is possible to predict some new outcomes and approaches from this unification. In (Kharal & Ahmad, 2009), in order to improve this fuzzy soft theory, The concept of mapping fuzzy soft sets was defined by Kharal and Ahmad and the properties of fuzzy soft images and fuzzy soft inverse images of fuzzy soft sets were studied.

Fuzzy soft topology on a fuzzy soft set over an initial universe was described by Tanay and Kandemir (Tanay & Kandemir, 2011). They defined some notions of fuzzy soft topological spaces and investigated some properties. Roy and Samanta (Roy & Samanta, 2012) defined a fuzzy soft topology over the initial universe, and they also introduced the base and subbase for this space , by giving some characterizations.

Smarandache (Smarandache, 2018) has implemented new uncertainty management methods. He extended the soft set to a hypersoft set by transforming the features into a multi-decision method. Hypersoft set structure is a more general form of soft set structure since it consists of elements selected from different attributes. Due to the fact that this new structure is more useful, a lot of research have been done in a short time (Gayen et al., 2020), (Martin & Smarandache, 2020), (Rana et al., 2019), (Saeed et al., 2020), (Saqlain et al., 2020a, 2020b). Yolcu and Ozturk (Yolcu & Ozturk, 2021) defined the fuzzy hypersoft set structure by combining fuzzy and hypersoft set structures. In this paper, we are trying to extend this notion to topological spaces. Then we define a fuzzy hypersoft topology, fuzzy hypersoft open(closed) sets, fuzzy hypersoft interior, fuzzy hypersoft closure, fuzzy hypersoft base and fuzzy hypersoft subspace topology and also here we established some important theorems related to this spaces.

2. PRELIMINARIES

Definition 2.1 (Zadeh, 1965) Let U be a initial universe. A fuzzy set Λ in U, $\Lambda = \{(u, \mu_\Lambda(u)) : u \in U\}$, where $\mu_\Lambda : U \to [0,1]$ is the membership function of the fuzzy set Λ; $\mu_\Lambda(u) \in [0,1]$ is the membership $u \in U$ in Λ. The set of all fuzzy sets ove U will be denoted by $FP(U)$.

Definition 2.2 (Molodtsov, 1999) Let U be an initial universe and E be a set of parameters. A pair (F, E) is called a soft set over U, where F is a mapping $F : E \to \mathrm{P}(U)$. In other words, the soft set is a parameterized family of subsets of the set U.

Definition 2.3 (Maji et al., 2001) Let U be a initial universe, E be a set of parameters and $FP(U)$ be the set of all fuzzy sets in U. Then a pair (f, E) is called a fuzzy soft set over U, where $f : E \to FP(U)$ is a mapping.

Definition 2.4 (Smarandache, 2018) Let U be the universal set and $P(U)$ be the power set of U. Consider $e_1, e_2, e_3, ..., e_n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attribute values are resspectively the sets $E_1, E_2, ..., E_n$ with $E_i \cap E_j = \varnothing$, for $i \neq j$ and $i, j \in \{1, 2, ..., n\}$, then the pair $(\Theta, E_1 \times E_2 \times ... \times E_n)$ is said to be Hypersoft set over U where $\Theta : E_1 \times E_2 \times ... \times E_n \to P(U)$.

Definition 2.5 (Yolcu & Ozturk, 2021) Let U be the universal set and $FP(U)$ be a family of all fuzzy set over U and $E_1, E_2, ..., E_n$ the pairwise disjoint sets of parameters. Let A_i be the nonempty subset of E_i for each $i = 1, 2, ..., n.$ A fuzzy hypersoft set defined as the pair $(\Theta, A_1 \times A_2 \times ... \times A_n)$ where; $\Theta : A_1 \times A_2 \times ... \times A_n \to FP(U)$ and

$\Theta(A_1 \times A_2 \times ... \times A_n) = \{< u, \Theta(\alpha)(u) >: u \in U,$
$\alpha \in A_1 \times A_2 \times ... \times A_n \subseteq E_1 \times E_2 \times ... \times E_n\}$

For sake of simplicity, we write the symbols Σ for $E_1 \times E_2 \times ... \times E_n$, Γ for $A_1 \times A_2 \times ... \times A_n$ and α for an element of the set Γ. The set of all fuzzy hypersoft sets over U will be denoted by $FHS(U, \Sigma)$. Here after, FHS will be used for short instead of fuzzy hypersoft sets.

Definition 2.6 (Yolcu & Ozturk, 2021)

i) A fuzzy hypersoft set (Θ, Σ) over the universe U is said to be null fuzzy hypersoft set and denoted by $0_{(U_{FH}, \Sigma)}$ if for all $u \in U$ and $\varepsilon \in \Sigma$, $\Theta(\varepsilon)(u) = 0$.

ii) A fuzzy hypersoft set (Θ, Σ) over the universe U is said to be absolute fuzzy hypersoft set and denoted by $1_{(U_{FH}, \Sigma)}$ if for all $u \in U$ and $\varepsilon \in \Sigma$, $\Theta(\varepsilon)(u) = 1$.

Definition 2.7 (Yolcu & Ozturk, 2021) Let U be an initial universe set $(\Theta_1, \Gamma_1), (\Theta_2, \Gamma_2)$ be two fuzzy hypersoft sets over the universe U. We say that (Θ_1, Γ_1) is a fuzzy hypersoft subset of (Θ_2, Γ_2) and denote $(\Theta_1, \Gamma_1) \tilde{\subseteq} (\Theta_2, \Gamma_2)$ if

i) $\Gamma_1 \subseteq \Gamma_2$

ii) For any $\varepsilon \in \Gamma_1$, $\Theta_1(\varepsilon) \subseteq \Theta_2(\varepsilon)$.

Definition 2.8 (Yolcu & Ozturk, 2021) The complement of fuzzy hypersoft set (Θ, Γ) over the universe U is denoted by $(\Theta, \Gamma)^c$ and defined as $(\Theta, \Gamma)^c = (\Theta^c, \Gamma)$, where $\Theta^c(\varepsilon)$ is complement of the set $\Theta(\varepsilon)$, for $\varepsilon \in \Gamma$.

Definition 2.9 (Yolcu & Ozturk, 2021) Let U be an initial universe set and $(\Theta_1, \Gamma_1), (\Theta_2, \Gamma_2)$ be two fuzzy hypersoft sets over the universe U. The union of (Θ_1, Γ_1) and (Θ_2, Γ_2) is denoted by $(\Theta_1, \Gamma_1) \tilde{\cup} (\Theta_2, \Gamma_2) = (\Theta_3, \Gamma_3)$ where $\Gamma_3 = \Gamma_1 \cup \Gamma_2$ and

$$\Theta_3(\varepsilon) = \begin{cases} \Theta_1(\varepsilon) & \text{if } \varepsilon \in \Gamma_1 - \Gamma_2 \\ \Theta_2(\varepsilon) & \text{if } \varepsilon \in \Gamma_2 - \Gamma_1 \\ \max\{\Theta_1(\varepsilon), \Theta_2(\varepsilon)\} & \text{if } \varepsilon \in \Gamma_1 \cap \Gamma_2 \end{cases}$$

Definition 2.10 Let U be an initial universe set and $(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2)$ be fuzzy hypersoft sets over the universe U. The intersection of (Θ_1,Γ_1) and (Θ_2,Γ_2) is denoted by $(\Theta_1,\Gamma_1)\tilde{\cap}(\Theta_2,\Gamma_2)=(\Theta_3,\Gamma_3)$ where $\Gamma_3=\Gamma_1\cap\Gamma_2$ and each $\varepsilon\in\Gamma_3, \Theta_3(\varepsilon)(u)=\min\{\Theta_1(\varepsilon)(u),\Theta_2(\varepsilon)(u)\}$. (Yolcu & Ozturk, 2021)

3. FUZZY HYPERSOFT TOPOLOGICAL SPACES

Definition 3.1 Let $FHS(U,\Sigma)$ be the set of all fuzzy hypersoft subsets of (U,Σ) over the universe U and $\tilde{\tau}$ be a subfamily of $FHS(U,\Sigma)$. Then $\tilde{\tau}$ is called a fuzzy hypersoft topology on U if the following condition are satisfied.

1. $0_{(U_{FH},\Sigma)}$ and $1_{(U_{FH},\Sigma)}$ belongs to $\tilde{\tau}$,
2. The union of any number of fuzzy hypersoft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$,
3. The intersection of any two fuzzy hypersoft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

The triple $(U,\tilde{\tau},\Sigma)$ is called a fuzzy hypersoft topological space over U. Every member of $\tilde{\tau}$ is called a fuzzy hypersoft open set in U.

Definition 3.2 Let $FHS(U,\Sigma)$ be the set of all fuzzy hypersoftsets over the universe U. Then,

1. If $\tilde{\tau}=\{0_{(U_{FH},\Sigma)},1_{(U_{FH},\Sigma)}\}$, then $\tilde{\tau}$ is called to be fuzzy hypersoft indiscrete topology and $(U,\tilde{\tau},\Sigma)$ is called to be fuzzy hypersoft indiscrete topological space over the universe U.
2. If $\tilde{\tau}=FHS(U,\Sigma)$, then $\tilde{\tau}$ is called to be fuzzy hypersoft discrete topology and $(U,\tilde{\tau},\Sigma)$ is called to be fuzzy hypersoft discrete topological space over the universe U.

Example 3.1 Let $U=\{u_1,u_2,u_3\}$ be the set of mobile phones and also consider the set of attributes E_1 = Screen Size, E_2 = RAM, E_3 = Operating System, E_4 = Battery Power are given as;

E_1 = Screen Size = $\{6.55in(\alpha_1), 6.78in(\alpha_2)\}$

E_2 = RAM = $\{12GB(\beta_1), 8GB(\beta_2), 4GB(\beta_3)\}$

E_3 = Operating System = $\{Android(\gamma_1), IOS(\gamma_2)\}$

E_4 = Battery Power = $\{4300mAH(\delta_1), 4115mAH(\delta_2), 5100mAH(\delta_3)\}$

Suppose that

$$A_1=\{\alpha_1\}, A_2=\{\beta_1,\beta_2\}, A_3=\{\gamma_2\}, A_4=\{\delta_1,\delta_3\}$$

$$B_1=\{\alpha_2\}, B_2=\{\beta_2,\beta_3\}, B_3=\{\gamma_1\}, B_4=\{\delta_1,\delta_2\}$$

Let

$$\tilde{\tau}=\{0_{(U_{FH},\Sigma)},1_{(U_{FH},\Sigma)},(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2),(\Theta_3,\Gamma_3)\}$$

be a subfamily of $FHS(U,\Sigma)$ where $(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2),(\Theta_3,\Gamma_3)$ for $\forall\Gamma\subseteq\Sigma$, fuzzy hypersoft sets over U and defined as follows;

$$(\Theta_1,\Gamma_1)=\left\{\begin{array}{l} <(\alpha_1,\beta_1,\gamma_2,\delta_1),\{\frac{u_1}{0,2},\frac{u_2}{0,5}\}>, \\ <(\alpha_1,\beta_1,\gamma_2,\delta_3),\{\frac{u_1}{0,5},\frac{u_2}{0,1},\frac{u_3}{0,7}\}>, \\ <(\alpha_1,\beta_2,\gamma_2,\delta_1),\{\frac{u_1}{0,6},\frac{u_3}{0,2}\}>, \\ <(\alpha_1,\beta_2,\gamma_2,\delta_3),\{\frac{u_2}{0,2},\frac{u_3}{0,3}\}> \end{array}\right\},$$

$$(\Theta_2,\Gamma_2)=\left\{\begin{array}{l} <(\alpha_2,\beta_2,\gamma_1,\delta_1),\{\frac{u_1}{0,4},\frac{u_2}{0,7},\frac{u_3}{0,2}\}>, \\ <(\alpha_2,\beta_2,\gamma_1,\delta_2),\{\frac{u_1}{0,8},\frac{u_2}{0,2},\frac{u_3}{0,4}\}>, \\ <(\alpha_2,\beta_3,\gamma_1,\delta_1),\{\frac{u_1}{0,3},\frac{u_2}{0,4}\}>, \\ <(\alpha_2,\beta_3,\gamma_1,\delta_2),\{\frac{u_2}{0,7},\frac{u_3}{0,8}\}> \end{array}\right\},$$

$$(\Theta_3,\Gamma_3)=\left\{\begin{array}{c} <(\alpha_1,\beta_1,\gamma_2,\delta_1),\{\frac{u_1}{0,2},\frac{u_2}{0,5}\}>,\\ <(\alpha_1,\beta_1,\gamma_2,\delta_3),\{\frac{u_1}{0,5},\frac{u_2}{0,1},\frac{u_3}{0,7}\}>,\\ <(\alpha_1,\beta_2,\gamma_2,\delta_1),\{\frac{u_1}{0,6},\frac{u_3}{0,2}\}>,\\ <(\alpha_1,\beta_2,\gamma_2,\delta_3),\{\frac{u_2}{0,2},\frac{u_3}{0,3}\}>,\\ <(\alpha_2,\beta_2,\gamma_1,\delta_1),\{\frac{u_1}{0,4},\frac{u_2}{0,7},\frac{u_3}{0,2}\}>,\\ <(\alpha_2,\beta_2,\gamma_1,\delta_2),\{\frac{u_1}{0,8},\frac{u_2}{0,2},\frac{u_3}{0,4}\}>,\\ <(\alpha_2,\beta_3,\gamma_1,\delta_1),\{\frac{u_1}{0,3},\frac{u_2}{0,4}\}>,\\ <(\alpha_2,\beta_3,\gamma_1,\delta_2),\{\frac{u_2}{0,7},\frac{u_3}{0,8}\}> \end{array}\right\}.$$

Then $\tilde{\tau}$ is a fuzzy hypersoft topology on U and hence $(U,\tilde{\tau},\Sigma)$ is a fuzzy hypersoft topological space.

Remark 3.1 It is clear that each fuzzy hypersoft topology is also fuzzy soft topology. We consider that Example 3.1. If we select the parameters from a single attribute set such as E_2 while creating fuzzy hypersoft topology, then the resulting topology becomes fuzzy soft topology. Therefore fuzzy hypersoft topology is also fuzzy soft topology. But the reverse is not true since there are no different attributes in the fuzzy soft set structure, such as the fuzzy hypersoft set structure. From here, it is seen that the fuzzy hypersoft structure is a more general structure than the fuzzy soft structure.

Proposition 3.1 Let $(U,\tilde{\tau}_1,\Sigma)$ and $(U,\tilde{\tau}_2,\Sigma)$ be two fuzzy hypersoft topological space over U. Then $(U,\tilde{\tau}_1\tilde{\cap}\tilde{\tau}_2,\Sigma)$ is a fuzzy hypersoft topological space over the universe U.

Proof. 1. Since $0_{(U_{FH},\Sigma)},1_{(U_{FH},\Sigma)}\in\tilde{\tau}_1$ and $0_{(U_{FH},\Sigma)},1_{(U_{FH},\Sigma)}\in\tilde{\tau}_2$, then $0_{(U_{FH},\Sigma)},1_{(U_{FH},\Sigma)}\in\tilde{\tau}_1\tilde{\cap}\tilde{\tau}_2$.

2. Suppose that $\{(\Theta_i,\Gamma_i):i\in I\}$ be a family of fuzzy hypersoft sets in $\tilde{\tau}_1\tilde{\cap}\tilde{\tau}_2$. Then $(\Theta_i,\Gamma_i)\in\tilde{\tau}_1$ and $(\Theta_i,\Gamma_i)\in\tilde{\tau}_2$ for all $i\in I$, so $\bigcup_{i\in I}(\Theta_i,\Gamma_i)\in\tilde{\tau}_1$ and $\bigcup_{i\in I}(\Theta_i,\Gamma_i)\in\tilde{\tau}_2$. Thus $\bigcup_{i\in I}(\Theta_i,\Gamma_i)\in\tilde{\tau}_1\tilde{\cap}\tilde{\tau}_2$.

3. Suppose that $\{(\Theta_i,\Gamma_i): i=\overline{1,n}\}$ be a family of the finite number of fuzzy hypersoft sets in $\tilde{\tau}_1 \tilde{\cap} \tilde{\tau}_2$. Then $(\Theta_i,\Gamma_i)\in\tilde{\tau}_1$ and $(\Theta_i,\Gamma_i)\in\tilde{\tau}_2$ for all $i=\overline{1,n}$, so $\bigcap_{i=1}^{n}(\Theta_i,\Gamma_i)\in\tilde{\tau}_1$ and $\bigcap_{i=1}^{n}(\Theta_i,\Gamma_i)\in\tilde{\tau}_2$. Thus $\bigcap_{i=1}^{n}(\Theta_i,\Gamma_i)\in\tilde{\tau}_1 \tilde{\cap} \tilde{\tau}_2$.

Remark 3.2 The union of two fuzzy hypersoft topologies over U may not be a fuzzy soft topology on U. This condition showed the following example.

Example 3.2 We consider the Example 3.1. Let

$\tilde{\tau}_1=\{0_{(U_{FH},\Sigma)},1_{(U_{FH},\Sigma)},(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2),(\Theta_3,\Gamma_3)\}$

$\tilde{\tau}_2=\{0_{(U_{FH},\Sigma)},1_{(U_{FH},\Sigma)},(\Theta_2,\Gamma_2),(\Theta_4,\Gamma_4)\}$

where

$$(\Theta_4,\Gamma_4)=\left\{\begin{array}{c} <(\alpha_2,\beta_2,\gamma_1,\delta_1),\{\frac{u_1}{0,5},\frac{u_2}{0,7},\frac{u_3}{0,4}\}>, \\ <(\alpha_2,\beta_2,\gamma_1,\delta_2),\{\frac{u_1}{0,9},\frac{u_2}{0,4},\frac{u_3}{0,6}\}>, \\ <(\alpha_2,\beta_3,\gamma_1,\delta_1),\{\frac{u_1}{0,5},\frac{u_2}{0,7}\}>, \\ <(\alpha_2,\beta_3,\gamma_1,\delta_2),\{\frac{u_2}{0,8},\frac{u_3}{0,9}\}> \end{array}\right\}.$$

It is clear that $\tilde{\tau}_1 \tilde{\cap} \tilde{\tau}_2$ is a fuzzy hypersoft topology. But $(\Theta_1,\Gamma_1)\cup(\Theta_4,\Gamma_4)\notin\tilde{\tau}_1\cup\tilde{\tau}_2$ and hence $\tilde{\tau}_1\cup\tilde{\tau}_2$ is not fuzzy hypersoft topology.

Proposition 3.2 Let $(U,\tilde{\tau},\Sigma)$ be a fuzzy hypersoft topological space over U. Then the collection $\tilde{\tau}=\{(\Theta,\Gamma):(\Theta,\Gamma)\in\tilde{\tau}\}$ defines a fuzzy soft topology and $(U,\tilde{\tau},\Sigma)$ is a fuzzy soft topological space over U.

Proof. Suppose that $(U,\tilde{\tau},\Sigma)$ be a fuzzy hypersoft topological space over U. Let us shoe that the collection $\tilde{\tau}=\{(\Theta,\Gamma):(\Theta,\Gamma)\in\tilde{\tau}\}$ provides the conditions of fuzzy soft topological spaces.

1. $0_{(U_{FH},\Sigma)},1_{(U_{FH},\Sigma)}\in\tilde{\tau}$,

2. Suppose that $\{(\Theta_i,\Gamma_i): i \in I\}$ be a family of fuzzy hypersoft sets in $\tilde{\tau}$. Since $\tilde{\tau} = \{(\Theta,\Gamma):(\Theta,\Gamma) \in \tilde{\tau}\}$, $\{(\Theta_i,\Gamma_i): i \in I\}$ is also a family of fuzzy soft sets in $\tilde{\tau}$. Then $\bigcup_{i\in I}(\Theta_i,\Gamma_i) \in \tilde{\tau}$ and hence $\bigcup_{i\in I}(\Theta_i,\Gamma_i) \in \tilde{\tau}$.

3. Suppose that $\{(\Theta_i,\Gamma_i): i = \overline{1,n}\}$ be a family of the finite number of fuzzy hypersoft sets in $\tilde{\tau}$. Then $\{(\Theta_i,\Gamma_i): i = \overline{1,n}\}$ is a family of fuzzy soft sets in $\tilde{\tau}$. Then $\bigcap_{i=1}^{n}(\Theta_i,\Gamma_i) \in \tilde{\tau}$ so $\bigcap_{i=1}^{n}(\Theta_i,\Gamma_i) \in \tilde{\tau}$.

Definition 3.3 Let $(U,\tilde{\tau},\Sigma)$ be a fuzzy hypersoft topological space over U and (Θ,Γ) be a fuzzy hypersoft set over U. Then (Θ,Γ) is said to be a fuzzy hypersoft closed set if its complement $(\Theta,\Gamma)^c$ belongs to $\tilde{\tau}$.

Example 3.3 We consider the Example 3.1. It is clear that $\left(0_{(U_{FH},\Sigma)}\right)^c$, $\left(1_{(U_{FH},\Sigma)}\right)^c$, $((\Theta_1,\Gamma_1))^c$, $((\Theta_2,\Gamma_2))^c$, $((\Theta_3,\Gamma_3))^c$ are fuzzy hypersoft closed sets.

$$\left(0_{(U_{FH},\Sigma)}\right)^c = 1_{(U_{FH},\Sigma)}, \left(1_{(U_{FH},\Sigma)}\right)^c = 0_{(U_{FH},\Sigma)}$$

$$((\Theta_1,\Gamma_1))^c = \left\{\begin{array}{l} <(\alpha_1,\beta_1,\gamma_2,\delta_1),\{\frac{u_1}{0,8},\frac{u_2}{0,5},\frac{u_3}{1}\}>, \\ <(\alpha_1,\beta_1,\gamma_2,\delta_3),\{\frac{u_1}{0,5},\frac{u_2}{0,9},\frac{u_3}{0,3}\}>, \\ <(\alpha_1,\beta_2,\gamma_2,\delta_1),\{\frac{u_1}{0,4},\frac{u_2}{1},\frac{u_3}{0,8}\}>, \\ <(\alpha_1,\beta_2,\gamma_2,\delta_3),\{\frac{u_1}{1},\frac{u_2}{0,8},\frac{u_3}{0,7}\}> \end{array}\right\},$$

$$((\Theta_2,\Gamma_2))^c = \left\{ \begin{array}{l} <(\alpha_2,\beta_2,\gamma_1,\delta_1),\{\frac{u_1}{0,6},\frac{u_2}{0,3},\frac{u_3}{0,8}\}>, \\ <(\alpha_2,\beta_2,\gamma_1,\delta_2),\{\frac{u_1}{0,2},\frac{u_2}{0,8},\frac{u_3}{0,6}\}>, \\ <(\alpha_2,\beta_3,\gamma_1,\delta_1),\{\frac{u_1}{0,7},\frac{u_2}{0,6},\frac{u_3}{1}\}>, \\ <(\alpha_2,\beta_3,\gamma_1,\delta_2),\{\frac{u_1}{1},\frac{u_2}{0,3},\frac{u_3}{0,2}\}> \end{array} \right\},$$

$$((\Theta_3,\Gamma_3))^c = \left\{ \begin{array}{l} <(\alpha_1,\beta_1,\gamma_2,\delta_1),\{\frac{u_1}{0,8},\frac{u_2}{0,5},\frac{u_3}{1}\}>, \\ <(\alpha_1,\beta_1,\gamma_2,\delta_3),\{\frac{u_1}{0,5},\frac{u_2}{0,9},\frac{u_3}{0,3}\}>, \\ <(\alpha_1,\beta_2,\gamma_2,\delta_1),\{\frac{u_1}{0,4},\frac{u_2}{1},\frac{u_3}{0,8}\}>, \\ <(\alpha_1,\beta_2,\gamma_2,\delta_3),\{\frac{u_1}{1},\frac{u_2}{0,8},\frac{u_3}{0,7}\}> \\ <(\alpha_2,\beta_2,\gamma_1,\delta_1),\{\frac{u_1}{0,6},\frac{u_2}{0,3},\frac{u_3}{0,8}\}>, \\ <(\alpha_2,\beta_2,\gamma_1,\delta_2),\{\frac{u_1}{0,2},\frac{u_2}{0,8},\frac{u_3}{0,6}\}>, \\ <(\alpha_2,\beta_3,\gamma_1,\delta_1),\{\frac{u_1}{0,7},\frac{u_2}{0,6},\frac{u_3}{1}\}>, \\ <(\alpha_2,\beta_3,\gamma_1,\delta_2),\{\frac{u_1}{1},\frac{u_2}{0,3},\frac{u_3}{0,2}\}> \end{array} \right\}.$$

Proposition 3.3 Let $(U,\tilde{\tau},\Sigma)$ be a fuzzy hypersoft topological space over U . Then the following properties are provide.

1. $0_{(U_{FH},\Sigma)}, 1_{(U_{FH},\Sigma)}$ are fuzzy hypersoft closed sets over U.
2. The intersection of any number of fuzzy hypersoft closed set is a fuzzy hypersoft set over U.
3. The union of any two fuzzy hypersoft closed set is a fuzzy hypersoft closed set over U.

Proof. Straightforward.

Definition 3.4 Let $(U,\tilde{\tau},\Sigma)$ be a fuzzy hypersoft topological space over U and (Θ,Γ) be a fuzzy hypersoft set over U. The fuzzy hypersoft closure of (Θ,Γ) denoted by $cl_{FH}(\Theta,\Gamma)$ is the intersection of all fuzzy hypersoft closed super sets of (Θ,Γ).

It is clear that $cl_{FH}(\Theta,\Gamma)$ is the smallest fuzzy hypersoft closed set over U which contain (Θ,Γ).

Theorem 3.1 Let $(U,\tilde{\tau},\Sigma)$ be a fuzzy hypersoft topological space over U and $(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2)\in FHS(U,\Sigma)$. Then,

1. $cl_{FH}\left(0_{(U_{FH},\Sigma)}\right)=0_{(U_{FH},\Sigma)}$ and $cl_{FH}\left(1_{(U_{FH},\Sigma)}\right)=1_{(U_{FH},\Sigma)}$,
2. $(\Theta_1,\Gamma_1)\subseteq cl_{FH}(\Theta_1,\Gamma_1)$,
3. (Θ_1,Γ_1) is a fuzzy hypersoft closed set if and only if $(\Theta_1,\Gamma_1)=cl_{FH}(\Theta_1,\Gamma_1)$,
4. $cl_{FH}\left(cl_{FH}(\Theta_1,\Gamma_1)\right)=cl_{FH}(\Theta_1,\Gamma_1)$,
5. If $(\Theta_1,\Gamma_1)\subseteq(\Theta_2,\Gamma_2)$, then $cl_{FH}(\Theta_1,\Gamma_1)\subseteq cl_{FH}(\Theta_2,\Gamma_2)$,
6. $cl_{FH}\left[(\Theta_1,\Gamma_1)\cup(\Theta_2,\Gamma_2)\right]=$ $cl_{FH}(\Theta_1,\Gamma_1)\cup cl_{FH}(\Theta_2,\Gamma_2)$.

Proof. 1 and 2 are clear from the definition of closure.

3. Let (Θ_1,Γ_1) be a fuzzy hypersoft closed set. By (2), we have $(\Theta_1,\Gamma_1)\subseteq cl_{FH}(\Theta_1,\Gamma_1)$. Since $cl_{FH}(\Theta_1,\Gamma_1)$ is the smallest fuzzy hypersoft closed set over U which contain (Θ_1,Γ_1), then $cl_{FH}(\Theta_1,\Gamma_1)\,\tilde{\subset}\,(\Theta_1,\Gamma_1)$. Hence $(\Theta_1,\Gamma_1)=cl_{FH}(\Theta_1,\Gamma_1)$. Conversely, suppose that $(\Theta_1,\Gamma_1)=cl_{FH}(\Theta_1,\Gamma_1)$. Since $cl_{FH}(\Theta_1,\Gamma_1)$ is a fuzzy hypersoft closed set, then (Θ_1,Γ_1) is closed.

4. Let $(\Theta_2,\Gamma_2)=cl_{FH}(\Theta_1,\Gamma_1)$. Then, (Θ_2,Γ_2) is a fuzzy hypersoft closed set, $(\Theta_2,\Gamma_2)=cl_{FH}(\Theta_2,\Gamma_2)$. So, we have $cl_{FH}\left(cl_{FH}(\Theta_1,\Gamma_1)\right)=cl_{FH}(\Theta_1,\Gamma_1)$.

5. If $(\Theta_1,\Gamma_1)\subseteq(\Theta_2,\Gamma_2)$, then
$$\begin{aligned}&(\Theta_2,\Gamma_2)=(\Theta_1,\Gamma_1)\cup(\Theta_2,\Gamma_2)\Rightarrow\\&cl_{FH}(\Theta_2,\Gamma_2)=cl_{FH}\left[(\Theta_1,\Gamma_1)\cup(\Theta_2,\Gamma_2)\right]=\\&cl_{FH}(\Theta_1,\Gamma_1)\cup cl_{FH}(\Theta_2,\Gamma_2)\Rightarrow\\&cl_{FH}(\Theta_1,\Gamma_1)\subseteq cl_{FH}(\Theta_2,\Gamma_2).\end{aligned}$$

6. Since $(\Theta_1,\Gamma_1)\subseteq(\Theta_1,\Gamma_1)\cup(\Theta_2,\Gamma_2)$ and $(\Theta_2,\Gamma_2)\subseteq(\Theta_1,\Gamma_1)\cup(\Theta_2,\Gamma_2)$, from the (5), $cl_{FH}(\Theta_1,\Gamma_1)\subseteq cl_{FH}\left[(\Theta_1,\Gamma_1)\cup(\Theta_2,\Gamma_2)\right]$ and $(\Theta_2,\Gamma_2)\subseteq cl_{FH}\left[(\Theta_1,\Gamma_1)\cup(\Theta_2,\Gamma_2)\right]$. Therefore

$cl_{FH}(\Theta_1,\Gamma_1)\cup cl_{FH}(\Theta_2,\Gamma_2)\subseteq cl_{FH}\left[(\Theta_1,\Gamma_1)\cup(\Theta_2,\Gamma_2)\right]$. Conversely, since $(\Theta_1,\Gamma_1)\subseteq cl_{FH}(\Theta,\Gamma)$ and $(\Theta_2,\Gamma_2)\subseteq cl_{FH}(\Theta_2,\Gamma_2)$ are fuzzy hypersoft closed sets, from the proposition 3, $cl_{FH}(\Theta_1,\Gamma_1)\cup cl_{FH}(\Theta_2,\Gamma_2)$ is a fuzzy hypersoft closed set over U being the union of two fuzzy hypersoft fuzzy soft closed sets. Then, $cl_{FH}\left[(\Theta_1,\Gamma_1)\cup(\Theta_2,\Gamma_2)\right]\subseteq cl_{FH}(\Theta_1,\Gamma_1)\cup cl_{FH}(\Theta_2,\Gamma_2)$. Hence $cl_{FH}\left[(\Theta_1,\Gamma_1)\cup(\Theta_2,\Gamma_2)\right] = cl_{FH}(\Theta_1,\Gamma_1)\cup cl_{FH}(\Theta_2,\Gamma_2)$ is obtained.

Definition 3.5 Let $(U,\tilde{\tau},\Sigma)$ be a fuzzy hypersoft topological space over U and (Θ,Γ) be a fuzzy hypersoft set over U. The fuzzy hypersoft interior of (Θ,Γ) denoted by $int_{FH}(\Theta,\Gamma)$ is the union of all fuzzy hypersoft open subsets of (Θ,Γ).

It is clear that $int_{FH}(\Theta,\Gamma)$ is the largest fuzzy hypersoft open set contained in (Θ,Γ).

Theorem 3.2 Let $(U,\tilde{\tau},\Sigma)$ be a fuzzy hypersoft topological space over U and $(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2)\in FHS(U,\Sigma)$. Then,

1. $int_{FH}\left(0_{(U_{FH},\Sigma)}\right) = 0_{(U_{FH},\Sigma)}$ and $int_{FH}\left(1_{(U_{FH},\Sigma)}\right) = 1_{(U_{FH},\Sigma)}$,
2. $int_{FH}(\Theta_1,\Gamma_1)\subseteq(\Theta_1,\Gamma_1)$,
3. (Θ_1,Γ_1) is a fuzzy hypersoft open set if and only if $int_{FH}(\Theta_1,\Gamma_1) = (\Theta_1,\Gamma_1)$,
4. $int_{FH}\left(int_{FH}(\Theta_1,\Gamma_1)\right) = int_{FH}(\Theta_1,\Gamma_1)$,
5. If $(\Theta_1,\Gamma_1)\subseteq(\Theta_2,\Gamma_2)$, then $int_{FH}(\Theta_1,\Gamma_1)\subseteq int_{FH}(\Theta_2,\Gamma_2)$,
6. $int_{FH}\left[(\Theta_1,\Gamma_1)\,\tilde{\cap}\,(\Theta_2,\Gamma_2)\right] = int_{FH}(\Theta_1,\Gamma_1)\,\tilde{\cap}\,int_{FH}(\Theta_2,\Gamma_2)$.

Proof. 1 and 2 are clear from the definition of interior.

3. Let (Θ_1,Γ_1) be a fuzzy hypersoft open set. Since $int_{FH}(\Theta_1,\Gamma_1)$ is the largest fuzzy hypersoft open set contained in (Θ_1,Γ_1), $int_{FH}(\Theta_1,\Gamma_1) = (\Theta_1,\Gamma_1)$. Conversely, suppose that $int_{FH}(\Theta_1,\Gamma_1) = (\Theta_1,\Gamma_1)$. Since $int_{FH}(\Theta_1,\Gamma_1)$ is a fuzzy hypersoft open set, (Θ_1,Γ_1) is also fuzzy hypersoft open set.

4. Let $int_{FH}(\Theta_1,\Gamma_1) = (\Theta_2,\Gamma_2)$. Since (Θ_2,Γ_2) is a fuzzy hypersoft open set $int_{FH}(\Theta_2,\Gamma_2) = (\Theta_2,\Gamma_2)$, so $int_{FH}\left(int_{FH}(\Theta_1,\Gamma_1)\right) = int_{FH}(\Theta_1,\Gamma_1)$ is obtained.

5. Let $(\Theta_1,\Gamma_1)\subseteq(\Theta_2,\Gamma_2)$. $int_{FH}(\Theta_1,\Gamma_1)\subseteq(\Theta_1,\Gamma_1)$ and hence $int_{FH}(\Theta_1,\Gamma_1)\subseteq(\Theta_2,\Gamma_2)$ also $int_{FH}(\Theta_2,\Gamma_2)$ is the largest fuzzy hypersoft open set contained in (Θ_2,Γ_2) and $int_{FH}(\Theta_1,\Gamma_1)\subseteq int_{FH}(\Theta_2,\Gamma_2)$.

6. $int_{FH}(\Theta_1,\Gamma_1)\subseteq(\Theta_1,\Gamma_1)$ and $int_{FH}(\Theta_2,\Gamma_2)\subseteq(\Theta_2,\Gamma_2)$. Hence $int_{FH}(\Theta_1,\Gamma_1)\tilde{\cap}\, int_{FH}(\Theta_2,\Gamma_2)\subseteq(\Theta_1,\Gamma_1)\tilde{\cap}(\Theta_2,\Gamma_2)$. Since the largest fuzzy hypersoft open set contained in $(\Theta_1,\Gamma_1)\tilde{\cap}(\Theta_2,\Gamma_2)$ is $int_{FH}\left[(\Theta_1,\Gamma_1)\tilde{\cap}(\Theta_2,\Gamma_2)\right]$, $int_{FH}(\Theta_1,\Gamma_1)\tilde{\cap}\, int_{FH}(\Theta_2,\Gamma_2)\subseteq int_{FH}\left[(\Theta_1,\Gamma_1)\tilde{\cap}(\Theta_2,\Gamma_2)\right]$.

Conversely $int_{FH}(\Theta_1,\Gamma_1)\tilde{\cap}\, int_{FH}(\Theta_2,\Gamma_2)\subseteq int_{FH}(\Theta_1,\Gamma_1)$ and $int_{FH}(\Theta_1,\Gamma_1)\tilde{\cap}\, int_{FH}(\Theta_2,\Gamma_2)\subseteq int_{FH}(\Theta_2,\Gamma_2)$. Hence $int_{FH}\left[(\Theta_1,\Gamma_1)\tilde{\cap}(\Theta_2,\Gamma_2)\right]\subseteq int_{FH}(\Theta_1,\Gamma_1)\tilde{\cap}\, int_{FH}(\Theta_2,\Gamma_2)$.

Example 3.4 We consider the attributes in Example 3.1. Let

$$(\Theta_1,\Gamma_1)=\left\{\begin{array}{c} <(\alpha_1,\beta_1,\gamma_2,\delta_1),\{\frac{u_1}{0,2},\frac{u_2}{0,5}\}>, \\ <(\alpha_1,\beta_1,\gamma_2,\delta_3),\{\frac{u_1}{0,5},\frac{u_2}{0,1},\frac{u_3}{0,7}\}>, \\ <(\alpha_1,\beta_2,\gamma_2,\delta_1),\{\frac{u_1}{0,6},\frac{u_3}{0,2}\}>, \\ <(\alpha_1,\beta_2,\gamma_2,\delta_3),\{\frac{u_2}{0,2},\frac{u_3}{0,3}\}> \end{array}\right\},$$

$$(\Theta_2,\Gamma_2)=\left\{\begin{array}{c} <(\alpha_1,\beta_1,\gamma_2,\delta_1),\{\frac{u_1}{0,3},\frac{u_2}{0,7},\frac{u_3}{0.3}\}>, \\ <(\alpha_1,\beta_1,\gamma_2,\delta_3),\{\frac{u_1}{0,6},\frac{u_2}{0,2},\frac{u_3}{0,7}\}>, \\ <(\alpha_1,\beta_2,\gamma_2,\delta_1),\{\frac{u_1}{0,7},\frac{u_3}{0,4}\}>, \\ <(\alpha_1,\beta_2,\gamma_2,\delta_3),\{\frac{u_2}{0,3},\frac{u_3}{0,5}\}> \end{array}\right\},$$

Obviously, $\tilde{\tau}=\{0_{(U_{FH},\Sigma)},1_{(U_{FH},\Sigma)},(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2)\}$ is a fuzzy hypersoft topology on U.

1. Suppose that any $(\Theta_3,\Gamma_3)\in FHS(U,\Sigma)$ be defined as follow;

$$(\Theta_3,\Gamma_3)=\left\{\begin{array}{l} <(\alpha_1,\beta_1,\gamma_2,\delta_1),\{\frac{u_1}{0,4},\frac{u_2}{0,7},\frac{u_3}{0.6}\}>, \\ <(\alpha_1,\beta_1,\gamma_2,\delta_3),\{\frac{u_1}{0,8},\frac{u_2}{0,4},\frac{u_3}{0,9}\}>, \\ <(\alpha_1,\beta_2,\gamma_2,\delta_1),\{\frac{u_1}{0,8},\frac{u_3}{0,7}\}>, \\ <(\alpha_1,\beta_2,\gamma_2,\delta_3),\{\frac{u_1}{0,4},\frac{u_2}{0,3},\frac{u_3}{0,5}\}> \end{array}\right\}$$

Then $0_{(U_{FH},\Sigma)},(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2)\subseteq(\Theta_3,\Gamma_3)$. Therefore $int_{FH}(\Theta_3,\Gamma_3)=0_{(U_{FH},\Sigma)}\cup(\Theta_1,\Gamma_1)\cup(\Theta_2,\Gamma_2)=(\Theta_2,\Gamma_2)$.

2. Suppose that any $(\Theta_4,\Gamma_4)\in FHS(U,\Sigma)$ be defined as follow;

$$(\Theta_4,\Gamma_4)=\left\{\begin{array}{l} <(\alpha_1,\beta_1,\gamma_2,\delta_1),\{\frac{u_1}{0,5},\frac{u_2}{0,2},\frac{u_3}{0,4}\}>, \\ <(\alpha_1,\beta_1,\gamma_2,\delta_3),\{\frac{u_1}{0,3},\frac{u_2}{0,4},\frac{u_3}{0,2}\}>, \\ <(\alpha_1,\beta_2,\gamma_2,\delta_1),\{\frac{u_1}{0,1},\frac{u_2}{0,2},\frac{u_3}{0,3}\}>, \\ <(\alpha_1,\beta_2,\gamma_2,\delta_3),\{\frac{u_1}{0,4},\frac{u_2}{0,5},\frac{u_3}{0,1}\}> \end{array}\right\}$$

Now we find the complement of $(\Theta_1,\Gamma_1),(\Theta_2,\Gamma_2)$,

$$(\Theta_1,\Gamma_1)^c=\left\{\begin{array}{l} <(\alpha_1,\beta_1,\gamma_2,\delta_1),\{\frac{u_1}{0,8},\frac{u_2}{0,5},\frac{u_3}{1}\}>, \\ <(\alpha_1,\beta_1,\gamma_2,\delta_3),\{\frac{u_1}{0,5},\frac{u_2}{0,9},\frac{u_3}{0,3}\}>, \\ <(\alpha_1,\beta_2,\gamma_2,\delta_1),\{\frac{u_1}{0,4},\frac{u_2}{1},\frac{u_3}{0,8}\}>, \\ <(\alpha_1,\beta_2,\gamma_2,\delta_3),\{\frac{u_1}{1},\frac{u_2}{0,8},\frac{u_3}{0,7}\}> \end{array}\right\},$$

$$(\Theta_2,\Gamma_2)^c = \left\{ \begin{array}{l} <(\alpha_1,\beta_1,\gamma_2,\delta_1),\{\frac{u_1}{0,7},\frac{u_2}{0,3},\frac{u_3}{0.7}\}>, \\ <(\alpha_1,\beta_1,\gamma_2,\delta_3),\{\frac{u_1}{0,4},\frac{u_2}{0,8},\frac{u_3}{0,3}\}>, \\ <(\alpha_1,\beta_2,\gamma_2,\delta_1),\{\frac{u_1}{0,3},\frac{u_2}{1},\frac{u_3}{0,6}\}>, \\ <(\alpha_1,\beta_2,\gamma_2,\delta_3),\{\frac{u_1}{1},\frac{u_2}{0,7},\frac{u_3}{0,5}\}> \end{array} \right\},$$

$$\left(0_{(U_{FH},\Sigma)}\right)^c = 1_{(U_{FH},\Sigma)}, \left(1_{(U_{FH},\Sigma)}\right)^c = 0_{(U_{FH},\Sigma)}$$

Obviously, $\left(0_{(U_{FH},\Sigma)}\right)^c, \left(1_{(U_{FH},\Sigma)}\right)^c, (\Theta_1,\Gamma_1)^c, (\Theta_2,\Gamma_2)^c$ are all fuzzy hypersoft closed sets over $(U,\tilde{\tau},\Sigma)$. Then $(\Theta_4,\Gamma_4) \subseteq \left(0_{(U_{FH},\Sigma)}\right)^c, (\Theta_1,\Gamma_1)^c, (\Theta_2,\Gamma_2)^c$. Therefore

$$\begin{aligned} cl_{FH}(\Theta_4,\Gamma_4) &= \left(0_{(U_{FH},\Sigma)}\right)^c \tilde{\cap} (\Theta_1,\Gamma_1)^c \tilde{\cap} (\Theta_2,\Gamma_2)^c \\ &= (\Theta_2,\Gamma_2)^c. \end{aligned}$$

Theorem 3.3 Let $(U,\tilde{\tau},\Sigma)$ be a fuzzy hypersoft topological space over U and $(\Theta,\Gamma) \in FHS(U,\Sigma)$. Then,

1. $\left(cl_{FH}(\Theta,\Gamma)\right)^c = int_{FH}\left((\Theta,\Gamma)^c\right)$,
2. $\left(int_{FH}(\Theta,\Gamma)\right)^c = cl_{FH}\left((\Theta,\Gamma)^c\right)$.

Proof. 1.

$$cl_{FH}(\Theta,\Gamma) = \tilde{\cap}\left\{(\Theta,\Gamma) \in \tilde{\tau}^c : (\Theta_2,\Gamma_2) \tilde{\subset} (\Theta,\Gamma)\right\}$$

$$\Rightarrow \left(cl_{FH}(\Theta,\Gamma)\right)^c = \left(\tilde{\cap}\left\{(\Theta,\Gamma) \in \tilde{\tau}^c : (\Theta_2,\Gamma_2) \tilde{\subset} (\Theta,\Gamma)\right\}\right)^c$$

$$= \cup\left\{(\Theta,\Gamma) \in \tilde{\tau} : (\Theta,\Gamma)^c \tilde{\subset} (\Theta_2,\Gamma_2)^c\right\} = int_{FH}\left((\Theta,\Gamma)^c\right)$$

2. $int_{FH}(\Theta,\Gamma) = \cup\left\{(\Theta,\Gamma) \in \tilde{\tau} : (\Theta,\Gamma) \tilde{\subset} (\Theta_2,\Gamma_2)\right\}$

$$\Rightarrow \left(int_{FH}(\Theta,\Gamma)\right)^c = \left(\cup\left\{(\Theta,\Gamma) \in \tilde{\tau} : (\Theta,\Gamma) \tilde{\subset} (\Theta_2,\Gamma_2)\right\}\right)^c$$

$$= \tilde{\cap}\left\{(\Theta,\Gamma) \in \tilde{\tau}^c : (\Theta_2,\Gamma_2)^c \tilde{\subset} (\Theta,\Gamma)^c\right\} = cl_{FH}\left((\Theta,\Gamma)^c\right).$$

Definition 3.6 Let $(U, \tilde{\tau}, \Sigma)$ be a fuzzy hypersoft topological space over U and $\tilde{B} \subseteq \tilde{\tau}$. $\tilde{B}$ is called a fuzzy hypersoft basis for the fuzzy hypersoft topology $\tilde{\tau}$ if every element of $\tilde{\tau}$ can be written as the fuzzy hypersoft union of elements of $\tilde{B}$.

Proposition 3.4 Let $(U, \tilde{\tau}, \Sigma)$ be a fuzzy hypersoft topological space over U and $\tilde{B}$ be fuzzy hypersoft basis for $\tilde{\tau}$. Then $\tilde{\tau}$ equals the collection of fuzzy hypersoft union of elements of $\tilde{B}$.

Proof. The proof is clear from definition of fuzzy hypersoft basis.

Example 3.5 We consider that the Example 3.1. Then $\tilde{B} = \{0_{(U_{FH},\Sigma)}, 1_{(U_{FH},\Sigma)}, (\Theta_1, \Gamma_1), (\Theta_2, \Gamma_2)\}$ is a fuzzy hypersoft basis for the fuzzy hypersoft topology $\tilde{\tau}$.

Theorem 3.4 Let $(U, \tilde{\tau}, \Sigma)$ be a fuzzy hypersoft topological space over U and (Θ, Γ) be a fuzzy hypersoft set over U Then the collection $\tilde{\tau}_{(\Theta,\Gamma)} = \{(\Theta,\Gamma) \cap (\Xi,\Delta) : (\Xi,\Delta) \in \tilde{\tau}\}$ is a fuzzy hypersoft topology on the fuzzy hypersoft subset (Θ, Γ) relative parameter set Γ.

Proof. 1. $0_{(U_{FH},\Sigma)}, 1_{(U_{FH},\Sigma)} \in \tilde{\tau}_{(\Theta,\Gamma)}$,

2. Suppose that $(\Theta_1, \Gamma_1), (\Theta_2, \Gamma_2) \in \tilde{\tau}_{(\Theta,\Gamma)}$. Then for each $i = 1, 2$, there exist $(\Delta_i, \Gamma_i) \in \tilde{\tau}$ such that $(\Theta_i, \Gamma_i) = (\Theta, \Gamma) \tilde{\cap} (\Delta_i, \Gamma_i)$. Then

$$(\Theta_1, \Gamma_1) \tilde{\cap} (\Theta_2, \Gamma_2) = \left[(\Theta,\Gamma) \tilde{\cap} (\Xi_1, \Delta_1)\right] \tilde{\cap}$$
$$\left[(\Theta,\Gamma) \tilde{\cap} (\Xi_2, \Delta_2)\right] = (\Theta,\Gamma) \tilde{\cap}$$
$$\left[(\Xi_1, \Delta_1) \tilde{\cap} (\Xi_2, \Delta_2)\right].$$

Since $(\Xi_1, \Delta_1) \tilde{\cap} (\Xi_2, \Delta_2) \in \tilde{\tau}$, we have $(\Theta_1, \Gamma_1) \tilde{\cap} (\Theta_2, \Gamma_2) \in \tilde{\tau}_{(\Theta,\Gamma)}$.

3. Let $\{(\Xi,\Delta)_k : k \in K\}$ be a subfamily of $\tilde{\tau}_{(\Theta,\Gamma)}$. Then for each $k \in K$, there is a fuzzy soft set $(\Xi_2, \Delta_2)_k$ of $\tilde{\tau}$ such that $(\Xi,\Delta)_k = (\Theta,\Gamma) \tilde{\cap} (\Xi_2, \Delta_2)_k$. Then we have,

$$\bigcup_{k\in K} (\Xi,\Delta)_k = \bigcup_{k\in K} \left[(\Theta,\Gamma) \tilde{\cap} (\Xi_2, \Delta_2)_k\right]$$

$$= (\Theta,\Gamma) \tilde{\cap} \left(\bigcup_{k\in K} (\Xi_2, \Delta_2)_k \right)$$

Since $\bigcup_{k\in K} (\Xi_2, \Delta_2)_k \in \tilde{\tau}$, we have $\bigcup_{k\in K} (\Xi,\Delta)_k \in \tilde{\tau}_{(\Theta,\Gamma)}$.

Definition 3.7 Let $(U,\tilde{\tau},\Sigma)$ be a fuzzy hypersoft topological space over U and (Θ,Γ) be a fuzzy hypersoft set over U. Then the fuzzy hypersoft topology $\tilde{\tau}_{(\Theta,\Gamma)} = \{(\Theta,\Gamma)\cap(\Xi,\Delta) : (\Xi,\Delta)\in\tilde{\tau}\}$ is called fuzzy hypersoft subspace topology and $\left((\Theta,\Gamma),\tilde{\tau}_{(\Theta,\Gamma)},\Gamma\right)$ is called a fuzzy hypersoft subspace of $(U,\tilde{\tau},\Sigma)$.

Example 3.6 Let $(U,\tilde{\tau},\Sigma)$ be a fuzzy hypersoft topological space over U and $(\Theta,\Gamma)\in FHS(U,\Sigma)$. We consider the fuzzy hypersoft topology in Example 4 and (Θ,Γ) be defined as follow;

$$(\Theta,\Gamma) = \left\{\begin{array}{c} <(\alpha_1,\beta_1,\gamma_2,\delta_1),\{\frac{u_1}{0,3},\frac{u_2}{0,3}\}>, \\ <(\alpha_1,\beta_1,\gamma_2,\delta_3),\{\frac{u_1}{0,6},\frac{u_2}{0,3},\frac{u_3}{0,8}\}>, \\ <(\alpha_1,\beta_2,\gamma_2,\delta_1),\{\frac{u_1}{0,2},\frac{u_3}{0,6}\}>, \\ <(\alpha_1,\beta_2,\gamma_2,\delta_3),\{\frac{u_2}{0,4},\frac{u_3}{0,5}\}> \end{array}\right\}$$

Then the collection

$$\tilde{\tau}_{(\Theta,\Gamma)} = \left\{\begin{array}{l} 0_{(U_{FH},\Sigma)}\,\tilde{\cap}\,(\Theta,\Gamma), 1_{(U_{FH},\Sigma)}\,\tilde{\cap}\,(\Theta,\Gamma), \\ (\Theta_1,\Gamma_1)\,\tilde{\cap}\,(\Theta,\Gamma), (\Theta_2,\Gamma_2)\,\tilde{\cap}\,(\Theta,\Gamma) \end{array}\right\}$$

is a fuzzy hypersoft subspace topology and $\left((\Theta,\Gamma),\tilde{\tau}_{(\Theta,\Gamma)},\Gamma\right)$ is a fuzzy hypersoft topological subspace of $(U,\tilde{\tau},\Sigma)$.

Theorem 3.5 Let $(U,\tilde{\tau},\Sigma)$ be a fuzzy hypersoft topological space over U, $\tilde{B}$ be a fuzzy soft basis for $\tilde{\tau}$ and $(\Theta,\Gamma)\in FHS(U,\Sigma)$. Then the collection

$$\tilde{B}_{(\Theta,\Gamma)} = \{(\Theta,\Gamma)\,\tilde{\cap}\,(\Xi,\Delta) \mid (\Xi,\Delta)\in\tilde{B}\}$$

is a fuzzy hypersoft basis for the subspace topology $\tilde{\tau}_{(\Theta,\Gamma)}$.

Proof. Let (Ξ,Δ) be in $\tilde{\tau}_{(\Theta,\Gamma)}$. Then there is a fuzzy hypersoft set (Ξ_2,Δ_2) in $\tilde{\tau}$ such that $(\Xi,\Delta) = (\Theta,\Gamma)\,\tilde{\cap}\,(\Xi_2,\Delta_2)$. Since $\tilde{B}$ is a base for $\tilde{\tau}$, we can find a subcollection of $\tilde{B}$ such that $(\Xi_2,\Delta_2) = \bigcup_{i\in I}(k,\Gamma_3)_i$. Hence we have that

$$(\Xi,\Delta) = (\Theta,\Gamma)\,\tilde{\cap}\,(\Xi_2,\Delta_2) = (\Theta,\Gamma)\,\tilde{\cap}\left(\bigcup_{i\in I}(k,\Gamma_3)_i\right)$$

$$= \bigcup_{i\in I}\left[(\Theta,\Gamma)\,\tilde{\cap}\,(k,\Gamma_3)_i\right]$$

which implies that $\tilde{B}_{(\Theta,\Gamma)}$ is a fuzzy hypersoft basis for the fuzzy hypersoft subspace topology $\tilde{\tau}_{(\Theta,\Gamma)}$.

4. CONCLUSION

Topology is an interesting and important field in mathematics and can provide many links between other fields of science and mathematical models. The soft set theory, initiated by Molodtsov (Molodtsov, 1999) and easily extended to many problems of social life uncertainties, has recently been studied and developed by many researchers. In the present paper, we first gave the concept of fuzzy hypersoft topology and presented its fundamental properties with some important examples. This paper also can be the initial step for studies on fuzzy hypersoft topology; for example, by using fuzzy hypersoft functions, one can study on fuzzy hypersoft continuity, separation axioms, compact spaces, connected spaces and since there are compact ties between hypersoft sets and information systems, the results deducted from the fuzzy hypersoft topological space studies can be used to strengthen these types of ties.

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