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Comparative Analysis of Ski Jumping In-run Hill Models Profile*

Ihor ZANEVSKYY[1], Lyudmyla ZANEVSKA[2]
[1] Lviv State University of Physical Culture, Lviv, Ukraine
https://orcid.org/ 0000-0002-3276-6057
[2] Lviv State University of Physical Culture, Lviv, Ukraine
https://orcid.org/ 0000-0001-9279-2373
Email: izanevsky@ukr.net, lzanevska@ukr.net

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Abstract

Background. The aim of this research was done to create calculating methods for virtual replacing of a circle arc segment on the in-run hill. This replacement should not change the angle of the in-run hill inclination, but could change the length of the straight-line segment to such an extent that it can follow geometrical parameters of the in-runs in vogue.
Materials and Methods. 38 in-runs hills certificated by International Ski Federation; mechanical and mathematical modelling of the in-run hill profile modelled with cycloid, hyperbola, or inclined quadratic parabola with decreased ratio of vertical to horizontal dimensions.
Results and Discussion. The decreased ratio of the vertical to horizontal dimensions was in the range of 2.71–0.73% when cycloid was used, 16.33–8.60% when hyperbola was used, and 4.58–0.90% when inclined quadratic parabola was used. When the circle arc was replaced with a quadratic parabola or an inclined cubic parabola, the ratio of the dimensions increased. If the difference between the angles of inclination of straight-line segments increased, this ratio increased too. For the certificated in-runs, the ratio varied in the range of 2.21–8.61% when a quadratic parabola was used and 14.64–19.04% when inclined cubic parabola was used.

Keywords: Ski jumping; In-run hill profile; Mathematical Modeling

Introduction

Ski jumping from an in-run has four phases: in-run, taking off, flight, and landing. For judging the competition results, the judges follow the Ski Jumping Rules and consider only the third and the fourth phases (International ski competition rules, 2008). They evaluate the technique of the flight and landing, and measure the distance of the jump. The effect of in-run and take-off (first and second phases) on the finishing phases, therefore, plays a decisive role in determining, to a great extent, the quantitative and qualitative parameters of the jump (Schwameder, 2008; Muller, 2009; Jung et al., 2014).

The ski jumper executes the in-run and take-off sliding down the in-run hill which ends with a take-off table. The most widely used in-runs are solid units consisting of the in-run hill and the take-off table (Figure 1). The profile of the in-run track includes three segments: two straight-line ones and a curvilinear one. The first straight-line segment, together with the curvilinear segment, serves as the in-run track, and the second straight-line segment as the take-off table. In terms of actual usage, the inclined part of the take-off table serves, to some extent, as the in-run track too. The inclination of the curvilinear segment at its highest point equals the inclination of the first straight-line segment, and the inclination at its lowest point equals that of the take-off table.

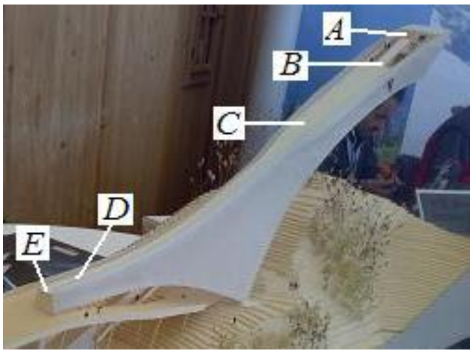

Figure 1. A model of an in-run hill (K-125) in Garmisch-Partenkirchen, Germany: *AC* is a straight-line in-run segment; *AB* is a segment of a start gate position; *CD* is a curvilinear in-run segment; *DE* is a straight-line take-off table (The new Olympiaschanze of Garmisch-Partenkirchen, 2017).

Sliding down the curvilinear segment can be considered a sub-phase of the in-run. The in-run curve starts when the ski jumper enters the radius, and ends as he reaches the take-off table. When the ski jumper enters the curvilinear segment the normal reaction force increases because of the centrifugal force. The take-off phase begins when the ski jumper initiates his take-off movement, and ends just as he leaves the take-off table (Banakh & Zanevskyy, 2010).

According to the norms of architecture, the curvilinear segment was being constructed, till date, as a circular arc (Neufert, 2004). Because of the sharp increase in the trajectory curvature at the junction of the first straight-line segment and the arc, the ski jumper's body will be affected by a centripetal force that equals about 87% of the normal reaction force value (Ettema et al., 2005). As the ski jumper slides along the arc, his body will be affected by a centripetal force that gradually increases in proportion to the squared speed of sliding, and dies down abruptly at the end of the take-off table. The normal force increased from 0.88 of the gravity on the first straight-line segment up to 1.65 on the arc. The exact value depends on the slope, speed, and radius of the arc. During the motion on straight-line segment, the normal force will be less than gravity because the ski jumper moves on the slope (Zanevskyy & Banakh, 2010).

For controlling the reaction force when the ski jumper moves along the curvilinear segment, researchers propose to use profiles with variable curvature: cycloid, parabola, hyperbola (Palej & Struk, 2003), and cubic parabola (Gasser, 2008). The last one was presented by International Ski Federation (ISF) as the standard profile for the in-run design. One in-run with cubic parabola profile of the in-run hill in Bischofshofen, Austria was certificated by ISF (Certificate of jumping hill, 2003).

Different profiles are proposed for different purposes: to reduce the reaction force at the end of the curvilinear segment, to stabilize its value or to reduce it to zero, to gradually increase the centripetal force at the very beginning of the curvilinear track, and so on. However, replacement of the circular arc with another profile causes major changes in some of the in-run hill parameters: the inclination of the straight-line segment, or horizontal and vertical dimensions of the curvilinear segment (Palej & Filipowska, 2009).

For reducing the value of the normal reaction force just near the take-off table, Palej & Struk (2004) proposed cycloid, parabolic, and hyperbolic profiles and considered cycloid profile the best. They formulated and solved an initial value problem for a nonlinear second order equation. They considered this approach as the simplest one, but cautioned that the normal reaction does not usually appear at the border with the take-off table.

Some researches tried to decrease the normal reaction force at the end of the curvilinear segment by using a family of even polynomial functions which possess the determined properties of the normal reaction (Filipowska, 2008; Jung et al., 2019). Considering the popular K125 power in-run Wielka Krokiew in Zakopane, Poland, they proposed to replace its straight-line and circle arc segments of the in-run hill with one polynomial curve. But, the implication of such replacement is the need to increase the inclination angle of the in-run hill to avoid the appearance of inflexion points. Unfortunately, the value of the increased incline should be greater than the maximum inclination of the in-run hills of the in-runs in vogue.

A weak point of these models is taking into account the air drag force and the force of friction between the skis and the in-run hill track. The corresponding models include empirical coefficients which are dependent on the ski jumper's body pose, speed, normal reaction force, temperature, dampness and other factors. Because the analytical functions used in modelling these factors do not ensure precision, it is considered better to create the profile model without necessarily taking into consideration the drag and friction forces. Therefore, from a practical point of view, using a geometrical model, which satisfies two fundamental conditions, was considered: smooth borders between the curvilinear and the straight-line segments of the in-run hill and the concave profile of the curvilinear segment (Zanevskyy & Banakh, 2010). With the frames of such a model, it would be possible to solve the problem with reasonable precision.

The objective of this research was to create calculating methods for virtual replacing of a circle arc segment on the in-run of the ski-jumping in-run with profiles of changeable curvature, based on the functions of cycloid, parabola, hyperbola, and cubic parabola. In the process of replacing, the angle of in-run hill inclination should not be changed, but the length of the straight-line segment could be changed to such an extent that it conforms to the geometrical parameters of the in-runs in vogue.

Materials and Methods

38 in-runs hills certificated by International Ski Federation; mechanical and mathematical modelling of the in-run hill profile modelled with a cycloid, a hyperbola, or an inclined quadratic parabola with decreased ratio of vertical to horizontal dimensions. Seven functions of a curvilinear segment profile of the in-run hill were investigated: arc, cycloid, quadratic parabola, cubic parabola, inclined quadratic parabola, inclined cubic parabola, and hyperbola (Gasser, 2018).

For smooth junctions between the arc and straight-line segments, their angles of inclination should be dependent on horizontal $(l = x_C - x_D)$ and vertical $(h = y_C - y_D)$ dimensions of the curvilinear segment defined by the following equations (Figure 2):

$$l = r(\sin\gamma - \sin\alpha); \qquad h = r(\cos\alpha - \cos\gamma), \tag{Eq.1}$$

where r is the radius of the arc, α the inclination angle of the second straight-line segment, i.e. take-off table, and γ the inclination angle of the first one, i.e. the in-run hill straight-line segment.

From equation (Eq.1), one gets the ratio between the circle arc dimensions by the following equation:

$$\left(\frac{h}{l}\right)_{circle} = tg\,\frac{\alpha + \gamma}{2}. \tag{Eq.2}$$

The length of the circle arc is:

$$S = r(\gamma - \alpha). \tag{Eq.3}$$

Using equations (1–3) and the inclination angles of straight-line segments (α, γ), one can calculate three of the four parameters of the arc segment (l, h, r, S), whereas the fourth one should be determined a priori.

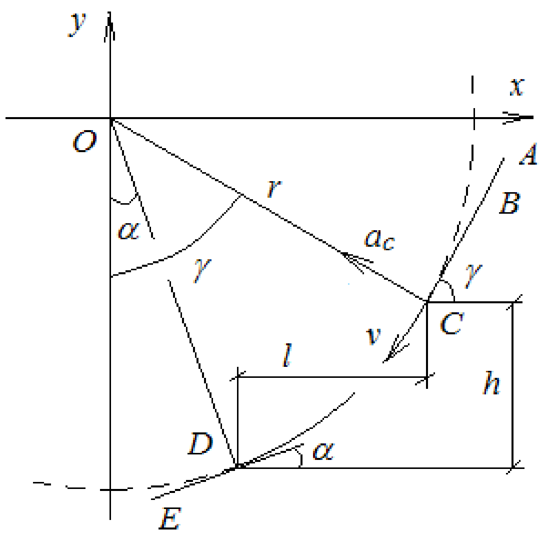

Figure 2. An in-run hill scheme model with a circle arc curvilinear segment.

The circle arc profile has one deficiency. Because of abrupt increase in the trajectory curvature at its junction with the first straight-line segment, the ski jumper's body is affected by a centripetal force whose magnitude is proportional to the body weight. The corresponding centripetal acceleration at the moment of entering the circle arc (point *C* on Figures 1, 2) is given by the following equation:

$$a_C = \frac{v_C^2}{r}, \qquad \text{(Eq.4)}$$

where v_C is the speed of sliding at point *C*.

Results

Dimensionless values of the circle arc curvature and the profile of a rather horizontal dimension of the in-run hill curvilinear segment with inclination angles of $\alpha = 11°$ and $\gamma = 35°$ are presented in the graph (Figure 3). These parameters were used, because among the 38 in-runs certificated by the ISF (Certificate of jumping hill, 2003) the in-run hill of seven in-runs had the same inclination angles and another five also had more or less the same inclination angle but for a difference $\pm 0.2°$ (Table 1: Numbers 20, 21, 25, 30, 31, 33, 34, and 9, 14, 15, 18, 32). These 12 in-runs present a full range in terms of power (K = 90–185) for high level competitions in ski jumping. The ratio of the circle arc dimensions (Eq.2) is $(h/l)_{circle} = 0.424$ and that of the dimensionless values of the curvature $(l/r)_{circle} = 0.383$.

Table 1. In-run hills which are certificated by ISF (Certificate of jumping hill, 2003)

No	Locality (country)	Size, K	γ	α	ρ
			Degree		m
1	Villach (AUT)	60	29.0	10.5	65
2	Wernigerode (GER)	63	35.0	9.5	59
3	Bischofsgrün (GER)	64	35.0	10.5	67
4	Namsos (NOR)	65	34.0	10.0	57
5	Bischofshofen (AUT)	65	35.0	10.0	65
6	Høydalsmo (NOR)	85	32.0	10.5	80
7	Villach (AUT)	90	35.0	10.5	64
8	Stryn (NOR)	90	30.0	10.5	85
9	Trondheim (NOR)	90	34.0	11.0	90
10	Örnsköldsvik (SWE)	90	36.0	10.5	90
11	Gällivare (SWE)	90	34.0	10.5	95
12	Heddal (NOR)	90	32.5	10.5	80
13	Mo I Rana (NOR)	90	36.5	10.5	80
14	Lillehammer (NOR)	90	35.0	11.2	90
15	Seefeld (AUT)	90	34.9	11.0	72
16	Lauscha (GER)	92	37.0	10.5	83
17	Oberwiesenthal (GER)	95	37.0	10.0	85
18	Hinterzarten (GER)	95	35.2	11.2	75
19	Gallio/Asiago (ITA)	95	30.0	11.0	90
20	Pragelato (ITA)	95	35.0	11.0	92
21	Sollefteå (SWE)	107	35.0	11.0	95
22	Ruhpolding (GER)	115	34.0	10.5	92
23	Zakopane (POL)	120	35.0	10.5	100

24	Engelberg (SUI)	120	35.0	10.5	110
25	Kuopio (FIN)	120	35.0	11.0	95
26	Kuusamo (FIN)	120	35.0	11.5	103
27	Trondheim (NOR)	120	34.0	11.0	105
28	Lillehammer (NOR)	120	34.0	11.0	107
29	Bischofshofen (AUT)	125	27.0	11.0	*
30	Klingenthal (GER)	125	35.0	11.0	105
31	Pragelato (ITA)	125	35.0	11.0	105
32	Whistler (CAN)	125	35.0	11.2	100
33	Garmisch-Partenkirchen (GER)	125	35.0	11.0	103
34	Willingen (GER)	130	35.0	11.0	105
35	Bad Mitterndorf (AUT)	185	35.0	10.7	147
36	Oberstdorf (GER)	185	39.0	10.5	120
37	Planica (SLO)	185	38.5	10.3	100
38	Vikersund (NOR)	185	40.4	11.0	105
	Max	185	40.4	11.5	147.0
	Min	60	27.0	9.5	57.0
	M	108.4	34.6	10.7	90.8
	SD	33.2	2.5	0.4	18.3

*Inclined cubic parabola

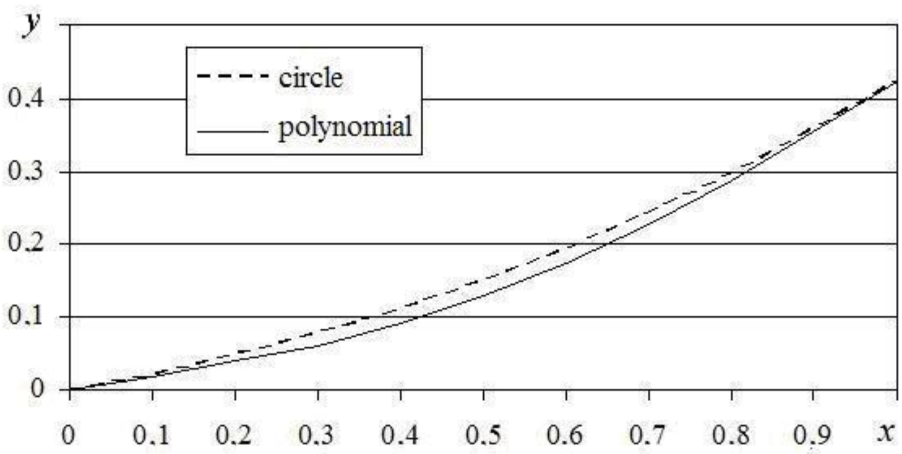

Figure 3. Dimensionless values of an arc curvature (---) and the profile (—) vs. a dimensionless distance of a skier to a take-off table relatively a horizontal dimension of the curvilinear segment of the hill.

Using the cubic parabola profile for the in-run with parameters of the in-run hill (α =11°, γ =35°), one can get almost the same ratio of curvilinear segment dimensions (h/l) as that of the arc profile, but for a difference of –0.7% (Table 2). After the cubic parabola, the nearest (based on the modulus of difference of the ratio with a circle profile) were the cycloid, inclined quadratic parabola, quadratic parabola, hyperbola, and inclined cubic parabola (Gasser, 2018). The maximum ratio of a curvilinear segment had an in-run hill profiled with an inclined cubic parabola and a minimum – hyperbola.

Table 2. Parameters of a curvilinear segment of an in-run hill

Curvilinear profile*	Curvature	Ratio between the vertical and horizontal

	dimensions			
	l/ρ_C	ρ_D/ρ_C	h/l	Difference from a circle arc, %
Circle	0.383	1	0.424	0
Cubic parabola	0.364	1.103	0.421	–0.7
Cycloid	0.428	1.198	0.417	–1.7
Inclined parabola	0.422	1.312	0.413	–2.7
Parabola	0.278	0.581	0.447	5.1
Hyperbola	0.691	3.973	0.369	–14.9
Incl.cub.parabola	0.614	1.312	0.500	15.2

*Angles of inclination of a take-off table (α =11°) and a top straight-line segment (γ =35°).

Only the circle arc profile has a constant curvature, whereas the other six functions, considered here as hypothetical profiles, have a variable curvature. The curvature increased down the hill when the curvilinear segment was profiled as a quadratic or cubic parabola, and decreased when profiled as an inclined quadratic parabola, hyperbola, cycloid, or cubic parabola.

The inclined cubic parabola profile gave zero value for the centripetal force at the junction of straight-line and curvilinear segments (point *C* in Figures 1, 2). No other investigated function (circle, cycloid, hyperbola, quadratic parabola, inclined quadratic parabola, and cubic parabola) could give zero value because of the abrupt increase in the trajectory curvature at the junction (see Figure 3 and Table 2).

Ratios of the dimensions of the in-run hill curvilinear segment, profiled with different functions, are presented in Table 3. The circle arc hill of the in-run K185 (α = 10.5°, γ = 39°) in Oberstdorf (GER) could be replaced by a cubic parabola profile, almost with the same ratio of the dimensions of curvilinear segments (the difference being 0.04%).

Table 3. Ratio of dimensions (vertical / horizontal) of an in-run hill curvilinear segment profiled with different functions

In-run*	h/l						
	Circle	Cycloid	Hyper-bola	Parabola	Cubic parabola	Inclined parabola	Inclined cub. par.
1	0.359	0.356	0.321	0.370	0.353	0.354	0.417
2	0.409	0.401	0.342	0.434	0.403	0.396	0.487
×	0.419	0.412	0.360	0.443	0.415	0.407	0.496
4	0.404	0.397	0.345	0.425	0.399	0.393	0.478
5	0.414	0.406	0.351	0.438	0.409	0.402	0.491
6	0.389	0.384	0.340	0.405	0.384	0.381	0.456
8	0.369	0.365	0.327	0.381	0.363	0.363	0.430
9	0.414	0.408	0.362	0.434	0.410	0.404	0.487
10	0.430	0.421	0.367	0.456	0.426	0.416	0.509
11, 22	0.409	0.402	0.354	0.430	0.404	0.399	0.482
12	0.394	0.388	0.344	0.411	0.389	0.385	0.463
13	0.435	0.426	0.370	0.463	0.432	0.420	0.515
14	0.427	0.419	0.372	0.449	0.424	0.416	0.502
15	0.423	0.416	0.368	0.446	0.420	0.412	0.499

16	0.440	0.431	0.374	0.469	0.438	0.425	0.522
17	0.435	0.425	0.365	0.465	0.431	0.418	0.517
18	0.429	0.421	0.374	0.452	0.426	0.417	0.504
19	0.374	0.370	0.335	0.386	0.369	0.369	0.434
+	0.424	0.417	0.369	0.447	0.421	0.413	0.500
26	0.430	0.422	0.377	0.452	0.427	0.419	0.504
27, 28	0.414	0.408	0.362	0.434	0.410	0.404	0.487
29	0.344	0.342	0.315	0.352	0.340	0.341	0.395
32	0.427	0.419	0.372	0.449	0.424	0.416	0.502
33	0.424	0.417	0.369	0.447	0.421	0.413	0.500
35	0.421	0.414	0.364	0.445	0.418	0.410	0.497
36	0.461	0.449	0.387	0.498	0.461	0.441	0.548
37	0.454	0.443	0.380	0.489	0.452	0.435	0.540
38	0.481	0.468	0.407	0.523	0.484	0.459	0.572

* See Table 1: × No 3, 7, 23, 24, + 20, 21, 30, 31, 33, 34

The curvilinear segment of the in-run hill of in-run K125 $(\gamma = 27^o,\ \alpha = 11^o)$ in Bischofshofen (AUT) was profiled with an inclined cubic parabola (see Table 1, No 29). The ratio of the dimensions of the in-run hill curvilinear segment would have been 0.340 if the segment was profiled with a simple cubic parabola, 0.395 if profiled with an inclined quadratic parabola, and 0.344 (Eq.2) if profiled with a circle arc (Table 4).

Table 4. Relative difference (%) of ratios dimensions of the in-run hill curvilinear segment profiled with a circle arc and six hypothetical functions (statistics of 38 trampolines listed in Table 1)

Hypothetic functions	Cycloid	Hyperbola	Parabola	Cub. parabola*	Inclined parabola	Inclined cubic parabola
Max	–0.73	–8.60	8.61	0.58	–0.90	19.04
Min	–2.71	–16.33	2.21	–1.57	–4.58	14.64
M	–1.73	–13.44	5.39	–0.89	–2.72	17.81
SD	0.38	1.63	1.24	0.41	0.74	0.88

* Statistics: *Max* = 1.57, *Min* = 0.04, *M* = 0.92, *SD* = 0.34

In-runs of similar size (K125), situated in Klingenthal (GER), Pragelato (ITA), and Garmisch-Partenkirchen (GER), had curvilinear segments which were profiled with circle arcs (see Table 3: No 30, 31, 33). The inclination angles of their take-off tables were the same $(\alpha = 11^o)$, but those of straight-line segments were greater $(\gamma = 35^o)$. And, their ratio of the dimensions of the in-run hill curvilinear segment was rather greater: 0.424 (Eq.2). If the curvilinear segment of the in-run hill of these in-runs was profiled with an inclined cubic parabola, the ratio would have been 0.500, and if profiled with a simple cubic parabola, it would have been h/l = 0.421 (Table 5).

Table 5. Angle of inclination of a strait-line for different profiles of a curvilinear segment of the in-run hill

In-run*	γ^o	$\frac{\gamma - \gamma_{circle}}{\gamma_{circle}} 100\%$
Circle	35.0	–
Cycloid	46.3	32.2
Parabola	49.2	40.5
Hyperbola	55.3	58.0

* Angle of inclination of a take-off table: α =10.5°

From a practical point of view, it is important to define the conditions when the ratio of curvilinear dimensions with hypothetical functions would equal the corresponding ratio with a circle arc profile of the in-run hill. These conditions were presented with equations constructed from the right parts of equations (Zanevskyy and Banakh, 2010) on the one hand, and the right part of equation (Eq.2) on the other. The conditions for hypothetical profile functions (cycloid, quadratic parabola, cubic parabola, inclined quadratic parabola, inclined cubic parabola, and hyperbola) were defined by the following equations (Table 6):

Cycloid: $$\frac{\cos 2\alpha - \cos 2\gamma}{2(\gamma - \alpha) + \sin 2\gamma - \sin 2\alpha} = tg\frac{\alpha + \gamma}{2}, \quad \text{(Eq.5)}$$

Quadratic parabola: $$\frac{tg\alpha + tg\gamma}{2} = tg\frac{\alpha + \gamma}{2}, \quad \text{(Eq.6)}$$

Table 6. Parameters of a real circle arc profile of an in-run hill and results of a virtual transformation to an inclined cubic parabola profile

In-run*	$tg\gamma$	$\frac{h_{circle}}{l_{circle}}$	$\frac{h}{l}$	$\frac{l - l_{circle}}{l_{circle}}$	$\frac{h - h_{circle}}{h_{circle}}$	$\frac{\Delta s}{l_{circle}}$
1	0.554	0.359	0.417	0.422	0.652	-0.483
2	0.700	0.409	0.487	0.365	0.625	-0.446
×	0.700	0.419	0.496	0.373	0.622	-0.455
4	0.675	0.404	0.478	0.378	0.631	-0.456
5	0.700	0.414	0.491	0.369	0.624	-0.450
6	0.625	0.389	0.456	0.399	0.641	-0.470
8	0.577	0.369	0.430	0.415	0.649	-0.479
9	0.675	0.414	0.487	0.385	0.627	-0.464
10	0.727	0.430	0.509	0.363	0.615	-0.449
11, 22	0.675	0.409	0.482	0.382	0.629	-0.460
12	0.637	0.394	0.463	0.394	0.638	-0.468
13	0.740	0.435	0.515	0.359	0.611	-0.446
14	0.700	0.427	0.502	0.378	0.620	-0.461
15	0.698	0.423	0.499	0.377	0.621	-0.460
16	0.754	0.440	0.522	0.354	0.606	-0.443
17	0.754	0.435	0.517	0.350	0.607	-0.438

18	0.705	0.429	0.504	0.376	0.619	-0.460
19	0.577	0.374	0.434	0.418	0.645	-0.482
+	0.700	0.424	0.500	0.376	0.621	-0.459
26	0.700	0.424	0.500	0.376	0.621	-0.459
27, 28	0.700	0.430	0.504	0.380	0.619	-0.464
29	0.675	0.414	0.487	Inclined cubic parabola		
32	0.510	0.344	0.395	0.439	0.650	-0.493
33	0.700	0.427	0.502	0.378	0.620	-0.461
35	0.700	0.421	0.497	0.374	0.622	-0.457
36	0.810	0.461	0.548	0.334	0.588	-0.430
37	0.795	0.454	0.540	0.338	0.592	-0.432
38	0.851	0.481	0.572	0.324	0.574	-0.426

* See Table 1: × 3, 7, 23, 24, + 20, 21, 30, 31, 33, 34

Cubic parabola: $$\frac{tg\alpha + tg\gamma + \sqrt{tg\alpha \times tg\gamma}}{3} = tg\frac{\alpha+\gamma}{2}, \qquad \text{(Eq.7)}$$

Inclined quadratic parabola: $$\frac{tg\alpha + tg\gamma + 2tg\alpha\ tg^2\gamma}{2 + tg\gamma(tg\alpha + tg\gamma)} = tg\frac{\alpha+\gamma}{2}, \qquad \text{(Eq.8)}$$

Inclined cubic parabola: $$\frac{tg\gamma - \frac{tg(\gamma-\alpha)}{3}}{1 + \frac{tg(\gamma-\alpha)}{3}tg\gamma} = tg\frac{\alpha+\gamma}{2}, \qquad \text{(Eq.9)}$$

Hyperbola: $$\sqrt{tg\alpha \times tg\gamma} = tg\frac{\alpha+\gamma}{2}. \qquad \text{(Eq.10)}$$

Discursions

According to the aim of this research, a calculating method for virtual replacing of a circle arc segment on the in-run hill has been created. Palej and Struk (2003) proposed to replace the straight-line (*BC*) and circle arc (*CD*) segments of the in-run hill with one curvilinear segment (*BD*) profiled as a polynomial of the second, fourth, sixth, and eighth power. The function was constructed on the condition that the effect of normal reaction on a ski jumper's body on the curvilinear segment had non-zero value. A form of the function was calculated as a solution to a nonlinear differential equation of the second order. The authors observed a positive consequence to this reconstruction: reduction in curvature as a result of decrease in normal reaction on a ski jumper's body. In general, as a result of this reconstruction, the straight-line segment of the in-run hill did not disappear; it only became shorter up to the straight-line segment *AB,* where the start gate was situated. Sometimes, although very seldom, the start gate can be placed at point *A*, and in such cases, the in-run should start at the very beginning of this curvilinear segment.

Some combinations of the inclination angles of a straight-line segment and take-off table (α, γ), and the ratio of dimensions of a curvilinear segment (h/l) can result in convexity that enables ski jumpers pull off the track. Because the values of three of these four parameters

(h,l,α) were restricted by the in-run size, Palej and Struk (2004) proposed to obtain the concave curvilinear segment by increasing the angle of inclination γ. Therefore, the implication of these functions was that it was necessary to increase of the inclination angle to avoid the inflexion points.

This model of in-run hill construction had a few defects which rendered this approach useless, in practical terms. First, according to this method, the angle of the increased incline should be greater than the maximum inclination of the in-run hills of the in-runs in vogue: γ = 29.0 – 40.4° (see Table 1). Second, it is doubtful if the curvature of the in-run hill can be decreased at its junction with the take-off table. Corresponding decrease in centripetal force causes similar decrease in take-off impulse at the very beginning of the phase. Third, although the problem under consideration was a dynamic one, air drag and ski jumper's friction were not taken into account in the frames of the model. These forces have significant influence on the dynamics of the ski jumper's in-run (Ettema et al., 2005).

Taking into account unequal $(h/l)_{parabola} < (h/l)_{circle}$, one can define that a difference between the ratios of a circle and a cubic parabola dimensions can be equal, less or greater a unit. Correlation between the angles of inclination of the in-run hill straight-line segment and the take-off table, when the ratios of the dimensions of a circle profile and a cubic parabola profile are equal, was calculated as a solution to equation. In the majority of the in-runs considered, the replacing of a circle arc with cubic parabola gave a greater ratio of vertical to horizontal dimensions, the difference being around 1.6%. (see Table 4). Following is the correlation equation between the angles of inclination:

$$\gamma = 52.3 - 1.26\alpha .$$

If the difference between the angles of inclination of straight-line segments $(\gamma - \alpha)$ is below 28.5° where $\alpha = 10.5°$, the ratio decreases, and when the difference is over 28.5°, the ratio decreases. For example, for the ski fly in-run K185 (see Table 1: No 36) in Vikersund (NOR), the difference in the ratios was 0.04% ($\gamma = 39.0°$, $\alpha = 10.5°$).

Considering the unequal parts of the model equations one can generalize that a circle arc profile can be replaced with a cycloid, a hyperbola, or an inclined quadratic parabola with decreased ratio of vertical to horizontal dimensions. If the difference between the angles of inclination of the straight-line segments increases, the ratio decreases. For example, for the 38 in-runs certificated by ISF (see Table 1), the ratio varied in the range of (2.71–0.73)% when cycloid was used, (16.33–8.60)% when hyperbola was used, and (4.58–0.90)% when inclined quadratic parabola was used (see Tables 3, 4). When the circle arc was replaced with a quadratic parabola or an inclined cubic parabola, the ratio of vertical to horizontal dimensions increased. If the difference between the angles of inclination of the straight-line segments increased, the ratio increased too. For the in-runs mentioned above, the ratio varied in the range of 2.21–8.61% when a quadratic parabola was used and 14.64–19.04% when an inclined cubic was used.

To control the force of inertia acting on a ski jumper's body during sliding Palej and Filipowska (2009) proposed to replace the first straight-line segment and the circle arc segment with one curvilinear segment of a hypothetical profile as a polynomial function. To avoid appearance of inflexion points, they were forced to increase the angle of inclination of the starting segment. For example, in K 120 in-run (see Table 1: No 23) at Zakopane (Poland), the angle of inclination in the circle arc ($\gamma = 35°$) was increased up to 41°80′–49°68′ corresponding to the power of the polynome that equaled 2–8. Palej and Struk (2003) used a cycloid, a quadratic parabola, or a hyperbola. They would have had to increase the angle of inclination up to 46°16′–55°19′ (see Table 5). These values are considered significantly high against the standard value of the in-run hill inclination.

An analytical method is proposed here for calculating the hypothetical in-run hill profile parameters, instead of the circle arc profile. The method allows for maintaining the inclination angles of the straight-line segment of the in-run hill and of the take-off table. The horizontal dimension (l) and vertical dimension (h) of a hypothetical profile and the corresponding dimensions of a circle arc profile $(l_{circle},\ h_{circle})$ should be dependent on the inclination of the in-run hill (see Figure 2).

If a quadratic parabola or an inclined cubic parabola replaces the circle in-run hill profile, its horizontal and vertical dimensions should be greater than the corresponding dimensions of the circle profile. The corrected length of the straight-line segment of the in-run hill should be smaller. If a cycloid, an inclined quadratic parabola, or a hyperbola profile is applied, the dimensions should be smaller. The corresponding corrected length of the straight-line segment should be greater. If a cubic parabola profile is applied, its dimensions should be greater than, smaller than, or equal to the circle dimensions depending on the angles of inclination of the in-run hill and take-off table.

The difference between the dimensions of a circle arc segment and the corresponding dimensions of a quadratic parabola, an inclined quadratic parabola, or an inclined cubic parabola depends more on the inclination angle of the in-run hill than of the take-off table. Conversely, the difference between the dimensions of a circle arc segment and corresponding dimensions of a hyperbola depends more on the inclination angle of the take-off table than of the in-run hill. There is no significant distinction in the dependence of difference between a cycloid and a cubic parabola. The same type of dependence could be noticed for the difference between the lengths of the in-run hill and straight-line segments (Jung et al., 2018).

The only profile that obtains zero centripetal acceleration at the top point of the curvilinear segment is the inclined cubic parabola. Therefore, virtual replacing of a real circle arc profile of an in-run hill with an inclined cubic parabola profile was considered in a special way. For this, the parameters of the real circle arc profile of an in-run hill and the results of its virtual transformation to an inclined cubic parabola profile are presented in Table 6. As in-run K 125 in Bischofshofen (AUT) was originally designed with an inclined cubic parabola profile, the corresponding line No 29 in the table was not completed. To equip the in-runs under consideration with an inclined cubic parabola profile, the horizontal dimension should be increased by 32.4–43.9%, and the vertical one by 54.7–65.2%. The relative (to the horizontal dimension) length of the straight-line segment of the in-run hill should be decreased by 42.6–49.3%.

For example, in-run K 120 (see Tables 1 and 3: No 23) Wielka Krokiew in Zakopane (POL) could be reconstructed and equipped with an inclined cubic parabola profile, instead of a circle arc profile, by increasing horizontal and vertical dimensions correspondingly to 37.3%

(14.59 m) and 62.2% (10.22 m); the relative length of the straight-line segment should be decreased by 45.5% (17.81 m). According to Palej and Struk's method (2003), the dimensions should be increased correspondingly by 53.8% (21.07 m) and 89.9% (14.75 m); the relative length of the straight-line segment should be decreased by 65.7% (25.72 m).

The proposed method of reconstructing in-run hill has three advantages: First, the angle of inclination of hill remains the same; second, there is no inflection of the curvilinear segment; third, only a significantly small part of the straight-line segment (69.2%) should be replaced by a curvilinear segment.

Palej and Struk (2004) tried to decrease the normal reaction force at the end of the curvilinear segment by using a family of even polynomial functions which possess the determined properties of the normal reaction. However, there are grounds to suppose that it is not reasonable to reduce a curvature of the curvilinear segment at the bottom point because a centripetal force acted a skier's body fall down to zero just entering to the table. On the contrary of negative influence on a skier's body of a momentary appearance of the in-run curvature, a momentary disappearance of the in-run curvature at the bottom is positive a sport result. On the curved area of the in-run hill, the weight of skier's body consists of sum of two forces. One of them is a normal (to the hill surface) component of a gravitation force, which equals a production of body mass and gravity acceleration and cosine of incline angle of the hill slope. The second one is a centrifugal force which equals production of body mass and centrifugal acceleration. The greater a body weight – more compact a body pose, then a jump length is greater. At the instant of running on the table, a centrifugal acceleration disappears, that in terms of dynamics means instantaneous reduction of the skier's body weight. This makes a motion of taking off more rapid and a jump length greater (Zanevskyy & Banakh, 2010).

So, it is reasonable to obtain a zero curvature only from the very beginning of the curvilinear track. An inclined cubic parabola is a simplest polynomial function of the in-run profile obtained zero centripetal acceleration at the top point of the curvilinear segment. This helps to avoid instantaneous increasing of the trajectory's curvature and make in-run more comfortable.

Conclusions

The calculating methods of virtual replacing of the in-run hill circle arc segment on a ski jumping in-run with profiles of changeable curvature, based on the functions of cycloid, parabola, hyperbola, and cubic parabola, allows for retaining the original angles of inclination. The length of the straight-line segment can be diminished to such an extent that it becomes suitable to the geometrical parameters of the in-runs in vogue.

The circle arc profile could be replaced with a cycloid, a hyperbola, or an inclined quadratic parabola with decreased ratio of vertical to horizontal dimensions. If the difference between the angles of inclination of straight-line segments increases, the ratio decreases. For the 38 in-runs certificated by International Ski Federation, the ratio varied in the range of 2.71–0.73% when cycloid was used, 16.33–8.60% when hyperbola was used, and 4.58–0.90% when inclined quadratic parabola was used. When a circle arc was replaced with a quadratic parabola or an inclined cubic parabola, the ratio of vertical to horizontal dimensions increased. If the difference between the angles of inclination of straight-line segments increased, the ratio increased too. For the certificated in-runs, the ratio varied in the range of

2.21–8.61% when a quadratic parabola was used and 14.64–19.04% when inclined cubic was used.

The difference between the dimensions of a circle arc segment and the corresponding dimensions of a quadratic parabola, an inclined quadratic parabola, or an inclined cubic parabola depends more on the angle of inclination of the in-run hill than of the take-off table. Conversely, the difference between the dimensions of a circle arc segment and the corresponding dimensions of a hyperbola depends more on the angle of inclination of the take-off table than of the in-run hill. There is no significant distinction in the dependence of difference between a cycloid and a cubic parabola. The same type of dependence could be noticed for the difference in the length of the in-run hill straight-line segment.

The only profile which obtains zero centripetal acceleration at the top point of the curvilinear segment is the inclined cubic parabola. This helps to avoid instantaneous increasing of the trajectory curvature and make in-run more comfortable. To equip the certificated in-runs with inclined cubic parabola profile, the horizontal dimension should be increased by 32.4–43.9%, and the vertical dimension by 54.7–65.2%. The relative (to the horizontal dimension) length of the straight-line segment of the in-run hill should be decreased by 42.6–49.3%.

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Conflict of Interests

Authors declare no conflict of interests regarding this paper.

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Appendix

Notation*

Symbol	Explanation
a	centripetal acceleration
f	transversal dimension of the inclined parabola profile
h	vertical dimension of an in-run hill curvilinear segment
k	coefficient of the quadratic and cubic parabolas
l	horizontal dimension of an in-run hill curvilinear segment
q	longitudinal dimension of the inclined parabola profile
r	radius of a circle arc in-run hill profile
v	sliding speed of a skier along an in-run hill
x	horizontal coordinate of the circle and parabolic profiles
y	vertical coordinate of the circle and parabolic profiles
H	coefficient of the hyperbola
K	In-run hill size
R	radius of the circumference circle of a cycloid
S	length of a circle arc in-run hill profile
α	angle of inclination of an in-run hill straight-line segment
β	angle parameter of the inclined parabolas
χ	horizontal coordinate of the cycloid profile
γ	angle of inclination of a take-off table
η	transversal coordinate of the inclined parabolas
φ	angle of inclination of a tangent line to the cycloid
κ	coefficient of the inclined quadratic and cubic parabola
ν	vertical coordinate of the hyperbolic profile
π	constant
ρ	in-run hill radius of curvature
τ	parameter of a cycloid
ξ	longitudinal coordinate of the inclined parabolas
ψ	horizontal coordinate of the hyperbolic profile
ζ	vertical coordinate of the cycloid profile
Δs	absolute difference in the length of the in-run straight-line segment

* Eastern Ski Jumping (2011)