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Subsethood measure for picture fuzzy sets and its applications on multicriteria decision making

Görüntü bulanık kümelerde altkümelik ve çok kriterli karar vermeye uygulanması

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Abstract
Picture fuzzy set is a direct generalization of intuitionistic fuzzy set and is therefore more capable of dealing with uncertainty while working on real life problems. The concept of inclusion is a subject that is frequently studied in family of fuzzy sets and has many applications in real life problems. Therefore, in this work, the measuring degree of inclusion between picture fuzzy sets is introduced. For this purpose, firstly axioms for subsethood measure are given and then a subsethood measure based on a distance measure for picture fuzzy sets is proposed. Then, a numerical example is provided to illustrate the applicability and usefulness of the presented measure. Finally, results are compared with the existing methods and aggregation operator to show validity of subsethood measure for PFS.

Keywords: MCDM, Picture fuzzy sets, Subsethood measure

Öz
Görüntü bulanık küme, sezgisel bulanık kümenin doğrudan bir genellemesidir ve bu nedenle gerçek hayat problemleri üzerinde çalışırken belirsizlikle başa çıkma konusunda daha yeteneklidir. Kapsama kavramı, bulanık kümeler ailesinde sıklıkla çalışılan ve gerçek hayat problemlerinde birçok uygulaması olan bir konudur. Bu nedenle, bu çalışmada, görüntü bulanık kümeleri arasındaki kapsama derecesinin ölçülmesi tanıtılmıştır. Bu amaçla, önce altkümelik ölçüsü için aksiyomlar verilmiş, ardından görüntü bulanık kümeleri için uzaklık ölçüsüne dayalı bir altküme ölçüsü önerilmiştir. Sonra, verilen ölçünün uygulanabilirliğini ve kullanışlılığını göstermek için sayısal bir örnek verilmiştir. Son olarak, sonuçlar PFS için altkümelik ölçüsünün geçerliliğini göstermek için mevcut yöntemler ve ortalama operatörleri ile karşılaştırılmıştır.

***Anahtar kelimeler**: ÇKKV, Görüntü bulanık kümeler, Altkümelik ölçüsü*

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1. Introduction
1. Giriş

Classic set theory is a solid branch of mathematic which forms a basis for other areas such as analysis, algebra, topology, etc. As expected, set theory starts with inclusions, intersections, unions and complements. These operations have their own duties in mathematical problems that can be applied to real life examples. One of the most important applications of these operators among many is subsethood measures. Generally, these measures are formulated with entropies or cardinalities. In classical approach, A set A is a subset of a set B if all elements in A are also in B, which is a well-known inclusion definition for many years. With the definition of fuzzy sets (Zadeh, 1965), point of view to the sets has changed. The membership function of fuzzy sets allows the degree to which an element belong to a set. In fuzzy sets, a fuzzy set A is a subset of a fuzzy set B if and only if membership degree of A is less than or equal to the membership degree of B for all elements. Thus, subsethood concept appeared for fuzzy sets (Sinha & Dougherty, 1993; Young, 1996). Fuzzy sets were generalized to intuitionistic fuzzy sets by adding a non-membership degree to the membership degree (Atanassov, 1986). Subsethood measure for IFSs was firstly introduced by Cornelis et al. (2003) based on the Sinha-Dougherty axioms in the unit square $[0,1]^2$. Then, Grzegorzewski and Mrowka (2004) suggested another approach for subsethood axioms with characterizing the degree of subsethood in unit interval $[0,1]$.

Cuong and Kreinovich (2014) introduced the picture fuzzy set (PFS) which is a direct extension of IFS. PFS has three components: degree of positive membership, degree of neutral membership and degree of negative membership where the sum of all degrees is in the interval of [0,1]. Obviously, it can deal with uncertain data more precisely in real life problems when compared to IFSs. Moreover, Cuong (2015) proposed interval-valued PFS (IvPFS) by enhancing every membership degree to a unit interval. PFSs have many applications in real life problems. Singh (2015) presented correlation coefficients for PFS. Wei (2018) proposed some similarity measures for PFS. Wang et al. (2017) investigated some aggregation operators and applied them into decision making problems. Son (2016) generalized distance measures for PFS and applied it to picture fuzzy clustering. Lately, Thao (2020) defined similarity measures based on entropy, and Ganie et al. (2020) introduced new correlation coefficients.

Subsethood measures have many applications on family of fuzzy sets and decision-making problems. Besides the former studies about them, these measures have been applied on many real-life problems lately for many different sets in the family of fuzzy sets (Köseoğlu & Şahin, 2019; Pękala et al., 2020; Peng et al., 2017; Şahin & Küçük, 2015; Şahin et al., 2015; Zadrożny et al., 2021; Köseoğlu, 2021).

In the light of the foregoing information, the need of such studies and the efficiency of their applications are clear. As far as we know, there is no research conducted on subsethood measure of PFS. Therefore, in this work, a subsethood measure for PFSs is introduced by giving a system of axioms adapted from Young (1996) and Grzegorzewski and Mrówka (2004). In addition, normalized Hamming distance measure based on Hausdorff metric is adapted to proposed subsethood measure. Some examples are given for the subsethood measure to be well understood. Furthermore, a real-life decision-making problem is presented to show the efficiency and applicability of the proposed measure. The rest of the paper is organized as follows: In Sect. 2, some basic definitions and operations are given. In Sect. 3, a new subsethood measure for PFS is proposed. Sect. 4 covers the numerical application of subsethood measure. Finally, conclusions about proposed subsethood measure are shared in Sect. 5.

2. Picture fuzzy sets
2. Görüntü bulanık kümeler

In this section, PFSs which will be used on other sections are presented.

Definition 1. (Atanassov, 1986) Let X be a non-empty set, then an IFS A in X is defined as

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\} \quad (1)$$

where $\mu_A, \nu_A : X \to [0,1]$ represents the degree of membership and the degree of non-membership of the element x such that for any $x \in X$,

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1. \tag{2}$$

Here, $\pi_A = 1 - (\mu_A(x) + \nu_A(x))$ is called hesitancy degree of the element x in the set A. Moreover, the pair of $(\mu_A(x), \nu_A(x))$ is called intuitionistic fuzzy number (IFS) and denoted as $a = (\mu_A, \nu_A)$.

Definition 2. (Cuong & Kreinovich, 2014) Let X be a universe of discourse. A PFS P in X is given by

$$P = \{\langle x, \mu_P(x), \eta_p(x), \nu_P(x)\rangle | x \in X\} \tag{3}$$

where $\mu_P(x)(\in [0,1])$ is called the "degree of positive membership of x in P", $\eta_P(x)(\in [0,1])$ is called the "degree of neutral membership of x in P", and $\nu_P(x)(\in [0,1])$ is called the "degree of negative membership of x in P" where

$$0 \leq \mu_P(x) + \eta_p(x) + \nu_P(x) \leq 1. \tag{4}$$

The degree of refusal is $\pi_P = 1 - \left(\mu_P(x) + \eta_p(x) + \nu_P(x)\right)$. For convenience in decision making problems, the triple $\left(\mu_P(x), \eta_p(x), \nu_P(x)\right)$ is called a Picture fuzzy number (PFN) denoted as $p = \left(\mu_P, \eta_p, \nu_P\right)$.

Picture fuzzy sets can express the answers to real life questions: yes, abstain, no and refusal. Voting on any condition in real life is a good example of such answers that Cuong (2015) suggested. Voters may be divided into four groups of those who vote for, abstain, vote against and refuse to vote. Moreover, PFS is reduced to IFS for $\eta_p(x) = \emptyset$ and thus it is a direct generalization of IFS.

Definition 3. (Cuong & Kreinovich, 2014) Let X be a fixed set and

$A = \{\langle x_i, \mu_A(x_i), \eta_A(x_i), \nu_A(x_i)\rangle | x_i \in X\}$
$B = \{\langle x_i, \mu_A(x_i), \eta_A(x_i), \nu_A(x_i)\rangle | x_i \in X\}$

be two PFSs. Then, some operations on PFSs are defined as:

1) $A \subseteq B \Leftrightarrow \mu_A(x_i) \leq \mu_B(x_i), \eta_A(x_i) \leq \eta_B(x_i)$ and $\nu_A(x_i) \geq \nu_B(x_i)$
2) $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$
3) $A \cap B = \{\langle x_i, \min\{\mu_A(x_i), \mu_B(x_i)\}, \min\{\eta_A(x_i), \eta_B(x_i)\}, \max\{\nu_A(x_i), \nu_B(x_i)\}\rangle\}$ (5)
4) $A \cup B = \{\langle x_i, \max\{\mu_A(x_i), \mu_B(x_i)\}, \min\{\eta_A(x_i), \eta_B(x_i)\}, \min\{\nu_A(x_i), \nu_B(x_i)\}\rangle\}$
5) $A^c = \{\langle x_i, \nu_A(x_i), \eta_A(x_i), \mu_A(x_i)\rangle | x_i \in X\}$

Proposition 1: The followings are valid for each PFS A:

1) $(A^c)^c = A$
2) $A \cap A = A$ (6)
3) $A \cup A = A$

Proposition 2: The following equalities hold for every PFSs A, B, and C:

1) $A \cup B = B \cup A$
2) $A \cap B = B \cap A$
3) $(A \cup B) \cup C = A \cup (B \cup C)$
4) $(A \cap B) \cap C = A \cap (B \cap C)$ (7)
5) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$
6) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$.

Definition 4. (Wei, 2017) Let $p_1 = \left(\mu_{P_1}, \eta_{P_1}, \nu_{P_1}\right), p_2 = \left(\mu_{P_2}, \eta_{P_2}, \nu_{P_2}\right)$ and $p = (\mu_P, \eta_P, \nu_P)$ be three PFNs. Then, for $\lambda > 0$ the corresponding operations are defined as follows:

$$\begin{aligned}
&\textbf{1)}\quad p_1 \oplus p_2 = \left(\mu_{P_1} + \mu_{P_2} - \mu_{P_1}\mu_{P_2}, \eta_{P_1}\eta_{P_2}, \nu_{P_1}\nu_{P_2}\right)\\
&\textbf{2)}\quad p_1 \otimes p_2 = \left(\mu_{P_1}\mu_{P_2}, \eta_{P_1} + \eta_{P_2} - \eta_{P_1}\eta_{P_2}, \nu_{P_1} + \nu_{P_2} - \nu_{P_1}\nu_{P_2}\right)\\
&\textbf{3)}\quad \lambda p = \left(1 - (1-\mu_P)^{\lambda}, \eta_P, \nu_P\right)\\
&\textbf{4)}\quad p^{\lambda} = \left(\mu_p^{\lambda}, 1 - (1-\eta_P)^{\lambda}, 1 - (1-\nu_P)^{\lambda}\right)
\end{aligned} \tag{8}$$

Definition 5. (Wei, 2017) For any PFN $p = (\mu_P, \nu_P)$, the score and the accuracy functions of p are defined as

$$s(p) = \mu_P - \nu_P \text{ and } a(p) = \mu_P + \eta_P + \nu_P \tag{9}$$

where $s(p) \in [-1,1]$ and $a(p) \in [0,1]$. For any PFNs p_1 and p_2

1. If $s(p_1) > s(p_2)$, then $p_1 > p_2$.
2. If $s(p_1) = s(p_2)$, then
   - **i.** If $a(p_1) > a(p_2) \Rightarrow p_1 > p_2$
   - **ii.** If $a(p_1) = a(p_2)$, then $p_1 \approx p_2$

Definition 6. (Wei, 2017) Let $p_i (i = 1,2,\dots,n)$ be a collection of PFNs, then the picture fuzzy weighted averaging (PFWA) operator is a mapping $P^n \to P$ such that

$$\mathrm{PFWA}(p_1, p_2, \dots, p_n) = \oplus_{i=1}^{n} (w_i p_i)$$

which can be described as

$$\mathrm{PFWA}(p_1, p_2, \dots, p_n) = \left(1 - \prod_{i=1}^{n}\left(1-\mu_{p_i}\right)^{w_i}, \prod_{i=1}^{n}(\eta_i)^{w_i}, \prod_{i=1}^{n}(\nu_i)^{w_i}\right) \tag{10}$$

where $w = (w_1, w_2, \dots w_n)^{\mathsf{T}}$ is the weight vector of α_i with $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$.

Definition 7. (Wei, 2017) Let $p_i (i = 1,2,\dots,n)$ be a collection of PFNs, then the picture fuzzy weighted geometric (PFWG) operator is a mapping $P^n \to P$ such that

$$\mathrm{PFWG}(p_1, p_2, \dots, p_n) = \otimes_{i=1}^{n} ({p_i}^{w_i})$$

which can be written as

$$\mathrm{PFWG}(p_1, p_2, \dots, p_n) = \left(\prod_{i=1}^{n}(\mu_i)^{w_i}, 1 - \prod_{i=1}^{n}\left(1-\eta_{p_i}\right)^{w_i}, 1 - \prod_{i=1}^{n}\left(1-\nu_{p_i}\right)^{w_i}\right) \tag{11}$$

where $w = (w_1, w_2, \dots w_n)$ is the weight vector of α_i with $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$.

Definition 8. (Cuong & Kreinovich, 2014) Let A and B be two PFSs, then the distance between these two sets is defined as:

$$\mathrm{d(A,B)} = \frac{1}{n}\sum_{i=1}^{n}\left(|\mu_A(x_i) - \mu_B(x_i)| + |\eta_A(x_i) - \eta_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|\right) \tag{12}$$

3. Subsethood measures for picture fuzzy sets

3. Görüntü bulanık kümelerde altkümelik

Firstly, Cornelis and Kerre (2003) introduced subsethood measures of IFSs as inclusion measures and then Grzegorzewski and Mrowka (2004) proposed a different approach by characterizing the degree of subsethood by a single number from the unit interval. By generalizing this approach, we propose a Subsethood measure for picture fuzzy sets based on a distance measure.

Definition 9. A mapping $S: PFS(X) \times PFS(X) \rightarrow [0,1]$ is called a picture fuzzy subsethood measure, if S satisfies the following conditions for all $A, B, C \in PFS(X)$:

$$\begin{aligned} &\textbf{1)}\ \ 0 \leq S(A,B) \leq 1 \\ &\textbf{2)}\ \ S(A,B) = 1 \text{ iff } A \subseteq B \\ &\textbf{3)}\ \ S(A,B) = 0 \text{ iff } A = S \text{ and } B = \emptyset \\ &\textbf{4)}\ \ \text{If } A \subseteq B \subseteq C, \text{ then } S(C,A) \leq S(B,A) \text{ and } S(C,A) \leq S(C,B) \end{aligned} \tag{13}$$

Let $\mathcal{L}: PFS(X) \times PFS(X) \rightarrow \mathbb{R}^+ \cup \{0\}$ be a metric in the family of PFSs in X. To show the degree of belonging of set A to set B, a normalized distance measure can be applied to form an inclusion indicator by calculating distance between A and $A \cap B$. Let $d: PFS(X) \times PFS(X) \rightarrow [0,1]$ is a metric for normalized distances. Then, the following a mapping:

$$S_d(A,B) = 1 - d(A, A \cap B) \tag{14}$$

is a subsethood measure based on distance for PFSs where:

$$\mathrm{d_H(A,B)} = \frac{1}{n}\sum_{i=1}^{n} \max\{|\mu_A(x_i) - \mu_B(x_i)|, |\eta_A(x_i) - \eta_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|\} \tag{15}$$

is the normalized Hamming distance based on the Hausdorff metric between PFSs A and B. Moreover, we extend this distance measure to weighted form. Then, normalized weighted Hamming distance based on Hausdorff metric for PFS is given as

$$\mathrm{d_{wH}(A,B)} = \frac{1}{n}\sum_{i=1}^{n} w_i(\max\{|\mu_A(x_i) - \mu_B(x_i)|, |\eta_A(x_i) - \eta_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|\}) \tag{16}$$

where w_i is the weight vector with $\sum_{i=1}^{n} w_i = 1$.

Theorem 1. Let $S: PFS(X) \times PFS(X) \rightarrow [0,1]$ be a mapping such that

$$S(A,B) = 1 - d_H(A, A \cap B) \tag{17}$$

where d_H is a normalized Hamming distance based on the Hausdorff metric between PFSs. Then, $S(A,B)$ is a subsethood measure indicating the degree to which B contains A.

Proof. Let $A = \{\langle x_i, \mu_A(x_i), \eta_A(x_i), \nu_A(x_i)\rangle | x_i \in X\}$, $B = \{\langle x_i, \mu_A(x_i), \eta_A(x_i), \nu_A(x_i)\rangle | x_i \in X\}$ and $C = \{\langle x_i, \mu_C(x_i), \eta_C(x_i), \nu_C(x_i)\rangle | x_i \in X\}$ be three PFSs.

1. It is clear from the definition of distance measures.

2. Using item 1 and 3 from Eq. (5), we have $S(A,B) = 1$

$$\begin{aligned} &\Leftrightarrow 1 - d(A, A \cap B) = 1 \Leftrightarrow d(A, A \cap B) \\ &\Leftrightarrow \tfrac{1}{n}\textstyle\sum_{i=1}^{n} \max\left\{ \begin{matrix} |\mu_A(x_i) - \min\{\mu_A(x_i), \mu_B(x_i)\}|, |\eta_A(x_i) - \min\{\eta_A(x_i), \eta_B(x_i)\}|, \\ |\nu_A(x_i) - \max\{\nu_A(x_i), \nu_B(x_i)\}| \end{matrix} \right\} = 0 \\ &\Leftrightarrow \mu_A(x_i) \leq \mu_B(x_i), \eta_A(x_i) \leq \eta_B(x_i) \text{ and } \nu_A(x_i) \geq \nu_B(x_i) \\ &\Leftrightarrow A \subseteq B \end{aligned}$$

3. Same as above, it is straightforward.

4. To prove $S(C,A) \leq S(B,A)$, we need to show $d_H(B, B \cap A) \leq d_H(C, C \cap A)$. From Eq. (15) we have

$$d_H(B, B\cap A) = \frac{1}{n}\sum_{i=1}^{n} \max\left\{ \begin{matrix} |\mu_B(x_i) - \min\{\mu_B(x_i), \mu_A(x_i)\}|, |\eta_B(x_i) - \min\{\eta_B(x_i), \eta_A(x_i)\}|, \\ |\nu_B(x_i) - \max\{\nu_B(x_i), \nu_A(x_i)\}| \end{matrix} \right\}$$

Since $A \subseteq B$, we have

$$= \frac{1}{n}\sum_{i=1}^{n} \max\{|\mu_B(x_i) - \mu_A(x_i)|, |\eta_B(x_i) - \eta_A(x_i)|, |\nu_B(x_i) - \nu_A(x_i)|\}$$

Since $B \subseteq C$, we obtain

$$\le \frac{1}{n}\sum_{i=1}^{n} \max\{|\mu_C(x_i) - \mu_A(x_i)|, |\eta_C(x_i) - \eta_A(x_i)|, |\nu_C(x_i) - \nu_A(x_i)|\}$$
$$= \frac{1}{n}\sum_{i=1}^{n} \max\left\{ \begin{matrix} |\mu_C(x_i) - \min\{\mu_C(x_i), \mu_A(x_i)\}|, |\eta_C(x_i) - \min\{\eta_C(x_i), \eta_A(x_i)\}|, \\ |\nu_C(x_i) - \max\{\nu_C(x_i), \nu_A(x_i)\}| \end{matrix} \right\}$$
$$= d_H(C, C\cap A)$$

Then $S(C,A) \le S(B,A)$ and it completes the proof. Similarly, $S(C,A) \le S(C,B)$ can be shown easily.

Example 1. Let $A = \{\langle x_1, 0.3, 0.4, 0.3\rangle, \langle x_2, 0.5, 0.1, 0.3\rangle\}$ and $B = \{\langle x_1, 0.4, 0.5, 0.1\rangle, \langle x_2, 0.6, 0.2, 0.1\rangle\}$ be two PFSs. Then, $S(A,B) = 1$ and $S(B,A) = 0$, which is consistent with Cuong and Kreinovich's subset definition.

Example 2. Let $A = \{\langle x_1, 1,0,0\rangle, \langle x_2, 0.6, 0.2, 0.2\rangle\}$ and $B = \{\langle x_1, 0, 0, 1\rangle, \langle x_2, 0.3, 0.3, 0.1\rangle\}$ be two PFSs. Then, $S(A,B) = 0{,}35$ and $S(B,A) = 0.95$. Clearly, either $A \subset B$ or $B \subset A$. But from subsethood measures, we can say that B is much more a subset of A.

Algorithm 1. Picture Fuzzy Subsethooding
Input: A set of PFSs $\{A_1, A_2, \ldots, A_m\}$
Steps:
1) Construct a decision matrix from given sets.
2) Determine an ideal point A^*.
3) Find the intersections of $A^* \cap A_i$ for each alternative using Eq. (5).
4) **Do**
   If weights are given, **then**
5) $S_i(A^*, A_i) = 1 - d_{wH}(A^*, A^* \cap A_i)$ using Eq. (16)
   Else
6) $S_i(A^*, A_i) = 1 - d_H(A^*, A^* \cap A_i)$ using Eq. (15)
   End
7) **Until** $i = m$

Output: Ranking of alternatives.

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4. Multicriteria decision making

4. Çok kriterli karar verme

In order to demonstrate the application of the proposed subsethood measure, a multicriteria decision making method is applied adapted from Ye (2010). All MCDM methods are based on choosing the best possible alternative by taking into consideration the criteria. For convenience in following section, let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of alternatives, $C = \{C_1, C_2, \ldots, C_n\}$ be a set of criteria and $w = [w_1, w_2, \ldots, w_n]$ be a weight vector with respect to criteria where $\sum_{j=1}^{n} w_j = 1$ and $w_j \ge 0$. In decision making, concept of ideal point is frequently used to obtain the best possible alternative. Ye (2010) gave the ideal alternative A^* as IFN. For the ranking order of the alternatives according to the decision-making problem, the ideal alternative is given as "excellence", and thus it is adapted to this example as $A^* = (1,0,0)$ since PFS is a generalization of IFS.

Suppose there is an investment company, which wants to invest a sum of money in the best option. There is a panel with four possible alternatives to invest the money:

A_1: car company
A_2: food company
A_3: computer company
A_4: arms company

The investment company considers the following three criteria to decide:
C_1: risk analysis
C_2: growth analysis
C_3: environmental impact analysis

The weights of criteria are determined by the tourist group as $w = [0.3560, 0.3613, 0.2827]$.

The decision matrix A is constructed as Table 1 according to their preferences with respect to criteria. The calculations are performed in MATLAB.

Table 1. Decision matrix A
Tablo 1. *A Karar matrisi*

	C_1	C_2	C_3
A_1	⟨0.45,0.15 0.35⟩	⟨0.50,0.10 0.30⟩	⟨0.20,0.05 0.55⟩
A_2	⟨0.65,0.05 0.25⟩	⟨0.65,0.10 0.25⟩	⟨0.55,0.15 0.15⟩
A_3	⟨0.45,0.20, 0.35⟩	⟨0.55,0.05, 0.35⟩	⟨0.55,0.10, 0.20⟩
A_4	⟨0.75,0.10 0.15⟩	⟨0.65,0.05, 0.20⟩	⟨0.35,0,30 0.15⟩

Taking $A^* = (1,0,0)$ is the ideal alternative, subsethood measure of each alternative is calculated using Eq. (17) as:

$S(A^*, A_1) = 0.3974$
$S(A^*, A_2) = 0.6217$
$S(A^*, A_3) = 0.5144$
$S(A^*, A_4) = 0.6008$

Then, the ranking forms as $A_2 > A_4 > A_3 > A_1$. It implies that A^* is much more subset of A_2. Therefore, A_2 (the food company) is the best option to invest. Furthermore, it is consistent with the Ye (2010)'s outputs. These results show that subsethood measure is much simpler and computationally easier than the other similarity measures.

4.1. Comparison of results

4.1. Sonuçların karşılaştırması

In order to compare this result with the existing aggregation operators given in Eq. (10) and Eq. (11), an analysis is conducted to calculate results with score function given in Eq. (9). Using Table 1, first PFWA operator is used to aggregate the decision matrix and then score function is used to rank the alternatives. Same operations are conducted for PFWG operator. The results are given as follow:

i. If PFWA operator is applied to decision matrix A, aggregated values are evaluated as:

$$PFWA(\mathrm{A}) = \begin{bmatrix} \langle 0.4092, 0.0950, 0.3762\rangle \\ \langle 0.6242, 0.0876, 0.2164\rangle \\ \langle 0.5167, 0.0996, 0.2988\rangle \\ \langle 0.6301, 0.1062, 0.1664\rangle \end{bmatrix}$$

Then, the score values are obtained as:

$$s(PFWA(A)) = [0.0331 \quad 0.4078 \quad 0.2179 \quad 0.4637]$$

According to score values, ranking of the alternatives is ordered as

$$A_4 > A_2 > A_3 > A_1$$

Then, A_4 (the arms company) is the best option to invest.

ii. If PFWG operator is applied to decision matrix A, aggregated values are evaluated as:

$$PFWA(\mathrm{A}) = \begin{bmatrix} \langle 0.3717, 0.1045, 0.3983\rangle \\ \langle 0.6200, 0.0972, 0.2230\rangle \\ \langle 0.5121, 0.1199, 0.3107\rangle \\ \langle 0.5742, 0.1452, 0.1684\rangle \end{bmatrix}$$

Then, the score values are obtained as:

$$s(PFWG(A)) = [-0.0266 \quad 0.3970 \quad 0.2014 \quad 0.4057]$$

According to score values, ranking of the alternatives is ordered as

$$A_4 > A_2 > A_3 > A_1$$

Then, A_4 (the arms company) is the best option to invest.

As can be seen in the Figure 1 and Table 2, ranking changes when subsethood measure is performed. Moreover, the ranking order for subsethood measure is in the agreement with the one Ye (2010) obtained. Both the change in the order for aggregation operators and the same as Ye's results show the effect and the stability of the proposed measure.

Table 2. Ranking comparison
Tablo 2. *Sıralama karşılaştırması*

Methods	Rankings
Subsethood Measure	$A_2 > A_4 > A_3 > A_1$
PFWA Operator	$A_4 > A_2 > A_3 > A_1$
PFWG Operator	$A_4 > A_2 > A_3 > A_1$
Ye (2010)'s Measure	$A_2 > A_4 > A_3 > A_1$

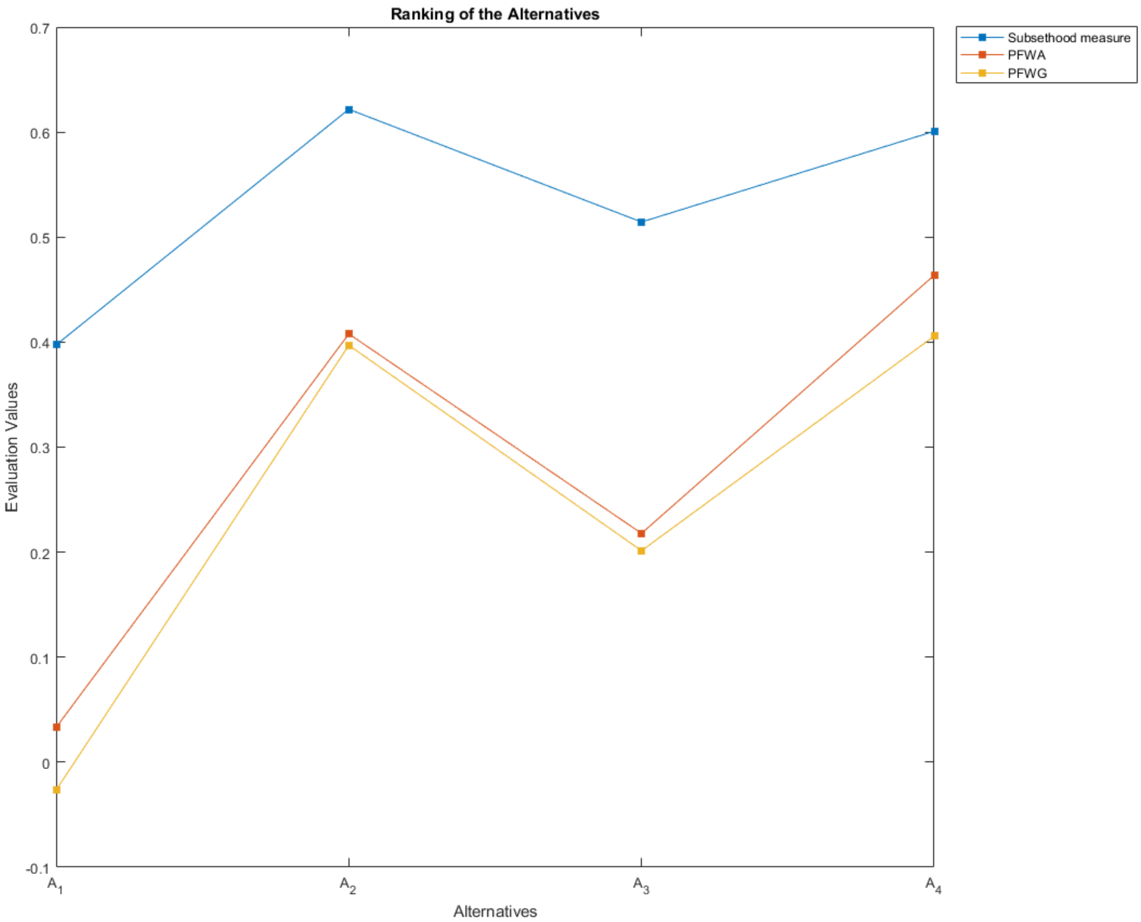

Figure 1. Ranking comparison of the alternatives with Subsethood measure, PFWA and PFWG.
Şekil 1. *Alternatiflerin Altkümelik ölçüsü, PFWA ve PFWG'ye göre sıralamalarının karşılaştırılması.*

5. Conclusion
5. Sonuç

In this study, a subsethood measure of PFS is proposed to show the degree of belonging of set A to set B. To make it practical, normalized Hamming distance based on the Hausdorff metric between PFSs A and B is implemented to subsethood measure. Then, this measure is applied to a real life problem adapted from Ye (2010). Moreover, the same example is performed with PFWA and PWWG aggregation operators with score functions. Later, all the results are compared with the Ye (2010)'s findings. It is shown that the proposed subsethood measure is consistent with Ye's results and has a small difference in the ranking when compared to aggregation operators. The fact that the results of the proposed method are the same with the results Ye obtained shows the consistency of the method, and the fact that it is different from that obtained by aggregation operators shows the effect of the method. Furthermore, when compared to other similarity measures in the literature, the proposed method comes into prominence with its simplicity and practicability. In future studies, entropy measures for PFSs can be applied to develop new subsethood measures.

Author contribution
Yazar katkısı

All authors contributed equally to the study.

Declaration of ethical code
Etik beyanı

The authors of this article declare that the materials and methods used in this study do not require any ethical committee approval and/or legal-specific permission.

Conflicts of interest
Çıkar çatışması beyanı

The authors declare that there is no conflict of interest.

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