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Application of an Epidemic Model to Turkey Data and Stability Analysis for the Covid-19 Pandemic

Nejdet KÖKER*¹, Ömer Faruk GÖZÜKIZIL¹

Abstract

An epidemic disease caused by a new coronavirus has spread all over the world with a high rate of transmission. The main purpose of this article is to define an epidemic model for the Covid-19 pandemic, to apply it to Turkey's data and to interpret it. Accordingly, a SEIR model was created to calculate the infected population and the number of deaths caused by this epidemic, and the stability of the model was examined. Since all the parameters affecting the stability cannot be calculated clearly, it cannot be expected to reach a realistic result. For this reason, a model was created with accessible parameters. Later, the diseased and non diseased equilibrium points of the model were discussed. The Hurwitz theorem is used to find the local stability of the model, while the Lyapunov function theory is used to investigate its global stability. Finally, some numerical results are given with the help of MATLAB program.

Keywords: SEIR Model, Covid-19 Pandemic, Stability.

1. INTRODUCTION

Ordinary Differential Equations are the types of equations that appear and are widely used in applications in many Applied Sciences as well as the Department of Mathematics. The concept of stability, which constitutes a wide research area in differential equations, was put forward by the Soviet mathematician A. M. Lyapunov in the early 1900s [1]. Stability has applications in many applied sciences such as physics, engineering, medicine. Mathematical biology is a broad branch with many applications. Viruses are the most abundant species in nature, some viruses cause serious infectious diseases in humans. One of these types is coronavirus. The epidemic caused by a new coronavirus (COVID-19) has spread rapidly all over the world. A study was conducted

by José M. Carcione et al. to determine the course of the epidemic in Italy in 2020 [2]. Although there are many studies on the subject around the world, there are not many studies specific to Turkey. In this direction, it is aimed to create and interpret an epidemic model by using Turkey Covid-19 data within the scope of this article.

2. MATHEMATICAL MODEL

2.1. Model Formation

We divided the population into five classes. $X(t)$, $E(t)$, $I(t)$, $N(t)$, $R(t)$ as susceptible vulnerable population class, exposed class, infected (infectious) class, intubated (severely ill) class, recovered class, Model created at time (t).

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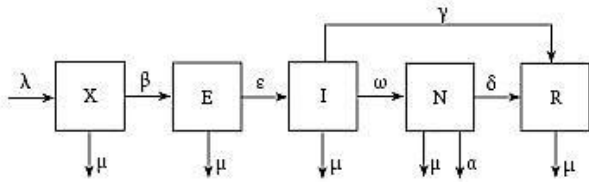


Figure 1 Schematic Diagram of the Model

The differential equations that make up the model are as follows and all of the parameters used are positive real numbers and are explained in Table 1.

$$\begin{cases} \frac{dX}{dt} = \lambda - \mu X - \beta X E \\ \frac{dE}{dt} = \beta X E - \varepsilon E I - \mu E \\ \frac{dI}{dt} = \varepsilon E I - (\mu + \omega + \gamma) I \\ \frac{dN}{dt} = \omega I - (\mu + \alpha + \delta) N \\ \frac{dR}{dt} = \gamma I + \delta N - \mu R \end{cases} \quad (1)$$

Table 1 Parameters and description.

symbols	Definition
λ	Birth rate per capita.
μ	Natural death rate per capita.
α	Average death rate from the virus.
β	The rate at which the susceptible population moves to the exposed class.
ε	Rate of progression from exposure to infection.
ω	The rate at which the infected become intubated (severely ill).
γ	The rate of recovery of those infected.
δ	Recovery rate of intubated patients.

Since the first four equations of system (1) are independent, we skip $R(t)$ without losing generality, and then system (1) is reduced to the following system of differential equations.

$$\begin{cases} \frac{dX}{dt} = \lambda - \mu X - \beta X E \\ \frac{dE}{dt} = \beta X E - \varepsilon E I - \mu E \\ \frac{dI}{dt} = \varepsilon E I - (\mu + \omega + \gamma) I \\ \frac{dN}{dt} = \omega I - (\mu + \alpha + \delta) N \end{cases} \quad (2)$$

2.2. Equilibrium Points of the Differential Equation System

2.2.1. Disease Free Balance Point

For the absence of a diseased individual; if $E = I = N = 0$ is taken and the expressions on the right side of the system are equal to zero, $\lambda - \mu X = 0 \rightarrow X = \frac{\lambda}{\mu}$ is obtained, and the equilibrium point $E_0 = (\frac{\lambda}{\mu}, 0, 0, 0)$ is obtained.

The Jacobian matrix of system (2);

$$J = \begin{bmatrix} -\mu - \beta E & -\beta X & 0 & 0 \\ \beta E & \beta X - \varepsilon I - \mu & -\varepsilon E & 0 \\ 0 & \varepsilon I & \varepsilon E - (\mu + \omega + \gamma) & 0 \\ 0 & 0 & \omega & -(\mu + \alpha + \delta) \end{bmatrix}$$

is found.

When the Jacobian matrix E_0 is at equilibrium point

$$J(E_0) = \begin{bmatrix} -\mu & -\frac{\beta\lambda}{\mu} & 0 & 0 \\ 0 & \frac{\beta\lambda}{\mu} - \mu & 0 & 0 \\ 0 & 0 & -(\mu + \omega + \gamma) & 0 \\ 0 & 0 & \omega & -(\mu + \alpha + \delta) \end{bmatrix}$$

The following matrices are written for the reproduction number.

$$F = \begin{pmatrix} \frac{\beta\lambda}{\mu} & 0 & 0 \\ \mu & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} \mu & 0 & 0 \\ 0 & \mu + \omega + \gamma & 0 \\ 0 & -\omega & \mu + \alpha + \delta \end{pmatrix}$$

Here $\frac{\beta\lambda}{\mu^2}$ is the dominant eigenvalue of the matrix FV^{-1} . That is, required reproduction number

$$R_0 = \frac{\beta\lambda}{\mu^2} \tag{3}$$

is obtained in the form [3-4].

2.2.2. Diseased Equilibrium Point

Theorem 2.1. If $R_0 > 1$, infectious disease has a positive equilibrium point.

Proof: Let's find the positive equilibrium point E_0^* , (2) by equating the left side of the system of differential equations to zero, the following statements are obtained.

(2) from the first equation of the system of differential equations,

$$\begin{aligned} \lambda - \mu X^* - \beta X^* E^* &= 0 \\ X^*(\mu + \beta E^*) &= \lambda \\ X^* &= \frac{\lambda}{\mu + \beta E^*} \end{aligned}$$

(2) from the second equation of the system of differential equations,

$$\begin{aligned} \beta X^* E^* - \varepsilon E^* I^* - \mu E^* &= 0 \\ E^*(\beta X^* - \varepsilon I^* - \mu) &= 0 \\ \beta X^* - \varepsilon I^* - \mu &= 0 \\ I^* &= \frac{\beta X^* - \mu}{\varepsilon} \end{aligned}$$

(2) from the third equation of the system of differential equations,

$$\begin{aligned} \varepsilon E^* I^* - (\mu + \omega + \gamma) I^* &= 0 \\ I^*[\varepsilon E^* - (\mu + \omega + \gamma)] &= 0 \\ \varepsilon E^* - (\mu + \omega + \gamma) &= 0 \\ E^* &= \frac{\mu + \omega + \gamma}{\varepsilon} \end{aligned}$$

(2) from the fourth equation of the system of differential equations

$$\begin{aligned} \omega I^* - (\mu + \alpha + \delta) N^* &= 0 \\ N^* &= \frac{\omega I^*}{\mu + \alpha + \delta} \end{aligned}$$

is found. From here,

$$\begin{aligned} I^* &= \frac{\beta X^* - \mu}{\varepsilon} = \frac{\beta \frac{\lambda}{\mu + \beta E^*} - \mu}{\varepsilon} = \\ &= \frac{\beta \frac{\lambda}{\mu + \beta \frac{\mu + \omega + \gamma}{\varepsilon}} - \mu}{\varepsilon} = \\ &= \frac{\beta \lambda \varepsilon}{\mu \varepsilon + \beta(\mu + \omega + \gamma)} - \mu = \\ &= \frac{\beta \lambda \varepsilon - \mu^2 \varepsilon - \beta \mu(\mu + \omega + \gamma)}{\varepsilon^2 \mu + \varepsilon \beta(\mu + \omega + \gamma)} = \\ &= \frac{\mu^2 \varepsilon \left(\frac{\beta \lambda}{\mu^2} - 1\right) - \beta \mu(\mu + \omega + \gamma)}{\varepsilon^2 \mu + \varepsilon \beta(\mu + \omega + \gamma)} = \\ &= \frac{\mu^2 \varepsilon(R_0 - 1) - \beta \mu(\mu + \omega + \gamma)}{\varepsilon^2 \mu + \varepsilon \beta(\mu + \omega + \gamma)} = \end{aligned}$$

It is obtained from (3).

For $\mu^2 \varepsilon(R_0 - 1) > \beta \mu(\mu + \omega + \gamma)$, $R_0 > 1$ must be present. Thus, the value of I^* becomes positive.

That is, if $R_0 > 1$, the required positive balance point

$$E_0^* = (X^*, E^*, I^*, N^*) = \left(\frac{\varepsilon I^* + \mu}{\beta}, \frac{\mu + \omega + \gamma}{\varepsilon}, \frac{\mu^2 \varepsilon(R_0 - 1) - \beta \mu(\mu + \omega + \gamma)}{\varepsilon^2 \mu + \varepsilon \beta(\mu + \omega + \gamma)}, \frac{\omega I^*}{\mu + \alpha + \delta} \right) \tag{4}$$

2.3. Stability of the Model

2.3.1. Local Stability

Theorem 2.2. If $R_0 < 1$, at equilibrium point E_0 , the system of differential equations (2) is locally stable. If $R_0 > 1$, the system of (2) differential equations is unstable.

Proof: Let's write the jacobian matrix of the system of differential equations when E_0 is at equilibrium point (2)

$$J(E_0) = \begin{bmatrix} -\mu & -\frac{\beta\lambda}{\mu} & 0 & 0 \\ 0 & \frac{\beta\lambda}{\mu} - \mu & 0 & 0 \\ 0 & 0 & -(\mu + \omega + \gamma) & 0 \\ 0 & 0 & \omega & -(\mu + \alpha + \delta) \end{bmatrix}$$

The eigenvalues of the matrix are as follows.

$$\lambda_1 = -\mu < 0$$

$$\lambda_2 = \frac{\beta\lambda}{\mu} - \mu = \mu^2(R_0 - 1)$$

$$\lambda_3 = -(\mu + \omega + \gamma) < 0$$

$$\lambda_4 = -(\mu + \alpha + \delta) < 0$$

If all eigenvalues are negative, the system of differential equations is stable. For $\lambda_2 < 0$ to be $R_0 < 1$. If $R_0 = 1$ or $R_0 > 1$, then $\lambda_2 < 0$ cannot be so (2) the system of differential equations becomes unstable. It is necessary proof.

Theorem 2.3. If $R_0 > \frac{\varepsilon\lambda}{\mu(\mu+\omega+\gamma)}$, at equilibrium point E_0^* , the system of differential equations (2) is locally stable. Otherwise, at equilibrium point E_0^* , the system of differential equations (2) is unstable.

Proof: When E_0^* is at the equilibrium point, let (2) Write the jacobian matrix of the system of differential equations.

$$J(E_0^*) = \begin{bmatrix} \mu - \beta E^* & -\beta X^* & 0 & 0 \\ \beta E^* & \beta X^* - \varepsilon I^* - \mu & -\varepsilon E^* & 0 \\ 0 & \varepsilon I^* & \varepsilon E^* - (\mu + \omega + \gamma) & 0 \\ 0 & 0 & \omega & -(\mu + \alpha + \delta) \end{bmatrix}$$

$$J(E_0^*) = \begin{bmatrix} \mu - \beta E^* & -\beta X^* & 0 & 0 \\ \beta E^* & 0 & -\varepsilon E^* & 0 \\ 0 & \varepsilon I^* & 0 & 0 \\ 0 & 0 & \omega & -(\mu + \alpha + \delta) \end{bmatrix} \begin{matrix} R_2 \rightarrow R_2 + R_1 \\ R_3 \rightarrow R_2 + R_3 \end{matrix}$$

$$J(E_0^*) = \begin{bmatrix} \mu - \beta E^* & -\beta X^* & 0 & 0 \\ -\mu & -\beta X^* & -\varepsilon E^* & 0 \\ \beta X^* & \varepsilon I^* & -\varepsilon E^* & 0 \\ 0 & 0 & \omega & -(\mu + \alpha + \delta) \end{bmatrix} \begin{matrix} R_2 \rightarrow R_2 + \frac{\mu R_3}{\beta E^*} \end{matrix}$$

$$J(E_0^*) = \begin{bmatrix} \mu - \beta E^* & -\beta X^* & 0 & 0 \\ 0 & -\beta X^* + \frac{\mu \varepsilon I^*}{\beta E^*} & -\varepsilon E^* - \frac{\mu \varepsilon}{\beta} & 0 \\ \beta X^* & \varepsilon I^* & -\varepsilon E^* & 0 \\ 0 & 0 & \omega & -(\mu + \alpha + \delta) \end{bmatrix}$$

To be simpler, we can write this matrix as follows.

$$J(E_0^*) = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$\text{Here, } A = \begin{bmatrix} \mu - \beta E^* & -\beta X^* \\ 0 & -\beta X^* + \frac{\mu \varepsilon I^*}{\beta E^*} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 \\ -\varepsilon E^* - \frac{\mu \varepsilon}{\beta} & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} \beta X^* & \varepsilon I^* \\ 0 & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} -\varepsilon E^* & 0 \\ \omega & -(\mu + \alpha + \delta) \end{bmatrix} \text{ [6].}$$

The eigenvalues of the $J(E_0^*)$ matrices vary depending on the eigenvalues of the A and D matrices. The eigenvalues of the matrix A are obtained as follows.

$$\lambda_1 = -\mu - \beta E^* < 0$$

$$\lambda_2 = -\beta X^* + \frac{\mu \varepsilon I^*}{\beta E^*} =$$

$$-\beta \frac{\varepsilon I^* + \mu}{\beta} + \frac{\mu \varepsilon I^*}{\beta E^*} =$$

$$-\varepsilon I^* - \mu + \frac{\mu \varepsilon I^*}{\beta E^*} =$$

$$-\mu + \frac{\mu \varepsilon I^*}{\beta E^* \lambda} \left(\lambda - \frac{\beta E^* \lambda}{\mu} \right) =$$

$$-\mu + \frac{\mu \varepsilon I^*}{\beta E^* \lambda} \left(\lambda - \frac{\beta (\mu + \omega + \gamma) \lambda}{\mu} \right) =$$

$$-\mu + \frac{\mu \varepsilon I^*}{\beta E^* \lambda} \left(\lambda - \frac{\mu (\mu + \omega + \gamma) \beta \lambda}{\varepsilon \mu^2} \right) =$$

$$-\mu + \frac{\mu \varepsilon I^*}{\beta E^* \lambda} \left(\lambda - \frac{\mu (\mu + \omega + \gamma)}{\varepsilon} R_0 \right)$$

$$\text{If } R_0 > \frac{\varepsilon\lambda}{\mu(\mu+\omega+\gamma)} \text{ then } \lambda_2 < 0.$$

The eigenvalues of the matrix D are obtained as follows.

$$\lambda_3 = -\varepsilon E^* < 0$$

$$\lambda_4 = -(\mu + \alpha + \delta) < 0$$

Thus, the proof is completed.

2.3.2. Global Stability

Theorem 2.4. If $R_0 < 1$ then (2) the system of differential equations is spherically stable.

Proof: To prove this theorem, let's first construct the Lyapunov function as L,

$$L = \ln \frac{X}{X_0} + \ln \frac{E}{E_0} + I + N \tag{4}$$

(4) if the derivative of the equation is taken according to time; according to the Lyapunov stability criterion ,

$$L' = \frac{X'}{X} + \frac{E'}{E} + I' + N'$$

$$L' = \frac{\lambda}{X} - \mu - \beta E + \beta X - \varepsilon I - \mu + \varepsilon EI - (\mu + \omega + \gamma)I + \omega I - (\mu + \alpha + \delta)N$$

$$L' = \frac{\lambda}{X} - 2\mu - \beta E + \beta X + \varepsilon EI - (\mu + \varepsilon + \gamma)I - (\mu + \alpha + \delta)N$$

, are obtained. Typing instead of E_0 in the above equation,

$$L' = \mu - 2\mu + \beta \frac{\lambda}{\mu} = \beta \frac{\lambda}{\mu} - \mu = \mu \left(\frac{\beta\lambda}{\mu^2} - 1 \right) = \mu(R_0 - 1)$$

can be found.

For $L' < 0$ it must be $R_0 < 1$. In this case (2) the system of differential equations is globally stable.

3. RESEARCH FINDINGS

3.1. Application Of Model To Turkey Data

In this part of our study, Turkey data dated 25.10.2020 and 03.07.2021 corresponding to the parameter values of the model created in Section 2 are given in Table 2 the classification of the population of Turkey is also given in table 3 [5,6,7,8].

Table 2 Parameters and Turkey data.

symbols	Definition	Turkey Data (25.10.2020)	Turkey Data (03.07.2021)
λ	Birth rate per capita..	0.015	0.013
μ	Natural death rate per capita.	0.053	0.053
α	Average death rate from the virus.	0.084	0.062
β	The rate at which the susceptible population moves to the exposed class.	0.192	0.078
ε	Rate of progression from exposure to infection.	0.166	0.023
ω	The rate at which the infected become intubated (severely ill).	0.033	0.010
γ	The rate of recovery of those infected.	0.843	0.887
δ	Recovery rate of intubated patients.	0.687	0.785

Table 3 Classification Of Turkish Population

class	Definition	Turkey Data (25.10.2020)	Turkey Data (03.07.2021)
X(t)	susceptible population	57.999.210 (0.987)	52.875.643 (0.829)
E(t)	exposed class	361.801 (0.006)	5.440.368 (0.085)
I(t)	infected (infectious) class	47.411 (0.001)	129.599 (0.002)

N(t)	intubated (severely ill) class	15.751 (0.0003)	75.293 (0.001)
R(t)	recovered class	314.390 (0.0057)	5.310.769 (0.083)

3.1.1. Disease-Free Equilibrium Point According To Turkey Covid-19 Data

Equilibrium point according to the data in Table 2 $E_0 = (\frac{\lambda}{\mu}, 0, 0, 0) = (\frac{0.013}{0.053}, 0, 0, 0) = (0.245, 0, 0, 0)$. Accordingly, the jacobian matrix of the system, at the equilibrium point, if the data dated 25.10.2020 is written

$$J(E_0) = \begin{bmatrix} -0.053 & -0.054 & 0 & 0 \\ 0 & 0.001 & 0 & 0 \\ 0 & 0 & -0.927 & 0 \\ 0 & 0 & 0.033 & -0.824 \end{bmatrix}$$

can be found.

From here, the reproduction number is found $R_0 = \frac{\beta\lambda}{\mu^2} = \frac{0.015 \cdot 0.192}{(0.053)^2} \cong 1.025$ according to (3).

If data dated 03.07.2021 is written

$$J(E_0) = \begin{bmatrix} -0.053 & -0.019 & 0 & 0 \\ 0 & -0.034 & 0 & 0 \\ 0 & 0 & -0.94 & 0 \\ 0 & 0 & 0.01 & -0.89 \end{bmatrix}$$

can be found.

From here, the reproduction number is found $R_0 = \frac{\beta\lambda}{\mu^2} = \frac{0.078 \cdot 0.013}{(0.053)^2} \cong 0.361$ according to (3).

3.1.2. Diseased Equilibrium Point According To Turkey Covid-19 Data

Since $R_0 \cong 1.025$, which we found according to data dated 25.10.2020 for Turkey, the system of differential equations has a positive equilibrium point.

This point

$$E_0^* = \left(\frac{\varepsilon I^* + \mu}{\beta}, \frac{\mu + \omega + \gamma}{\varepsilon}, \frac{\mu^2 \varepsilon (R_0 - 1) - \beta \mu (\mu + \omega + \gamma)}{\varepsilon^2 \mu + \varepsilon \beta (\mu + \omega + \gamma)}, \frac{\omega I^*}{\mu + \alpha + \delta} \right)$$

$E_0^* \cong (0.277, 5.596, -0.125, 0.000037)$ can be found.

According to the data dated 03.07.2021 for Turkey, since $R_0 \cong 0.361 < 1$, The System (2) does not have a positive equilibrium point.

3.2. Stability Of The Model

3.2.1. Local stability

According to the data dated 03.07.2021 for Turkey, at the equilibrium point $E_0 = (0.245, 0, 0, 0)$, the jacobian matrix of the (2) system $J(E_0) =$

$$\begin{bmatrix} -0.053 & -0.019 & 0 & 0 \\ 0 & -0.034 & 0 & 0 \\ 0 & 0 & -0.94 & 0 \\ 0 & 0 & 0.01 & -0.89 \end{bmatrix} \text{ found.}$$

Eigenvalues of this matrix;

$$\lambda_1 = -0.053 < 0$$

$$\lambda_1 = -0.034 < 0$$

$$\lambda_1 = -0.940 < 0$$

$$\lambda_1 = -0.890 < 0$$

it is calculated in the form of. Since $R_0 \cong 0.361 < 1$ is what we find, the (2) system is locally stable according to Theorem 2.2.

For the values $R_0 \cong 1.025$ and $\frac{\varepsilon\lambda}{\mu(\mu+\omega+\gamma)} = \frac{0.166 \cdot 0.015}{0.053 \cdot 0.929} \cong 0.051$ that we find at the equilibrium point $E_0^* \cong$

$(0.277, 5.596, -0.125, 0.000037)$ according to the data dated 25.10.2020 for Turkey, the system (2) is locally stable, since $R_0 > \frac{\varepsilon\lambda}{\mu(\mu+\omega+\gamma)}$ according to Theorem 2.3.

3.2.2. Global Stability

By Lyapunov function,

$$L' = \frac{X'}{X} + \frac{E'}{E} + I' + N'$$

$$L' = \frac{\lambda}{x} - \mu - \beta E + \beta X - \varepsilon I - \mu + \varepsilon EI - (\mu + \omega + \gamma)I + \omega I - (\mu + \alpha + \delta)N$$

$$L' = \frac{\lambda}{x} - 2\mu - \beta E + \beta X + \varepsilon EI - (\mu + \varepsilon + \gamma)I - (\mu + \alpha + \delta)N$$

By writing instead of $E_0 = (0.245, 0, 0, 0)$ in the equation, $L' = \beta \frac{\lambda}{\mu} - \mu = \beta \cdot 0.245 - \mu$ is found, where $L' = \beta \frac{\lambda}{\mu} - \mu = \beta \cdot 0.245 - \mu$ is obtained if the values β and μ are written. According to the Lyapunov stability criterion, the (2) system of differential equations is globally stable.

Also, since $R_0 \cong 0.361 < 1$ according to Theorem 2.4, the system (2) is globally stable.

4. CONTROVERSY AND CONCLUSION

One of the major concerns with any infectious disease is its ability to invade the population. Many epidemiological models have a disease-free balance, in which the population remains disease-free. These models usually have a threshold parameter, R_0 , known as the basic breeding number, is defined as this threshold parameter. The R_0 reproduction number of a mathematical model submits us very important information about the increase and decrease of the epidemic. R_0 consists of the parameters of the model. In our model, $R_0 = \frac{\beta\lambda}{\mu^2}$ is found. If $R_0 < 1$, the epidemic is close to ending over time. If $R_0 = 1$, the epidemic never disappears, it is in balance. If $R_0 > 1$, contagion increases as time passes. R_0 also provides us with information about the stability of the model. If $R_0 < 1$, then the system of differential equations is stable. If $R_0 = 1$ or $R_0 > 1$, the system is unstable. The number of R_0 reproductions to be calculated for the Covid-19 global pandemic can guide us in the measures to be taken. The R_0 value can be analyzed for decisions on increasing or reducing constraints.

4.1. Appendices

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Authors' Contribution

The authors contributed equally to the study.

The Declaration of Ethics Committee Approval

This study does not require ethics committee permission or any special permission.

The Declaration of Research and Publication Ethics

The authors of the paper declare that they comply with the scientific, ethical and quotation rules of SAUJS in all processes of the paper and that they do not make any falsification on the data collected. In addition, they declare that Sakarya University Journal of Science and its editorial board have no responsibility for any ethical violations that may be encountered, and that this study has not been evaluated in any academic publication environment other than Sakarya University Journal of Science.

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