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STOCHASTIC INTEGRATION WITH RESPECT TO A CYLINDRICAL SPECIAL SEMI-MARTINGALE

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Abstract. In this research, we introduce the stochastic integration with respect to a cylindrical special semi-martingale, which is a specific case of general integration, with specific properties of special semi-martingales.

1. Introduction

Cylindrical semi-martingales play a key role in application, specially in stochastic partial differential equations. Among the wide class of cylindrical semi-martingales, cylindrical Brownian motions are used widely as models in stochastic analysis [3, 5, 8, 9, 11, 14, 18, 19]. Although Brownian motions work as good models, motivation of using other classes of cylindrical semi-martingales appears in recent research. Interesting examples of such a view can be found in [1, 2, 6, 12, 13, 15]. In spite of the fact that most of the past articles have an applied view to extend the concepts and utilities the stochastic integration, none of these works considers stochastic integration with respect to cylindrical special semi-martingales.

In this work, our main objective is to introduce a theory of stochastic integration for cylindrical special semi-martingales, which are a particular family of semi-martingales with complex behavior in relation with the measure of the space, defined on. P is a special semi-martingale if P can be decomposed into $P = M + A$ where M is a local martingale and A a process with predictable finite variation, with $A_0 = 0$. Such a decomposition is then unique and is called canonical decomposition.

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On the other hand, for a Banach space $\mathscr{X}$, the cylindrical σ-algebra is defined to be the coarsest σ-algebra, i.e. the one with the fewest measurable sets, such that every continuous linear function on $\mathscr{X}$ is a measurable function. That is important to note that in general, the cylindrical σ-algebra is not the same as the Borel σ-algebra on $\mathscr{X}$, which is the coarsest σ-algebra that contains all open subsets of $\mathscr{X}$.

In the following, we study the cylindrical special semi-martingale $M : \mathscr{X}^* \to \mathcal{S}^{\mathrm{SP}}$ from the dual of a separable Banach space $\mathscr{X}$ to the space of special semi-martingales. Moreover, we define the integral of a progressive process with respect to a cylindrical special semi-martingale.

2. Preliminaries

Let $\mathscr{X}, \mathscr{Y}$ be two Banach spaces. We will denote the space of all bilinear operators from $\mathscr{X} \times \mathscr{Y}$ to $\mathbb{R}$ as $\mathscr{B}(\mathscr{X}, \mathscr{Y})$. Note that for a continuous $b \in \mathscr{B}(\mathscr{X}, \mathscr{Y})$ there exists an operator $\mathcal{B} \in \mathscr{L}(\mathscr{X}, \mathscr{Y}^*)$ such that

$$b(x, y) = \langle \mathcal{B}x, y\rangle = \mathcal{B}x(y), \quad x \in X, y \in Y. \tag{1}$$

An operator $\mathcal{B} : \mathscr{X} \to \mathscr{X}^*$ is called self-adjoint, if for each $x, y \in \mathscr{X}$

$$\langle \mathcal{B}x, y\rangle = \langle \mathcal{B}y, x\rangle.$$

and is called positive, if $\mathcal{B}$ is self-adjoint and $\mathcal{B}_x(x) = \langle \mathcal{B}x, x\rangle \geq 0$ for all $x \in \mathscr{X}$.

Recall that if $\mathcal{B} : \mathscr{X} \to \mathscr{X}^*$ is a positive self-adjoint operator, then the Cauchy-Schwartz inequality holds for the bilinear form $\langle \mathcal{B}x, y\rangle$. In a natural way in functional analysis, the norm of $\mathcal{B}$ is defined as

$$\|\mathcal{B}\| = \sup_{x\in\mathscr{X}, \|x\|=1} |\langle \mathcal{B}x, x\rangle| \tag{2}$$

Note that if $\mathscr{X}$ is a Hilbert space, then (2) would be coincides with the induced norm of the inner product defined on $\mathscr{X}$.

Let $(\Omega, \mathscr{F}, \mu)$ be a measure space and $\mathscr{X}$ a Banach space. A function $f : \Omega \to \mathscr{X}$ is called simple if there exist $x_1, x_2, \ldots x_n \in \mathscr{X}$ and $E_1, E_2, \ldots, E_n \in \mathscr{F}$ such that $f = \sum_{i=1}^n x_i \chi_{E_i}$, where $\chi_{E_i}(\omega) = 1$ if $\omega \in E_i$ and $\chi_{E_i}(\omega) = 0$ if $\omega \neq E_i$. A function $f : \Omega \to \mathscr{X}$ is called strong measurable if there exists a sequence of simple functions (f_n) with $\lim_n \|f_n - f\| = 0$, μ-almost everywhere. A function $f : \Omega \to \mathscr{X}$ is called scalar measurable if for each $x^* \in \mathscr{X}^*$ the numerical function $x^* f$ is strong measurable.

Further we will need the following lemma.

Lemma 1. *[15, Proposition 32] Let (S, Σ) be a measurable space, H be a separable Hilbert space, $f : S \to \mathscr{L}(\mathscr{H})$ be a scalar measurable self-adjoint operator-valued function. Let $F : \mathbb{R} \to \mathbb{R}$ be locally bounded measurable. Then $F(f) : S \to \mathscr{L}(\mathscr{H})$ is a scalar measurable self-adjoint operator-valued function.*

That is trivial to think about the square root of a positive operator. It would be appreciated if the square root drops us in to a Hilbert space, even in a special case.

Lemma 2. *[19, Lemma 2.4] Let $\mathscr{X}$ be a reflexive separable Banach space, $\mathcal{B} : \mathscr{X} \to \mathscr{X}^*$ be a positive operator. Then there exists a separable Hilbert space $\mathscr{H}$ and an operator $\mathcal{B}^{1/2} : \mathscr{X} \to \mathscr{H}$ such that $\mathcal{B} = \mathcal{B}^{1/2*}\mathcal{B}^{1/2}$.*

A scalar-valued process M is called a continuous local martingale if there exists a sequence of stopping times $(\tau_n)_{n\geq 1}$ such that $\tau_n \uparrow \infty$ almost surely as $n \to \infty$ and $1_{\tau_n > 0} M^{\tau_n}$ is a continuous martingale.

We denote by $\mathscr{M}$ and $\mathscr{M}^{\mathrm{loc}}$ the class of continuous and continuous local martingales, respectively. It is well known that $\mathscr{M}^{\mathrm{loc}}$ is a vector space with respect to usual operations. Several topologies can be defined on $\mathscr{M}^{\mathrm{loc}}$, for example UCP, which is based on convergence in probability, or Emery topology [4, 7]. Although, we can define a norm on $\mathscr{M}^{\mathrm{loc}}$ as

$$\|M\|_{\mathscr{M}^{\mathrm{loc}}} = \sum_{n=1}^{\infty} 2^{-n} E[1 \wedge \sup_{t\in[0,n]} |M_t|]. \tag{3}$$

It can be seen that the topology induced by the norm in (3) in coincides with the UCP and Emery topology (because of the continuity property). That is proved in several articles that $\mathscr{M}^{\mathrm{loc}}$ equipped with the norm (3) is a complete metric space.

Let $\mathscr{X}$ be a Banach space. In general, a cylindrical semi-martingale on $\mathscr{X}$ is a continuous linear mapping $\varphi : \mathscr{X}^* \to S^0$, where S^0 denotes the space of real semi-martingales with respect to a common stochastic basis $(\Omega, \mathscr{F}, (\mathscr{F}_t)_{0\leq t\leq 1}, P)$, endowed with the Emery topology. The general case is studied before in literature. (see for example [10]). As a special case, a continuous linear mapping $M : \mathscr{X}^* \to \mathscr{M}^{\mathrm{loc}}$ is called a cylindrical continuous local martingale.

In the following, we interested to study the continuous linear mapping $M : X^* \to \mathcal{S}$ where $\mathcal{S}$ is the collection of locally integrable semi-martingales. Our motivation comes from the collection of particular type of martingales, called as *Special Semi-martingales* $\mathcal{S}^{\mathrm{SP}}$, coincides with $\mathcal{S}$.

A processes $P = M + A$ which can be decomposed, by Doob decomposition, into a local martingale M and a predictable cádlag locally finite variation process A is known as special semimartingales. On the space of special semimartingales, we can define p-norm for $p > 0$ as follows and denote the semimartingales with finite p norm by $\mathbb{H}^p$:

$$\|P\|_{\mathbb{H}^p} = \left(E\left[[M, M]_\infty^{p/2} + (\int_0^\infty |dA|)^p \right] \right)^{1/p}.$$

One of the most interesting properties of special semi-martingales is compatibility of integration with the canonical decomposition in the construction of the stochastic integrals. That is, for a special semi-martingale $P = M + A$ and a predictable process ξ we have

$$\int \xi dP = \int \xi dM + \int \xi dA$$

3. Cylindrical Special Martingales

In this section, we define the notion of a cylindrical special martingale and integration with respect to a cylindrical special martingale.

Definition 1. *Let $\mathscr{X}$ be a Banach space. A continuous linear mapping $P : \mathscr{X}^* \to \mathcal{S}^{\mathrm{SP}}$ is called a cylindrical continuous special martingale. In this way, $Px^* = Mx^* + Ax^*$, where Mx^* is a local martingale and Ax^* is a finite variation process, for any $x^* \in X^*$,*

For a cylindrical continuous special martingale P and a stopping time τ, one can define $P^\tau : \mathscr{X}^* \to \mathcal{S}^{\mathrm{SP}}$ by $P^\tau x^*(t) = Px^*(t \wedge \tau)$. Clearly P^τ is also a cylindrical continuous special martingale.

We expect that our definition of a cylindrical continuous special martingale be a generalization of a cylindrical continuous local martingale. A characteristic property of a local martingale is its quadratic variation. Thanks to the finite variation part of P, which has the zero quadratic variation, we can easily define the quadratic variation $[[P]]$ of P similar to the quadratic variation of mapping to its local martingale part M.

Recall that If M is a continuous local martingale with values in a Hilbert space, then it is well known that it has a classical quadratic variation $[M]$ in the sense that there exists an a.s. unique increasing continuous process $[M]$ starting at zero such that $\|M\|^2 - [M]$ is a continuous local martingale again.

Definition 2. *Let $P : \mathscr{X}^* \to \mathcal{S}^{\mathrm{SP}}$ be a linear mapping. The quadratic variation $[[P]]$ of P is defined as*

$$[[P]]_t = \sup_{N \in \mathbb{N}} \sum_{n=1}^{N} \sup_m ([Mx_m^*]_{t_i+1} - [Mx_m^*]_{t_i}), \quad t \geq 0,$$

where Mx^* is the local martingale part of Px^* and the limit is taken over all rational partitions $0 = t_0 < \cdots < t_N = t$ and $(x_m^*)_{m \geq 1}$ is a dense subset of the unit ball in X^*.

Note that existence of $(x_m^*)_{m \geq 1}$ follows from the separability of $\mathscr{X}^*$. For a cylindrical special semi-martingale P on a Banach space $\mathscr{X}$, one can think about covariance $[Px^*, Py^*]_t$ for any $x^*, y^* \in X^*$. However, by the ineffectiveness of finite variation part A of P, we have $[Px^*, Py^*]_t = [Mx^*, My^*]_t$. Therefore, by the polar decomposition, there exists a process $Q_P : \mathbb{R}_+ \times \Omega \to \mathscr{L}(\mathscr{X}^*, \mathscr{X}^{**})$ such that for almost surly $t > 0$

$$[Px^*, Py^*]_t = \int_0^t Q_P x^*(y^*) d[[P]]_s, \quad x^*, y^* \in X^*.$$

The process Q_P is self-adjoint and $\|Q_P(t)\| = 1$.

Let $\mathscr{X}, \mathscr{Y}$ be two Banach spaces. For any $x^* \in \mathscr{X}^*, y \in \mathscr{Y}$, we can define the linear operator $x^* \circledast y \in \mathscr{L}(\mathscr{X}, \mathscr{Y})$ such that $x^* \circledast y : x \mapsto x^*(x)y$. Using the defined operator, the process $\phi : \mathbb{R}_+ \times \Omega \to \mathscr{L}(\mathscr{H}, \mathscr{X})$ is called elementary progressive with respect to the filtration $\mathscr{F} = (\mathscr{F}_t)_{t \in R_+}$ if it is of the form

$$\phi(t, \omega) = \sum_{n=1}^{N} \sum_{m=1}^{M} \mathbf{1}_{(t_{n-1}, t_n] \times B_{mn}}(t, \omega) \sum_{k=1}^{K} v_k \circledast x_{kmn},$$

where $0 \leq t_0 < \cdot < t_n < \infty$, for each $n = 1, \cdot, N$ the sets $B_{1n}, \ldots, B_{Mn} \in \mathscr{F}_{t_{n-1}}$ and vectors $v_1, \ldots, v_K$ are orthogonal.

For each elementary progressive ϕ we define the stochastic integral with respect to $\mathscr{X} \in \mathscr{M}^{\mathrm{sp}}_{\mathrm{var}}(\mathscr{H})$ as an element of $L_0(\Omega; C_b(\mathbb{R}_+; \mathscr{X}))$ as

$$\int_0^t \phi(s)\, \mathrm{d}P(s) = \sum_{n=1}^{N} \sum_{m=1}^{M} \mathbf{1}_{B_{mn}} \sum_{k=1}^{K} (M(t_n \wedge t)v_k - M(t_{n-1} \wedge t)v_k + \mathrm{V}_n(A)v_k) x_{kmn}, \tag{4}$$

where $V_n(A)$ is the total variation of process A in the n-th interval, $[t_{n-1}, t_n]$, and C_b is the set of all continuous and bounded mappings. This is usual to use the notation $\phi \cdot P$ for the process $\int_0^{\cdot} \phi(s)\, \mathrm{d}P(s)$.

Clearly, the definition in (4) is a generalization of integration with respect to a cylindrical local martingale.

Lemma 3. *For all progressively measurable processes* $\phi : \mathbb{R}_+ \times \Omega \to \mathscr{L}(\mathscr{H}, \mathbb{R})$ *with* $\phi Q_P^{1/2} \in L^2(\mathbb{R}_+, [[P]]; \mathscr{L}(\mathscr{H}, \mathbb{R}))$ *we have*

$$\left[\int_0^{\cdot} \phi\, \mathrm{d}P\right]_t = \int_0^t \phi(s) Q_P(s) \phi^*(s)\, \mathrm{d}[[P]]_s. \tag{5}$$

Proof. Note that our definition of quadratic variation for cylindrical special semi-martingales P is reduced to its local martingale part M. Therefore, the proof is similar to the proof of [13, Theorem 14.7.4]. □

It is important to note that for any (t, ω) in $\mathbb{R}_+ \times \Omega$, the mapping $Q_P(t, \omega)$ is a positive mapping from $\mathscr{X}^*$ to $\mathscr{X}^{**}$. Therefore, there exists a Hilbert space $\mathscr{H}$ such that $Q_P^{1/2}(t, \omega)$ maps $\mathscr{X}^*$ to $\mathscr{H}$ and $Q_P(t, \omega) = Q_P^{1/2*}(t, \omega) Q_P^{1/2}(t, \omega)$. Moreover, $\phi(t, \omega) Q_P^{1/2}(t, \omega)$ is an operator and we may think about $(\phi(t, \omega) Q_P^{1/2}(t, \omega))^* = Q_P^{1/2}(t, \omega)^* \phi(t, \omega)^*$. On the other hand, $\phi(t, \omega)$ is an operator from $\mathscr{H}$ to $\mathbb{R}$ and

$\phi(t,\omega)^*$ is well defined. Breaking the Q_P appears in (5) to its roots and have an inner product scheme can make a transparent illustration of the idea behind the lemma.

Theorem 1. *Let $\mathscr{H}$ be a Hilbert space and $P \in \mathscr{M}^{\mathrm{sp}}_{\mathrm{var}}(\mathscr{H})$. Let $\phi : \mathbb{R}_+ \times \Omega \to \mathscr{L}(\mathscr{H},\mathscr{X})$ be such that $\phi^* x^*$ is progressively measurable for each $x^* \in \mathscr{X}^*$, and assume $\phi(\omega)Q_P(\omega)\phi^*(\omega)x^*(x^*) \in L^1_{\mathrm{loc}}(\mathbb{R}_+, [[P]](\omega))$, for all $x^* \in \mathscr{X}^*, \omega \in \Omega$. Set $M := \phi \cdot P$ by*

$$Mx^*(t) := \int_0^t \phi^* x^* \, \mathrm{d}P, \quad x^* \in \mathscr{X}^*. \tag{6}$$

If $\|\phi Q_P \phi^*\|_\infty < \infty$ then $M \in \mathscr{M}^{\mathrm{sp}}_{\mathrm{var}}(\mathscr{X})$.

Proof. It is clear that for each $x^* \in \mathscr{X}^*$, mapping Mx^* is a continuous local martingale. We need just to show theta the mapping $x^* \mapsto Mx^*$ is continuous in the UPC topology. Fix $T > 0$ and set Ω_0 be a subset of Ω such that for almost every $\omega \in \Omega_0$ we have

$$t \mapsto \langle \phi(t,\omega) Q_N(t,\omega)^* \phi(t,\omega)^* x^*, x^* \rangle \in \mathscr{L}^1(0,T).$$

Therefore, we have a bounded operator and there exists a constant C such that

$$\|\langle \phi(\cdot,\omega) Q_N(\cdot,\omega)^* \phi(\cdot,\omega)^* x^*, y^* \rangle\|_{L^1(0,T,[[N]](\omega))} \le C\|x^*\| \, \|y^*\|.$$

Moreover, we have

$$[Mx^*]_t = \int_0^t \langle \phi(s) Q_P \phi^*(s) x^*, x^* \rangle \, \mathrm{d}[[P]], \quad \text{for all } x^* \in \mathscr{X}^*.$$

Note that $\|\phi(s){Q_P}^{1/2}\|_\infty < \infty$ by definition of ϕ and Q_P. Now let (x_n^*) be a sequence in $\mathscr{X}^*$ and $\lim_{n\to\infty} x_n = x$. We have

$$\begin{aligned}
&\|[Mx_n^*]_t - [Mx_n]_t\| \\
&= \left\| \int_0^t \langle \phi(s) Q_P \phi^*(s) x_n^*, x_n^* \rangle \, \mathrm{d}[[P]] - \int_0^t \langle \phi(s) Q_P \phi^*(s) x^*, x^* \rangle \, \mathrm{d}[[P]] \right\|_1 \\
&= \left\| \int_0^t \langle \phi(s) Q_P \phi^*(s) x_n^*, x_n^* \rangle - \langle \phi(s) Q_P \phi^*(s) x^*, x^* \rangle \, \mathrm{d}[[P]] \right\|_1 \\
&\le \|\phi(s) Q_P \phi^*(s)\|_\infty \|x_n - x\| \to 0
\end{aligned}$$

□

Corollary 1. *Let M be the cylindrical continuous local martingale defined in Theorem 1. Then we have*

$$[[M]]_t = \int_0^t \|\phi(s) Q_P(s) \phi^*(s)\| \, \mathrm{d}[[P]], \quad t \ge 0.$$

Proof. To prove the equivalence it suffices to observe that

$$
\begin{aligned}
[[M]]_t &= \lim \sum_{j=1}^{J} \sup_{x^* \in \mathscr{X}^*, \|x^*\|=1} ([Px^*]_{t_j} - [Px^*]_{t_{j-1}}) \\
&= \lim \sum_{j=1}^{J} \sup_{x^* \in \mathscr{X}^*, \|x^*\|=1} \int_{t_{j-1}}^{t_j} \langle \phi(s) Q_P(s) \phi^*(s) x^*, x^* \rangle \, \mathrm{d}[[P]]_s \\
&= \int_0^t \|\phi(s) Q_P(s) \phi^*(s)\| \, \mathrm{d}[[P]]_s.
\end{aligned}
$$

The limit takes when the partition of $0 = t_0 < t_1 < \cdots < t_n = t$ of $[0,t]$ becomes refined, when n tends to infinity. Note that the space $\mathscr{X}^*$ is assumed to be a separable space which helps us to justify the last equation. □

Corollary 2. *Let M be the cylindrical continuous local martingale defined in Theorem 1. Then we have*

$$\phi(s) Q_P(s) \phi^*(s) = Q_M(s) \|\phi(s) Q_P(s) \phi^*(s)\|$$

Proof. By the Corollary 1, we have

$$
\begin{aligned}
[[M]]_t &= \int_0^t \|\phi(s) Q_P(s) \phi^*(s)\| \, \mathrm{d}[[P]] \\
&\Rightarrow \frac{\mathrm{d}}{\mathrm{d}[[P]]}[[M]]_t = \frac{\mathrm{d}}{\mathrm{d}[[P]]} \left(\int_0^t \|\phi(s) Q_P(s) \phi^*(s)\| \, \mathrm{d}[[P]] \right) \\
&\Rightarrow \mathrm{d}[[M]]_s = \|\phi(s) Q_N(s) \phi^*(s)\| \, \mathrm{d}[[P]]_s. \qquad (7)
\end{aligned}
$$

In the other way,

$$
\begin{aligned}
[Mx^*, My^*]_t &= \int_0^t \langle Q_P(s) \phi^*(s) x^*, \phi^*(s) y^* \rangle \, \mathrm{d}[[P]]_s \\
&= \int_0^t \langle \phi(s) Q_P(s) \phi^*(s) x^*, y^* \rangle \, \mathrm{d}[[P]]_s. \qquad (8)
\end{aligned}
$$

Replacing (7) in (8) implies the statement. □

Conclusion

The stochastic integration with respect to a cylindrical Semi-martingale is studied before in general case. In this research, we specified the general case to special semi-martingales and used their specific properties to refine the definition. Since the case of semi-martingales would be studied in relation with the Banach space and some convergence theorems, our refined definition would affect the convergence accuracy.

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