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Research Article

Norm attaining multilinear forms on the spaces c_0 or l_1

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ABSTRACT. $T \in \mathcal{L}(^nE)$ is called a *norming attaining* if there are $x_1, \ldots, x_n \in E$ such that $\|x_1\| = \cdots = \|x_n\| = 1$ and $|T(x_1, \ldots, x_n)| = \|T\|$, where $\mathcal{L}(^nE)$ denotes the space of all continuous n-linear forms on E. We investigate norm attaining multilinear forms on c_0 or l_1.

Keywords: Norming attaining multilinear forms, norming points, norming sets.

2020 Mathematics Subject Classification: 46A22.

1. INTRODUCTION

Let us sketch a brief history of norm attaining multilinear mappings and polynomials on Banach spaces. In 1961, Bishop and Phelps [3] initiated and showed that the set of norm attaining functionals on a Banach space is dense in the dual space. Shortly after, attention was paid to possible extensions of this result to more general settings, specially bounded linear operators between Banach spaces. The problem of denseness of norm attaining functions has moved to other types of mappings like multilinear forms or polynomials. The first result about norm attaining multilinear forms appeared in a joint work of Aron, Finet and Werner [2], where they showed that the Radon-Nikodym property is sufficient for the denseness of norm attaining multilinear forms. Choi and Kim [4] showed that the Radon-Nikodym property is also sufficient for the denseness of norm attaining polynomials. Jimenez-Sevilla and Paya [7] studied the denseness of norm attaining multilinear forms and polynomials on preduals of Lorentz sequence spaces. Acosta and Dávila [1] characterized real Banach spaces Y such that the pair (l_∞^n, Y) has the Bishop-Phelps-Bollobás property for operators. Recently, Dantas et al. [5] introduced and studied a concept of norm-attainment in the space of nuclear operators and in the projective tensor product space of given two Banach spaces.

Let $n \in \mathbb{N}$. We write B_E and S_E for the unit ball and sphere of a Banach space E. We denote by $\mathcal{L}(^nE)$ the Banach space of all continuous n-linear forms on E endowed with the norm $\|T\| = \sup_{(x_1,\cdots,x_n)\in S_E\times\cdots\times S_E} |T(x_1, \cdots, x_n)|$. $\mathcal{L}_s(^nE)$ denotes the closed subspace of all continuous symmetric n-linear forms on E. An element $(x_1, \ldots, x_n) \in E^n$ is called a *norming point* of T if $\|x_1\| = \cdots = \|x_n\| = 1$ and $|T(x_1, \ldots, x_n)| = \|T\|$. For $T \in \mathcal{L}(^nE)$, we define

$$\mathrm{Norm}(T) = \{(x_1, \ldots, x_n) \in E^n : (x_1, \ldots, x_n) \text{ is a norming point of } T\}.$$

$\mathrm{Norm}(T)$ is called the *norming set* of T. Notice that $(x_1, \ldots, x_n) \in \mathrm{Norm}(T)$ if and only if $(\epsilon_1 x_1, \ldots, \epsilon_n x_n) \in \mathrm{Norm}(T)$ for some $\epsilon_k = \pm 1$ $(k = 1, \ldots, n)$. Indeed, if $(x_1, \ldots, x_n) \in \mathrm{Norm}(T)$

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then

$$|T(\epsilon_1 x_1, \ldots, \epsilon_n x_n)| = |\epsilon_1 \cdots \epsilon_n T(x_1, \ldots, x_n)| = |T(x_1, \ldots, x_n)| = \|T\|$$

which shows that $(\epsilon_1 x_1, \ldots, \epsilon_n x_n) \in \mathrm{Norm}(T)$. If $(\epsilon_1 x_1, \ldots, \epsilon_n x_n) \in \mathrm{Norm}(T)$ for some $\epsilon_k = \pm 1$ $(k = 1, \ldots, n)$, then

$$(x_1, \ldots, x_n) = (\epsilon_1(\epsilon_1 x_1), \ldots, \epsilon_n(\epsilon_n x_n)) \in \mathrm{Norm}(T).$$

For $m \in \mathbb{N}$, let $l_\infty^m := \mathbb{R}^m$ with the supremum norm. Notice that for every $T \in \mathcal{L}(^n l_\infty^m)$, $\mathrm{Norm}(T) \neq \emptyset$ since $S_{l_\infty^m}$ is compact. Kim [10] classified $\mathrm{Norm}(T)$ for every $T \in \mathcal{L}_s(^2 l_\infty^2)$. If $\mathrm{Norm}(T) \neq \emptyset$, $T \in \mathcal{L}(^n E)$ is called ([2, 4]) a *norm attaining* n-linear form and we denote by

$$\mathrm{NA}(\mathcal{L}(^n E)) = \{T \in \mathcal{L}(^n E) : T \text{ is norm attaining }\}.$$

If S_E is compact, then $\mathrm{NA}(\mathcal{L}(^n E)) = \mathcal{L}(^n E)$. Notice that if $T \in \mathrm{NA}(\mathcal{L}(^n E))$, then $\lambda T \in \mathrm{NA}(\mathcal{L}(^n E))$ for every $\lambda \in \mathbb{R}$. A mapping $P : E \to \mathbb{R}$ is a continuous n-homogeneous polynomial if there exists a continuous n-linear form L on the product $E \times \cdots \times E$ such that $P(x) = L(x, \ldots, x)$ for every $x \in E$. We denote by $\mathcal{P}(^n E)$ the Banach space of all continuous n-homogeneous polynomials from E into $\mathbb{R}$ endowed with the norm $\|P\| = \sup_{\|x\|=1} |P(x)|$.

An element $x \in E$ is called a *norming point* of $P \in \mathcal{P}(^n E)$ if $\|x\| = 1$ and $|P(x)| = \|P\|$. For $P \in \mathcal{P}(^n E)$, we define

$$\mathrm{Norm}(P) = \{x \in E : x \text{ is a norming point of } P\}.$$

$\mathrm{Norm}(P)$ is called the *norming set* of P. Notice that $\mathrm{Norm}(P) = \emptyset$ or a finite set or an infinite set. Kim [9] classify $\mathrm{Norm}(P)$ for every $P \in \mathcal{P}(^2 l_\infty^2)$. If $\mathrm{Norm}(P) \neq \emptyset$, $P \in \mathcal{P}(^n E)$ is called [4] a *norm attaining* n-homogeneous polynomial.

For more details about the theory of multilinear mappings and polynomials on a Banach space, we refer to [6].

It seems to be natural and interesting to study about $\mathrm{NA}(\mathcal{L}(^n E))$. In this paper, we investigate $\mathrm{NA}(\mathcal{L}(^n E))$ for $E = c_0$ or l_1, where

$$c_0 = \{(x_j)_{j \in \mathbb{N}} : x_j \in \mathbb{R}, \lim_{j \to \infty} x_j = 0\},$$

$$l_1 = \{(x_j)_{j \in \mathbb{N}} : x_j \in \mathbb{R}, \sum_{j=1}^{\infty} |x_j| < \infty\}.$$

2. Results

Throughout the paper, we let $n \in \mathbb{N}, n \geq 2$. For a real sequence $(x_j)_{j \in \mathbb{N}}$, we denote by $\mathrm{supp}((x_j)_{j \in \mathbb{N}}) = \{j \in \mathbb{N} : x_j \neq 0\}$. For $T \in \mathcal{L}(^n c_0)$ or $\mathcal{L}(^n l_1)$ with

$$T((x_j^{(1)})_{j \in \mathbb{N}}, \ldots, (x_j^{(n)})_{j \in \mathbb{N}}) = \sum_{(j_1, \ldots, j_n) \in \mathbb{N}^n} a_{j_1 \cdots j_n} \, x_{j_1}^{(1)} \cdots x_{j_n}^{(n)}$$

for some $a_{j_1 \cdots j_n} \in \mathbb{R}$, we denote by $\mathrm{supp}(T) = \{(j_1, \ldots, j_n) \in \mathbb{N}^n : a_{i_1 \cdots i_n} \neq 0\}$. Notice that if $\mathrm{supp}(T)$ is finite, then T is norm attaining. Without loss of generality, we may restrict T such that $\mathrm{supp}(T)$ is infinite.

The following theorem presents a sufficient condition that the norm of $T \in \mathcal{L}(^n c_0)$ is less than of the sum of the absolute values of its coefficients.

Theorem 2.1. *Let $T \in \mathcal{L}(^n c_0)$ be such that*

$$T((x_j^{(1)})_{j \in \mathbb{N}}, \ldots, (x_j^{(n)})_{j \in \mathbb{N}}) = \sum_{(j_1, \ldots, j_n) \in \mathbb{N}^n} a_{j_1 \cdots j_n} \, x_{j_1}^{(1)} \cdots x_{j_n}^{(n)}$$

for some $a_{j_1 \cdots j_n} \in \mathbb{R}$. If $T \in NA(\mathcal{L}(^n c_0))$ and $\mathrm{supp}(T)$ is infinite, then $\|T\| < \sum_{(j_1, \ldots, j_n) \in A} |a_{j_1 \cdots j_n}|$.

Proof. Assume the contrary. Let $((x_j^{(1)})_{j\in\mathbb{N}},\dots,(x_j^{(n)})_{j\in\mathbb{N}})\in \mathrm{Norm}(T)$. Let $A=\mathrm{supp}(T)$ and $A_l:=\{i_l\in\mathbb{N}:(i_1,\dots,i_l,\dots,i_n)\in A\}$ for $l=1,\dots,n$. There is $1\le l\le n$ such that $\mathrm{supp}((x_j^{(l)})_{j\in\mathbb{N}})\cap A_l$ is infinite. Without loss of generality, we may assume that $\mathrm{supp}((x_j^{(1)})_{j\in\mathbb{N}})\cap A_1$ is infinite. Choose $i_1^{'}\in A_1$ such that $|x_{i_1^{'}}^{(1)}|<\frac{1}{2}$. Let $(i_1^{'},\dots,i_n^{'})\in A$. It follows that

$$\begin{aligned}
\|T\| &= \Big|T((x_j^{(1)})_{j\in\mathbb{N}},\dots,(x_j^{(n)})_{j\in\mathbb{N}})\Big|\\
&= \Big|\sum_{(j_1,\dots,j_n)\in A} a_{j_1\cdots j_n}\, x_{j_1}^{(1)}\cdots x_{j_n}^{(n)}\Big|\\
&\le \sum_{(j_1,\dots,j_n)\in A} |a_{j_1\cdots j_n}|\, |x_{j_1}^{(1)}|\cdots|x_{j_n}^{(n)}|\\
&= \sum_{(j_1,\dots,j_n)\in A\setminus\{(i_1^{'},\dots,i_n^{'})\}} |a_{j_1\cdots j_n}|\, |x_{j_1}^{(1)}|\cdots|x_{j_n}^{(n)}| + |a_{i_1^{'}\cdots i_n^{'}}|\ |x_{i_1^{'}}^{(1)}|\cdots|x_{i_n^{'}}^{(n)}|\\
&\le \sum_{(j_1,\dots,j_n)\in A\setminus\{(i_1^{'},\dots,i_n^{'})\}} |a_{j_1\cdots j_n}| + \frac{1}{2}|a_{i_1^{'}\cdots i_n^{'}}|\\
&< \sum_{(j_1,\dots,j_n)\in A} |a_{j_1\cdots j_n}| \le \|T\|
\end{aligned}$$

which is a contradiction. Therefore, $\|T\|<\sum_{(j_1,\dots,j_n)\in A}|a_{j_1\cdots j_n}|$. □

Remark 2.1. *The converse of Theorem 2.1 is not true in general.*

In fact, let

$$T((x_j)_{j\in\mathbb{N}},(y_j)_{j\in\mathbb{N}})=\frac{1}{2}(x_1y_1-x_2y_2+x_1y_2+x_2y_1)+\sum_{k=3}^{\infty}\frac{1}{2^{k-1}}x_ky_k\in\mathcal{L}(^2c_0).$$

Obviously, supp$(T)=\{(k,k),(1,2),(2,1):k\in\mathbb{N}\}$. Let $A=$ supp(T).

Claim 1. $1=\|T\|<\sum_{(i,j)\in A}|a_{ij}|=\frac{5}{2}$.
We may consider the bilinear form $x_1y_1-x_2y_2+x_1y_2+x_2y_1$ as an element of $\mathcal{L}(^2l_\infty^2)$. It was shown [8] *that for $T((x_1,x_2),(y_1,y_2))=ax_1y_1+bx_2y_2+cx_1y_2+dx_2y_1\in\mathcal{L}(^2l_\infty^2)$,*

$$\|T\|=\max\{|a+b|+|c+d|,|a-b|+|c-d|\}. \tag{2.1}$$

By (2.1), $\|x_1y_1-x_2y_2+x_1y_2+x_2y_1\|=1$. It follows that

$$\begin{aligned}
\|T\| &\le \frac{1}{2}\Big\|x_1y_1-x_2y_2+x_1y_2+x_2y_1\Big\|+\sum_{k=3}^{\infty}\Big\|\frac{1}{2^{k-1}}x_ky_k\Big\|\\
&=\frac{1}{2}+\frac{1}{2}=1.
\end{aligned}$$

For $n\in\mathbb{N}$,

$$\|T\|\ge|T(e_1+\sum_{k=3}^{n+2}e_k,\ e_1+\sum_{k=3}^{n+2}e_k)|=1-\frac{1}{2^{n+1}}\to 1$$

as $n\to\infty$. Hence, $\|T\|=1$. Obviously, $\sum_{(i,j)\in A}|a_{ij}|=\frac{5}{2}$.

Claim 2. $T\notin NA(\mathcal{L}(^2c_0))$.
Assume the contrary. Let $((x_j)_{j\in\mathbb{N}},(y_j)_{j\in\mathbb{N}})\in\mathrm{Norm}(T)$. Notice that

$$S:=\mathit{supp}((x_j)_{j\in\mathbb{N}})\cap\mathit{supp}((y_j)_{j\in\mathbb{N}})$$

is infinite because if S is finite, then $\|T\| < 1$ by the above argument. Choose $i_0 \in S\backslash\{1,2\}$ such that $|x_{i_0}| < \frac{1}{2}$. It follows that

$$\begin{aligned}
1 = \|T\| &= \Big|\frac{1}{2}(x_1y_1 - x_2y_2 + x_1y_2 + x_2y_1) + \sum_{k\in S\backslash\{1,2\}} \frac{1}{2^{k-1}} x_k y_k\Big| \\
&\le \frac{1}{2}\Big|x_1y_1 - x_2y_2 + x_1y_2 + x_2y_1\Big| + \sum_{k\in S\backslash\{1,2\}} \Big|\frac{1}{2^{k-1}} x_k y_k\Big| \\
&\le \frac{1}{2} + \sum_{k\in S\backslash\{1,2,i_0\}} \frac{1}{2^{k-1}}|x_k|\,|y_k| + \frac{1}{2^{i_0-1}}|x_{i_0}|\,|y_{i_0}| \text{ (by (2.1))} \\
&< \frac{1}{2} + \sum_{k\in S\backslash\{1,2,i_0\}} \frac{1}{2^{k-1}} + \frac{1}{2^{i_0}} < 1
\end{aligned}$$

which is a contradiction. Hence, $T \notin NA(\mathcal{L}(^2c_0))$.

Lemma 2.1. *Let $T \in NA(\mathcal{L}(^2c_0))$ and $(x_1, x_2) \in \mathrm{Norm}(T)$ with $x_k = (x_j^{(k)})_{j\in\mathbb{N}}$ for $k = 1, 2$. Then, there is $N \in \mathbb{N}$ such that*
(1) if $n \ge N$ and $|x_j^{(1)}| < 1$ for some $j \in \mathbb{N}$, then $T(e_j, e_n) = 0$,
(2) if $n \ge N$ and $|x_j^{(2)}| < 1$ for some $j \in \mathbb{N}$, then $T(e_n, e_j) = 0$.

Proof. (1) Since $x_1, x_2 \in S_{c_0}$, there are $N \in \mathbb{N}$ and $0 < \delta < \frac{1}{2}$ such that if $n \ge N$, then $|x_n^{(k)}| < \delta$ for $k = 1, 2$. It follows that for $0 < \lambda < 1 - |x_j^{(1)}|$ and $0 < \beta < 1 - \delta$,

$$\begin{aligned}
\|T\| &\ge \max\{|T(x_1 \pm \lambda e_j,\ x_2 \pm \beta e_n)|\} \\
&= \max\{|T(x_1,\ x_2) \pm \beta T(x_1, e_n) \pm \lambda T(e_j, x_2) \pm \lambda\beta T(e_j, e_n)|\} \\
&= |T(x_1,\ x_2)| + \beta|T(x_1, e_n)| + \lambda|T(e_j, x_2)| + \lambda\beta|T(e_j, e_n)| \\
&= \|T\| + \beta|T(x_1, e_n)| + \lambda|T(e_j, x_2)| + \lambda\beta|T(e_j, e_n)|
\end{aligned}$$

which shows that $|T(x_1, e_n)| = |T(e_j, x_2)| = |T(e_j, e_n)| = 0$.
(2) follows by the similar argument as in the proof of (1). □

The following theorem presents a sufficient condition that $T \in \mathrm{NA}(\mathcal{L}(^2c_0))$ is a finite-type bilinear form.

Theorem 2.2. *Let $T \in NA(\mathcal{L}(^2c_0))$ and $(x_1, x_2) \in \mathrm{Norm}(T)$ with $x_k = (x_j^{(k)})_{j\in\mathbb{N}}$ for $k = 1, 2$. Suppose that $|\{j \in \mathbb{N} : |x_j^{(k)}| = 1\}| = 1$ for $k = 1, 2$. Then $T((x_j)_{j\in\mathbb{N}}, (y_j)_{j\in\mathbb{N}}) = \sum_{1\le i,j\le N} a_{ij} x_i y_j$ for some $a_{ij} \in \mathbb{R}$ and $N \in \mathbb{N}$. Hence, $\mathrm{supp}(T)$ is finite.*

Proof. Let $N \in \mathbb{N}$ be the number in the proof of Lemma 2.1. Let $j_1, j_2 \in \mathbb{N}$ be such that $|x_{j_k}^{(k)}| = 1$ and $|x_j^{(k)}| < 1$ for all $j \ne j_k$. By the proof of Lemma 2.1, $T(x_1, e_n) = T(e_j, e_n) = 0$ for every $j \ne j_1$ and $n \ge N$. It follows that

$$\begin{aligned}
0 = T(x_1, e_n) &= T\Big(\sum_{1\le k\le N} x_k^{(1)} e_k,\ e_n\Big) \\
&= \sum_{1\le k\le N} x_k^{(1)}\, T(e_k, e_n) = x_{j_1}^{(1)}\, T(e_{j_1}, e_n)
\end{aligned}$$

which implies that $T(e_{j_1}, e_n) = 0$. Hence, $T(e_j, e_n) = 0$ for all $j \in \mathbb{N}$ and $n \geq N$. By the proof of Lemma 2.1, $T(e_n, x_2) = T(e_n, e_j) = 0$ for every $j \neq j_2$ and $n \geq N$. It follows that

$$\begin{aligned} 0 = T(e_n, x_2) &= T(e_n, \sum_{1\leq k\leq N} x_k^{(2)} e_k) \\ &= \sum_{1\leq k\leq N} x_k^{(2)}\ T(e_n, e_k) = x_{j_2}^{(2)}\ T(e_n, e_{j_2}) \end{aligned}$$

which implies that $T(e_n, e_{j_2}) = 0$. Hence, $T(e_n, e_j) = 0$ for all $j \in \mathbb{N}$ and $n \geq N$. Therefore, $T((x_j)_{j\in\mathbb{N}}, (y_j)_{j\in\mathbb{N}}) = \sum_{1\leq i,j\leq N} a_{ij} x_i y_j$ for some $a_{ij} \in \mathbb{R}$. $\square$

Motivated by Theorem 2.2, we propose some question.
Question. *Is it true that $NA(\mathcal{L}(^2c_0)) = \{T \in \mathcal{L}(^2c_0) : \operatorname{supp}(T)$ is finite$\}$?*
The following theorem characterizes $\mathrm{NA}(\mathcal{L}(^nl_1))$.

Theorem 2.3. *Let $T \in \mathcal{L}(^nl_1)$ be such that*

$$T((x_j^{(1)})_{j\in\mathbb{N}}, \ldots, (x_j^{(n)})_{j\in\mathbb{N}}) = \sum_{(j_1,\ldots,j_n)\in\mathbb{N}^n} a_{j_1\cdots j_n}\ x_{j_1}^{(1)} \cdots x_{j_n}^{(n)}$$

for some $a_{j_1\cdots j_n} \in \mathbb{R}$. Then $T \in NA(\mathcal{L}(^nl_1))$ if and only if there are $j_1^{'}, \ldots, j_n^{'} \in \mathbb{N}$ such that $\|T\| = \left|a_{j_1^{'}\cdots j_n^{'}}\right|$.

Proof. Without loss of generality, we may assume that $T \neq 0$.
($\Rightarrow$) Assume the contrary. Let $((x_j^{(1)})_{j\in\mathbb{N}}, \ldots, (x_j^{(n)})_{j\in\mathbb{N}}) \in \mathrm{Norm}(T)$. Let $B = \operatorname{supp}(T)$. We claim that B is infinite. Assume that B is finite. Let $\delta := \max\{|a_{j_1\cdots j_n}| : (j_1, \ldots, j_n) \in B\} < \|T\|$. It follows that

$$\begin{aligned} \|T\| &= \left|T((x_j^{(1)})_{j\in\mathbb{N}}, \ldots, (x_j^{(n)})_{j\in\mathbb{N}})\right| = \Big|\sum_{(j_1,\ldots,j_n)\in B} a_{j_1\cdots j_n}\ x_{j_1}^{(1)} \cdots x_{j_n}^{(n)}\Big| \\ &\leq \sum_{(j_1,\ldots,j_n)\in B} |a_{j_1\cdots j_n}|\ |x_{j_1}^{(1)}| \cdots |x_{j_n}^{(n)}| \leq \delta \sum_{(j_1,\ldots,j_n)\in\mathbb{N}^n} |x_{j_1}^{(1)}| \cdots |x_{j_n}^{(n)}| = \delta < \|T\| \end{aligned}$$

which is a contradiction. Hence, B is infinite. Since $T \neq 0$, there are $(j_1^{'}, \ldots, j_n^{'}) \in B$ such that $j_k^{'} \in \operatorname{supp}((x_j^{(k)})_{j\in\mathbb{N}})$ for $k = 1, \ldots, n$. Then

$$\begin{aligned} \|T\| &= \left|T((x_j^{(1)})_{j\in\mathbb{N}}, \ldots, (x_j^{(n)})_{j\in\mathbb{N}})\right| \\ &= \Big|\sum_{(j_1,\ldots,j_n)\in B} a_{j_1\cdots j_n}\ x_{j_1}^{(1)} \cdots x_{j_n}^{(n)}\Big| \\ &\leq \sum_{(j_1,\ldots,j_n)\in B} |a_{j_1\cdots j_n}|\ |x_{j_1}^{(1)}| \cdots |x_{j_n}^{(n)}| \\ &= |a_{j_1^{'}\cdots j_n^{'}}|\ |x_{j_1^{'}}^{(1)}| \cdots |x_{j_n^{'}}^{(n)}| + \sum_{(j_1,\ldots,j_n)\in B\setminus\{(j_1^{'},\ldots,j_n^{'})\}} |a_{j_1\cdots j_n}|\ |x_{j_1}^{(1)}| \cdots |x_{j_n}^{(n)}| \end{aligned}$$

$$< \|T\|\, |x^{(1)}_{j'_1}|\cdots|x^{(n)}_{j'_n}| + \sum_{(j_1,\ldots,j_n)\in B\setminus\{(j'_1,\ldots,j'_n)\}} |a_{j_1\cdots j_n}|\, |x^{(1)}_{j_1}|\cdots|x^{(n)}_{j_n}|$$
$$\leq \|T\| \sum_{(j_1,\ldots,j_n)\in\mathbb{N}^n} |x^{(1)}_{j_1}|\cdots|x^{(n)}_{j_n}|$$
$$= \|T\|(\sum_{j_1\in\mathbb{N}} |x^{(1)}_{j_1}|)\cdots(\sum_{j_n\in\mathbb{N}} |x^{(n)}_{j_n}|) = \|T\|$$

which is a contradiction.
($\Leftarrow$) Since $\|T\| = |T(e_{j'_1},\ldots,e_{j'_n})|$ for some $(j'_1,\ldots,j'_n) \in \mathbb{N}^n$, $(e_{j'_1},\ldots,e_{j'_n}) \in \mathrm{Norm}(T)$ and $T \in \mathrm{NA}(\mathcal{L}(^n l_1))$. We complete the proof. $\square$

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