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The Journal of Operations Research, Statistics, Econometrics and Management Information Systems

Volume 3, Issue 1, 2015

2015.03.01.STAT.02

A SOLUTION TO RATIONAL DECISION MAKING VIA COMPOSITIONAL DATA ANALYSIS: A CASE STUDY USING STUDENTS CELLULAR PHONE TENDENCIES

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Received: 17 March 2015
Accepted: 29 June 2015

Abstract

Decisions can be simple or complex depending on the alternatives available to the decision maker, and also, to the different state of the worlds. In classical decision theory, a pay-off table is analyzed in order to make an optimal decision. Here, the decision process involves the definition of different alternatives and state of the worlds, cost or profit calculations of the pay-off table, and probability values, if any, related to state of the worlds. In decision problems, sometimes, it may be necessary to involve emotional attachments in order to create pay-off table. In this setting, rather than using the usual cost or profit values, some satisfaction values can be assigned to the criteria and the rationality of different alternatives can be investigated. In this study, the rationality approach to decision pay-off matrix is demonstrated using a real life example regarding a cellular phone purchase. Furthermore, a compositional data analysis approach is also suggested, and the contributions of the compositional data analysis to decision theory are given.

Keywords: Decision Theory, Rationality, Compositional Data, Cell Phone, Principal Components Analysis, Biplot

Jel Code: C44, C10

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RASYONEL KARAR VERME PROBLEMİ İÇİN COMPOSİTİONAL VERİ ANALİZİ YAKLAŞIMI: ÖĞRENCİ TELEFON TERCİHLERİ ÜZERİNE BİR UYGULAMA

Özet

Karar vericinin içinde bulunduğu doğal durumlar ve karar seçeneklerine bağlı olarak, Kararlar basit veya karmaşık olabilir. Klasik karar teorisinde en iyi kararın verilmesi için sonuç matrisi analiz edilir. Bu analizde karar süreci, farklı karar seçeneklerinin belirlenmesi, doğal durumların tespiti, beklenen kar veya zarar değerlerinin hesaplanması ile sonuç matrisinin hesaplanması, ve var ise doğal durumlara ilişkin olasılık değerlerinin tespitini içerir. Sonuç matrisi oluşturulurken kimi karar verme süreçlerinde karar verici duygusal yargılarını da sürece eklemek isteyebilir. Böyle bir durumda, sıradan kar-zarar hesabı yerine, kriterlere bazı memnuniyet değerleri ataması yapılarak, farklı karar seçeneklerinin rasyonel olup olmadıkları araştırılabilir. Bu çalışmada, karar sonuç matrisi için rasyonellik yaklaşımı, cep telefonu alımı örneği üzerinden gösterilmiştir. Ek olarak, çözüm için Compositional (bütüne tamamlayan) veri analizi yaklaşımı da önerilerek, Compositional veri analizinin karar teorisine katkıları sunulmuştur.

Anahtar Kelimeler : Karar teorisi, Rasyonellik, Compositional (bütüne tamamlayan) veri, Cep Telefonu, Asal Bileşenler Analizi, Bi-Plot.
Jel Kodu : C44, C10

1. INTRODUCTION

Decision theory in business and statistics is usually concerned with identifying the values, uncertainties and other issues relevant in a given decision and the resulting optimal decision [1]. Decision analysis provides structure and guidance for thinking systematically about hard decisions. With decision analysis, a decision maker can take action with confidence gained through a clear understanding of the problem. Along with a conceptual framework for thinking about hard problems, decision analysis provides analytical tools that can make the required hard thinking easier. Although every decision may have its own special problems, there are four basic sources of difficulty. First, a decision can be hard simply because of its complexity. Second, a decision can be difficult because of the inherent uncertainty in the situation. Third, a decision maker may be interested in working toward multiple objectives, but progress in one direction may impede progress in others. Fourth, and finally, a problem may be difficult if different perspectives lead to different conclusions [2]. Modern decision theory has developed since the middle of the 20th century through contributions from several academic disciplines. Although it is now clearly an academic subject of its own right, decision theory is typically pursued by researchers who identify themselves as economists, statisticians, psychologists, political and social scientists or philosophers [3].

A good decision is the one that gives the best outcome, but one can make a good decision but still have an unlucky outcome. On the other hand one may prefer to have lucky outcomes rather than make good decisions. Figure 1. shows a flowchart for the decision analysis process [2]. A careful decision maker may cycle through the process shown in Figure 1. several times as the analysis is refined. Emotional attachments also may play a role in decision making. Such decisions can be real and can have serious consequences for the organization or the decision maker. They can lead to poor decisions due to biases and lack of objectivity. The negatives of emotional decisions will be minimized if the decision maker, firstly, becomes aware of biases and makes allowances for them and, secondly, seeks out independent opinions – ask the opinion of some person who has no vested interest in the decision [4].

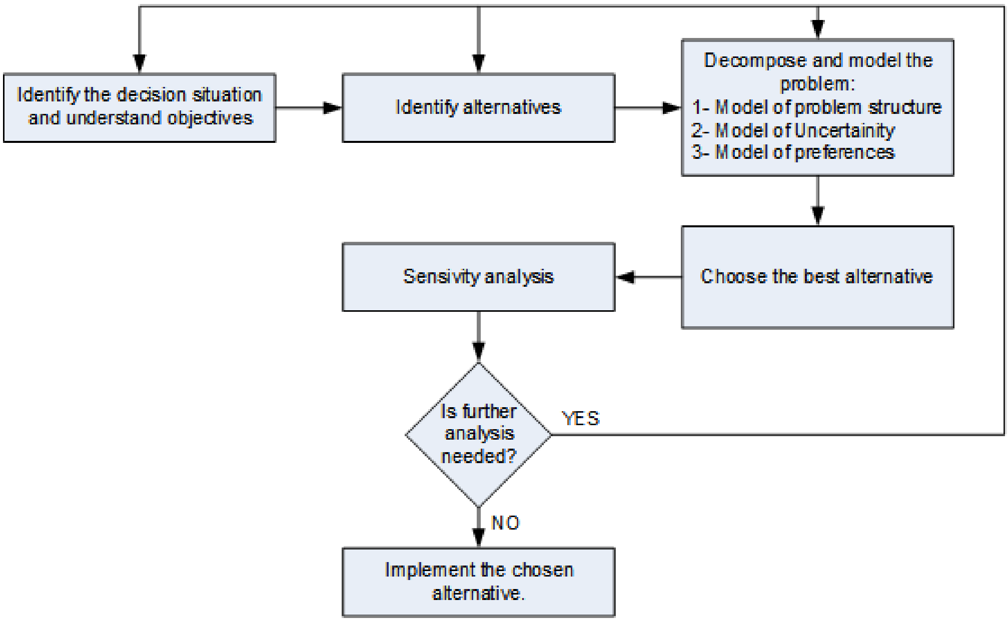


Figure 1. A decision analysis process flowchart

In this study, a daily life decision regarding a cellular phone purchase is investigated for university students. The decision problem is investigated via two different techniques in this context. First, the decision problem is considered as a classical decision problem, but once the structure of the problem is laid out, it is seen that the data matrix is not the usual pay-off table used in the classical decision theory. For this purpose, the decision matrix is specially designed to be solved by the rationality approach to the decision theory. This new setting of the decision problem allowed the author to further investigate the data set using robust principal components analysis for compositional data. Therefore in the second part of the investigation, some further results obtained from the output of robust principal components analysis are shown. The second section of the study is a review of the materials and methods, and also, the definition of the data matrix obtained from the university students is given.

2. Materials and Methods

In this section, the materials and methods used in this study are shortly reviewed. Before the review of the techniques, the data set used in the analysis is described in detail.

2.1. Data Set

The main aim of this study is to show the relationship between rational decision making and create a solution using correspondence analysis. Therefore, rather then using a big population the study is restricted to a small population and a small sample size. For this purpose, the population of the study is defined to be the students in Faculty of Economics and Administrative Sciences, Anadolu University, Turkey. In order to obtain the sample size the following well known equation is used:

$$n = p\left(1-p\right)\left(\frac{z}{E}\right)^2 \tag{1}$$

Since there are not any previous studies on the subject, p value is taken as 0.50 and the margin of error is assigned as 0.15. If the level of confidence to be used in the study is defined as 95%, then the sample size equation yields a value equal to:

$$n = 0.50\left(1-0.50\right)\left(\frac{1.96}{0.15}\right)^2 = 42.68 \tag{2}$$

Therefore, in order to get the projected values, the minimum sample size proposed from the equation is 43. In this study, 50 university students are chosen randomly from Department of Business, Anadolu University. First, all the students are asked "if they needed to buy a new cellular phone, what characteristics of the phone they would be interested in". Six major characteristics are chosen by students and these are Wi-fi, 3G, Video Calling, MP3/Radio, Touch Screen and Price. The other characteristics given by students are color, sound quality, technical support, and robustness of the phone material. The students, then, are asked to weight their preferences among the characteristics from an interval of 1 to 6, where 1 being least preferred and 6 most preferred. For each phone characteristic, the arithmetic mean of the preference scores of students is calculated and assigned as the weights to be used in this investigation. The weights are Wi-Fi = 4.03 , 3G = 3.60, Camera = 3.92, MP3/Radio = 3.64, Touch Screen = 4.65, Price = 5.16. At this stage of the investigation, the students are provided with computers with Internet connection and asked to browse the internet for different phone companies and their products. The purpose of the students is to find the different phones offered by the companies and weight the performance (from 1 up to 6 again) of the companies in terms of the characteristics they would prefer in a new phone. Consequently, if a telephone company produces a number of products with MP3/radio characteristic, then the firm will receive a score of 6. The students are left for browsing for 2 hours. Table 1. shows the phone companies chosen by students and their average grades for the six characteristics given above. Note that the company names are the trademarks of the respective companies and to avoid any copyright infringement, the names of the companies are not given in this study but presented by numbers.

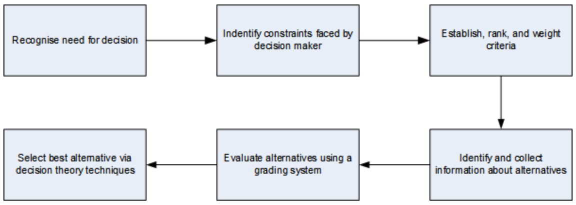
Table 1. Grades and weights of different phone companies

Weights	**4.03**	**3.60**	**3.92**	**3.64**	**4.65**	**5.16**
Criteria / Phone Company	WI-FI	3G	CAMERA	MP3/RADIO	TOUCH SCREEN	PRICE
1.	4	4	1	2	2	2
2.	5	3	2	2	2	2
3.	4	4	1	3	3	3
4.	1	1	1	4	2	2
5.	2	2	1	4	1	4
6.	1	1	1	4	1	2
7.	1	1	1	4	1	3
8.	2	2	2	4	2	3
9.	1	1	1	5	1	4
10.	1	1	1	5	1	4
11.	1	1	1	5	1	4
12.	1	1	1	5	2	4
13.	2	1	2	5	2	3
14.	1	1	1	5	1	4
15.	4	4	4	5	2	3
16.	3	4	4	5	4	4
17.	5	5	4	5	3	6
18.	1	1	1	5	1	4
19.	4	5	5	5	5	3
20.	3	1	1	5	2	4
21.	2	1	1	5	4	4
22.	5	5	1	6	6	2
23.	5	1	1	6	2	3
24.	2	5	5	6	1	4

It can easily be seen from Table 1. that this data set is not usual pay-off table used in classical decision theory. In order to analyses this data set, the technique to be used is the rationality approach to the decision theory. The details of the rationality in decision theory are given in the following subsection. Also, once the rationality matrix is designed, a further statistical technique can be applied to the data set, namely, robust principal components analysis.

The concept of rationality has an action-guiding dimension. In its most basic sense, the concept of rationality applies to resolutions of decision problems. Some ways of resolving decision problems may be rational. Others not. In some cases, there may be more than one rational resolution. And nor, at this stage, should we rule out the possibility that in some circumstances there may not be any [5]. Physical sciences provide an alternative approach to decision making which is readily adaptable to business decisions. The rational approach to decision making is shown in Figure 2. Once the need for making a decision has been recognized, the firm has to identify its constraints and establish criteria [4].

Figure 2. The rational approach to decision making.



Criteria and alternatives used in the analysis should be carefully selected in order to avoid a bias on the results. Once a decision maker decides on necessary criteria, the next step is to rank and weight the criteria according to their relative importance. The number of criteria defines the scale to be used for the weighting scheme, if there are k different criteria available for the decision maker, then the weights should be given to each criterion from a scale of 1 up to k, where the small values represent smaller importance and the higher values or values closer the k show higher importance to the criteria. The next step is to identify and collect information about alternatives and then evaluate each one of them via a grading system subject to all criteria and their weights. In the last step, the weights and the grades of each alternative are multiplied to give a total score for the pair (of alternative, criteria). Then for each alternative, the row sums are calculated to get the highest score so as to represent the best alternative of the decision problem. In Table 2., the rational approach to pay-off table is shown.

Table 2. Pay-off table for rational approach to decision problem.

Weights	w_1	w_2		w_k	
Criteria / Alternatives	*Criteria 1*	*Criteria 2*	…	*Criteria k*	TOTAL
Alternative 1	$Grade_{11}$; $Grade_{11}$ x w_1	$Grade_{12}$; $Grade_{12}$ x w_2	…	$Grade_{1k}$; $Grade_{1k}$ x w_k	Row Sum 1
….	….	….	…	….	….
….	….	….	…	….	….
Alternative m	$Grade_{m1}$; $Grade_{m1}$ x w_1	$Grade_{m2}$; $Grade_{m2}$ x w_2	…	$Grade_{mk}$; $Grade_{mk}$ x w_k	Row Sum m

Once the pay-off table is constructed for the decision problem, then the decision maker selects the alternative as the best alternative with the highest row sum total. In each row, the total represents the alternative in a whole.

2.2. Compositional Data Analysis

Compositional data are vectors of proportions describing the relative contributions of each of k categories to the whole. Compositional data occur frequently in official statistics (tax components in tax data, income components, wage components, expenditures, etc.), in environmental and technical sciences, and in many other fields. An observation $x = (x_1, \dots, x_D)^t$ is by definition a D-part composition if, and only if, all its components are strictly positive real numbers, and if all the relevant information is contained in the ratios between them [6]. As a consequence of this formal definition $(x_1, \dots, x_D)^t$ and its $c > 0$ multiple $(cx_1, \dots, cx_D)^t$ contain essentially the same information. One can thus define the simplex, which is the sample space of D-part compositions, as

$$x = (x_1, \dots, x_D)^t,\ \ x_i > 0,\ \ i = 1, \dots D, \sum_{i=1}^{D} x_i = \tau \qquad (3)$$

the constant τ represents the sum of the parts.

The application of standard statistical methods such as correlation analysis, principal component analysis or factor analysis directly to compositional data can thus lead to meaningless results [7-9]. This is also the case for imputation methods. Thus, this type of data needs to be transformed first with an appropriate transformation before any statistical analysis can be applied.

Filzmoser [9] introduce the idea of multivariate methods such as (robust) factor analysis, (robust) principal components to be used for analysis and procedures for outlier detection in compositional data. For compositional data, a special transformation is needed prior to applying multivariate data analysis tools. If this is not done, the correlation structure can be completely biased and results become useless. If robust multivariate techniques like principal component analysis (PCA) are used, the transformed data need to have full rank, and thus a transformation like the centered logratio transformation cannot be used. Principal component analysis (PCA) is one of the most important multivariate statistical methods. It is widely applied for data pre-processing and dimension reduction, and the resulting PCs are then used for plotting or for subsequent multivariate analyses [10]. Filzmoser [9] shows how Principal component analysis for compositional data is analysed. Once a principal components analyses of a data set is obtained, then it is not too diffucult to show graphical form of the results via Bi-Plot [11].

In this study, the compositional data analysis connection is obtained through each row total. Remember that the data set used in this study is designed as a pay-off table in which the pay-off table is a row sum number representing the alternative as a whole. It can be clearly seen from this that each alternative in the study is a D-part composition. Therefore, the techniques suitable for the compositional data can be applied to this rather quirky pay-off table.

3. Results

In Table 1., grades and weights of different phone companies obtained from the university students are given. Now, a compositional or rationality pay-off table can be constructed. For this purpose, each alternative value is multiplied by its corresponding weight. The resulting pay-off table is given in Table 3.

Table 3. Final pay-off table for phone companies

Weights	4.03	3.60	3.92	3.64	4.65	5.16	
	WI-FI	3G	CAMERA	MP3/RADIO	TOUCH SCREEN	PRICE	SUM
1.	16.1	14.4	3.92	7.28	9.3	10.3	61.34
2.	20.1	10.8	7.84	7.28	9.3	10.3	65.69
3.	16.1	14.4	3.92	10.9	13.9	15.4	74.79
4.	4.03	3.6	3.92	14.5	9.3	10.3	45.73
5.	8.06	7.2	3.92	14.5	4.65	20.6	59.03
6.	4.03	3.6	3.92	14.5	4.65	10.3	41.08
7.	4.03	3.6	3.92	14.5	4.65	15.4	46.24
8.	8.06	7.2	7.84	14.5	9.3	15.4	62.44
9.	4.03	3.6	3.92	18.2	4.65	20.6	55.04
10.	4.03	3.6	3.92	18.2	4.65	20.6	55.04
11.	4.03	3.6	3.92	18.2	4.65	20.6	55.04
12.	4.03	3.6	3.92	18.2	9.3	20.6	59.69
13.	8.06	3.6	7.84	18.2	9.3	15.4	62.48
14.	4.03	3.6	3.92	18.2	4.65	20.6	55.04
15.	16.1	14.4	15.6	18.2	9.3	15.4	89.18
16.	12.0	14.4	15.6	18.2	18.6	20.6	99.61
17.	20.1	18	15.6	18.2	13.9	30.9	116.9
18.	4.03	3.6	3.92	18.2	4.65	20.6	55.04
19.	16.1	18	19.6	18.2	23.2	15.4	110.6
20.	12.0	3.6	3.92	18.2	9.3	20.6	67.75
21.	8.06	3.6	3.92	18.2	18.6	20.6	73.02
22.	20.1	18	3.92	21.8	27.9	10.3	102.1
23.	20.1	3.6	3.92	21.8	9.3	15.4	74.29
24.	8.06	18	19.6	21.8	4.65	20.6	92.79

Now in order to choose the best alternative, the best phone company according to the students' preferences is given by the maximum value of the total column in Table 3. It can be seen that company 17 is the first company in the list with an overall score of 116.94 and the runner-up is company 19 with 110.65 points. The third company is company 22 with a score of 135.

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According to the students' opinions, company 6 gets the last place in this investigation. As a result, it looks like company 17 provides such a good selection of phones that the students may be satisfied with the strengths of the phones.

In the next step, robust principal components analysis, based on the approach suggested by Filzmoser [9], is applied to the pay-off table. The resulting Bi-Plot of the analysis is given in Figure 3.

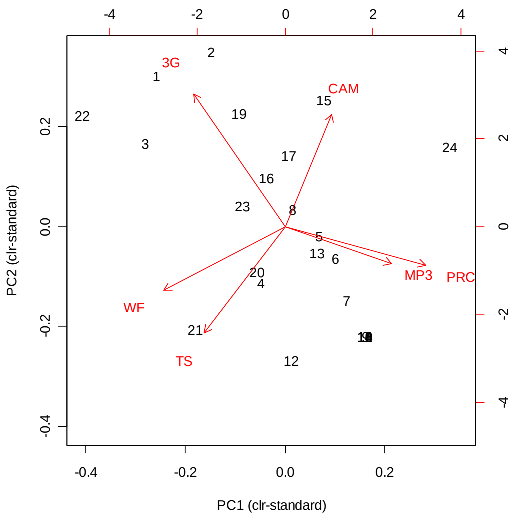

Figure 3. Bi-Plot from robust principal components analysis

From Figure 3., it can be seen that the company 24 might be an outlier. Also, the first phone company 17 is around the center of the figure. This shows that the phone company in the center meets different criteria for a phone purchase. The surprise of this figure is the company 8, which sits almost in the middle of the figure. This happens since the results for this company for each criterion is close to average values. The main result that can be obtained from the Biplot is that the three criteria are found to be very important in terms of first principal component since their arrow makes a tight angle with this component. These criteria are 3G, Wi-Fi and Price. As it can be seen, these criteria are the result of the new technologies in mobile networking. Therefore, it can be said that the students pay attention to the new technologies offered by the phone companies. Also when the size of the variable arrows is investigated in detail, it can be seen that the longest arrow is for 3G criteria. This shows the quality of this variable or criteria in our problem. The second longest arrow is drawn, almost equally, for the price and touch screen criteria. This is not a surprising result since in the original pay-off table, the weights given to these criteria are also high values.

4. Conclusion

Decision making is an important aspect for any company. While good decisions may increase the prosperity of a company, a bad decision can equally become a doomsday scenario for the company. One of the easiest techniques to be used for decision analysis is to analysis a pay-off table, but in pay-off tables, usually a decision maker puts numerical values obtained from profit or cost for the given state of the worlds and alternative pairs. If the emotional attachments are given to the decision, the usual pay-off table is of no use. In this study, the emotional attachments to the different alternatives are introduced in to the pay-off table via rationality approach. In this new setting of the pay-off table, the best alternative can be chosen as the highest score obtained from a row sum. The new structure of the pay-off table allows a decision maker to deal with the problem as a compositional data problem where a row represents the components of a given situation. Once the pay-off table is recognized as compositional data matrix, further statistical analysis of the problem becomes possible providing extra information such as the positions of different alternatives in terms of the criteria investigated. In this study, the phone company preferences of the university students are investigated and their preference tendencies are shown using robust principal components analysis. This extra knowledge for any alternative may influence the decision makers' decision where a decision maker may further investigate the improvements possible for a given alternative. Then, the analysis can be repeated for new findings until an optimal decision is made.

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