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Experimental And Theoretical Work On The Dynamic Characteristics Of A Continuous-Flow Agitated Tank Cooled By Jacked (III)

by

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Experimental And Theoretical Work On The Dynamic Characteristics Of A Continuous-Flow Agitated Tank Cooled By Jacked (III)

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Summary

The dynamic proporties of the tank was investigated with the step change given to the feed and cooling flow rates. The developed models were linearized and than solved with Laplace transform (1,2) and digital computer with the aid of matrices. Besides of them, related models without linearization were solved with the Runge-Kutta integration method. Theoretical results were compared with the experimental data.

INTRODUCTION

Alpbaz and Erdoğan (1,2,3) have investigated dynamic proporties of a well mixed heating vessel with a cooling jacked in which glycerin and water were mixed. They have developed the mathematical models for this system and solved linearized models with the aid of Laplace transform. They have changed the viscosity of the tank content and observed the effect of the viscosity change on the transient behaviour of the tank and cooling output temperature.

Melsa and Jones (4) outlined thirteen programs which perform the analysis, desing and simulation of a wide range of lineer control problems.

Özdemir (5) has work on the test-running of the programs which have given on the work of Melsa and Jones (4) formed a basis for the well-mixed heating vessel cooled by jacked. Study was carried out by utilizing these programs. He also has presented a set of analitical solutions performed to provide guidance in the analysis of the results obtained from the computer tests.

Franks (6) has given the digital computer solution of the mathematical models for well mixed tank.

Hiçşaşmaz (7) has investigated the dynamics of an agitated insulated vessel by pulse test method to find out the adequate pulse characteristics and the best manipulated controlled variable couples.

MATHEMATICAL MODEL

Necessary approximations and related mathematical models for continuous-flow agitated tank cooled by jacked were given in the previous work of Alpbaz and Erdoğan (1,2). The mathematical models for two different case were given below.

i. The case in which glycerin was given as an input.

The transient energy balance for tank,

$$Q + M_P C_P T^o{}_{P_i} = M_P C_P T_{Po} + UA \left[T_{Po} - \left(\frac{T_{co} + T^o{}_{ci}}{2} \right) \right] + M_v C_P \frac{dT_{Po}}{dt} \quad (1)$$

Coolent energy balance,

$$M_c C_c T^o{}_{ci} = M_c C_c T_{co} - UA \left[T_{Po} - \left(\frac{T_{co} + T^o{}_{ci}}{2} \right) \right] + M_j C_c \frac{dT_{co}}{dt} \quad (2)$$

Steady-state balance,

$$Q + M^o{}_P C_P T^o{}_{P_i} = M^o{}_P C_P T^o{}_{Po} + UA \left[T^o{}_{Po} - \left(\frac{T^o{}_{co} + T^o{}_{ci}}{2} \right) \right] \quad (3)$$

$$M^o{}_c C_c T^o{}_{ci} = M^o{}_c C_c T^o{}_{co} - UA \left[T^o{}_{Po} - \left(\frac{T^o{}_{co} + T^o{}_{ci}}{2} \right) \right] \quad (4)$$

ii. The case in which glycerin and water were given as an input.

The transient energy balance for tank,

$$Q + (M_P C_P)_G T^o{}_{P_i G} + (M_P C_P)_S T^o{}_{P_i S} = (M_P C_P)_{GL} T_{Po}$$

$$+ \mathrm{UA}\left[\mathrm{T_{Po}} - \left(\frac{\mathrm{T_{co}} + \mathrm{T^o_{ci}}}{2}\right)\right] + \mathrm{M_V C_P}\frac{\mathrm{dT_{Po}}}{\mathrm{dt}} \tag{5}$$

Steady-state balance,

$$\mathrm{Q} + (\mathrm{M_P C_P})^o{}_{\mathrm{G}}\mathrm{T^o_{PiG}} + (\mathrm{M_P C_P})_{\mathrm{s}}{}^o\mathrm{T^o_{Pis}} = (\mathrm{M_P C_P})^o{}_{\mathrm{GL}}\mathrm{T^o_{Po}}$$

$$+ \mathrm{UA}\left[\mathrm{T^o_{Po}} - \left(\frac{\mathrm{T^o_{co}} + \mathrm{T^o_{ci}}}{2}\right)\right] \tag{6}$$

Similar coolant energy balance (1) was used for this case.

THE SOLUTION OF MATHEMATICAL MODEL WITH AID OF MATRICES

The mathematical model was linearized in order to solve with Laplace Transform and matrices. The linearization method for this mathematical model was given in the previous work of the Alpbaz and Erdoğan (1,2). Here only linearized form of models are given.

For glycerin,

$$\frac{\mathrm{dT'_{Po}}}{\mathrm{dt}} = \left(\frac{\mathrm{T^o_{Pi}} - \mathrm{T^o_{Po}}}{\mathrm{M_V}}\right)\mathrm{M'_P} - \left(\frac{\mathrm{UA} + \mathrm{M_P C_P}}{\mathrm{M_V C_P}}\right)\mathrm{T'_{Po}} + \left(\frac{\mathrm{UA}}{2\mathrm{M_V C_P}}\right)\mathrm{T'_{co}} \tag{7}$$

For coolant

$$\frac{\mathrm{dT'_{co}}}{\mathrm{dt}} = \left(\frac{\mathrm{T^o_{ci}} - \mathrm{T^o_{co}}}{\mathrm{M_j C_c}}\right)\mathrm{M'_c} - \left(\frac{\mathrm{M_c C_c} + \frac{\mathrm{UA}}{2}}{\mathrm{M_j C_c}}\right)\mathrm{T'_{co}} + \left(\frac{\mathrm{UA}}{\mathrm{M_j C_c}}\right)\mathrm{T'_{Po}} \tag{8}$$

For glycerin and water,

$$\frac{\mathrm{dT'_{Po}}}{\mathrm{dt}} = \left(\frac{\mathrm{T^o_{PiG}}}{\mathrm{M_V C_P}}\right)(\mathrm{M_P C_P})'_{\mathrm{G}}$$

$$+ \left(\frac{\mathrm{T^o_{Pis}}}{\mathrm{M_V C_P}}\right)(\mathrm{M_P C_P})'_{\mathrm{s}} - \left[\frac{(\mathrm{M_P C_P})_{\mathrm{GL}} + \mathrm{UA}}{\mathrm{M_V C_P}}\right]\mathrm{T'_{Po}} \tag{9}$$

$$- \left(\frac{\mathrm{T^o_{Po}}}{\mathrm{M_V C_P}}\right)(\mathrm{M_P C_P})'_{\mathrm{GL}} + \left(\frac{\mathrm{UA}}{2\mathrm{M_V C_P}}\right)\mathrm{T'_{co}}$$

The state of a system can be described by the set of first order differential equations written in terms of the state variable, and than this set of simultaneous differential equations may be written in matrix form as follows.

$$\dot{\underline{X}}(t) = A\,\underline{X}(t)$$

This type of equation (10) is also known as ' the plant equation'.

The solution of this type of equation can be shown as below,

$$X(t) = e^{A(t)}\,X(0) \tag{11}$$

$$X(t) = \varnothing(t)\,X(0) \tag{12}$$

The development of the state equations for the case in which glycerin was given as an input was chosen as a typical example. State variable representation was obtained from the differential equations (7,8) in the linearized form, if step change was given on the feed flow rate,

$$\begin{bmatrix} \dot{T}'_{Po} \\ \dot{T}'_{co} \\ \dot{M}'_{P} \end{bmatrix} = \begin{bmatrix} -\dfrac{UA + M_P C_P}{M_V C_P} & \dfrac{UA}{2M_V C_P} & \dfrac{T^o_{Pi} - T^o_{Po}}{M_V} \\ \dfrac{'UA}{M_j C_c} & -\dfrac{M_c C_c + \dfrac{UA}{2}}{M_j Cc} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T'_{Po} \\ T'_{co} \\ M'_{p} \end{bmatrix} \tag{13}$$

For the solution of equation (13), initial conditions and step change, A, given to the feed flow rate are shown below,

$$\begin{bmatrix} T'_{Po}(0) \\ T'_{co}(0) \\ M'_{p}(0) \end{bmatrix} = \begin{bmatrix} 0.0 \\ 0.0 \\ A \end{bmatrix} \tag{14}$$

The Basmat (Basic Matrix) program used to solve the equation (13) can compute the determinant of A, detA, the inverse of A, A^{-1}, the characteristic polinomial det (sI-A) and eigen values of A, λ_i, as well as the state transition matrix $\Phi(t) = \exp(At)$ and the resolvent matrix $\Phi(s) = (sI-A)^{-1}$. The numerical values of equations (13, 14) are shown below. The related data was taken frcm Table. 1.

Table. 1. The operating conditions for glycerin input.

$C_1(\%)$	$C_l(\%)$	$M^o{}_p$	$M^o{}_c$	$T^o{}_{pi}$	T_{ci}	UA	M_p	M_c
95	95	8.61	17	54	17	5.94	8.61	89
95	95	5.23	17	66.5	15	6.40	5.23	73
95	95	5.23	21	65	16	6.78	8.61	85

$$\begin{bmatrix} \dot{T}'_{Po} \\ \dot{T}'_{co} \\ \dot{M}'_p \end{bmatrix} = \begin{bmatrix} -0.000530 & 0.0001394 & -0.000337 \\ 0.0012188 & -0.004062 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} T'_{Po} \\ T'_{co} \\ M'_p \end{bmatrix} \qquad (15)$$

$$\begin{bmatrix} T'_{Po}\ (0) \\ T'_{co}\ (0) \\ M'_P\ (0) \end{bmatrix} = \begin{bmatrix} 0.0 \\ 0.0 \\ 2.98 \end{bmatrix} \qquad (16)$$

The results obtained from the solution of equation (15, 16) is shown in Fig. 1.

THE SOLUTION OF MATHEMATICAL MODELS WITH RUNGE KUTTA INTEGRATION METHOD

The equations (1,2,5) described the unsteady-state energy balance for tank and coolant were solved with the 4'th order Runge-Kutta integration method. The related results are shown in all figures for comparing with other solution methods. The solution of related linearized equations (7,8,9) with Laplace transform were given in the work of Alpbaz and Erdoğan (1,2)

COMPARISON OF THE EXPERIMENTAL RESPONSE WITH THEORETICAL RESULTS

Experimental procedure, description of equipment, the physical properties of mixture of glycerin and water and solution of mathematical models were given in the previous work of the Alpbaz and Erdoğan (1,2) Comparisons have been done between experimental response and theoretical results obtained from Laplace transform (1,2), matrices and 4'th order Runge-Kutta integration methods. These comparisons have been done for two cases.

i- The case in which glycerin was given as an input

When the system was in the first steady-state with definite glycerin and water flow rate, a step change was given to the feed or cooling water flow rate, than the variation of output temperatures with time were calculated and compared with experimental results. The value of UA was changed with operating conditions. The related operating conditions and the values of UA are given in Table. 1.

When the system came to the steady-state having input conditions, ($M^o_P = 3.61$ g/sec, $T^o_{Pi} = 54.0$ °C, $M^o_c = 17.0$ g/sec, $T^o_{ci} = 17.0$ °C) a step change was given to the cooling flow rate ($M_C = 89.0$ g/sec). The time response of the output temperatures are shown in Fig. 1. The experimental temperature response are compared with calculated results.

When the feed flow rate having input conditions ($M^o_P = 5.22$ g/sec, $T^o_{Pi} = 64.0$ °C) was given to the tank with cooling water ($M^o_c = 17.0$ g/sec, $T^o_{ci} = 17.0$ g/sec °C). On the first steady-state conditons, a step change was given to the feed flow rate ($M_P = 8.20$ g/sec) and comparison between theoretical and experiment results of the time response for output temperatures are shown in Fig. 2.

When the system was in the steady-state having input conditions ($M^o_P = 5.23$ g/sec, $T^o_{Pi} = 71.\ 5$°C, $M°_C = 21.0$ g/sec, $T^o_{ci} = 17.0$ °C) shown in Table. 1. a step change was given to the feed and cooling flow rate ($M_P = 8.62$ g/sec, $M_C = 85.0$ g/sec). Similar comparisons are shown in Fig. 3.

ii- The case in which water and glycerin were given as an input For this case the operating conditions, the values of UA and concentra tion of glycerin with different operating conditions are given in Table. 2. Similar experimental and theoretical work has been done in this part of the work. Comparison between theoretical calculations and experimental results are shown in Figs. 4-6 and related input conditions are given in Table. 2.

Table. 2. The operating conditions for glycerin and water input.

C_l(%)	C_o(%)	M^o_{PG}	M^o_{PS}	M^o_c	T_{PiG}	T_{PiS}	T_{ci}	UA	M_{PG}	M_c
80	95	8.54	1.58	17	50.5	57	29.5	11.20	8.54	72.5
60	95	8.61	5.06	17	64	54	19	13.14	8.61	69.7
25	35	10.52	4.16	17	57	57	22	19.14	10.52	69.7

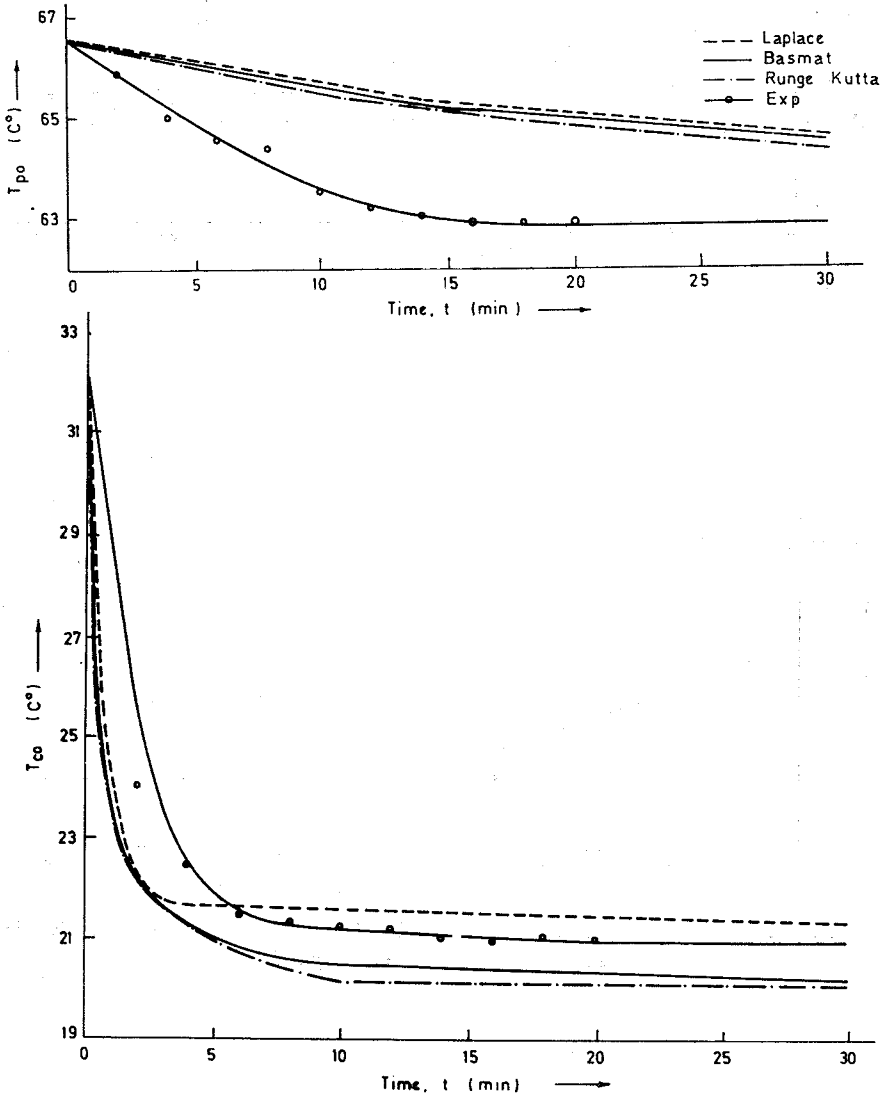
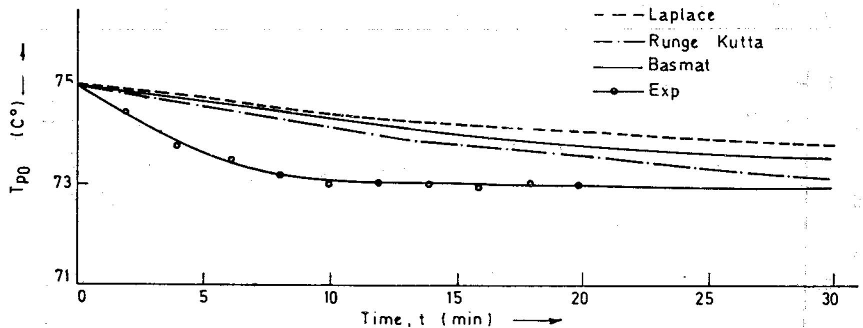


Figure. 1. The transient response of the output and coolant temperature (C_l= 95 % Glycerin, M^o_c= 17 g/sec, M_c= 89 g/sec)



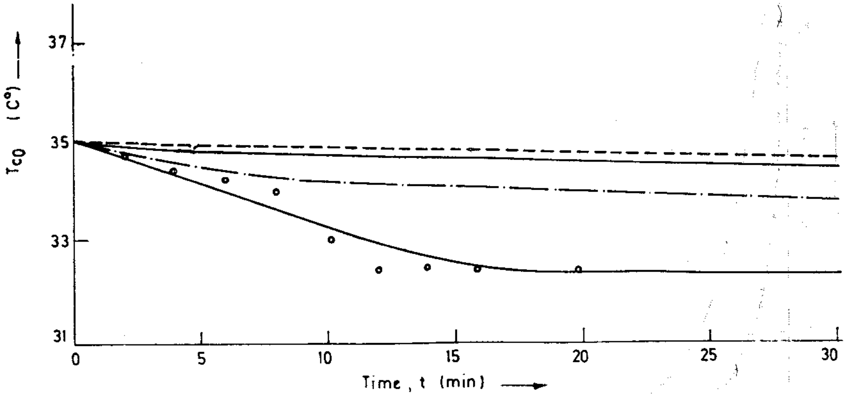


Figure. 2. The transient response of the output and coolant temperature ($C_l = 95$ % Glycerin $M^o{}_c = 5.22$ g/sec, $M_p = 8.20$ g/sec)

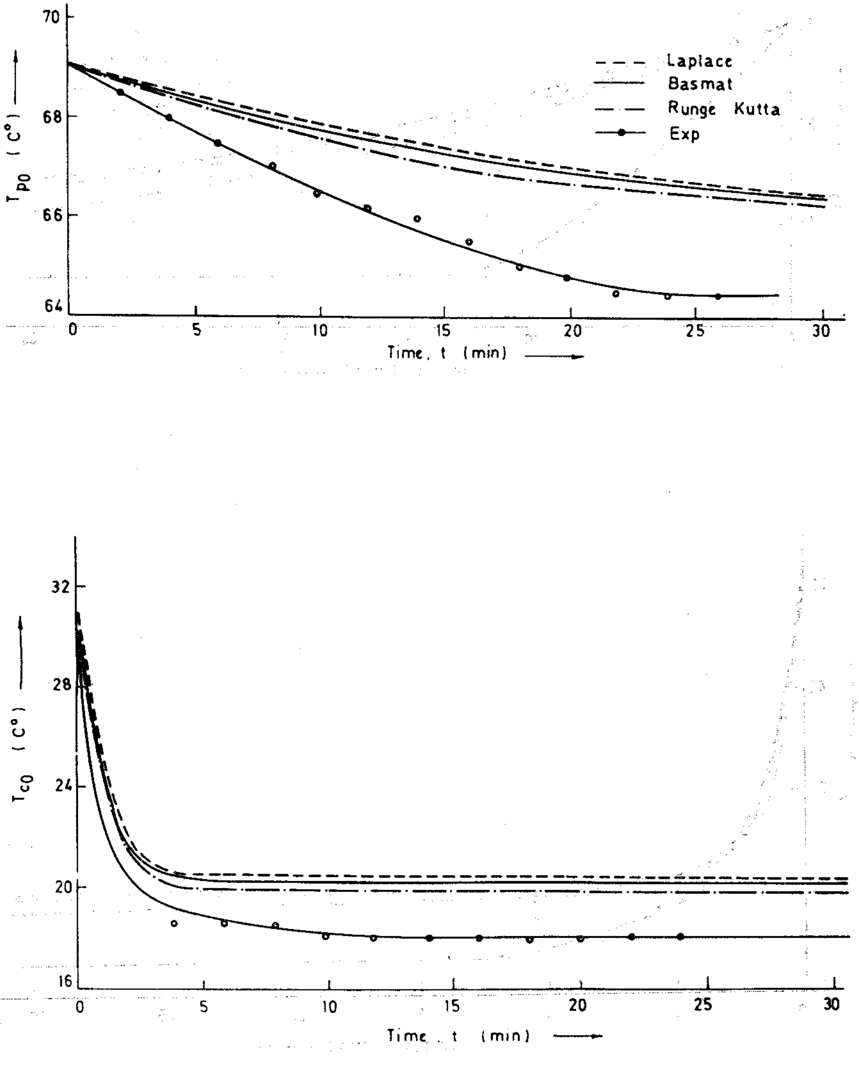


Figure. 3. The transient response of the output and coolant temperature (C_1= 95 % Glycerin, M^o_p= 5.22 g/sec, M_p= 8.61 g/sec, M^o_c= 21 g/sec M_c= 85 g/sec)

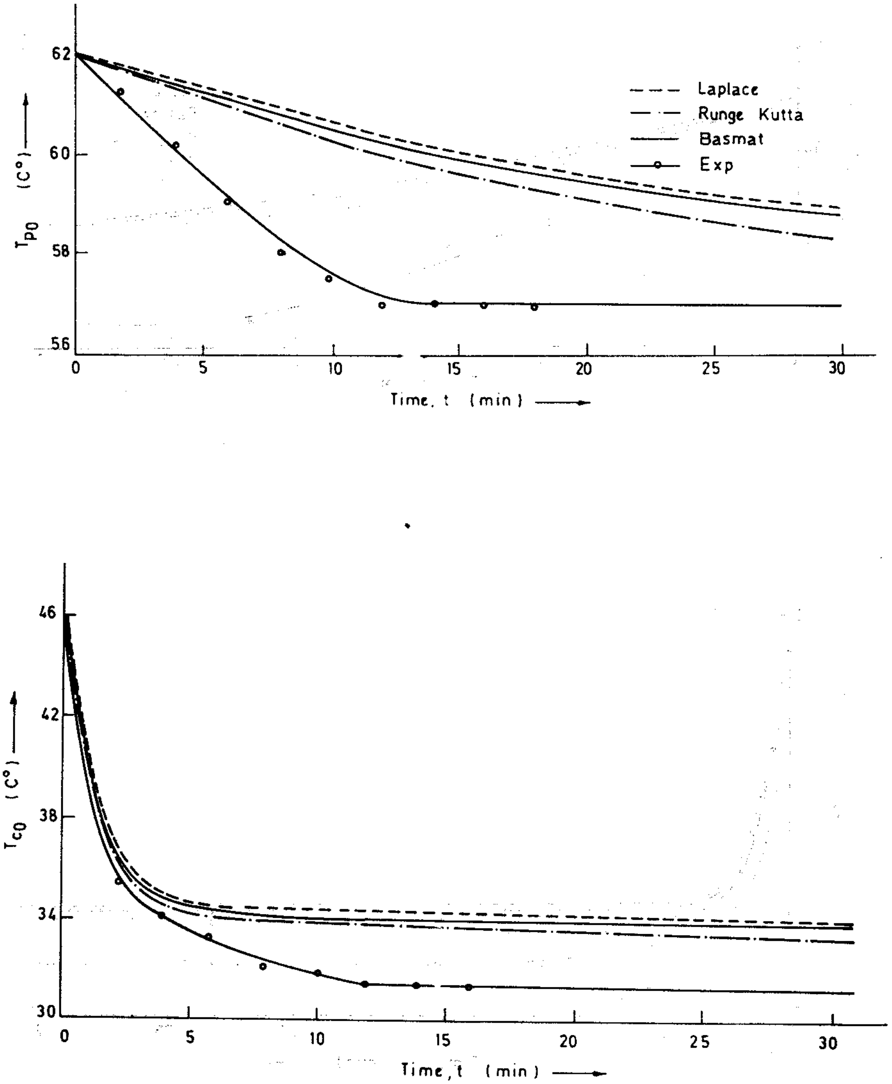
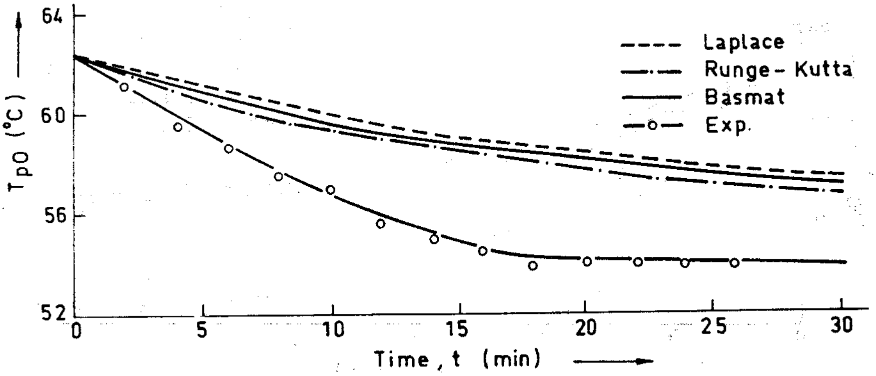


Figure. 4. The transient response of the output and coolant temperature (C_1= 80 % Glycerin, M^o_c= 17 g/sec, M_c= 72.5 g/sec)



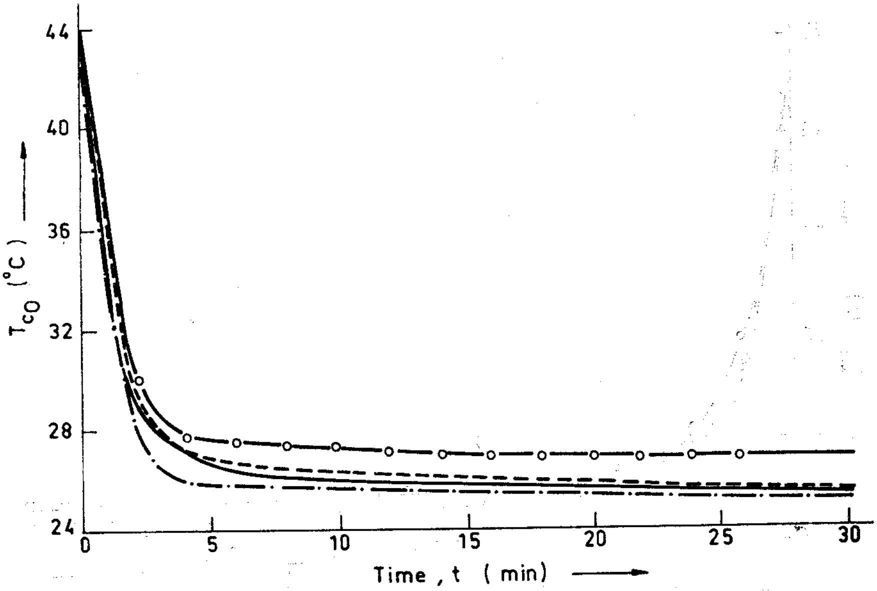
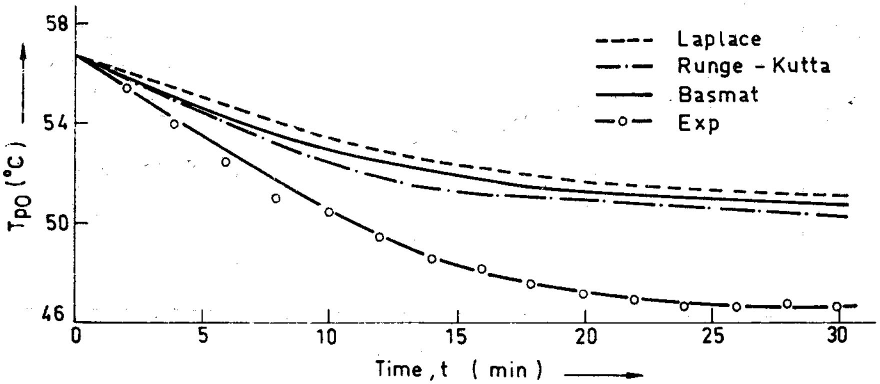


Figure. 5. The transient response of the output and coolant temperature (C_l= 80 % Glycerin, M^o_c= 17 g/sec, M_c= 69.7 g/sec)



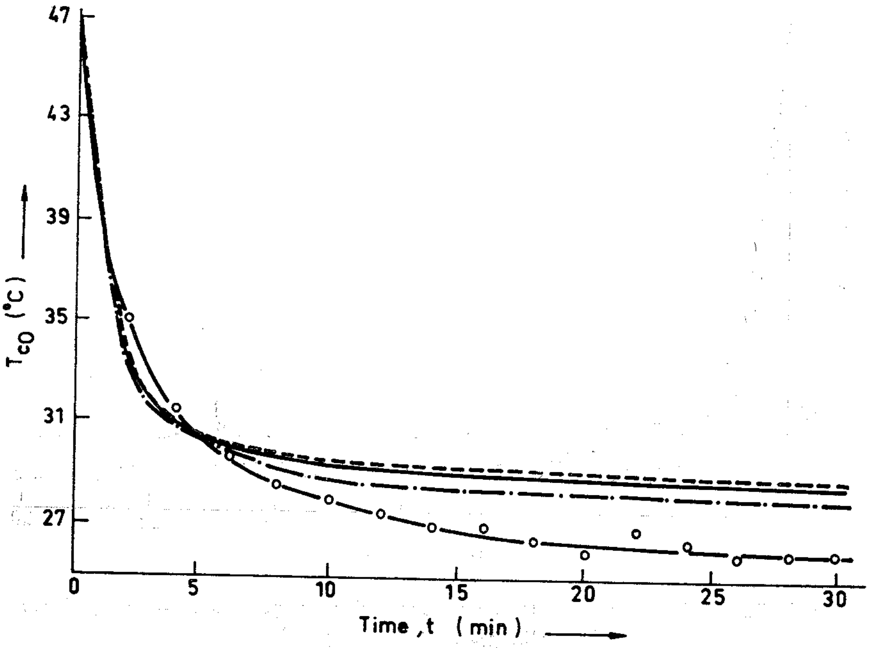


Figure. 6. The transient response of the output and coolant temperature ($C_1 = 25$ % Glycerin, $M^o_c = 17$ g/sec, $M_c = 69.7$ g/sec)

NOMENCLATURE

A	Heat transfer surface (cm^2)
$\underline{A}$	Plant matrix
C_c	Specific heat of coolant (cal/g°C)
C_P	Specific heat of tank content (cal/g°C)
M_c	Coolant mass flow rate (g/sec)
M_P	Feed mass flow rate (g/sec)
M_{PG}	Feed mass flow rate for glycerin (g/sec)
M_{PS}	Feed mass flow rate for water (g/sec)
M_V	Mass hold up in the tank (g)
M_j	Mass hold up in the jacket (g)
s	Laplace operator
Q	Heat output from immertion heaters (Cal/sec)
T_{PiG}	Feed temperature for glycerin (°C)
T_{Pis}	Feed temperature for water (°C)
T_{P_i}	Feed temperature (°C)
T_{Po}	Output temperature (°C)
T_{co}	Output coolant temperature (°C)
t	Time
U	Overall heat transfer coefficient (Cal/sec °C)
X	System state variables
$\bar{\rho}$	Density (g/cm^3)
U	Viscosity (g/cm sec)

Özet

Tank dinamik özellikleri, besleme ve soğutma suyuna kademe değişimi verilmesi ile incelenmiştir. Geliştirilen modeller doğrusallaştırılarak, Laplace dönüşümü (1,2) ve sayısal bilgi sayarda matris kullanımı ile çözülmüşlerdir. Bunun yanı sıra modeller, doğrusallaştırılma yapılmadan Runge-Kutta yöntemi ile bilgisayarda çözülmüşlerdir. Teorik sonuçlar deneysel veriler ile karşılaştırılmıştır.

REFERENCES

1– **Erdoğan Yüceer,** S. Ph. D Thesis, Ankara University (1984)

2– **Alpbaz, M and Erdoğan,** S Comm. Fac. Sci. Ankara B 1984 inpress

3– **Alpbaz, M and Erdoğan,** S Comm. Fac. Sci. Ankara B 1984 inpress

4– **Melsa. J.L and Jones, S.K** "Computer programs for computational asistance in the study of lineer control theory" McGraw-Hill Co. (1973)

5– **Özdemir, E.** MSc Thesis, Aston University England (1978)

6– **Franks, R.G.** "Modelling and simulation in chemical engineering" Willey Interscience (1972)

7– **Hiçşaşmaz, Z.** MSc. Thesis M.E.T.U., (1982)