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RESEARCH ARTICLE / ARAŞTIRMA MAKALESİ

Using Chaos Theory to Determine Average Prediction Times of Different Meteorological Variables: A Case Study in Sivas

Farklı Meteorolojik Değişkenlerin Ortalama Öngörü Sürelerini Belirlemek İçin Kaos Teorisinin Kullanımı: Sivas Örneği

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Abstract
Processes in the atmosphere can be described by nonlinear approaches since they depend on a large number of independent variables. Even a slight change in initial conditions can cause unpredictable results. Therefore, long-term prediction is not possible to obtain. This is usually called "sensitive dependence on initial conditions". In this study, average prediction times were determined for different meteorological variables by using a nonlinear approach. Daily values of relative humidity, air temperature, and wind speed in Sivas for the period 2006-2010 were used. To implement the method, the first step is to reconstruct the phase space. Phase space has two embedding parameters, namely time delay and embedding dimension. Mutual Information Function (MIF) can be used to determine the optimal value of the time delay. It considers both linear and nonlinear dependencies in a time series. To define phase space, embedding dimension, which is the number of state variables that define the dynamics of a system, must be identified correctly. The algorithm to describe the dimension is called False Nearest Neighbors (FNN). In the study, average prediction times of variables were calculated by using maximum Lyapunov exponents. Average prediction times for relative humidity, temperature, and wind speed were determined as 6.2, 5.8, and 2.5 days, respectively. In addition, it is found that the sensitivity of measurements increases the prediction time. For relative humidity, the average prediction time can have a 50% increase with 10 times increase of sensitivity.
Keywords: Chaos, Lyapunov exponent, Meteorology, Phase space, Prediction

Öz
Atmosferdeki süreçler çok sayıda bağımsız değişkene bağlı oldukları için doğrusal olmayan yaklaşımlarla tanımlanabilir. Başlangıç koşullarındaki küçük bir değişiklik bile, öngörülemeyen sonuçlara neden olabilir. Bu nedenle, uzun vadeli öngörü elde etmek mümkün değildir. Buna genellikle "başlangıç koşullarına hassas bağımlılık" denir. Bu çalışmada, doğrusal olmayan yaklaşım kullanılarak farklı meteorolojik değişkenler için ortalama öngörü süreleri belirlenmiştir. Çalışmada Sivas ilinde 2006-2010 dönemine ait günlük bağıl nem, hava sıcaklığı ve rüzgar hızı verileri kullanılmıştır. Yöntemi uygulamak için ilk adım, faz uzayının yeniden oluşturulmasıdır. Faz uzayının zaman gecikmesi ve embedding (gömme) boyutu olmak üzere iki gömme parametresi vardır. Zaman gecikmesinin optimum değerini belirlenmek için Karşılıklı Bilgi Fonksiyonu (MIF) kullanılabilir. MIF, bir zaman serisinde doğrusal ve doğrusal olmayan bağımlılıkları hesaba katar. Faz uzayını tanımlamak için, bir sistemin dinamiklerini tanımlayan durum değişkenlerinin sayısı olan gömme boyutu doğru bir şekilde tanımlanmalıdır. Bu boyutu tanımlayan algoritmaya Yanlış En Yakın Komşular (FNN) denir. Çalışmada maksimum Lyapunov üstelleri kullanılarak meteorolojik değişkenlerin ortalama öngörü süreleri hesaplanmıştır. Bağıl nem, sıcaklık ve rüzgar hızı için ortalama öngörü süreleri sırasıyla 6.2, 5.8 ve 2.5 gün olarak belirlenmiştir. Ayrıca ölçüm hassasiyetinin, öngörü süresini arttırdığı tespit edilmiştir. Bağıl nem için ortalama öngörü süresi, ölçüm hassasiyetinin 10 kat artmasıyla, %50 artışa sahip olabilmektedir.
Anahtar kelimeler: Kaos, Lyapunov üsteli, Meteoroloji, Faz uzayı, Öngörü

I. INTRODUCTION

Prediction of atmospheric variables is an important issue all over the world. There are several approaches in the literature for the prediction of meteorological parameters. However, most of the approaches are linear. On the other hand, as a generally known fact that atmospheric processes have complexity and are controlled by many different mechanisms. The studies of chaotic analysis in atmospheric sciences are limited because of their abovementioned complexity. Some of the nonlinear approaches that are used in meteorology are artificial neural networks, support vector regression algorithms, and chaos theory. Among these approaches, chaos theory seems to be more practical than the other methods in terms of being easily applicable [1].

Prediction of meteorological parameters is crucial for a great number of different areas. For example, prediction of air temperature is important for climate change, agricultural purposes, epidemic diseases control, etc. [2-5]. In addition, the prediction of wind speed is substantial for energy production in the renewable energy sector [6, 7].

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Chaos theory was applied to a wide range of meteorological and hydrological variables such as streamflow [1, 8, 9], wind speed [10], evapotranspiration [11, 12], and temperature [13]. Most of the studies focused on fractal dimension and Lyapunov exponents which are the most important criteria of chaotic properties. The time series of the meteorological variables may be produced by a chaotic process since atmospheric variables can be controlled by many independent variables. If trajectories of dynamical systems merge in a specific subspace independent with initial conditions, it is called “attractor”. If it is not possible to make long-term predictions, it can be said that there is a dependency on initial conditions [14, 15].

Although chaos is defined as complexity in the literature, chaotic systems are not complex systems. Complex systems are those whose behavior cannot be controlled under any circumstances because they are not dependent on initial conditions. However, chaotic systems are systems that can be analyzed and controlled. The most important property of chaotic systems is their sensitivity to initial conditions. The chaotic behavior of a system is not only dependent on external factors but also closely related to the system's own internal dynamics and initial conditions. Chaotic systems show irregular behavior in the time dimension. If the initial state of any deterministic system and its equations are known, the next behavior of the system can be defined. In chaotic systems, it is crucial to know the initial values with accurate precision to fully determine the development of any system over time. Since the chaotic systems are nonlinear, the error will increase exponentially over time [16].

This study aimed to define the average prediction times of three different meteorological variables (relative humidity, temperature, and wind speed) by using a chaotic approach. Firstly, the phase space was reconstructed for each variable by using their observed data. Then, the average prediction time of selected meteorological variables was determined using the Lyapunov exponent which is one of the most important chaos criteria. Finally, the possible changes in prediction sensitivity with changing measurement sensitivity were also investigated.

II. MATERIAL AND METHOD

2.1. Data and Study Area

Before giving the properties of the study area, it is useful to explain the general characteristics of the climate of Turkey and the Central Anatolian Region. Turkey is affected by a subtropical climate regime that is called as “Mediterranean”. In summer, there are two dominant air masses (maritime polar (mP) and continental polar (cP)) in the country. On the other hand, northern and eastern parts of the country are affected by the Siberian high-pressure system in winter. In the Central Anatolian Region, continental climate character is observed because of the topographic effect [17, 18].

In the study, daily relative humidity, temperature, and wind speed data of Sivas station obtained from the Turkish State Meteorological Service (MGM in Turkish acronym) were used. The altitude of the station is 1294 m asl. The observation period was taken to be 2006-2010. The coordinates of the meteorological station in Sivas are 39°44'37.3"N and 37°00'7.2"E (Figure 1). Figure 2 shows the time series of the variables used in the study.

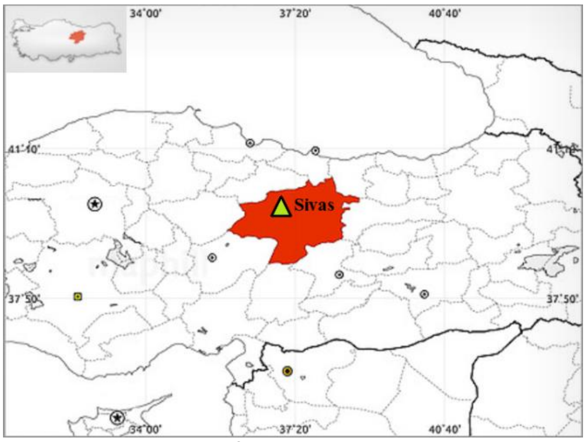

Figure 1. Location of the meteorological station used in the study

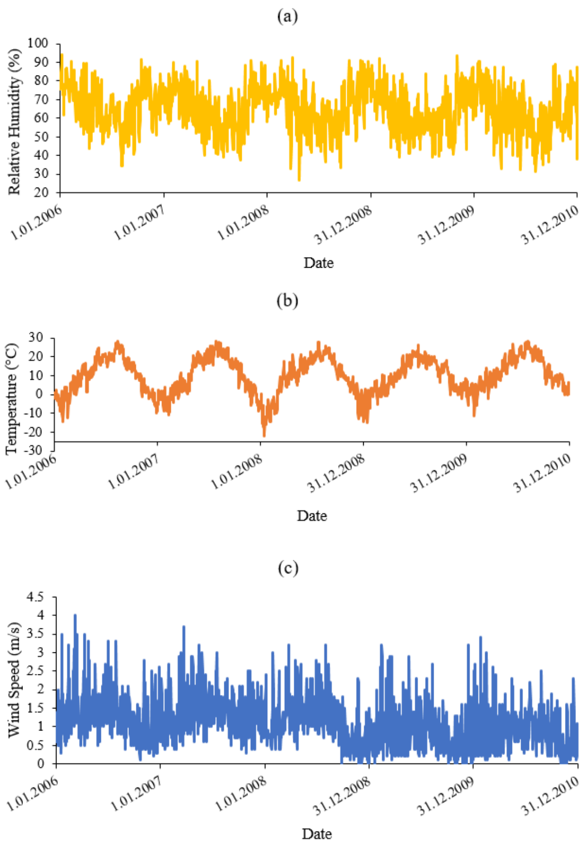

Figure 2. Time series of daily (a) relative humidity, (b) temperature, and (c) wind speed for 2006-2010

2.2. Methodology

2.2.1. Explanation of Lyapunov exponents

The reason why chaotic systems exhibit aperiodic dynamics is that each phase space curve has different exponential growth rates at almost the same initial conditions. This is called "sensitive dependence on initial conditions". The Lyapunov exponent, also called stability exponent, is a measure of sensitivity to initial conditions. It is defined as the average of the local degrees of separation of neighboring curves in the phase space [19]. Figure 3 represents the trajectories of two attractors starting with different initial conditions.

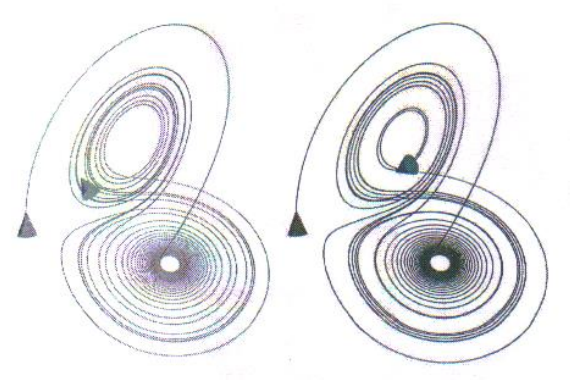

Figure 3. Divergence of two orbits that have different initial points [20]

Although the difference between initial conditions is too little, an apparent difference between the final forms of the attractor can be easily seen [21].

Figure 4 shows the development of phase space for two weather predictions in a specific period. For initial conditions given in Figure 4(a), the predictions were stable and the reliability was very high. According to Figure 4(b), it can be said that a slight difference in initial conditions leads to larger differences in prediction. Predictions spread very wide areas in phase space. Thus, the atmosphere behaves chaotically and in this condition, it is hard to make a reliable prediction [22].

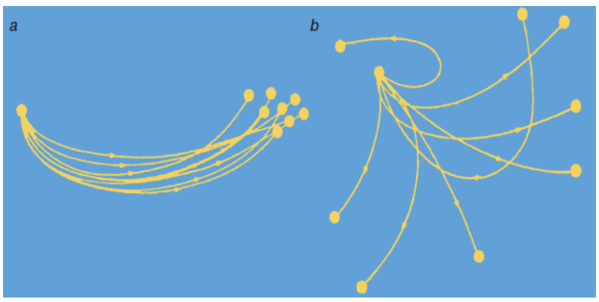

Figure 4. Time evolution of phase space for different weather predictions

Lyapunov exponents present the amount of convergence or divergence of nearby trajectories). Positive Lyapunov exponents exhibit divergence of the trajectories and chaotic process. In addition, the bigger Lyapunov exponent is an indicator of the shorter prediction time [23].

Lyapunov exponent represents the divergence degree of two nearby points in trajectory. The distance between two points is d_0 in t=0, and change of distance with time can be expressed by using Equation (1):

$$d_0(t) = d_0 e^{\lambda t} \tag{1}$$

where λ parameter is named as Lyapunov exponent [24]. If any Lyapunov exponent of a system is greater than zero, this system is accepted to be chaotic. It was said that the higher positive Lyapunov exponent leads to lower future prediction reliability.

2.2.2. Reconstruction of phase space

Phase space reconstruction is a crucial process in time series prediction. Reconstructing a phase space of any system from observed data is a common procedure in nonlinear time series analysis [25]. Attractors of any dynamical system can be reconstructed by using the measured data. In order to reconstruct the phase space from any observed data, attractor information should be estimated from a given time series [14]. The time delay (τ) and embedding dimension (m) are the two key parameters in the above-mentioned reconstruction process [26].

It is important to define the time delay accurately. If the time delay is chosen smaller than the typical period of the time series, x[n] and x[n+τ] values are almost equal and the system does not have enough time to change its dynamics. On the other hand, if the time delay is defined as bigger, the difference between x[n] and x[n+τ]. Thus, this leads to randomly distributed situations in the phase space. In order to obtain the optimum time delay, the mutual information function which was proposed by Fraser and Swinney [27] was applied to time series. Mutual information is defined as a quantity that measures the nonlinear interdependence between two random variables [10].

After obtaining the optimum time delay, the embedding dimension can be defined. The embedding dimension is the number of state variables that define the dynamics of any system. The most common method to obtain embedding dimension is the False Nearest Neighbors (FNN) approach. If any attractor is embedded in a phase space that has a lower dimension, some no-neighboring points of any single point seem like a neighbor. Besides, since the dimension is increasing, the percentage of false neighbors decreases. By plotting percentages of false neighbors and embedding dimensions, the minimum percentage value is defined as the optimum embedding dimension [28].

2.2.3. Obtaining average prediction time by using Lyapunov exponent

As mentioned before, at least one of Lyapunov exponents must be greater than zero for a chaotic system. It will be hard to make a forward prediction while the Lyapunov exponent is getting bigger. The average prediction time of any meteorological variable can be given as Equation (2) below [29]:

$$T = 1/\lambda_{max} \ln(L/\varepsilon) \quad (2)$$

where T is the prediction time, L is the attractor dimension, ε is the measurement sensitivity. In the measurement system of our country, sensitivity can be accepted as 0.1. The dimension of an attractor is derived from the time series itself. Firstly, the original time series (x-axis) and lagged time series (y-axis) are plotted. The time lag is found by using the mutual information function algorithm. After plotting the original time series vs. lagged time series, the y-axis gives the attractor dimension of the parameter.

The embedding parameters, namely time delay (τ) and embedding dimension (m) were computed by using TISEAN (TIme SEries ANalysis) program [30]. Maximum Lyapunov exponents for each variable were calculated by using Nonlinear Dynamics Toolbox (NDT) program [31].

III. RESULTS AND DISCUSSION

Table 1 represents the required parameters found in the study to define average prediction time. It is seen in Table 1 that embedding dimensions for relative humidity, temperature, and wind speed were obtained as 8, 12, and 10, respectively. Time delays of these three variables were attained as 6, 3, and 3, as well.

Table 1. Parameters required to calculate average prediction time

Variable	m (Embedding dimension)	τ (Time delay)	λ_{max} (Maximum value of Lyapunov exponent)	L (Attractor dimension)
Relative Humidity	8	6	1.03	62
Air Temperature	12	3	1.04	43
Wind Speed	10	3	1.45	3.5

Figure 5 illustrates the mutual information function of wind speed values. The x-value of the first minimum of the mutual information function is accepted as the optimum time delay. Thus, it is clearly said that the time delay of wind speed can be accepted as 3.

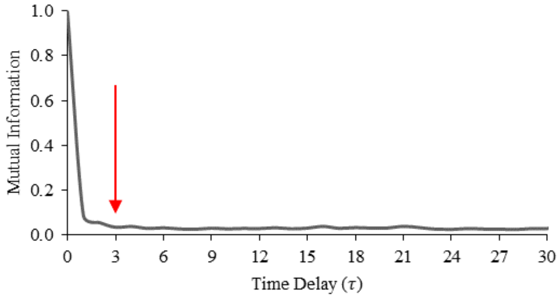

Figure 5. Mutual information function of wind speed values

Figure 6 shows the optimum embedding dimension of relative humidity values. The x-value of the first minimum of false nearest neighbor percentage can be accepted as embedding dimension. Therefore, the embedding dimension for relative humidity can be accepted as 8, as can be seen in Figure 6. After finding embedding parameters, maximum Lyapunov exponents for relative humidity, air temperature, and wind speed were 1.03, 1.04, and 1.45, respectively.

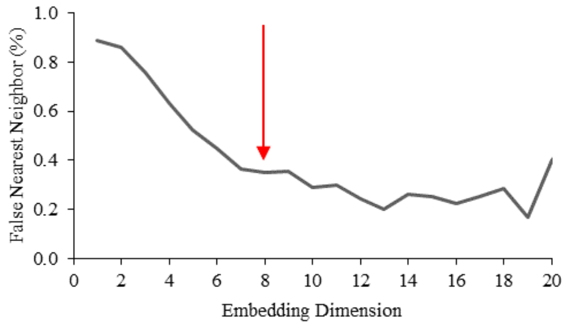

Figure 6. Embedding dimension vs false nearest neighbor for relative humidity values

Figure 7 represents the Lyapunov spectrums for air temperature. While looking at Figure 7, it is easily specified that the maximum Lyapunov value is fixed at a certain value. For air temperature, this value was obtained as 1.04.

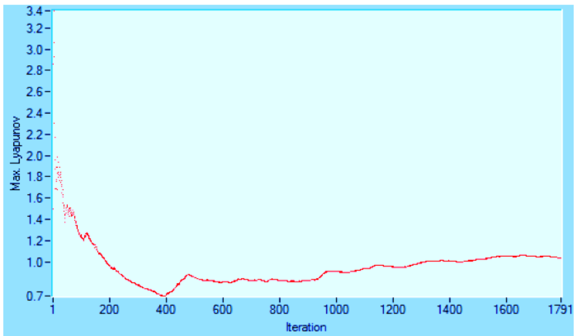

Figure 7. Maximum Lyapunov exponent for air temperature values

Using Equation (2), it is possible to obtain the average prediction time for any variables. Table 2 represents the average prediction time for all variables. By using the findings above, the average prediction time for relative humidity, temperature, and wind speed was found as 6.2, 5.8, and 2.5 days, as well.

According to Table 2, relative humidity has the longest prediction time with nearly one week. A good relationship between maximum Lyapunov exponent value and average prediction time has been observed. While the Lyapunov exponent value is increasing, the average prediction time is decreasing, as well.

Table 2. Average prediction time for all variables

Variable	λ_{max}	L	T
Relative Humidity	1.03	62	6.2 days
Air Temperature	1.04	43	5.8 days
Wind Speed	1.45	3.5	2.5 days

Finally, the average prediction time for different sensitivity was calculated while changing the sensitivity step by step. Figure 8 shows the average prediction time in relative humidity for different sensitivity values. As the sensitivity of the measurement system decrease, long-term prediction becomes more impossible.

The average prediction time is 6.2 days in 0.1 sensitivity for relative humidity, although that is 4.01 days in a 1.0 sensitivity value.

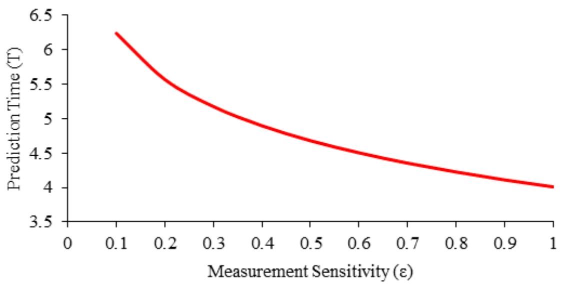

Figure 8. An example of average prediction time for different measurement sensitivity (relative humidity)

IV. CONCLUSION

As a generally known fact, long-term weather prediction is not possible because of the uncertainties in meteorological variables. Most of the variables in the atmosphere are controlled by a lot of independent variables. Thus, it is not easy to make a long-term prediction. A small difference in initial conditions of a variable produces higher differences in increasing time. This property is one of the most obvious criteria for chaotic behavior. Lyapunov exponent is a key element to determine predictability in any variable. The results obtained from the study will contribute to decision-makers for planning and managing data-based sectors such as energy production, weather forecast, agricultural areas, etc. The findings obtained from this study might be helpful for a long-term prediction of any meteorologic variable. The magnitude of the positive Lyapunov exponent can be a good indicator for prediction time length. Besides, other factors such as measurement sensitivity should not be ignored for making an accurate prediction, as well. Finally, detailed studies should be conducted with more stations and more meteorological variables since the atmosphere has chaotic properties. As a future study, it is planned to apply the given method for other stations and other meteorological variables to obtain regional results all over the country.

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