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Araştırma Makalesi - Research Article

Dynamic Stability of Cracked Composite Plates

Çatlaklı Kompozit Plakaların Dinamik Kararlılığı

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ABSTRACT

In this study, the dynamic instability region of cracked composite plates subjected to periodic axial loading has been investigated numerically by using the finite element method. A composite plate based on classical lamination theory is modeled by using the finite element method. It is assumed that the boundary condition of the composite plate is fixed on one side and the other sides are free. The composite plate is divided into square plate elements, each having four nodes. Each node has one translational and two rotational degrees of freedom, therefore, the square plates have twelve degrees of freedom. Mathieu-Hill type motion equation, which is used to calculate the dynamic instability region of the plate is created by using energy expressions obtained from the Lagrange equation. The developed MATLAB finite element code is used to examine the effect of crack on dynamic instability region for composite plates.

Keywords- Dynamic Instability Region, Cracked Composite Plates, Finite Element Method

ÖZ

Bu çalışmada, sonlu elemanlar yöntemi kullanılarak periyodik eksenel yüklemeye maruz kalan çatlaklı kompozit plakaların dinamik kararsızlık bölgesi sayısal olarak incelenmiştir. Klasik laminasyon teorisine dayanan kompozit plaka, sonlu elemanlar yöntemi kullanılarak modellenmiştir. Kompozit plakanın sınır şartlarının bir taraftan sabitlendiği, diğer tarafların serbest olduğu varsayılmaktadır. Kompozit plaka, her biri dört düğüme sahip kare plaka elemanlarına bölünmüştür. Her düğümün bir vektörel ve iki dönme serbestlik derecesine sahiptir, bu yüzden bu kare plaka elemanlar on iki serbestlik derecesine sahiptir. Plakanın dinamik kararsızlık bölgesi hesaplamak için kullanılan Mathieu-Hill tipi hareket denklemi, Lagrange denkleminden elde edilen enerji ifadeleri kullanılarak oluşturulmuştur. Geliştirilen MATLAB sonlu elemanlar kodu kullanılarak çatlağın kompozit plakalar için dinamik kararsızlık bölgesi üzerindeki etkisi incelenmiştir.

Anahtar Kelimeler- Dinamik Kararsızlık Bölgesi, Çatlaklı Kompozit Plakalar, Sonlu Elemanlar Metodu

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I. INTRODUCTION

In today's world, reliable structures with high performance, toughness, strength, being lightweight, together with low-cost sustainable products and processes are some of the requisitions. Therefore, in many branches of engineering, composite plates are widely used such as the automotive, aerospace, spacecraft industries and solar panels. Composite plates are subjected to various loading types such as static and dynamic loads which cause loss of static and dynamic stability. Furthermore, cracks can occur on composite plates due to various reasons such as mechanical damage and fatigue. Many studies about composite material, crack, and dynamic stability features of plates have been carried out by other researchers. Some of them can be summarized as follows:

Chen and Yang [1] investigated the effects of various parameters on the dynamic instability region of laminated composite plates owing to periodic loads by using the finite element method. Reddy [2] examined the mechanics of the composite plate, namely the bending, buckling and vibration of composite plates were analyzed. Voyiadjis and Kattan [3] studied on mechanics of composite materials by using the MATLAB program. MATLAB code was written to create the composite materials in this study. Daniel and Ishai [4] studied the classical lamination theory. Classical lamination theory has been expanded to include various applications to sandwich plates. Aggarwal [5] investigated the method for determining the optimal ply angle distributions in variable stiffness laminates by using the finite element method.

Krawczuk and Ostachowicz [6] studied a method of creating internal, non-propagating and open crack on the stiffness matrix of a finite plate element. A cracked stiffness matrix was created by using the Castigliano theory. Avadutala [7] studied vibrational responses from a specimen which different types of fractures in it at different locations on the composite plate. Khoei [8] studied on extended finite element method theory. In the study, special functions were added to the finite element approximation using the partition of unity in the extended finite element method.

Timoshenko and Gere [9] investigated the theory of elastic stability on elastic systems. Bolotin [10] studied on the dynamic stability of elastic systems. This study has examined the effects of various parameters on dynamic instability region by applying various methods. Dey and Singha [11] examined Dynamic stability analysis of composite skew plates subjected to periodic in-plane load. In this study, the dynamic stability characteristics of simply supported laminated composite skew plates subjected to a periodic in-plane load are investigated using the finite element approach. Stability analysis of a cantilever composite beam on elastic supports have investigated by Ozturk and Sabuncu [12]. Goren Kiral et al. [13] examined the dynamic stability of composite cantilever beams subjected to periodic axial loading with delaminations at pre-set locations by using finite element method. Nonlinear vibration and dynamic stability analysis of composite plates were investigated by using the shear deformable finite element method by Singha and Duripa [14]. Hutt [15] studied on dynamic instability region of plates by using the finite element method. This study investigated the development of the governing set of matrix differential equations and the procedure for the development of the elemental matrices. Srivastava et al. [16] investigated dynamic instability of stiffened plates subjected to non-uniform harmonic in-plane edge loading Assessment of dynamic instability of laminated composite-sandwich plates have examined by Sahoo and Singh [17]. Radu and Chattopadhyay [18] investigated the dynamic stability of composite plates with delaminations. The results of natural frequencies and critical buckling loads were calculated and compared with NASTRAN results. The effect of edge crack and crack growth on the stability was examined in this study. Abramovich [19] studied on vibrations and stability of composite plate structures. The natural frequencies, the critical buckling loads, dynamic stability of composite structures were examined in this study. Dynamic stability of rotating cantilever composite thin walled twisted plate with initial geometric imperfection under in-plane load were investigated by Gu et al. [20]. Sayer [21] investigated dynamic stabiliy of cracked composite plates by using the finite element method In this paper, the effects of ply angles, static and dynamic load parameters, and crack location on the dynamic instability region of cracked composite plates subjected to periodic axial loading have been investigated by using the finite element method. Through this study, composite plate, crack and dynamic stability issues, which were examined separately in the literature, were combined and all of these issues were examined together.

II. THE CRACKED COMPOSITE PLATE MODELS

In this study, the plate is divided into square elements using the finite element method. These square elements are connected to each other by nodes. Degrees of freedom for these square elements are the displacements at the nodes. As shown in figure 1, each square element has one translational and two rotational degrees of freedom at each corner. The two rotational components are situated on the x and y axes. The one translational component is situated on the z axes. Each square plate element has a total of twelve degrees of freedom.

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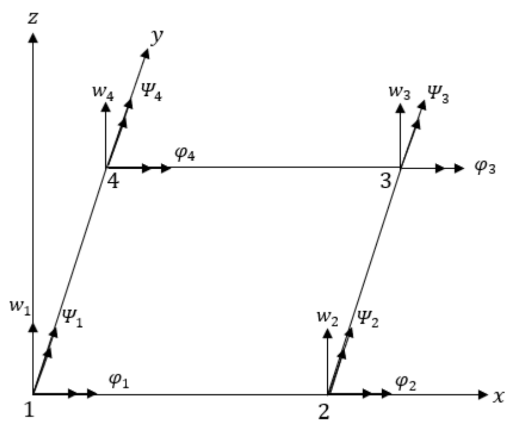

Figure 1. Generalized coordinates of a square plate element

Ψ is symbol of rotational displacement on y axis, φ is symbol of rotational displacement on x axis and w is symbol of translational displacement on z axis

The rotational components at corner 1 are

$$\Psi_1 = \frac{\partial w_1}{\partial y} \tag{1}$$

$$\varphi_1 = \frac{\partial w_1}{\partial x} \tag{2}$$

The displacement vector for an element square element is

$$\{q\}^{\mathrm{T}} = [w_1, \Psi_1, \varphi_1, w_2, \Psi_2, \varphi_2, w_3, \Psi_3, \varphi_3, w_4, \Psi_4, \varphi_4] \tag{3}$$

In this paper, a composite plate based on classical lamination theory (CLT) is modelled by using the finite element method. Calculation of the stresses functions in terms of strains functions;

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \Upsilon_{12} \end{Bmatrix} \tag{4}$$

The stress is obtained;

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} \frac{E_1}{(1-\ v_{12}v_{21})} & \frac{v_{21}E_1}{(1-\ v_{12}v_{21})} & 0 \\ \frac{v_{12}E_2}{(1-\ v_{12}v_{21})} & \frac{E_2}{(1-\ v_{12}v_{21})} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \Upsilon_{12} \end{Bmatrix} \tag{5}$$

E_1 is symbol of longitudinal modulus of the composite plate, E_2 is symbol of transverse modulus of composite plate and G_{12} is symbol shear modulus of the composite plate. E_1 can be written as

$$E_1 = \ E_f V_f + \ E_m V_m \tag{6}$$

Where E_f is Young's modulus of fiber material and E_m is Young's modulus of matrix material. V_f is volume fraction of fiber material and V_m is volume fraction of matrix material. The total volume fraction is

$$V_f + V_m = 1 \tag{7}$$

The major Poisson's ratio, v_{12}, is given by

$$v_{12} = \ v_f V_f + \ v_m V_m \tag{8}$$

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As seen in figure 2, two coordinate systems are described. The coordinate system with indexes 1 and 2 describes the layer coordinate system, this coordinate system is rotated by angle Θ and obtaining in a new coordinate system containing *x* and *y*.

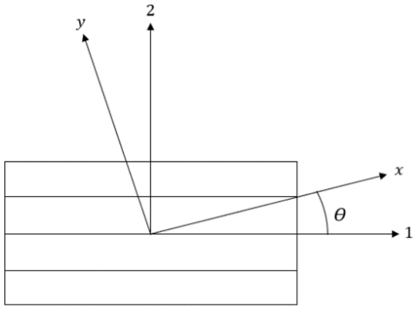

Figure 2. The coordinate system of the composite plate

The stresses and strains are multiplied by the transformation matrix $[T]$ for the transformation of the stresses and strains on the *x*, *y* coordinate system. [19]

$$\begin{Bmatrix}\sigma_{11}\\\sigma_{22}\\\tau_{12}\end{Bmatrix}^{\mathrm{k}} = [T]\begin{Bmatrix}\sigma_{xx}\\\sigma_{yy}\\\tau_{xy}\end{Bmatrix}^{\mathrm{k}} \tag{9}$$

$$\begin{Bmatrix}\varepsilon_{11}\\\varepsilon_{22}\\\frac{\Upsilon_{12}}{2}\end{Bmatrix}^{\mathrm{k}} = [T]\begin{Bmatrix}\varepsilon_{xx}\\\varepsilon_{yy}\\\frac{\Upsilon_{xy}}{2}\end{Bmatrix}^{\mathrm{k}} \tag{10}$$

The number of the ply is described by k and the transformation matrix $[T]$ is given by

$$[T] = \begin{bmatrix}c^2 & s^2 & 2cs\\ s^2 & c^2 & -2cs\\ -cs & cs & c^2-s^2\end{bmatrix} \tag{11}$$

Where $c = cos\Theta$ and $s = sin\Theta$

The ply strain-stress relationship is [19]

$$\begin{Bmatrix}\sigma_{11}\\\sigma_{22}\\\tau_{12}\end{Bmatrix}^{\mathrm{k}} = [T]^{-1}[Q]^{\mathrm{k}}[T]\begin{Bmatrix}\varepsilon_{xx}\\\varepsilon_{yy}\\\Upsilon_{xy}\end{Bmatrix}^{\mathrm{k}} \tag{12}$$

Where

$$[Q]^{\mathrm{k}} = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & 2Q_{66}\end{bmatrix}^{\mathrm{k}} \tag{13}$$

This equation is obtained

$$\begin{Bmatrix}\sigma_{11}\\\sigma_{22}\\\tau_{12}\end{Bmatrix}^{\mathrm{k}} = [\bar{Q}]^{\mathrm{k}}\begin{Bmatrix}\varepsilon_{11}\\\varepsilon_{22}\\\Upsilon_{12}\end{Bmatrix}^{\mathrm{k}} \tag{14}$$

Where

$$[\bar{Q}]^{\mathrm{k}} = \begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}^{\mathrm{k}} \tag{15}$$

$$\bar{Q}_{11} = Q_{11}cos^4\Theta + 2(Q_{12}+2Q_{66})sin^2\Theta cos^2\Theta + Q_{22}sin^4\Theta \tag{16}$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})sin^2\theta cos^2\theta + Q_{12}(sin^4\theta + cos^4\theta) \quad (17)$$

$$\bar{Q}_{22} = Q_{11}sin^4\theta + 2(Q_{12} + 2Q_{66})sin^2\theta cos^2\theta + Q_{22}cos^4\theta \quad (18)$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})sin\theta cos^3\theta + (Q_{12} - Q_{22} + 2Q_{66})sin^3\theta cos\theta \quad (19)$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})sin^3\theta cos\theta + (Q_{12} - Q_{22} + 2Q_{66})sin\theta cos^3\theta \quad (20)$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})sin^2\theta cos^2\theta + Q_{66}(sin^4\theta + cos^4\theta) \quad (21)$$

At the ply level, the stresses are given by

$$\{\sigma\}^k = [\bar{Q}]^k\{\varepsilon^0\} + z[\bar{Q}]^k\{K\} \quad (22)$$

A function of the strain on the midplane is ε^0 and curvature is *K.*

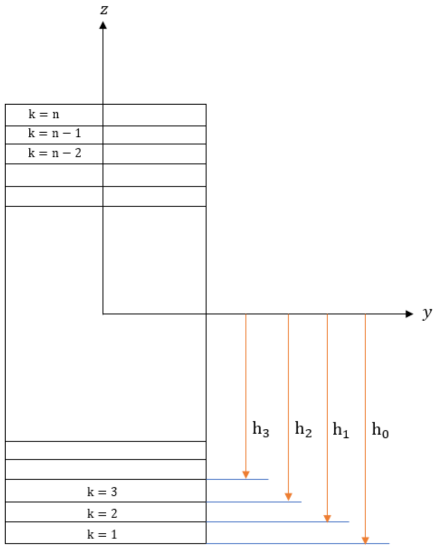

Figure 3. Layer numbering used for a typical laminated plate

As shown in figure 3, the laminates have the same thickness and *y* is the axis of symmetry. The total thickness of the composite plate is h. The force and moment matrix expressions are

$$\begin{Bmatrix} \begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} \\ \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} \end{Bmatrix} = \begin{bmatrix} \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} & \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \\ \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} & \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \end{bmatrix} \begin{Bmatrix} \begin{Bmatrix} \varepsilon^0_{xx} \\ \varepsilon^0_{yy} \\ \gamma^0_{xy} \end{Bmatrix} \\ \begin{Bmatrix} K_{xx} \\ K_{yy} \\ K_{xy} \end{Bmatrix} \end{Bmatrix} \quad (23)$$

Where the various constant defined as

$$A_{ij} = \int_{-h/2}^{h/2} \bar{Q}^k_{ij}\, dz = \sum_{k=1}^{n} \bar{Q}^k_{ij}\, (h_k - h_{k-1}) \quad (24)$$

$$B_{ij} = \int_{-h/2}^{h/2} \bar{Q}^k_{ij}\, zdz = \frac{1}{2}\sum_{k=1}^{n} \bar{Q}^k_{ij}\, (h^2_k - h^2_{k-1}) \quad (25)$$

$$D_{ij} = \int_{-h/2}^{h/2} \bar{Q}^k_{ij}\, z^2dz = \frac{1}{3}\sum_{k=1}^{n} \bar{Q}^k_{ij}\, (h^3_k - h^3_{k-1}) \quad (26)$$

Where *i*, *j* = 1, 1; 1, 2; 2, 2; 1, 6; 2, 6; 6, 6

It has been assumed that the various defects cause a crack formation on the composite plate. The finite element method is used to design the crack in the structure. This method is similar to the extended finite element method. The creation of cracks in the finite element method is basically the disruption of the unity of the structure assuming that there is a crack in a certain area of the composite plate.

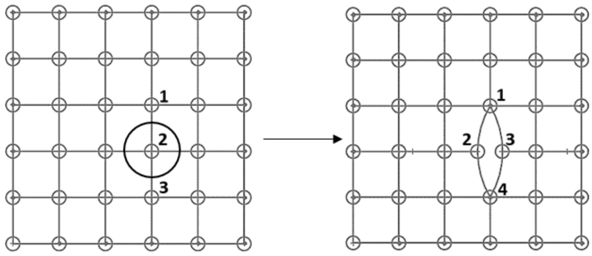

Figure 4. Finite element model converts into crack finite element model

According to this theory, firstly, the finite element method is applied to the composite plate. The composite plate is divided into elements appropriate to the shape of the composite plate and these elements are connected to each other at certain points known as nodes. As shown in figure 4, the node in the circle and the four elements that contact this node are removed from the composite plate. And then, two nodes and four elements are added to this removed region. Consequently, a void is created in the structure by using the finite element method and performing these operations. In this way, a cracked effect is achieved in the composite plate.

III. THE STABILITY THEORY

Complex structures are subjected to dynamic loads and the problem of elastic instability has arisen in complex structures. Stability formulations have been developed for the elements (such as plates, beams and bars) of the complex structure to prevent these problems. Stability problems can be defined by using energy conservation law [21].

Element geometric stiffness $[K_g]^e$, element elastic stiffness $[K_e]^e$, element mass matrix $[M]^e$ and displacement matrix $\{q\}$ are associated with each other using the following energy equations.

The potential energy of the system;

$$\mathrm{U} = \frac{1}{2}\{q\}^{\mathrm{T}}[K_e]^e\{q\} \tag{27}$$

The strain energy of the external forces

$$\Lambda = \frac{1}{2}\{q\}^{\mathrm{T}}[K_g]^e\{q\} \tag{28}$$

The kinetic energy of the system;

$$V = \frac{1}{2}\{\dot{q}\}^{\mathrm{T}}[M]^e\{\dot{q}\} \tag{29}$$

The dynamic response of a plate for a conservative system can be formulated by means of Lagrange's equation of motion in which the external forces are expressed in terms of time-dependent potentials, and then performing the required operations the entire system leads to the governing matrix equation of motion.

Mass and stiffness matrices of each plate element are used to form global mass matrix $[M]$, global geometric stiffness matrix $[K_g]$, global elastic stiffness matrix $[K_e]$

$$[M]\{\ddot{q}\} + [K_e]\{q\} - \mathrm{N}_x[K_g]\{q\} = 0 \tag{30}$$

The periodic force is

$$\mathrm{N}_x = \mathrm{N}_s + \mathrm{N}_d ø(t) \tag{31}$$

The static and time dependent components of the load can be represented as a fraction of the fundamental static buckling load N_{cr}, and periodic force is rewritten $\mathrm{N}_x = \alpha\mathrm{N}_{cr} + \beta\mathrm{N}_{cr}ø(t)$ equation (30) becomes [9]

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$$[M]\{\ddot{q}\}+\left[[K_e]-\alpha \mathrm{N}_{cr}[K_{g_s}]-\beta \mathrm{N}_{cr}\phi(t)[K_{g_d}]\right]\{q\}=0 \quad (32)$$

The periodic force is

$$\mathrm{N}_x=\mathrm{N}_s+\mathrm{N}_d\cos\Omega t \quad (33)$$

Where Ω is the exciting frequency, equation (32) becomes [10]

$$[M]\{\ddot{q}\}+\left[[K_e]-\alpha \mathrm{N}_{cr}[K_{g_s}]-\beta \mathrm{N}_{cr}\cos\Omega t\,[K_{g_d}]\right]\{q\}=0 \quad (34)$$

Bolotin proved that solutions with a period of $2T_p$ are considerably of practical importance and the boundaries of the dynamic instability regions can be determined with Equation (35).

$$\left[[K_e]-\alpha \mathrm{N}_{cr}[K_{g_s}]\pm\frac{1}{2}\beta \mathrm{N}_{cr}[K_{g_d}]-\frac{\Omega^2}{4}[M]\right]\{q\}=0 \quad (35)$$

When the static and time dependent components of the loads are applied in the same manner, $[K_{g_s}]$ and $[K_{g_d}]$ will be identical. If $[K_{g_s}]\equiv[K_{g_d}]\equiv[K_g]$, in this situation equation (35) becomes

$$\left[[K_e]-(\alpha\pm\frac{1}{2}\beta)\mathrm{N}_{cr}[K_g]-\frac{\Omega^2}{4}[M]\right]\{q\}=0 \quad (36)$$

Equation (36) represents solutions to three related problems

1) Free vibration with ω, in case of $\mathrm{N}_{cr}=0$

$$\left[[K_e]-\omega^2[M]\right]\{q\}=0 \quad (37)$$

2) Static stability with $\alpha=1,\ \beta=0$ and $\Omega=0$

$$\left[[K_e]-\mathrm{N}_{cr}[K_g]\right]\{q\}=0 \quad (38)$$

3) Dynamic stability when all terms are present

$$\left[[K_e]-(\alpha\pm\frac{1}{2}\beta)\mathrm{N}_{cr}[K_g]-\frac{\Omega^2}{4}[M]\right]\{q\}=0 \quad (39)$$

IV. RESULTS AND DISCUSSIONS

As shown in figure 5, it is assumed for the composite plate that the boundary condition of the thin plate is fixed on one side and the other sides are free. It is also assumed for the composite plate that the cantilever composite plate's length (l_p) and width (w_p) are 400 mm. The cantilever composite plate is formed four layers and each of the layer's thickness is 1 mm so that the cantilever composite plate's thickness (h) is 4 mm. The composite plates have been investigated for the five different orientation angles. The orientation angles are denoted by C1, C2, C3, C4, C5 as follows:

C1= [0°/0°/0°/0°]
C2= [0°/30°/-30°/0°]
C3= [0°/45°/-45°/0°]
C4= [0°/60°/-60°/0°]
C5= [0°/90°/90°/0°]

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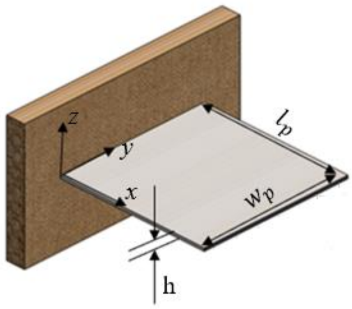

Figure 5. Cantilever composite plate

The material properties of the cantilever composite plate are given in table 1. The material properties are used for each lamina of the composite plate.

Table 1. Material properties of the composite plate

Properties	Quantities
Elastic modulus of matrix, $\boldsymbol{E_m}$ (GPa)	73
Elastic modulus of fiber, $\boldsymbol{E_f}$ (GPa)	190
Poisson's ratio of matrix, $\boldsymbol{\upsilon_m}$	0.33
Poisson's ratio of fiber, $\boldsymbol{\upsilon_f}$	0.29
Fiber volume fraction, $\boldsymbol{V_f}$ %	70
Density, $\boldsymbol{\rho}$ (kg/m^3)	8000

The first and second exciting frequency values are given approximately the same for each of five different orientations of laminates in composite plates. Therefore, C1 is examined for the composite plate in figures 6 and 7. The effect of static and dynamic load factor on the first dynamic instability region of cantilever composite plate is examined in figure 6. When the static load factor (α) is equal to 0, 0.5 and dynamic load factor (β) is equal to 1, 2, respectively, the first exciting frequencies constructing the lower border of the unstable region is equal to 0. When the static load factor (α) is equal to 0.5, the dynamic load factor (β) is bounded between the values of 0 and 1. If the static load factor (α) is equal to 0, the dynamic load factor (β) is bounded between the values of 0 and 2. Figure 7 shows the effect of static and dynamic load factor on the second dynamic instability region of cantilever composite plate. As seen from these figures, the dynamic load factor (β) increases, the unstable region widens. The unstable region of the second exciting frequency narrower than the unstable region of the first exciting frequency. When the dynamic load factor (β) is equal to 0, the first exciting frequency is less than the second exciting frequency. When the static load factor is 0, the first and second exciting frequency frequencies are higher than when the static load factor is 0.5.

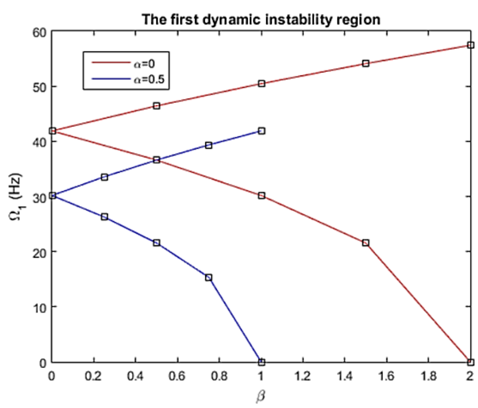

Figure 6. The effect of static and dynamic load factors on the first dynamic instability region of the composite plate [0°/0°/0°/0°]

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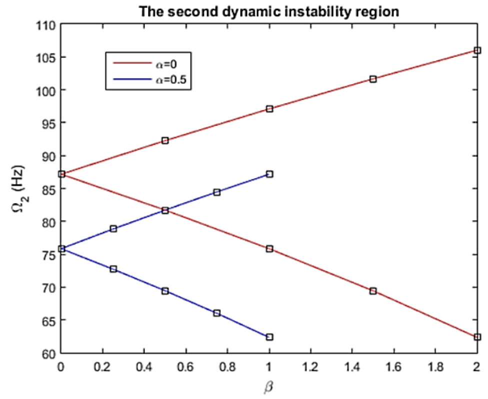

Figure 7. The effect of static and dynamic load factors on the second dynamic instability region of the composite plate [0°/0°/0°/0°]

As seen from figure 8, crack is located in the middle of y the axis and crack is moved along the x axis. It is assumed that the crack occurs in the plate and the crack's length is 32 mm. In this paper, the dynamic stability is analyzed for each of the different crack locations on the cantilever composite plate.

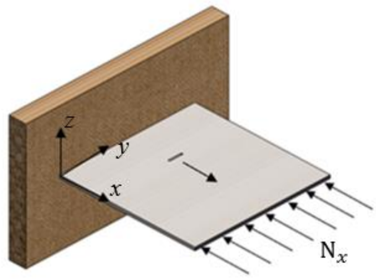

Figure 8. Cracked composite plate

Figures from 9 to 28 show the effects of the dynamic load factor, static load factor and the crack location on the first and second dynamic instability regions of cantilever composite plate, as 3D plots. The composite plates have been examined for the five different orientation angles which are denoted by C1, C2, C3, C4, C5. In these figures, the crack is located in the middle of y the axis and the crack is moved along the x axis of the cantilever composite plate. As seen from these figures, when the static load factor (α) is equal to 0.5, the dynamic load factor (β) is bounded between the values of 0 and 1. If the static load factor (α) is equal to 0, the dynamic load factor (β) is bounded between the values of 0 and 2. If the dynamic load factor (β) increases, the unstable region widens for C1, C2, C3, C4, C5. If the static load factor (α) is equal to 0, 0.5 and dynamic load factor (β) is equal to 2, 1, respectively, the first exciting frequencies constructing the lower border of the unstable region reach zero. It is observed that the crack location does not change these situations.

In these figures, the crack changes the unstable region of cantilever composite plate for C1, C2, C3, C4, C5. It is observed for C2, C3, C4, C5 orientation angles that if the crack is moved away from the fixed support on the x axis direction, the first exciting frequency increases for each static load factor (α). It is also observed that when the crack is moved away from the fixed support on the x axis direction and static load factor (α) is equal to 0, the first exciting frequency increases for C1 orientation. As seen from figure 10, if the crack is moved away from the fixed support to 100 mm on the x axis direction and static load factor (α) is equal to 0.5, the effect of crack is less on the first exciting frequency for C1 orientation. If the crack is moved away from 100 to 400 mm, the first exciting frequency increases. In these figures, if static load factor (α) is equal to 0, the effect of crack is less on the second exciting frequency. If the crack is moved away from the fixed support to 350 mm on the x axis direction and static load factor (α) is equal to 0.5, the crack decreases the second exciting frequency. When the crack is moved away from 350 to 400 mm, the effect of crack is less on the second exciting frequency for C1, C2, C3, C4, C5.

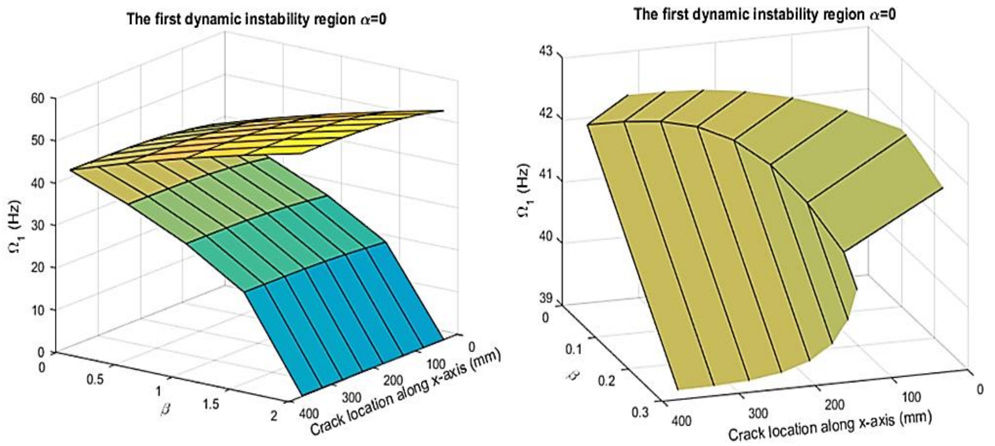

Figure 9. The effect of crack on the first dynamic instability region of the laminated composite plate [0°/0°/0°/0°] for α=0

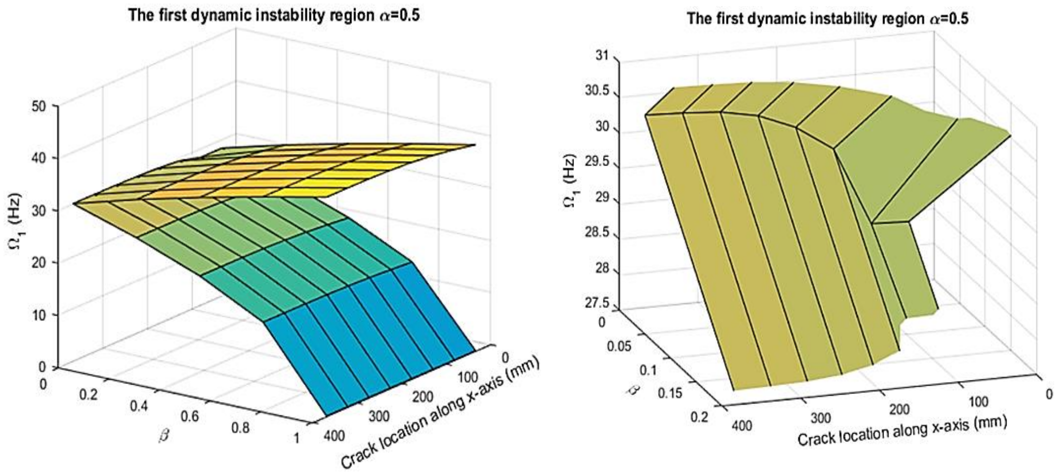

Figure 10. The effect of crack on the first dynamic instability region of the laminated composite plate [0°/0°/0°/0°] for α=0.5

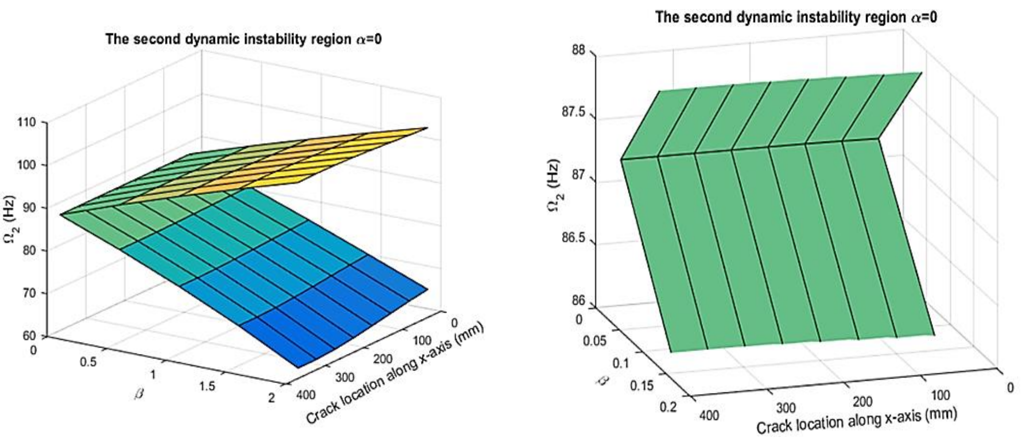

Figure 11. The effect of crack on the second dynamic instability region of the laminated composite plate [0°/0°/0°/0°] for α=0

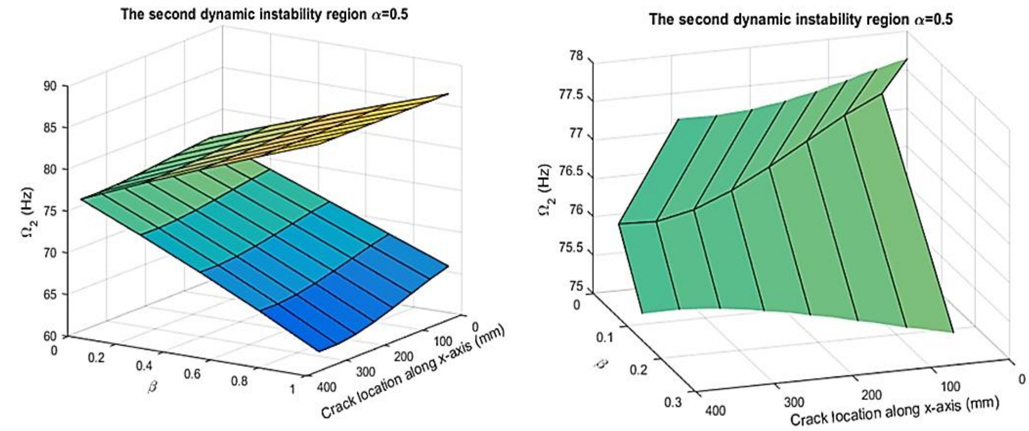

Figure 12. The effect of crack on the second dynamic instability region of the laminated composite plate [0°/0°/0°/0°] for α=0.5

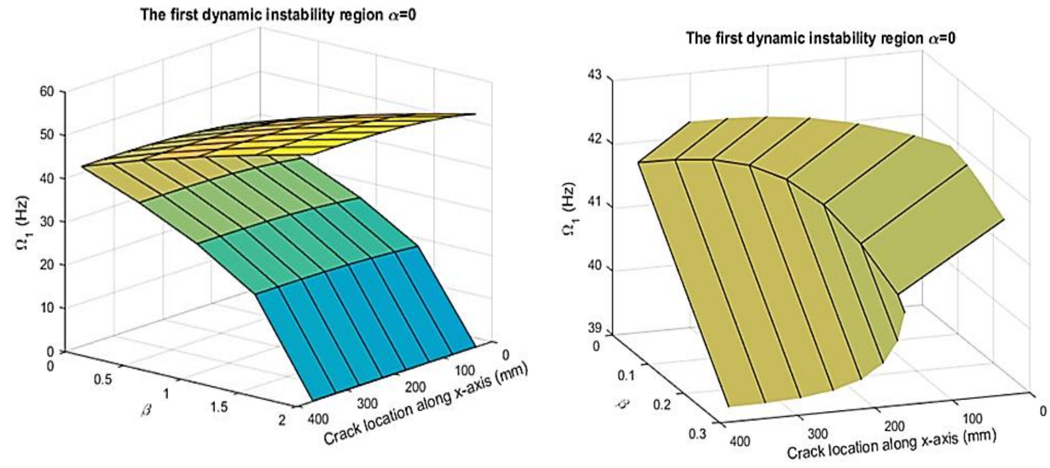

Figure 13. The effect of crack on the first dynamic instability region of the laminated composite plate [0°/30°/-30°/0°] for α=0

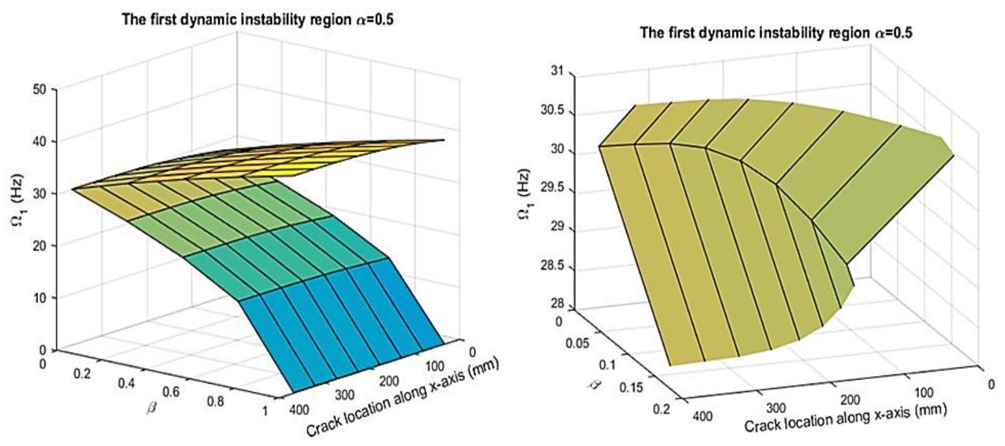

Figure 14. The effect of crack on the first dynamic instability region of the laminated composite plate [0°/30°/-30°/0°] for α=0.5

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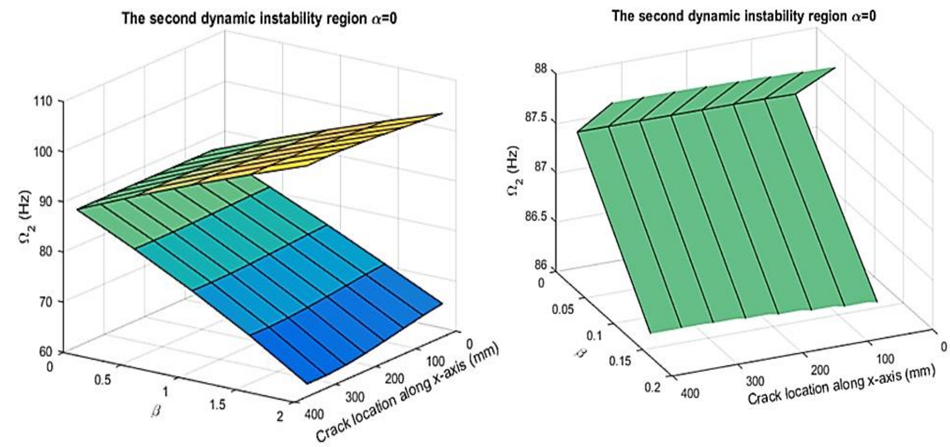

Figure 15. The effect of crack on the second dynamic instability region of the laminated composite plate [0°/30°/-30°/0°] for α=0

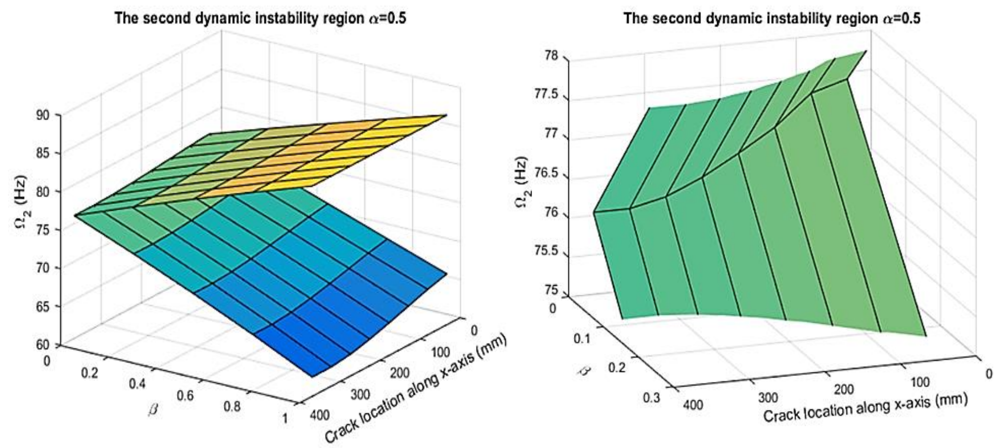

Figure 16. The effect of crack on the second dynamic instability region of the laminated composite plate [0°/30°/-30°/0°] for α=0.5

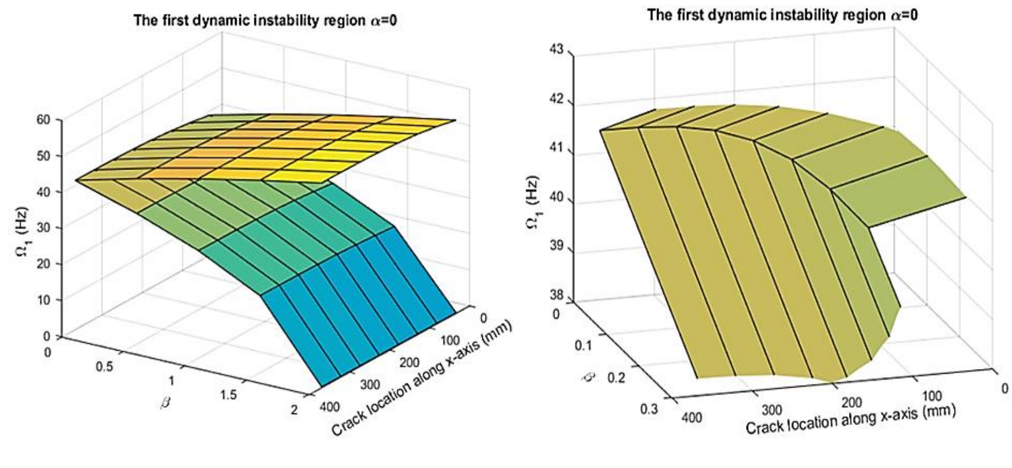

Figure 17. The effect of crack on the first dynamic instability region of the laminated composite plate [0°/45°/-45°/0°] for α=0

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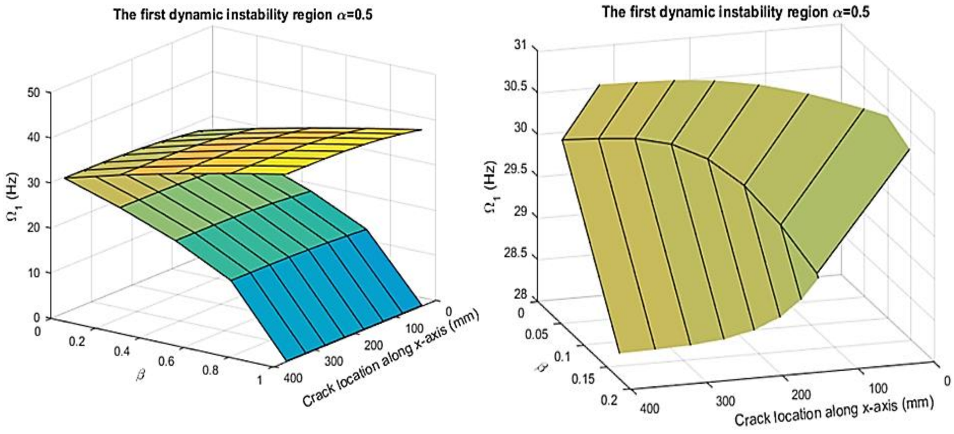

Figure 18. The effect of crack on the first dynamic instability region of the laminated composite plate [0°/45°/-45°/0°] for α=0.5

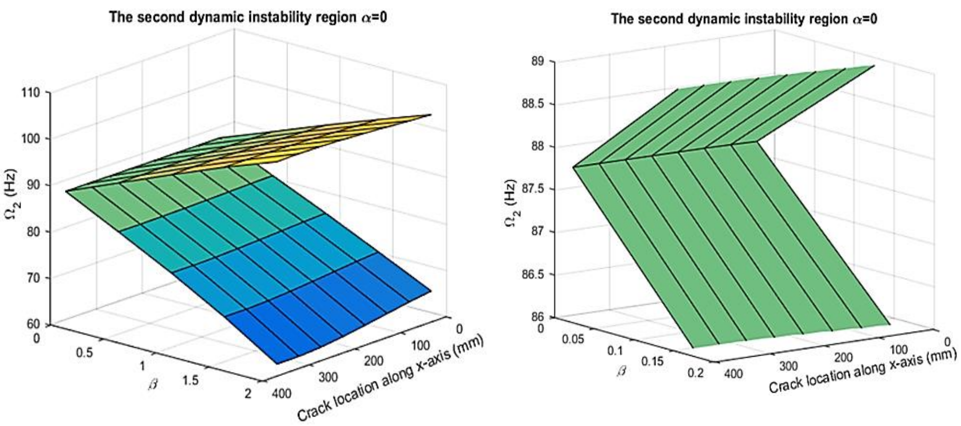

Figure 19. The effect of crack on the second dynamic instability region of the laminated composite plate [0°/45°/-45°/0°] for α=0

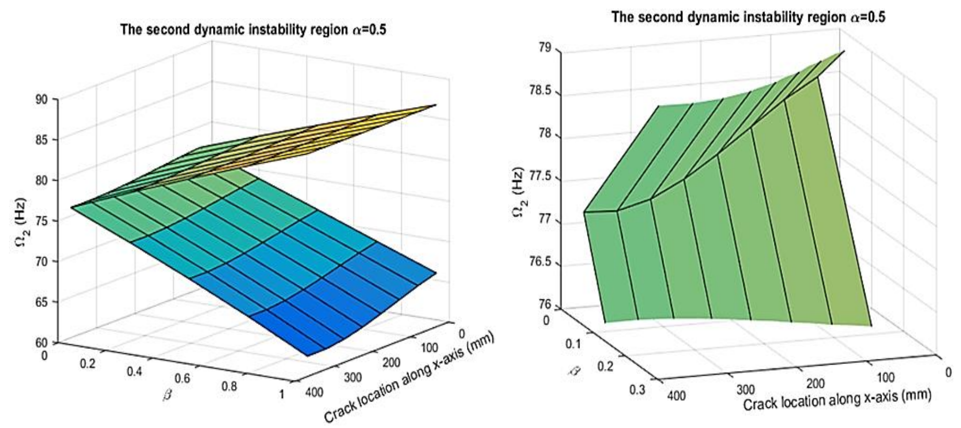

Figure 20. The effect of crack on the second dynamic instability region of the laminated composite plate [0°/45°/-45°/0°] for α=0.5

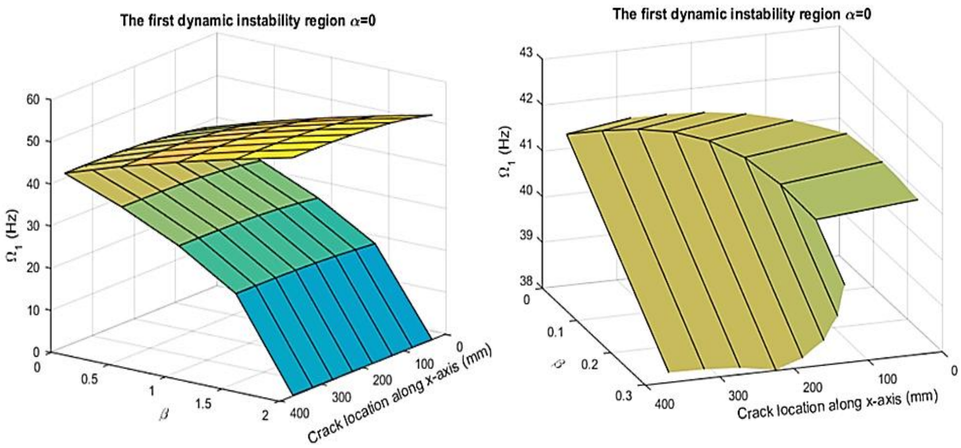

Figure 21. The effect of crack on the first dynamic instability region of the laminated composite plate [0°/60°/-60°/0°] for α=0

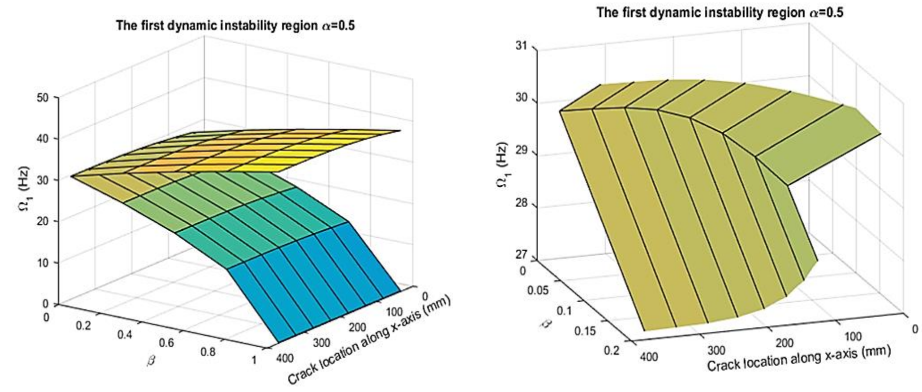

Figure 22. The effect of crack on the first dynamic instability region of the laminated composite plate [0°/60°/-60°/0°] for α=0.5

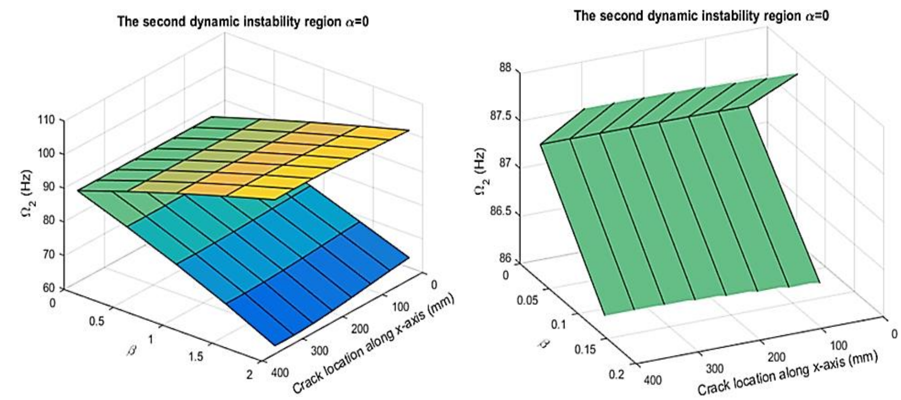

Figure 23. The effect of crack on the second dynamic instability region of the laminated composite plate [0°/60°/-60°/0°] for α=0

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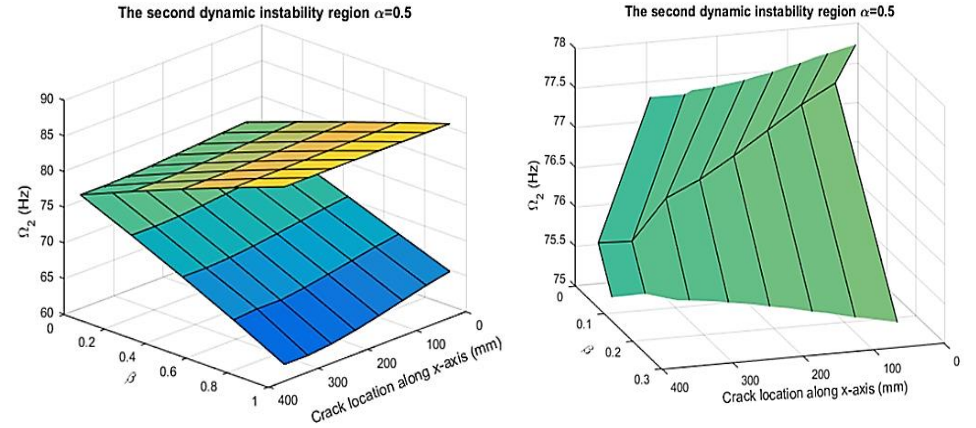

Figure 24. The effect of crack on the second dynamic instability region of the laminated composite plate [0°/60°/-60°/0°] for α=0.5

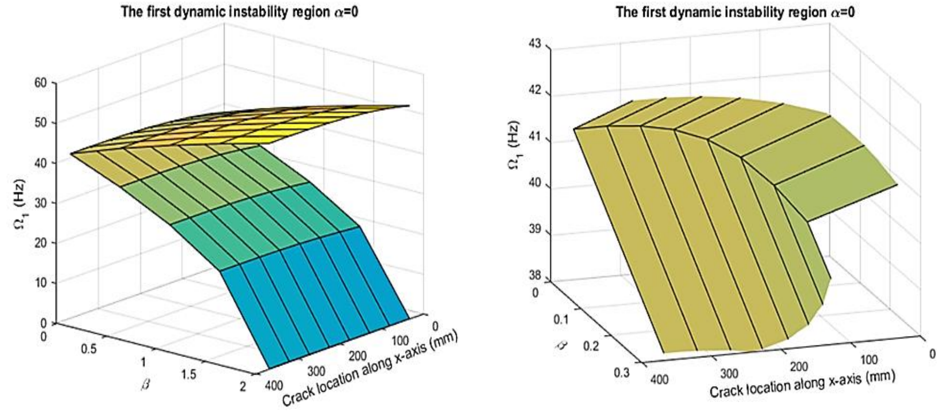

Figure 25. The effect of crack on the first dynamic instability region of the laminated composite plate [0°/90°/90°/0°] for α=0

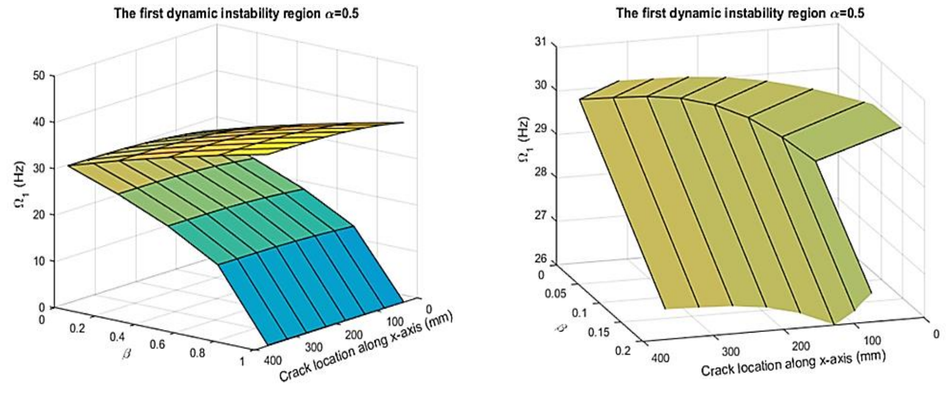

Figure 26. The effect of crack on the first dynamic instability region of the laminated composite plate [0°/90°/90°/0°] for α=0.5

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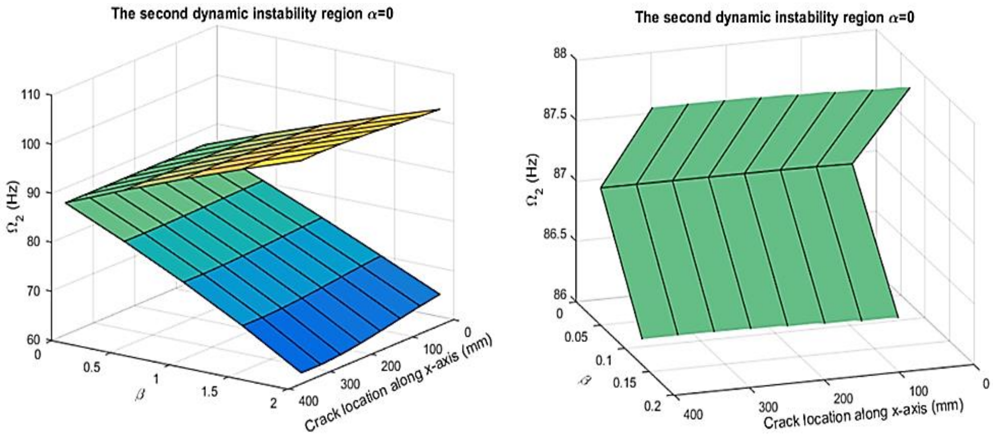

Figure 27. The effect of crack on the second dynamic instability region of the laminated composite plate [0°/90°/90°/0°] for α=0

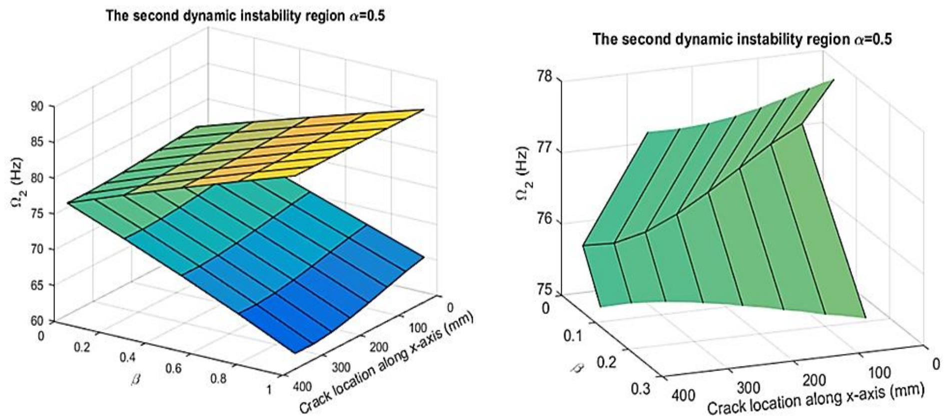

Figure 28. The effect of crack on the second dynamic instability region of the laminated composite plate [0°/90°/90°/0°] for α=0.5

V. CONCLUSIONS

In this study, the effect of crack on the free vibration, buckling and dynamic stability analysis of isotropic and composite plates for the five different orientation angles have been investigated numerically by using the finite element method. The following conclusions are drawn:

- The effect of the crack is different for each orientation angle.
- When the static load factor (α) is equal to 0, the dynamic load factor (β) is bounded between the values of 0 and 2. If the static load factor (α) is equal to 0.5, the dynamic load factor (β) is bounded between the values of 0 and 1.
- If the dynamic load factor (β) increases, the unstable region widens.
- If the crack is moved from the fixed end to the free end on the *x* axis direction for C2, C3, C4, C5, the first exciting frequency increases for each static load factor (α). When the crack is moved from the fixed end to free end on the *x* axis direction and the static load factor (α) is equal to 0, the first exciting frequency increases for C1 orientation. If the crack is moved from the free end to the fixed end on the *x* axis and the static load factor (α) is equal to 0.5, the effect of the crack is less at the first excitation frequency for the C1 orientation than for the other four orientations.
- If the static load factor (α) is equal to 0, the effect of the crack is less on the second exciting frequency. If the crack is moved from the fixed end to free end on the *x* axis direction and static load factor (α) is equal to 0.5, the second exciting frequency decreases. When the crack location is moved from the fixed end to the free end on the *x* axis, the effect of the crack is less on the second exciting frequency for C1, C2, C3, C4, C5.

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