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Theory of Generalized Compactness in Generalized Topological Spaces: Part II. Countable, Sequential and Local Properties

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Abstract: In a recent paper, a novel class of generalized compact sets (briefly, $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-compact sets*) in generalized topological spaces (briefly, $\mathscr{T}_{\mathfrak{g}}$*-spaces*) has been studied. In this paper, the concept is further studied and, other derived concepts called countable, sequential, and local generalized compactness (*countable, sequential, local* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-compactness*) in $\mathscr{T}_{\mathfrak{g}}$-spaces are also studied relatively. The study reveals that $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness implies local $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness and countable $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness, sequential $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness implies countable $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness and, $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness is a generalized topological property (briefly, $\mathscr{T}_{\mathfrak{g}}$*-property*). Diagrams establish the various relationships amongst these types of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness presented here and in the literature, and a nice application supports the overall theory.

Keywords: Generalized topological space ($\mathscr{T}_{\mathfrak{g}}$*-space*), generalized compactness ($\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-compactness*), countable generalized compactness (*countable* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-compactness*), sequential generalized compactness (*sequential* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-compactness*), local generalized compactness (*local* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-compactness*).

1. Introduction

Since the study of such fundamental topological invariants as ordinary and generalized compactness in ordinary and generalized topological spaces (briefly, $\mathfrak{T}$, $\mathfrak{g}$-$\mathfrak{T}$*-compactness in* $\mathscr{T}$*-spaces* and $\mathfrak{T}_{\mathfrak{g}}$, $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-compactness in* $\mathscr{T}_{\mathfrak{g}}$*-spaces*), a variety of weaker and stronger forms of $\mathfrak{T}$, $\mathfrak{g}$-$\mathfrak{T}$compactness in $\mathscr{T}$-spaces and $\mathfrak{T}_{\mathfrak{g}}$, $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness in $\mathscr{T}_{\mathfrak{g}}$-spaces have been introduced and investigated [1–3, 5–8, 13–19].

Bacon [2] studied a class of $\mathscr{T}$-spaces in which closed countably $\mathfrak{T}$-compact subsets are always $\mathfrak{T}$-compact. Butcher and Joseph [3] gave theorems embracing known characterizations of many of the $\mathfrak{g}$-$\mathfrak{T}$-compactness properties. El-Monsef et al. [6] generalized and studied the notions

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of $\mathfrak{T}$-compactness, para $\mathfrak{T}$-compactness, and many weak forms of such types of $\mathfrak{T}$-compactness. Greever [7] studied the extent to which Hausdorff $\mathscr{T}$-spaces with various combinations of $\mathfrak{T}$-compactness can exist, just to name a few.

Having studied a novel class of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact sets in $\mathscr{T}_{\mathfrak{g}}$-spaces recently [12], it is proposed in this paper to advance the study a step further by studying other properties and other derived concepts called countable, sequential, local $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness in $\mathscr{T}_{\mathfrak{g}}$-spaces relatively.

The paper is organized as follows: In Section 2, preliminary notions are described in Subsection 2.1 and the main results of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness in a $\mathscr{T}_{\mathfrak{g}}$-space are reported in Section 3. In Section 4, the establishment of the relationships among various types of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness are discussed in Subsection 4.1. To support the work, a nice application of the concept of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness in a $\mathscr{T}_{\mathfrak{g}}$-space is presented in Subsection 4.2. Finally, Subsection 4.3 provides concluding remarks and future directions of the notion of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness in a $\mathscr{T}_{\mathfrak{g}}$-space.

2. Theory

2.1. Preliminaries

Standard references for notations and concepts are [9–12]. The mathematical structures $\mathfrak{T} \stackrel{\text{def}}{=} (\Omega, \mathscr{T})$ and $\mathfrak{T}_{\mathfrak{g}} \stackrel{\text{def}}{=} (\Omega, \mathscr{T}_{\mathfrak{g}})$, respectively, are $\mathscr{T}$, $\mathscr{T}_{\mathfrak{g}}$-spaces [9], on both of which no separation axioms are assumed unless otherwise mentioned [4, 10]. A $\mathscr{T}_{\mathfrak{g}}$-space $\mathfrak{T}_{\mathfrak{g}} = (\Omega, \mathscr{T}_{\mathfrak{g}})$ endowed with a $\mathfrak{g}$-$\mathrm{T}_{\mathfrak{g},\mathrm{H}}$-axiom is called a $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}$-space $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}^{(\mathrm{H})} \stackrel{\text{def}}{=} \big(\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}\big)$ [9–11]. The sets I_n^0, I_n^* and I_∞^0, I_∞^*, respectively, are finite and infinite index sets [9]. Sets of the class $\mathscr{T}_{\mathfrak{g}}$ and of its complement class $\neg\mathscr{T}_{\mathfrak{g}}$, respectively, are called $\mathscr{T}_{\mathfrak{g}}$-open and $\mathscr{T}_{\mathfrak{g}}$-closed sets [9]. The class $\mathfrak{g}$-ν-$\mathrm{S}\big[\mathfrak{T}_{\mathfrak{g},\Lambda}\big] = \bigcup_{\mathrm{E}\in\{\mathrm{O},\mathrm{K}\}} \mathfrak{g}$-$\nu$-$\mathrm{K}\big[\mathfrak{T}_{\mathfrak{g}}\big]$ is called the class of all $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-sets of category $\nu \in I_3^0$ (briefly, $\mathfrak{g}$-ν-$\mathfrak{T}_{\mathfrak{g}}$*-sets*) [9, 12]. Accordingly, the class of all $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-sets [9] are

$$\mathfrak{g}\text{-}\mathrm{S}\left[\mathfrak{T}_{\mathfrak{g}}\right] = \bigcup_{\nu\in I_3^0} \mathfrak{g}\text{-}\nu\text{-}\mathrm{S}\left[\mathfrak{T}_{\mathfrak{g}}\right] \quad = \bigcup_{(\nu,\mathrm{E})\in I_3^0\times\{\mathrm{O},\mathrm{K}\}} \mathfrak{g}\text{-}\nu\text{-}\mathrm{O}\left[\mathfrak{T}_{\mathfrak{g}}\right] = \bigcup_{\mathrm{E}\in\{\mathrm{O},\mathrm{K}\}} \mathfrak{g}\text{-}\mathrm{E}\left[\mathfrak{T}_{\mathfrak{g}}\right]. \tag{1}$$

Definition 2.1 (**$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-Map** *[9]*) *A map $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \longrightarrow \mathfrak{T}_{\mathfrak{g},\Sigma}$ from a $\mathscr{T}_{\mathfrak{g}}$-space $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathscr{T}_{\mathfrak{g},\Omega})$ into a $\mathscr{T}_{\mathfrak{g}}$-space $\mathfrak{T}_{\mathfrak{g},\Sigma} = (\Sigma, \mathscr{T}_{\mathfrak{g},\Sigma})$ is called a $(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-map.*

Definition 2.2 (**$\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-Map** *[9]*) *Let $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathscr{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Sigma} = (\Sigma, \mathscr{T}_{\mathfrak{g},\Sigma})$ be $\mathscr{T}_{\mathfrak{g}}$-spaces, and let $\mathbf{op}_{\mathfrak{g}}(\cdot) \in \mathscr{L}_{\mathfrak{g}}[\Sigma]$. Then, $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \longrightarrow \mathfrak{T}_{\mathfrak{g},\Sigma}$ is called a $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-map if and only if, for every $(\mathscr{O}_{\mathfrak{g},\omega}, \mathscr{K}_{\mathfrak{g},\omega}) \in \mathscr{T}_{\mathfrak{g},\Omega} \times \neg\mathscr{T}_{\mathfrak{g},\Omega}$ there corresponds $(\mathscr{O}_{\mathfrak{g},\sigma}, \mathscr{K}_{\mathfrak{g},\sigma}) \in \mathscr{T}_{\mathfrak{g},\Sigma} \times \neg\mathscr{T}_{\mathfrak{g},\Sigma}$ such* *that:*

$$\left[\pi_{\mathfrak{g}}\left(\mathscr{O}_{\mathfrak{g},\omega}\right)\subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathscr{O}_{\mathfrak{g},\sigma}\right)\right]\vee\left[\pi_{\mathfrak{g}}\left(\mathscr{K}_{\mathfrak{g},\omega}\right)\supseteq\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathscr{K}_{\mathfrak{g},\sigma}\right)\right]. \tag{2}$$

A $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$*-map is of category* ν *if and only if it is in the class of* $\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$*-maps:*

$$\begin{aligned}\mathfrak{g}\text{-}\nu\text{-}\mathrm{M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]\stackrel{\mathrm{def}}{=}\big\{\pi_{\mathfrak{g}}:\ &\left(\forall\mathscr{O}_{\mathfrak{g},\omega},\mathscr{K}_{\mathfrak{g},\omega}\right)\left(\exists\mathscr{O}_{\mathfrak{g},\sigma},\mathscr{K}_{\mathfrak{g},\sigma},\mathbf{op}_{\mathfrak{g},\nu}\left(\cdot\right)\right)\\&\left[\left(\pi_{\mathfrak{g}}\left(\mathscr{O}_{\mathfrak{g},\omega}\right)\subseteq\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{O}_{\mathfrak{g},\sigma}\right)\right)\vee\left(\pi_{\mathfrak{g}}\left(\mathscr{K}_{\mathfrak{g},\omega}\right)\supseteq\neg\,\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{K}_{\mathfrak{g},\sigma}\right)\right)\right]\big\}.\end{aligned} \tag{3}$$

Definition 2.3 *The classes of* $\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$*-open and* $\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$*-closed maps, respectively, are:*

$$\begin{aligned}\mathfrak{g}\text{-}\nu\text{-}\mathrm{M}_{\mathrm{O}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]&\stackrel{\mathrm{def}}{=}\big\{\pi_{\mathfrak{g}}:\left(\forall\mathscr{O}_{\mathfrak{g},\omega}\right)\left(\exists\mathscr{O}_{\mathfrak{g},\sigma},\mathbf{op}_{\mathfrak{g},\nu}\left(\cdot\right)\right)\left[\pi_{\mathfrak{g}}\left(\mathscr{O}_{\mathfrak{g},\omega}\right)\subseteq\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{O}_{\mathfrak{g},\sigma}\right)\right]\big\},\\ \mathfrak{g}\text{-}\nu\text{-}\mathrm{M}_{\mathrm{K}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]&\stackrel{\mathrm{def}}{=}\big\{\pi_{\mathfrak{g}}:\left(\forall\mathscr{K}_{\omega}\right)\left(\exists\mathscr{K}_{\mathfrak{g},\sigma},\mathbf{op}_{\mathfrak{g},\nu}\left(\cdot\right)\right)\left[\pi_{\mathfrak{g}}\left(\mathscr{K}_{\mathfrak{g},\omega}\right)\supseteq\neg\,\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{K}_{\mathfrak{g},\sigma}\right)\right]\big\}.\end{aligned} \tag{4}$$

Accordingly, the class of all $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-maps [9] are

$$\begin{aligned}\mathfrak{g}\text{-}\mathrm{M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]&=\bigcup_{\nu\in I_3^0}\mathfrak{g}\text{-}\nu\text{-}\mathrm{M}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]\\&=\bigcup_{(\nu,\mathrm{E})\in I_3^0\times\{\mathrm{O},\mathrm{K}\}}\mathfrak{g}\text{-}\nu\text{-}\mathrm{M}_{\mathrm{E}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]=\bigcup_{\mathrm{E}\in\{\mathrm{O},\mathrm{K}\}}\mathfrak{g}\text{-}\mathrm{M}_{\mathrm{E}}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right].\end{aligned} \tag{5}$$

Definition 2.4 ($\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-Continuous *[9]***)** *Let* $\mathfrak{T}_{\mathfrak{g},\Omega}=(\Omega,\mathscr{T}_{\mathfrak{g},\Omega})$ *and* $\mathfrak{T}_{\mathfrak{g},\Sigma}=(\Sigma,\mathscr{T}_{\mathfrak{g},\Sigma})$ *be* $\mathscr{T}_{\mathfrak{g}}$*-spaces, and let* $\mathbf{op}_{\mathfrak{g}}(\cdot)\in\mathscr{L}_{\mathfrak{g}}[\Omega]$. *Then,* $\pi_{\mathfrak{g}}:\mathfrak{T}_{\mathfrak{g},\Omega}\longrightarrow\mathfrak{T}_{\mathfrak{g},\Sigma}$ *is said to be* $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$*-continuous if and only if, for every* $(\mathscr{O}_{\mathfrak{g},\sigma},\mathscr{K}_{\mathfrak{g},\sigma})\in\mathscr{T}_{\mathfrak{g},\Sigma}\times\neg\mathscr{T}_{\mathfrak{g},\Sigma}$ *there corresponds* $(\mathscr{O}_{\mathfrak{g},\omega},\mathscr{K}_{\mathfrak{g},\omega})\in\mathscr{T}_{\mathfrak{g},\Omega}\times\neg\mathscr{T}_{\mathfrak{g},\Omega}$ *such that:*

$$\left[\pi_{\mathfrak{g}}^{-1}\left(\mathscr{O}_{\mathfrak{g},\sigma}\right)\subseteq\mathrm{op}_{\mathfrak{g}}\left(\mathscr{O}_{\mathfrak{g},\omega}\right)\right]\vee\left[\pi_{\mathfrak{g}}^{-1}\left(\mathscr{K}_{\mathfrak{g},\sigma}\right)\supseteq\neg\,\mathrm{op}_{\mathfrak{g}}\left(\mathscr{K}_{\mathfrak{g},\omega}\right)\right]. \tag{6}$$

A $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$*-continuous map is of category* ν *if and only if it is in the class of* $\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$*-continuous maps:*

$$\begin{aligned}\mathfrak{g}\text{-}\nu\text{-}\mathrm{C}\left[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}\right]\stackrel{\mathrm{def}}{=}\big\{\pi_{\mathfrak{g}}:\ &\left(\forall\mathscr{O}_{\mathfrak{g},\sigma},\mathscr{K}_{\mathfrak{g},\sigma}\right)\left(\exists\mathscr{O}_{\mathfrak{g},\omega},\mathscr{K}_{\mathfrak{g},\omega},\mathbf{op}_{\mathfrak{g},\nu}\left(\cdot\right)\right)\\&\left[\left(\pi_{\mathfrak{g}}^{-1}\left(\mathscr{O}_{\mathfrak{g},\sigma}\right)\subseteq\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{O}_{\mathfrak{g},\omega}\right)\right)\vee\left(\pi_{\mathfrak{g}}^{-1}\left(\mathscr{K}_{\mathfrak{g},\sigma}\right)\supseteq\neg\,\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{K}_{\mathfrak{g},\omega}\right)\right)\right]\big\}.\end{aligned} \tag{7}$$

Definition 2.5 ($\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-Irresolute *[9]***)** *Let* $\mathfrak{T}_{\mathfrak{g},\Omega}=(\Omega,\mathscr{T}_{\mathfrak{g},\Omega})$ *and* $\mathfrak{T}_{\mathfrak{g},\Sigma}=(\Sigma,\mathscr{T}_{\mathfrak{g},\Sigma})$ *be* $\mathscr{T}_{\mathfrak{g}}$*-spaces, and let* $\mathbf{op}_{\mathfrak{g}}(\cdot)\in\mathscr{L}_{\mathfrak{g}}[\Omega]$. *Then,* $\pi_{\mathfrak{g}}:\mathfrak{T}_{\mathfrak{g},\Omega}\longrightarrow\mathfrak{T}_{\mathfrak{g},\Sigma}$ *is said to be* $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$*-irresolute if and only if, for every* $(\mathscr{O}_{\mathfrak{g},\sigma},\mathscr{K}_{\mathfrak{g},\sigma})\in\mathscr{T}_{\mathfrak{g},\Sigma}\times\neg\mathscr{T}_{\mathfrak{g},\Sigma}$ *there corresponds* $(\mathscr{O}_{\mathfrak{g},\omega},\mathscr{K}_{\mathfrak{g},\omega})\in$

$\mathscr{T}_{\mathfrak{g},\Omega} \times \neg \mathscr{T}_{\mathfrak{g},\Omega}$ *such that:*

$$\left[\pi_{\mathfrak{g}}^{-1}\left(\mathrm{op}_{\mathfrak{g}}\left(\mathscr{O}_{\mathfrak{g},\sigma}\right)\right) \subseteq \mathrm{op}_{\mathfrak{g}}\left(\mathscr{O}_{\mathfrak{g},\omega}\right)\right] \vee \left[\pi_{\mathfrak{g}}^{-1}\left(\neg\, \mathrm{op}_{\mathfrak{g}}\left(\mathscr{K}_{\mathfrak{g},\sigma}\right)\right) \supseteq \neg\, \mathrm{op}_{\mathfrak{g}}\left(\mathscr{K}_{\mathfrak{g},\omega}\right)\right]. \tag{8}$$

A $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$*-irresolute map is of category* ν *if and only if it is in the class of* $\mathfrak{g}$-ν-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$*-irresolute maps:*

$$\begin{aligned} \mathfrak{g}\text{-}\nu\text{-}\mathrm{I}\left[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Sigma}\right] \stackrel{\mathrm{def}}{=} \big\{\pi_{\mathfrak{g}} :\ & \left(\forall \mathscr{O}_{\mathfrak{g},\sigma}, \mathscr{K}_{\mathfrak{g},\sigma}\right)\left(\exists \mathscr{O}_{\mathfrak{g},\omega}, \mathscr{K}_{\mathfrak{g},\omega}, \mathbf{op}_{\mathfrak{g},\nu}(\cdot)\right) \\ & \left[\left(\pi_{\mathfrak{g}}^{-1}\left(\mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{O}_{\mathfrak{g},\sigma}\right)\right) \subseteq \mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{O}_{\mathfrak{g},\omega}\right)\right) \vee \left(\pi_{\mathfrak{g}}^{-1}\left(\neg\, \mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{K}_{\mathfrak{g},\sigma}\right)\right) \supseteq \neg\, \mathrm{op}_{\mathfrak{g},\nu}\left(\mathscr{K}_{\mathfrak{g},\omega}\right)\right)\right]\big\}. \end{aligned} \tag{9}$$

The classes of $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-continuous and $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-irresolute maps, respectively, are:

$$\mathfrak{g}\text{-}\mathrm{C}\left[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Sigma}\right] = \bigcup_{\nu \in I_3^0} \mathfrak{g}\text{-}\nu\text{-}\mathrm{C}\left[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Sigma}\right], \quad \mathfrak{g}\text{-}\mathrm{I}\left[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Sigma}\right] = \bigcup_{\nu \in I_3^0} \mathfrak{g}\text{-}\nu\text{-}\mathrm{I}\left[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Sigma}\right]. \tag{10}$$

By a $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}$-open set and a $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}$-closed set are meant a $\mathscr{T}_{\mathfrak{g}}$-open set $\mathscr{O}_{\mathfrak{g}} \in \mathscr{T}_{\mathfrak{g}}$ and a $\mathscr{T}_{\mathfrak{g}}$-closed set $\mathscr{K}_{\mathfrak{g}} \in \neg \mathscr{T}_{\mathfrak{g}}$ satisfying $\mathscr{O}_{\mathfrak{g}} \subseteq \mathrm{op}_{\mathfrak{g}}(\mathscr{O}_{\mathfrak{g}})$ and $\mathscr{K}_{\mathfrak{g}} \supseteq \neg\, \mathrm{op}_{\mathfrak{g}}(\mathscr{K}_{\mathfrak{g}})$, respectively. Likewise, by a $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}$-open set of category ν and a $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}$-closed set of category ν are meant a $\mathscr{T}_{\mathfrak{g}}$-open set $\mathscr{O}_{\mathfrak{g}} \in \mathscr{T}_{\mathfrak{g}}$ and a $\mathscr{T}_{\mathfrak{g}}$-closed set $\mathscr{K}_{\mathfrak{g}} \in \neg \mathscr{T}_{\mathfrak{g}}$ satisfying $\mathscr{O}_{\mathfrak{g}} \subseteq \mathrm{op}_{\mathfrak{g},\nu}(\mathscr{O}_{\mathfrak{g}})$ and $\mathscr{K}_{\mathfrak{g}} \supseteq \neg\, \mathrm{op}_{\mathfrak{g},\nu}(\mathscr{K}_{\mathfrak{g}})$, respectively; $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}$-sets of category ν will be called $\mathfrak{g}$-ν-$\mathscr{T}_{\mathfrak{g}}$-sets [9].

Given the $\mathfrak{T}_{\mathfrak{g}}$-sets $\mathscr{R}_{\mathfrak{g}}, \mathscr{S}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g}}$, $\mathscr{R}_{\mathfrak{g}}$ is said to be *equivalent* to $\mathscr{S}_{\mathfrak{g}}$, written $\mathscr{R}_{\mathfrak{g}} \sim \mathscr{S}_{\mathfrak{g}}$, if and only if, there exists a $\mathfrak{T}_{\mathfrak{g}}$-map $\pi_{\mathfrak{g}} : \mathscr{R}_{\mathfrak{g}} \longrightarrow \mathscr{S}_{\mathfrak{g}}$ which is bijective. A $\mathfrak{T}_{\mathfrak{g}}$-set $\mathscr{S}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g}}$ is *finite* if and only if $\mathscr{S}_{\mathfrak{g}} = \emptyset$ or $\mathscr{S}_{\mathfrak{g}} \sim I_{\mu}^{*}$ for some $\mu \in I_{\infty}^{*}$; otherwise, the $\mathfrak{T}_{\mathfrak{g}}$-set $\mathscr{S}_{\mathfrak{g}}$ is said to be *infinite*. A $\mathfrak{T}_{\mathfrak{g}}$-set $\mathscr{R}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g}}$ is *denumerable* and satisfies the condition $\mathrm{card}(\mathscr{R}_{\mathfrak{g}}) = \aleph_0$ (*aleph-null*) if and only if $\mathscr{S}_{\mathfrak{g}} \sim I_{\infty}^{*}$. The $\mathfrak{T}_{\mathfrak{g}}$-set $\mathscr{R}_{\mathfrak{g}}$ is called *countable* if and only if it is *finite* or *denumerable* [9].

The symbol $\langle \mathscr{S}_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-}\nu\text{-}\mathrm{S}[\mathfrak{T}_{\mathfrak{g}}] \rangle_{\alpha \in I_{\sigma}^{*}}$ denotes a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-sets sequence of category ν in $\mathfrak{T}_{\mathfrak{g}}$ [9, 11]. The sequences $\langle \mathscr{S}_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-}\mathrm{S}[\mathfrak{T}_{\mathfrak{g}}] \rangle_{\alpha \in I_{\sigma}^{*}}$, $\langle \mathscr{U}_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-}\mathrm{O}[\mathfrak{T}_{\mathfrak{g}}] \rangle_{\alpha \in I_{\sigma}^{*}}$, and $\langle \mathscr{V}_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-}\mathrm{K}[\mathfrak{T}_{\mathfrak{g}}] \rangle_{\alpha \in I_{\sigma}^{*}}$, respectively, are simply said to be a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-covering*, a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-open covering*, and a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-closed covering* of a $\mathfrak{T}_{\mathfrak{g}}$-set $\mathscr{S}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g}}$ whose cardinality is at most $\sigma \in I_{\infty}^{*}$ if and only if the corresponding relations $\mathscr{S}_{\mathfrak{g}} \subseteq \bigcup_{\alpha \in I_{\sigma}^{*}} \mathscr{S}_{\mathfrak{g},\alpha}$, $\mathscr{S}_{\mathfrak{g}} \subseteq \bigcup_{\alpha \in I_{\sigma}^{*}} \mathscr{U}_{\mathfrak{g},\alpha}$ and $\mathscr{S}_{\mathfrak{g}} \subseteq \bigcup_{\alpha \in I_{\sigma}^{*}} \mathscr{V}_{\mathfrak{g},\alpha}$ hold true [9, 11]. The map

$$\vartheta : \langle \mathscr{S}_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-}\mathrm{S}[\mathfrak{T}_{\mathfrak{g}}] \rangle_{\alpha \in I_{\sigma}^{*}} \longrightarrow \langle \mathscr{S}_{\mathfrak{g},\vartheta(\alpha)} \in \mathfrak{g}\text{-}\mathrm{S}[\mathfrak{T}_{\mathfrak{g}}] \rangle_{(\alpha,\vartheta(\alpha)) \in I_{\sigma}^{*} \times I_{\vartheta(\sigma)}^{*}} \tag{11}$$

is said to realise a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-subcovering $\langle \mathscr{S}_{\mathfrak{g},\vartheta(\alpha)} \rangle_{(\alpha,\vartheta(\alpha)) \in I_{\sigma}^{*} \times I_{\vartheta(\sigma)}^{*}}$ of $\mathscr{S}_{\mathfrak{g}}$ from the $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-covering $\langle \mathscr{S}_{\mathfrak{g},\alpha} \rangle_{\alpha \in I_{\sigma}^{*}}$ if and only if $\mathscr{S}_{\mathfrak{g}} \subseteq \bigcup_{(\alpha,\vartheta(\alpha)) \in I_{\sigma}^{*} \times I_{\vartheta(\sigma)}^{*}} \mathscr{S}_{\mathfrak{g},\vartheta(\alpha)}$ [9, 11]. The $\mathfrak{T}_{\mathfrak{g}}$-set $\mathscr{S}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g}}$ of a

$\mathscr{T}_{\mathfrak{g}}$-space $\mathfrak{T}_{\mathfrak{g}}$ is $\mathfrak{g}$-ν-$\mathfrak{T}_{\mathfrak{g}}$-compact if and only if, for every $\langle \mathscr{U}_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-}\nu\text{-}\mathrm{O}[\mathfrak{T}_{\mathfrak{g}}] \rangle_{\alpha \in I_{\sigma}^{*}}$,

$$\exists \langle \mathscr{U}_{\mathfrak{g},\vartheta(\alpha)} \rangle_{(\alpha,\vartheta(\alpha)) \in I_{\sigma}^{*} \times I_{\vartheta(\sigma)}^{*}} : \quad \mathscr{S}_{\mathfrak{g}} \subseteq \bigcup_{(\alpha,\vartheta(\alpha)) \in I_{\sigma}^{*} \times I_{\vartheta(\sigma)}^{*}} \mathscr{U}_{\mathfrak{g},\vartheta(\alpha)}, \tag{12}$$

where $\vartheta(\sigma) = \mathrm{card}\big(I_{\vartheta(\sigma)}^{*}\big) \leq \mathrm{card}(I_{\sigma}^{*}) = \sigma$ [9, 11]. The class of all $\mathfrak{g}$-ν-$\mathfrak{T}_{\mathfrak{g}}$-compact sets is:

$$\mathfrak{g}\text{-}\nu\text{-}\mathrm{A}[\mathfrak{T}_{\mathfrak{g}}] \stackrel{\mathrm{def}}{=} \Big\{ \mathscr{S}_{\mathfrak{g}} : \big[\forall \langle \mathscr{U}_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-}\nu\text{-}\mathrm{O}[\mathfrak{T}_{\mathfrak{g}}] \rangle_{\alpha \in I_{\sigma}^{*}}\big] \big[\exists \langle \mathscr{U}_{\mathfrak{g},\vartheta(\alpha)} \rangle_{(\alpha,\vartheta(\alpha)) \in I_{\sigma}^{*} \times I_{\vartheta(\sigma)}^{*}}\big] \Big(\mathscr{S}_{\mathfrak{g}} \subseteq \bigcup_{(\alpha,\vartheta(\alpha)) \in I_{\sigma}^{*} \times I_{\vartheta(\sigma)}^{*}} \mathscr{U}_{\mathfrak{g},\vartheta(\alpha)} \Big) \Big\}. \tag{13}$$

A $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-covering $\langle \mathscr{S}_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-}\mathrm{S}[\mathfrak{T}_{\mathfrak{g}}] \rangle_{\alpha \in I_{\sigma}^{*}}$ of a $\mathfrak{T}_{\mathfrak{g}}$-set $\mathscr{S}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g}}$ of a $\mathscr{T}_{\mathfrak{g}}$-space $\mathfrak{T}_{\mathfrak{g}} = (\Omega, \mathscr{T}_{\mathfrak{g}})$ is a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-refinement [9, 11] of another $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-covering $\langle \mathscr{R}_{\mathfrak{g},\beta} \in \mathfrak{g}\text{-}\mathrm{S}[\mathfrak{T}_{\mathfrak{g}}] \rangle_{\beta \in I_{\mu}^{*}}$ of the same $\mathfrak{T}_{\mathfrak{g}}$-set $\mathscr{S}_{\mathfrak{g}}$ if and only if:

$$\big(\forall \alpha \in I_{\sigma}^{*}\big)\big(\exists \beta \in I_{\mu}^{*}\big)\big[\mathscr{S}_{\mathfrak{g},\alpha} \subseteq \mathscr{R}_{\mathfrak{g},\beta}\big]. \tag{14}$$

Definition 2.6 ($\mathfrak{g}$-ν-$\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$-Space *[9, 11]*) *A $\mathscr{T}_{\mathfrak{g}}$-space $\mathfrak{T}_{\mathfrak{g}} = (\Omega, \mathscr{T}_{\mathfrak{g}})$ is called a $\mathfrak{g}$-ν-$\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$-space denoted $\mathfrak{g}$-ν-$\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]} \stackrel{\mathrm{def}}{=} (\Omega, \mathfrak{g}\text{-}\nu\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]})$ if and only if each $\mathfrak{g}$-ν-$\mathfrak{T}_{\mathfrak{g}}$-open covering $\langle \mathscr{U}_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-}\nu\text{-}\mathrm{O}[\mathfrak{T}_{\mathfrak{g}}] \rangle_{\alpha \in I_{\sigma}^{*}}$ of $\mathfrak{T}_{\mathfrak{g}}$ has a finite $\mathfrak{g}$-ν-$\mathfrak{T}_{\mathfrak{g}}$-open subcovering.*

By $\mathfrak{g}$-ν-$\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{CA}]} \stackrel{\mathrm{def}}{=} (\Omega, \mathfrak{g}\text{-}\nu\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{CA}]})$, $\mathfrak{g}$-ν-$\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{SA}]} \stackrel{\mathrm{def}}{=} (\Omega, \mathfrak{g}\text{-}\nu\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{SA}]})$, and $\mathfrak{g}$-ν-$\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{LA}]} \stackrel{\mathrm{def}}{=} (\Omega, \mathfrak{g}\text{-}\nu\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{LA}]})$, respectively, are meant *countably*, *sequentially*, and *locally* $\mathfrak{g}$-ν-$\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$-spaces; by a $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{[\mathrm{E}]}$-space $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{E}]} = (\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{E}]})$ is meant $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{E}]} = \bigvee_{\nu \in I_{3}^{0}} \mathfrak{g}\text{-}\nu\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{E}]} = (\Omega, \bigvee_{\nu \in I_{3}^{0}} \mathfrak{g}\text{-}\nu\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{E}]}) = (\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{E}]})$, where $\mathrm{E} \in \{\mathrm{A}, \mathrm{CA}, \mathrm{SA}, \mathrm{LA}\}$.

Definition 2.7 (Finite Intersection Property *[9, 11]*) *A sequence $\langle \mathscr{S}_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-}\mathrm{S}[\mathfrak{T}_{\mathfrak{g}}] \rangle_{\alpha \in I_{\sigma}^{*}}$ of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-sets is said to have the "finite intersection property" if and only if every finite subsequence of the type $\langle \mathscr{S}_{\mathfrak{g},\beta(\alpha)} \rangle_{(\alpha,\beta(\alpha)) \in I_{\sigma}^{*} \times I_{n}^{*}}$ has a non-empty intersection:*

$$\forall \langle \mathscr{S}_{\mathfrak{g},\beta(\alpha)} \rangle_{(\alpha,\beta(\alpha)) \in I_{\sigma}^{*} \times I_{n}^{*}} \prec \langle \mathscr{S}_{\mathfrak{g},\alpha} \rangle_{\alpha \in I_{\sigma}^{*}} : \bigcap_{(\alpha,\beta(\alpha)) \in I_{\sigma}^{*} \times I_{n}^{*}} \mathscr{S}_{\mathfrak{g},\beta(\alpha)} \neq \emptyset. \tag{15}$$

Definition 2.8 ($\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-Accumulation Point *[9, 11]*) *A point $\xi \in \mathfrak{T}_{\mathfrak{g}}$ of a $\mathscr{T}_{\mathfrak{g}}$-space $\mathfrak{T}_{\mathfrak{g}} = (\Omega, \mathscr{T}_{\mathfrak{g}})$ is called a "$\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-accumulation point" (or "$\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-limit point", "$\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-cluster point", "$\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-derived point") of a $\mathfrak{T}_{\mathfrak{g}}$-set $\mathscr{S}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g}}$ of $\mathfrak{T}_{\mathfrak{g}}$ if and only if every $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open set $\mathscr{U}_{\mathfrak{g},\xi} \in \mathfrak{g}\text{-}\mathrm{O}[\mathfrak{T}_{\mathfrak{g}}]$*

containing ξ *(whether* $\xi \in \mathscr{S}_{\mathfrak{g}}$ *or* $\xi \notin \mathscr{S}_{\mathfrak{g}}$*) contains at least a point* $\zeta \in \mathscr{S}_{\mathfrak{g}} \setminus \{\xi\}$*:*

$$\xi \in \mathscr{U}_{\mathfrak{g},\xi} \in \mathfrak{g}\text{-O}\left[\mathfrak{T}_{\mathfrak{g}}\right] \;\Rightarrow\; \mathscr{S}_{\mathfrak{g}} \cap \left(\mathscr{U}_{\mathfrak{g},\xi} \setminus \{\xi\}\right) \neq \emptyset. \tag{16}$$

The set $\mathrm{der}_{\mathfrak{g}}\left(\mathscr{S}_{\mathfrak{g}}\right) \subset \mathfrak{T}_{\mathfrak{g}}$ *of all* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-accumulation points is called the "*$\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-derived set of* $\mathscr{S}_{\mathfrak{g}}$*".*

Definition 2.9 (Countably $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-Compact *[9, 11]***)** *A* $\mathfrak{T}_{\mathfrak{g}}$*-set* $\mathscr{S}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g}}$ *of a* $\mathscr{T}_{\mathfrak{g}}$*-space* $\mathfrak{T}_{\mathfrak{g}} = (\Omega, \mathscr{T}_{\mathfrak{g}})$ *is said to be "countably* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-compact" if and only if every infinite* $\mathfrak{T}_{\mathfrak{g}}$*-subset* $\mathscr{R}_{\mathfrak{g}} \subset \mathscr{S}_{\mathfrak{g}}$ *of* $\mathscr{S}_{\mathfrak{g}}$ *has at least one* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-accumulation point* $\xi \in \mathscr{S}_{\mathfrak{g}}$*.*

Definition 2.10 (Sequentially $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact *[9, 11]***)** *A* $\mathfrak{T}_{\mathfrak{g}}$*-set* $\mathscr{S}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g}}$ *of a* $\mathscr{T}_{\mathfrak{g}}$*-space* $\mathfrak{T}_{\mathfrak{g}} = (\Omega, \mathscr{T}_{\mathfrak{g}})$ *is "sequentially* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-compact" if and only if every sequence* $\langle \xi_{\alpha} \in \mathscr{S}_{\mathfrak{g}} \rangle_{\alpha \in I_{\infty}^{*}}$ *in* $\mathscr{S}_{\mathfrak{g}}$ *contains a subsequence* $\langle \xi_{\vartheta(\alpha)} \rangle_{(\alpha,\vartheta(\alpha)) \in I_{\infty}^{*} \times I_{\infty}^{*}} \prec \langle \xi_{\alpha} \rangle_{\alpha \in I_{\infty}^{*}}$ *which converges to a point* $\xi \in \mathscr{S}_{\mathfrak{g}}$*.*

Definition 2.11 ($\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-Neighborhood *[9, 11]***)** *Let* $\xi \in \mathfrak{T}_{\mathfrak{g}}$ *be a point in a* $\mathscr{T}_{\mathfrak{g}}$*-space* $\mathfrak{T}_{\mathfrak{g}} = (\Omega, \mathscr{T}_{\mathfrak{g}})$*. A* $\mathfrak{T}_{\mathfrak{g}}$*-subset* $\mathscr{N}_{\mathfrak{g}} \subseteq \mathfrak{T}_{\mathfrak{g}}$ *of* $\mathfrak{T}_{\mathfrak{g}}$ *is a "*$\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-neighborhood of* ξ*" if and only if* $\mathscr{N}_{\mathfrak{g}}$ *is a* $\mathfrak{T}_{\mathfrak{g}}$*-superset of a* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-open set* $\mathscr{U}_{\mathfrak{g},\xi} \in \mathfrak{g}\text{-O}\left[\mathfrak{T}_{\mathfrak{g}}\right]$ *containing* ξ*:*

$$\left(\xi, \mathscr{N}_{\mathfrak{g}}, \mathscr{U}_{\mathfrak{g},\xi}\right) \in \mathfrak{T}_{\mathfrak{g}} \times \mathfrak{T}_{\mathfrak{g}} \times \mathfrak{g}\text{-O}\left[\mathfrak{T}_{\mathfrak{g}}\right]: \quad \xi \in \mathscr{U}_{\mathfrak{g},\xi} \subseteq \mathscr{N}_{\mathfrak{g}}. \tag{17}$$

The class of all $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-neighborhoods of* $\xi \in \mathfrak{T}_{\mathfrak{g}}$*, defined as*

$$\mathfrak{g}\text{-N}\left[\xi\right] \stackrel{\text{def}}{=} \left\{\mathscr{N}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g}} : \; \left(\exists \mathscr{U}_{\mathfrak{g},\xi} \in \mathfrak{g}\text{-O}\left[\mathfrak{T}_{\mathfrak{g}}\right]\right)\left[\xi \in \mathscr{U}_{\mathfrak{g},\xi} \subseteq \mathscr{N}_{\mathfrak{g}}\right]\right\}, \tag{18}$$

is called the "$\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-neighborhood system of* ξ*".*

Definition 2.12 (Locally $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-Compact *[9, 11]***)** *A* $\mathfrak{T}_{\mathfrak{g}}$*-set* $\mathscr{S}_{\mathfrak{g}} \subseteq \mathfrak{T}_{\mathfrak{g}}$ *of a* $\mathscr{T}_{\mathfrak{g}}$*-space* $\mathfrak{T}_{\mathfrak{g}} = (\Omega, \mathscr{T}_{\mathfrak{g}})$ *is said to be "locally* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-compact" if and only if, given any* $\left(\xi, \mathscr{N}_{\mathfrak{g},\xi}\right) \in \mathscr{S}_{\mathfrak{g}} \times \mathfrak{g}\text{-N}\left[\xi\right]$*, there is a* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-neighborhood* $\hat{\mathscr{N}}_{\mathfrak{g},\xi} \in \mathfrak{g}\text{-N}\left[\xi\right]$ *of* ξ *such that* $\hat{\mathscr{N}}_{\mathfrak{g},\xi} \subset \mathscr{N}_{\mathfrak{g},\xi}$ *and* $\hat{\mathscr{N}}_{\mathfrak{g},\xi} \cup \mathrm{der}_{\mathfrak{g}}\left(\hat{\mathscr{N}}_{\mathfrak{g},\xi}\right) \in \mathfrak{g}\text{-A}\left[\mathfrak{T}_{\mathfrak{g}}\right]$*.*

By omitting the subscript $\mathfrak{g}$ in almost all symbols of the above definitions, we obtain very similar definitions but in a $\mathscr{T}_{\Lambda}$-space; see [9, 11, 12].

3. Main Results

The main results of the theory of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness are presented in this section.

Lemma 3.1 *If* $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-A}\left[\mathfrak{T}_{\mathfrak{g}}\right]$ *is a* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-compact set of a* $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}$*-space* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}^{(\mathrm{H})} = \left(\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}\right)$ *and suppose* $\xi \notin \mathscr{S}_{\mathfrak{g}}$*, then there exists* $\left(\mathscr{U}_{\mathfrak{g},\alpha}, \mathscr{U}_{\mathfrak{g},\beta}\right) \in \mathfrak{g}\text{-O}\left[\mathfrak{T}_{\mathfrak{g}}\right] \times \mathfrak{g}\text{-O}\left[\mathfrak{T}_{\mathfrak{g}}\right]$ *such that* $\left(\{\xi\}, \mathscr{S}_{\mathfrak{g}}\right) \subseteq \left(\mathscr{U}_{\mathfrak{g},\alpha}, \mathscr{U}_{\mathfrak{g},\beta}\right)$ *and* $\bigcap_{\mu=\alpha,\beta} \mathscr{U}_{\mathfrak{g},\mu} = \emptyset$*.*

Proof Let $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}]$ be a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact set of a $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}$-space $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{(\mathrm{H})} = (\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})})$ and suppose $\xi \notin \mathscr{S}_{\mathfrak{g}}$. Since $\xi \notin \mathscr{S}_{\mathfrak{g}}$, it results that $\zeta \in \mathscr{S}_{\mathfrak{g}}$ implies $\xi \notin \{\zeta\}$. But by hypothesis, $\mathfrak{T}_{\mathfrak{g}}$ is a $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}$-space $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{(\mathrm{H})} = (\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})})$ and therefore, there exists $(\mathscr{U}_{\mathfrak{g},\zeta}, \hat{\mathscr{U}}_{\mathfrak{g},\zeta}) \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}] \times \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]$ such that $(\xi, \zeta) \in \mathscr{U}_{\mathfrak{g},\zeta} \times \hat{\mathscr{U}}_{\mathfrak{g},\zeta}$ and $\mathscr{U}_{\mathfrak{g},\zeta} \cap \hat{\mathscr{U}}_{\mathfrak{g},\zeta} = \emptyset$. Hence, it follows that $\mathscr{S}_{\mathfrak{g}} \subseteq \bigcup_{\zeta \in \mathscr{S}_{\mathfrak{g}}} \hat{\mathscr{U}}_{\mathfrak{g},\zeta}$, meaning that $\langle \hat{\mathscr{U}}_{\mathfrak{g},\zeta} \rangle_{\zeta \in \mathscr{S}_{\mathfrak{g}}}$ is a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open covering of $\mathscr{S}_{\mathfrak{g}}$. But $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}]$. Consequently, there exists $\langle \hat{\mathscr{U}}_{\mathfrak{g},\zeta(\mu)} \rangle_{(\mu,\zeta(\mu)) \in I_{\sigma}^{*} \times \mathscr{S}_{\mathfrak{g}}} \prec \langle \hat{\mathscr{U}}_{\mathfrak{g},\zeta} \rangle_{\zeta \in \mathscr{S}_{\mathfrak{g}}}$ such that $\mathscr{S}_{\mathfrak{g}} \subseteq \bigcup_{(\mu,\zeta(\mu)) \in I_{\sigma}^{*} \times \mathscr{S}_{\mathfrak{g}}} \hat{\mathscr{U}}_{\mathfrak{g},\zeta(\mu)}$. Now let

$$\mathscr{U}_{\mathfrak{g},\alpha} = \bigcap_{(\mu,\zeta(\mu)) \in I_{\sigma}^{*} \times \mathscr{S}_{\mathfrak{g}}} \mathscr{U}_{\mathfrak{g},\zeta(\mu)}, \quad \mathscr{U}_{\mathfrak{g},\beta} = \bigcup_{(\mu,\zeta(\mu)) \in I_{\sigma}^{*} \times \mathscr{S}_{\mathfrak{g}}} \hat{\mathscr{U}}_{\mathfrak{g},\zeta(\mu)}.$$

It is evidently that, $(\mathscr{U}_{\mathfrak{g},\alpha}, \mathscr{U}_{\mathfrak{g},\beta}) \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}] \times \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]$, since $(\mathscr{U}_{\mathfrak{g},\zeta(\mu)}, \hat{\mathscr{U}}_{\mathfrak{g},\zeta(\mu)}) \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}] \times \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]$ for every $(\mu, \zeta(\mu)) \in I_{\sigma}^{*} \times \mathscr{S}_{\mathfrak{g}}$. Furthermore, $(\{\xi\}, \mathscr{S}_{\mathfrak{g}}) \subseteq (\mathscr{U}_{\mathfrak{g},\alpha}, \mathscr{U}_{\mathfrak{g},\beta})$, since $\xi \in \mathscr{U}_{\mathfrak{g},\zeta(\mu)}$ for every $(\mu, \zeta(\mu)) \in I_{\sigma}^{*} \times \mathscr{S}_{\mathfrak{g}}$. Lastly, let it be claimed that $\bigcap_{\mu=\alpha,\beta} \mathscr{U}_{\mathfrak{g},\mu} = \emptyset$. Then, $\mathscr{U}_{\mathfrak{g},\zeta(\mu)} \cap \hat{\mathscr{U}}_{\mathfrak{g},\zeta(\mu)} = \emptyset$ for every $(\mu, \zeta(\mu)) \in I_{\sigma}^{*} \times \mathscr{S}_{\mathfrak{g}}$ which, in turn, implies that $\mathscr{U}_{\mathfrak{g},\alpha} \cap \hat{\mathscr{U}}_{\mathfrak{g},\zeta(\mu)} = \emptyset$ for every $(\mu, \zeta(\mu)) \in I_{\sigma}^{*} \times \mathscr{S}_{\mathfrak{g}}$. Hence,

$$\begin{aligned} \bigcap_{\mu=\alpha,\beta} \mathscr{U}_{\mathfrak{g},\mu} = \mathscr{U}_{\mathfrak{g},\alpha} \cap \left(\bigcup_{(\mu,\zeta(\mu)) \in I_{\sigma}^{*} \times \mathscr{S}_{\mathfrak{g}}} \hat{\mathscr{U}}_{\mathfrak{g},\zeta(\mu)} \right) &= \bigcup_{(\mu,\zeta(\mu)) \in I_{\sigma}^{*} \times \mathscr{S}_{\mathfrak{g}}} (\mathscr{U}_{\mathfrak{g},\alpha} \cap \hat{\mathscr{U}}_{\mathfrak{g},\zeta(\mu)}) \\ &= \bigcup_{(\mu,\zeta(\mu)) \in I_{\sigma}^{*} \times \mathscr{S}_{\mathfrak{g}}} \emptyset = \emptyset. \end{aligned}$$

This completes the proof of the lemma. □

Theorem 3.2 *Suppose* $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}]$ *be a* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-compact set of a* $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}$*-space* $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{(\mathrm{H})} = (\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})})$. *If* $\xi \notin \mathscr{S}_{\mathfrak{g}}$, *then there exists a* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-open set* $\mathscr{U}_{\mathfrak{g}} \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]$ *such that* $\xi \in \mathscr{U}_{\mathfrak{g}} \subseteq \complement(\mathscr{S}_{\mathfrak{g}})$.

Proof Let $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}]$ be a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact set of a $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}$-space $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{(\mathrm{H})} = (\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})})$ and suppose $\xi \notin \mathscr{S}_{\mathfrak{g}}$. Since $\mathfrak{T}_{\mathfrak{g}}$ is a $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}$-space $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{(\mathrm{H})} = (\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})})$, there exists then $(\mathscr{U}_{\mathfrak{g}}, \hat{\mathscr{U}}_{\mathfrak{g}}) \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}] \times \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]$ such that $(\{\xi\}, \mathscr{S}_{\mathfrak{g}}) \subseteq (\mathscr{U}_{\mathfrak{g}}, \hat{\mathscr{U}}_{\mathfrak{g}})$ and $\mathscr{U}_{\mathfrak{g}} \cap \hat{\mathscr{U}}_{\mathfrak{g}} = \emptyset$. Hence, $\mathscr{U}_{\mathfrak{g}} \cap \mathscr{S}_{\mathfrak{g}} = \emptyset$ and consequently, $\xi \in \mathscr{U}_{\mathfrak{g}} \subseteq \complement(\mathscr{S}_{\mathfrak{g}})$. This proves the theorem. □

Proposition 3.3 *Suppose* $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}]$ *be a* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-compact set of a* $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}$*-space* $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{(\mathrm{H})} = (\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})})$, *then* $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-K}[\mathfrak{T}_{\mathfrak{g}}]$ *in* $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{(\mathrm{H})}$.

Proof Let $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}]$ be a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact set of a $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}$-space $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{(\mathrm{H})} = \left(\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}\right)$. It must be proved that $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-K}[\mathfrak{T}_{\mathfrak{g}}]$ which is equivalent to prove that $\complement(\mathscr{S}_{\mathfrak{g}}) \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]$ in $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{(\mathrm{H})}$. Let $\xi \in \complement(\mathscr{S}_{\mathfrak{g}})$; that is, $\xi \notin \mathscr{S}_{\mathfrak{g}}$. Since $\xi \notin \mathscr{S}_{\mathfrak{g}}$ there exists a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open set $\mathscr{U}_{\mathfrak{g},\xi} \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]$ such that $\xi \in \mathscr{U}_{\mathfrak{g},\xi} \subseteq \complement(\mathscr{S}_{\mathfrak{g}})$. Consequently, $\complement(\mathscr{S}_{\mathfrak{g}}) = \bigcup_{\xi \in \complement(\mathscr{S}_{\mathfrak{g}})} \mathscr{U}_{\mathfrak{g},\xi}$. Therefore, $\complement(\mathscr{S}_{\mathfrak{g}}) \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]$, since $\mathscr{U}_{\mathfrak{g},\xi} \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]$ for every $\xi \in \complement(\mathscr{S}_{\mathfrak{g}})$. Hence, $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-K}[\mathfrak{T}_{\mathfrak{g}}]$ in $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{(\mathrm{H})}$. This proves the proposition. □

Lemma 3.4 *If $\mathfrak{T}_{\mathfrak{g}} = (\Omega, \mathscr{T}_{\mathfrak{g}})$ is a $\mathscr{T}_{\mathfrak{g}}$-space whose $\mathfrak{g}$-topology $\mathscr{T}_{\mathfrak{g}} : \mathscr{P}(\Omega) \longrightarrow \mathscr{P}(\Omega)$ is cofinite on Ω, then $\mathfrak{T}_{\mathfrak{g}}$ is a $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$-space $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]} = \left(\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}\right)$.*

Proof Let $\mathfrak{T}_{\mathfrak{g}} = (\Omega, \mathscr{T}_{\mathfrak{g}})$ be a $\mathscr{T}_{\mathfrak{g}}$-space whose $\mathfrak{g}$-topology $\mathscr{T}_{\mathfrak{g}} : \mathscr{P}(\Omega) \longrightarrow \mathscr{P}(\Omega)$ is cofinite on Ω and suppose $\left\langle \mathscr{U}_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]\right\rangle_{\alpha \in I_{\sigma}^{*}}$ be a $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}$-open covering of Ω. Then, $\complement(\mathscr{U}_{\mathfrak{g},\alpha}) \in \mathfrak{g}\text{-K}[\mathfrak{T}_{\mathfrak{g}}]$ for any chosen $\alpha \in I_{\sigma}^{*}$. Furthermore, since $\mathscr{T}_{\mathfrak{g}} : \mathscr{P}(\Omega) \longrightarrow \mathscr{P}(\Omega)$ is cofinite on Ω, $\mathscr{U}_{\mathfrak{g},\alpha}$, it follows that, for every $\alpha \in I_{\sigma}^{*}$, $\complement(\mathscr{U}_{\mathfrak{g},\alpha})$ is a finite $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}$-closed set. Set $\complement(\mathscr{U}_{\mathfrak{g},\alpha}) = \left\{\xi_{\beta(\alpha)} : (\alpha, \beta(\alpha)) \in I_{\sigma}^{*} \times I_{\vartheta(\sigma)}^{*}\right\}$. Since $\left\langle \mathscr{U}_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]\right\rangle_{\alpha \in I_{\sigma}^{*}}$ is a $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}$-open covering of Ω, for every $(\alpha, \beta(\alpha)) \in I_{\sigma}^{*} \times I_{\vartheta(\sigma)}^{*}$, $\xi_{\beta(\alpha)} \in \complement(\mathscr{U}_{\mathfrak{g},\alpha})$ implies the existence of $\mathscr{U}_{\mathfrak{g},\gamma(\alpha)}$, where $\left\langle \mathscr{U}_{\mathfrak{g},\gamma(\alpha)}\right\rangle_{(\alpha,\gamma(\alpha)) \in I_{\sigma}^{*} \times I_{\gamma(\sigma)}^{*}} \prec \left\langle \mathscr{U}_{\mathfrak{g},\alpha}\right\rangle_{\alpha \in I_{\sigma}^{*}}$, satisfying $\xi_{\beta(\alpha)} \in \mathscr{U}_{\mathfrak{g},\gamma(\alpha)}$. Hence, it follows that $\complement(\mathscr{U}_{\mathfrak{g},\alpha}) \subseteq \bigcup_{(\alpha,\gamma(\alpha)) \in I_{\sigma}^{*} \times I_{\gamma(\sigma)}^{*}} \mathscr{U}_{\mathfrak{g},\gamma(\alpha)}$ and therefore,

$$\Omega = \mathscr{U}_{\mathfrak{g},\alpha} \cup \complement(\mathscr{U}_{\mathfrak{g},\alpha}) = \mathscr{U}_{\mathfrak{g},\alpha} \cup \left(\bigcup_{(\alpha,\gamma(\alpha)) \in I_{\sigma}^{*} \times I_{\gamma(\sigma)}^{*}} \mathscr{U}_{\mathfrak{g},\gamma(\alpha)} \right).$$

Thus, $\mathfrak{T}_{\mathfrak{g}}$ is a $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$-space $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]} = \left(\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}\right)$. This completes the proof of the lemma. □

Theorem 3.5 *If $(\mathscr{R}_{\mathfrak{g}}, \mathscr{S}_{\mathfrak{g}}) \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}] \times \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}]$ is a pair of disjoint $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}$-compact sets of a $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}$-space $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{(\mathrm{H})} = \left(\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}\right)$, then there exists a pair $(\mathscr{U}_{\mathfrak{g},\alpha}, \mathscr{U}_{\mathfrak{g},\beta}) \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}] \times \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]$ of disjoint $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}$-open sets such that $(\mathscr{R}_{\mathfrak{g}}, \mathscr{S}_{\mathfrak{g}}) \subseteq (\mathscr{U}_{\mathfrak{g},\alpha}, \mathscr{U}_{\mathfrak{g},\beta})$.*

Proof Let $(\mathscr{R}_{\mathfrak{g}}, \mathscr{S}_{\mathfrak{g}}) \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}] \times \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}]$ be a pair of disjoint $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}$-compact sets of a $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}$-space $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{(\mathrm{H})} = \left(\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}\right)$ and suppose $\xi \in \mathscr{R}_{\mathfrak{g}}$. Then, since $\mathscr{R}_{\mathfrak{g}} \cap \mathscr{S}_{\mathfrak{g}} = \emptyset$, it results that $\xi \notin \mathscr{S}_{\mathfrak{g}}$. But by hypothesis, $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}]$ and consequently, there exists $\left(\mathscr{U}_{\mathfrak{g},\xi}, \hat{\mathscr{U}}_{\mathfrak{g},\xi}\right) \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}] \times \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]$ such that $\left(\{\xi\}, \mathscr{S}_{\mathfrak{g}}\right) \subseteq \left(\mathscr{U}_{\mathfrak{g},\xi}, \hat{\mathscr{U}}_{\mathfrak{g},\xi}\right)$ and $\mathscr{U}_{\mathfrak{g},\xi} \cap \hat{\mathscr{U}}_{\mathfrak{g},\xi} = \emptyset$. Since $\xi \in \mathscr{U}_{\mathfrak{g},\xi}$, it follows that $\left\langle \mathscr{U}_{\mathfrak{g},\xi}\right\rangle_{\xi \in \mathscr{R}_{\mathfrak{g}}}$

is a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open covering of $\mathscr{R}_{\mathfrak{g}}$. Since $\mathscr{R}_{\mathfrak{g}} \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}]$, a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open subcovering

$$\left\langle \mathscr{U}_{\mathfrak{g},\upsilon(\xi)} \right\rangle_{(\xi,\upsilon(\xi))\in\hat{\mathscr{R}}_{\mathfrak{g}}\times\mathscr{R}_{\mathfrak{g}}} \prec \left\langle \mathscr{U}_{\mathfrak{g},\xi} \right\rangle_{\xi\in\mathscr{R}_{\mathfrak{g}}},$$

where $\hat{\mathscr{R}}_{\mathfrak{g}} \subseteq \mathscr{R}_{\mathfrak{g}}$ is finite, can be selected so that $\mathscr{R}_{\mathfrak{g}} \subseteq \bigcup_{(\xi,\upsilon(\xi))\in\hat{\mathscr{R}}_{\mathfrak{g}}\times\mathscr{R}_{\mathfrak{g}}} \mathscr{U}_{\mathfrak{g},\upsilon(\xi)}$. Furthermore, $\mathscr{S}_{\mathfrak{g}} \subseteq \bigcap_{(\zeta,\vartheta(\zeta))\in\hat{\mathscr{S}}_{\mathfrak{g}}\times\mathscr{S}_{\mathfrak{g}}} \hat{\mathscr{U}}_{\mathfrak{g},\vartheta(\xi)}$, where $\hat{\mathscr{S}}_{\mathfrak{g}} \subseteq \mathscr{S}_{\mathfrak{g}}$ is finite, since $\mathscr{S}_{\mathfrak{g}} \subseteq \hat{\mathscr{U}}_{\mathfrak{g},\vartheta(\xi)}$ for every $(\zeta,\vartheta(\zeta)) \in \hat{\mathscr{S}}_{\mathfrak{g}} \times \mathscr{S}_{\mathfrak{g}}$. Now let

$$\mathscr{U}_{\mathfrak{g},\alpha} = \bigcup_{(\xi,\upsilon(\xi))\in\hat{\mathscr{R}}_{\mathfrak{g}}\times\mathscr{R}_{\mathfrak{g}}} \mathscr{U}_{\mathfrak{g},\upsilon(\xi)}, \quad \mathscr{U}_{\mathfrak{g},\beta} = \bigcap_{(\zeta,\vartheta(\zeta))\in\hat{\mathscr{S}}_{\mathfrak{g}}\times\mathscr{S}_{\mathfrak{g}}} \hat{\mathscr{U}}_{\mathfrak{g},\vartheta(\zeta)}.$$

Observe that $(\mathscr{R}_{\mathfrak{g}}, \mathscr{S}_{\mathfrak{g}}) \subseteq (\mathscr{U}_{\mathfrak{g},\alpha}, \mathscr{U}_{\mathfrak{g},\beta})$. Moreover, $(\mathscr{U}_{\mathfrak{g},\alpha}, \mathscr{U}_{\mathfrak{g},\beta}) \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}] \times \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]$, since $\mathscr{U}_{\mathfrak{g},\upsilon(\xi)} \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]$ for every $(\xi,\upsilon(\xi)) \in \hat{\mathscr{R}}_{\mathfrak{g}} \times \mathscr{R}_{\mathfrak{g}}$ and $\hat{\mathscr{U}}_{\mathfrak{g},\vartheta(\zeta)} \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]$ for every $(\zeta,\vartheta(\zeta)) \in \hat{\mathscr{S}}_{\mathfrak{g}} \times \mathscr{S}_{\mathfrak{g}}$. The proof of the theorem is complete when the statement $\mathscr{U}_{\mathfrak{g},\alpha} \cap \mathscr{U}_{\mathfrak{g},\beta} = \emptyset$ is proved. First observe that, for every $(\xi,\zeta,\upsilon(\xi),\vartheta(\zeta)) \in \hat{\mathscr{R}}_{\mathfrak{g}} \times \hat{\mathscr{S}}_{\mathfrak{g}} \times \mathscr{R}_{\mathfrak{g}} \times \mathscr{S}_{\mathfrak{g}}$, the relation $\mathscr{U}_{\mathfrak{g},\upsilon(\xi)} \cap \hat{\mathscr{U}}_{\mathfrak{g},\vartheta(\zeta)} = \emptyset$ implies $\mathscr{U}_{\mathfrak{g},\upsilon(\xi)} \cap \mathscr{U}_{\mathfrak{g},\beta} = \emptyset$. Consequently,

$$\begin{aligned}
\bigcap_{\mu=\alpha,\beta} \mathscr{U}_{\mathfrak{g},\mu} = \Bigg(\bigcup_{(\xi,\upsilon(\xi))\in\hat{\mathscr{R}}_{\mathfrak{g}}\times\mathscr{R}_{\mathfrak{g}}} \mathscr{U}_{\mathfrak{g},\upsilon(\xi)} \Bigg) \cap \mathscr{U}_{\mathfrak{g},\beta} &= \bigcup_{(\xi,\upsilon(\xi))\in\hat{\mathscr{R}}_{\mathfrak{g}}\times\mathscr{R}_{\mathfrak{g}}} \left(\mathscr{U}_{\mathfrak{g},\upsilon(\xi)} \cap \mathscr{U}_{\mathfrak{g},\beta} \right) \\
&= \bigcup_{(\xi,\upsilon(\xi))\in\hat{\mathscr{R}}_{\mathfrak{g}}\times\mathscr{R}_{\mathfrak{g}}} \emptyset = \emptyset.
\end{aligned}$$

This proves the theorem. □

Theorem 3.6 *Let* $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathscr{T}_{\mathfrak{g},\Omega})$ *and* $\mathfrak{T}_{\mathfrak{g},\Sigma} = (\Sigma, \mathscr{T}_{\mathfrak{g},\Sigma})$ *be* $\mathscr{T}_{\mathfrak{g}}$*-spaces. If* $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \longrightarrow \mathfrak{T}_{\mathfrak{g},\Sigma}$ *is a* $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$*-continuous map and* $\mathscr{S}_{\mathfrak{g},\omega} \in \mathfrak{g}\text{-A}\big[\mathfrak{T}_{\mathfrak{g},\Omega}\big]$ *in* $\mathfrak{T}_{\mathfrak{g},\Omega}$*, then* $\mathrm{im}\big(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}}\big) \in \mathfrak{g}\text{-A}\big[\mathfrak{T}_{\mathfrak{g},\Sigma}\big]$ *in* $\mathfrak{T}_{\mathfrak{g},\Sigma}$*.*

Proof Let $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathscr{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Sigma} = (\Sigma, \mathscr{T}_{\mathfrak{g},\Sigma})$ be given $\mathscr{T}_{\mathfrak{g}}$-spaces, $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-C}[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Sigma}]$, $\mathscr{S}_{\mathfrak{g},\omega} \in \mathfrak{g}\text{-A}\big[\mathfrak{T}_{\mathfrak{g},\Omega}\big]$ in $\mathfrak{T}_{\mathfrak{g},\Omega}$ and suppose $\left\langle \mathscr{U}_{\mathfrak{g},\alpha} \right\rangle_{\alpha\in I_{\sigma}^{*}}$ be a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open covering of $\mathrm{im}\big(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}}\big)$ in $\mathfrak{T}_{\mathfrak{g},\Sigma}$. Then,

$$\mathscr{S}_{\mathfrak{g},\omega} \subseteq \pi_{\mathfrak{g}}^{-1} \circ \pi_{\mathfrak{g}}(\mathscr{S}_{\mathfrak{g},\omega}) \subseteq \pi_{\mathfrak{g}}^{-1}\Big(\bigcup_{\alpha\in I_{\sigma}^{*}} \mathscr{U}_{\mathfrak{g},\alpha} \Big) \subseteq \bigcup_{\alpha\in I_{\sigma}^{*}} \pi_{\mathfrak{g}}^{-1}(\mathscr{U}_{\mathfrak{g},\alpha}).$$

Thus, $\left\langle \pi_{\mathfrak{g}}^{-1}(\mathscr{U}_{\mathfrak{g},\alpha}) \right\rangle_{\alpha\in I_{\sigma}^{*}}$ is a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open covering of $\mathscr{S}_{\mathfrak{g},\omega}$ in $\mathfrak{T}_{\mathfrak{g},\Sigma}$, because $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-C}[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Sigma}]$ and for every $\alpha \in I_{\sigma}^{*}$, $\mathscr{U}_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g},\Omega}]$ implies $\pi_{\mathfrak{g}}^{-1}(\mathscr{U}_{\mathfrak{g},\alpha}) \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g},\Sigma}]$. But, the relation $\mathscr{S}_{\mathfrak{g},\omega} \in \mathfrak{g}\text{-A}\big[\mathfrak{T}_{\mathfrak{g},\Omega}\big]$ holds and consequently, there exists $\left\langle \pi_{\mathfrak{g}}^{-1}\left(\mathscr{U}_{\mathfrak{g},\vartheta(\alpha)}\right) \right\rangle_{(\alpha,\vartheta(\alpha))\in I_{\sigma}^{*}\times I_{\vartheta(\sigma)}^{*}} \prec$

$\langle\pi_{\mathfrak{g}}^{-1}(\mathscr{U}_{\mathfrak{g},\alpha})\rangle_{\alpha\in I_{\sigma}^{*}}$ such that the relation $\mathscr{S}_{\mathfrak{g},\omega}\subseteq\bigcup_{(\alpha,\vartheta(\alpha))\in I_{\sigma}^{*}\times I_{\vartheta(\sigma)}^{*}}\pi_{\mathfrak{g}}^{-1}(\mathscr{U}_{\mathfrak{g},\vartheta(\alpha)})$ holds. Accordingly,

$$\pi_{\mathfrak{g}}(\mathscr{S}_{\mathfrak{g},\omega})\subseteq\pi_{\mathfrak{g}}\circ\pi_{\mathfrak{g}}^{-1}\Bigg(\bigcup_{(\alpha,\vartheta(\alpha))\in I_{\sigma}^{*}\times I_{\vartheta(\sigma)}^{*}}\mathscr{U}_{\mathfrak{g},\vartheta(\alpha)}\Bigg)=\bigcup_{(\alpha,\vartheta(\alpha))\in I_{\sigma}^{*}\times I_{\vartheta(\sigma)}^{*}}\mathscr{U}_{\mathfrak{g},\vartheta(\alpha)}.$$

Thus, $\langle\mathscr{U}_{\mathfrak{g},\vartheta(\alpha)}\rangle_{(\alpha,\vartheta(\alpha))\in I_{\sigma}^{*}\times I_{\vartheta(\sigma)}^{*}}$ is a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open subcovering of $\mathrm{im}(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}})$ and hence, it follows that $\mathrm{im}(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}})\in\mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g},\Sigma}]$ in $\mathfrak{T}_{\mathfrak{g},\Sigma}$. The proof of the theorem is complete. □

Theorem 3.7 *Let $\mathscr{S}_{\mathfrak{g},\omega}\subset\mathfrak{T}_{\mathfrak{g},\Omega}$ be a $\mathfrak{T}_{\mathfrak{g}}$-set and let $\pi_{\mathfrak{g}}\in\mathfrak{g}\text{-I}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$ be a $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-irresolute map, where $\mathfrak{T}_{\mathfrak{g},\Omega}=(\Omega,\mathscr{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Sigma}=(\Sigma,\mathscr{T}_{\mathfrak{g},\Sigma})$ are $\mathscr{T}_{\mathfrak{g}}$-spaces. If $\mathscr{S}_{\mathfrak{g},\omega}\in\mathfrak{g}\text{-A}[\mathfrak{T}_{\Omega}]$, then $\mathrm{im}(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}})\in\mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g},\Sigma}]$.*

Proof Let $\mathscr{S}_{\mathfrak{g},\omega}\subset\mathfrak{T}_{\mathfrak{g},\Omega}$ be a $\mathfrak{T}_{\mathfrak{g}}$-set and let $\pi_{\mathfrak{g}}\in\mathfrak{g}\text{-I}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$ be a $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega},\mathfrak{T}_{\mathfrak{g},\Sigma})$-irresolute map, where $\mathfrak{T}_{\mathfrak{g},\Omega}=(\Omega,\mathscr{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Sigma}=(\Sigma,\mathscr{T}_{\mathfrak{g},\Sigma})$ are $\mathscr{T}_{\mathfrak{g}}$-spaces. Suppose $\mathscr{S}_{\mathfrak{g},\omega}\in\mathfrak{g}\text{-A}[\mathfrak{T}_{\Omega}]$, let $\langle\mathscr{U}_{\mathfrak{g},\alpha}\in\mathfrak{g}\text{-O}[\mathfrak{T}_{\Sigma}]\rangle_{\alpha\in I_{\sigma}^{*}}$ be any $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open covering of $\pi_{\mathfrak{g}}(\mathscr{S}_{\mathfrak{g},\omega})\subset\mathfrak{T}_{\mathfrak{g},\Sigma}$. Then, since $\pi_{\mathfrak{g}}\in\mathfrak{g}\text{-I}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, it follows, evidently, that the relation $\mathscr{S}_{\mathfrak{g},\omega}\bigcup_{\alpha\in I_{\sigma}^{*}}\pi_{\mathfrak{g}}^{-1}(\mathscr{U}_{\mathfrak{g},\alpha})$ holds. On the other hand, since $\mathscr{S}_{\mathfrak{g},\omega}\in\mathfrak{g}\text{-A}[\mathfrak{T}_{\Omega}]$, it results that, a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open subcovering $\langle\mathscr{U}_{\mathfrak{g},\vartheta(\alpha)}\rangle_{(\alpha,\vartheta(\alpha))\in I_{\sigma}^{*}\times I_{\vartheta(\sigma)}^{*}}\prec\langle\mathscr{U}_{\mathfrak{g},\alpha}\rangle_{\alpha\in I_{\sigma}^{*}}$ exists such that the relation $\mathscr{S}_{\mathfrak{g},\omega}\subseteq\bigcup_{(\alpha,\vartheta(\alpha))\in I_{\sigma}^{*}\times I_{\vartheta(\sigma)}^{*}}\pi_{\mathfrak{g}}^{-1}(\mathscr{U}_{\mathfrak{g},\vartheta(\alpha)})$ holds. Consequently, it follows, then, that $\pi_{\mathfrak{g}}(\mathscr{S}_{\mathfrak{g},\omega})\subseteq\bigcup_{(\alpha,\vartheta(\alpha))\in I_{\sigma}^{*}\times I_{\vartheta(\sigma)}^{*}}\mathscr{U}_{\mathfrak{g},\vartheta(\alpha)}$ and hence, $\mathrm{im}(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}})\in\mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g},\Sigma}]$. The proof of the theorem is complete. □

Lemma 3.8 *Let $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}=(\Omega,\mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]})$ be a $\mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$-space. If $\mathscr{S}_{\mathfrak{g}}\in\mathfrak{g}\text{-K}[\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}]$, then $\mathscr{S}_{\mathfrak{g}}\in\mathfrak{g}\text{-A}[\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}]$ in $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}$.*

Proof Let $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}=(\Omega,\mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]})$ be a $\mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$-space and suppose $\mathscr{S}_{\mathfrak{g}}\in\mathfrak{g}\text{-K}[\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}]$. Suppose $\langle\mathscr{U}_{\mathfrak{g},\alpha}\in\mathfrak{g}\text{-O}[\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}]\rangle_{\alpha\in I_{\sigma}^{*}}$ be a $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}$-open covering of $\mathscr{S}_{\mathfrak{g}}$, then $\Omega=(\bigcup_{\alpha\in I_{\sigma}^{*}}\mathscr{U}_{\mathfrak{g},\alpha})\cup\complement(\mathscr{S}_{\mathfrak{g}})=\bigcup_{\alpha\in I_{\sigma}^{*}}(\mathscr{U}_{\mathfrak{g},\alpha}\cup\complement(\mathscr{S}_{\mathfrak{g}}))$, meaning that $\langle\mathscr{U}_{\mathfrak{g},\alpha}\cup\complement(\mathscr{S}_{\mathfrak{g}})\rangle_{\alpha\in I_{\sigma}^{*}}$ is a $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}$-open covering of $\mathscr{S}_{\mathfrak{g}}$ because, $\mathscr{S}_{\mathfrak{g}}\in\mathfrak{g}\text{-K}[\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}]$ implies $\complement(\mathscr{S}_{\mathfrak{g}})\in\mathfrak{g}\text{-O}[\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}]$. On the other hand, $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}$ is, by hypothesis, a $\mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$-space. Thus, there exists $\langle\mathscr{U}_{\mathfrak{g},\vartheta(\alpha)}\rangle_{(\alpha,\vartheta(\alpha))\in I_{\sigma}^{*}\times I_{\vartheta(\sigma)}^{*}}\prec\langle\mathscr{U}_{\mathfrak{g},\alpha}\rangle_{\alpha\in I_{\sigma}^{*}}$ such that $\Omega=(\bigcup_{(\alpha,\vartheta(\alpha))\in I_{\sigma}^{*}\times I_{\vartheta(\sigma)}^{*}}\mathscr{U}_{\mathfrak{g},\vartheta(\alpha)})\cup\complement(\mathscr{S}_{\mathfrak{g}})$. But $\mathscr{S}_{\mathfrak{g}}\cap\complement(\mathscr{S}_{\mathfrak{g}})=\emptyset$ and hence, $\mathscr{S}_{\mathfrak{g}}\subseteq\bigcup_{(\alpha,\vartheta(\alpha))\in I_{\sigma}^{*}\times I_{\vartheta(\sigma)}^{*}}\mathscr{U}_{\mathfrak{g},\vartheta(\alpha)}$. This shows that any $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}$-open covering $\langle\mathscr{U}_{\mathfrak{g},\alpha}\cup\complement(\mathscr{S}_{\mathfrak{g}})\rangle_{\alpha\in I_{\sigma}^{*}}$ of $\mathscr{S}_{\mathfrak{g}}$

contains a finite $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}$-open subcovering $\langle \mathscr{U}_{\mathfrak{g},\vartheta(\alpha)} \rangle_{(\alpha,\vartheta(\alpha))\in I_{\sigma}^{*}\times I_{\vartheta(\sigma)}^{*}}$ and hence, $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-}\mathrm{A}\big[\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}\big]$ in $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}$. The proof of the lemma is complete. □

Theorem 3.9 *Let* $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]} = \big(\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]}\big)$ *be a* $\mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$*-space and let* $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Sigma}^{(\mathrm{H})} = \big(\Sigma, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g},\Sigma}^{(\mathrm{H})}\big)$ *be a* $\mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}$*-space. If the* $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}$*-map* $\pi_{\mathfrak{g}} : \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]} \longrightarrow \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Sigma}^{(\mathrm{H})}$ *is a one-one* $\mathfrak{g}\text{-}\big(\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]}, \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Sigma}^{(\mathrm{H})}\big)$*-continuous map, then* $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]} \cong \pi_{\mathfrak{g}}\big(\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]}\big)$.

Proof Let $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]} = \big(\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]}\big)$ be a $\mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$-space and let $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Sigma}^{(\mathrm{H})} = \big(\Sigma, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g},\Sigma}^{(\mathrm{H})}\big)$ be a $\mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}$-space, and suppose $\pi_{\mathfrak{g}} : \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]} \longrightarrow \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Sigma}^{(\mathrm{H})}$ is a one-one $\mathfrak{g}\text{-}\big(\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]}, \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Sigma}^{(\mathrm{H})}\big)$-continuous map. Clearly, $\pi_{\mathfrak{g}} : \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]} \longrightarrow \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Sigma}^{(\mathrm{H})}$ is onto, and since it is, by hypothesis a one-one $\mathfrak{g}\text{-}\big(\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]}, \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Sigma}^{(\mathrm{H})}\big)$-continuous map, it follows that $\pi_{\mathfrak{g}}^{-1} : \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Sigma}^{(\mathrm{H})} \longrightarrow \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]}$ exists. It must be shown that $\pi_{\mathfrak{g}}^{-1} \in \mathfrak{g}\text{-}\mathrm{C}\big[\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Sigma}^{(\mathrm{H})}; \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]}\big]$. Recall that $\pi_{\mathfrak{g}}^{-1} : \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Sigma}^{(\mathrm{H})} \longrightarrow \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]}$ is $\mathfrak{g}\text{-}\big(\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Sigma}^{(\mathrm{H})}, \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]}\big)$-continuous if and only if, for every $\mathscr{K}_{\mathfrak{g},\omega} \in \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]}$, $\big(\pi_{\mathfrak{g}}^{-1}\big)^{-1}(\mathscr{K}_{\mathfrak{g},\omega}) = \pi_{\mathfrak{g}}(\mathscr{K}_{\mathfrak{g},\omega}) \in \mathfrak{g}\text{-}\mathrm{K}\big[\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{(\mathrm{H})}\big]$ and $\pi_{\mathfrak{g}}(\mathscr{K}_{\mathfrak{g},\omega}) \subseteq \mathrm{im}\big(\pi_{\mathfrak{g}|\Sigma}\big)$. Clearly, $\mathscr{K}_{\mathfrak{g},\omega} \supseteq \neg\,\mathrm{op}_{\mathfrak{g}}(\mathscr{K}_{\mathfrak{g},\omega})$, so $\mathscr{K}_{\mathfrak{g},\omega} \in \mathfrak{g}\text{-}\mathrm{K}\big[\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}\big]$. But, $\mathscr{K}_{\mathfrak{g},\omega} \in \mathfrak{g}\text{-}\mathrm{K}\big[\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}\big]$ implies $\mathscr{K}_{\mathfrak{g},\omega} \in \mathfrak{g}\text{-}\mathrm{A}\big[\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}\big]$ in $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}$. Furthermore, since $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-}\mathrm{C}\big[\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]}; \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Sigma}^{(\mathrm{H})}\big]$, it follows that $\pi_{\mathfrak{g}}(\mathscr{K}_{\mathfrak{g},\omega}) \in \mathfrak{g}\text{-}\mathrm{A}\big[\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{(\mathrm{H})}\big]$ and $\pi_{\mathfrak{g}}(\mathscr{K}_{\mathfrak{g},\omega}) \subseteq \mathrm{im}\big(\pi_{\mathfrak{g}|\Sigma}\big)$. But, $\pi_{\mathfrak{g}}(\mathscr{K}_{\mathfrak{g},\omega}) \in \mathfrak{g}\text{-}\mathrm{A}\big[\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{(\mathrm{H})}\big]$ implies $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-}\mathrm{K}\big[\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{(\mathrm{H})}\big]$. Accordingly, $\pi_{\mathfrak{g}}^{-1} \in \mathfrak{g}\text{-}\mathrm{C}\big[\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Sigma}^{(\mathrm{H})}; \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]}\big]$ and hence, $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]} \cong \pi_{\mathfrak{g}}\big(\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]}\big)$. The proof of the theorem is complete. □

Proposition 3.10 *Let* $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]} = \big(\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}\big)$ *be a* $\mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$*-space and let* $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{(\mathrm{H})} = \big(\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}\big)$ *be a* $\mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}$*-space. If* $\mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]} \supseteq \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g},}^{(\mathrm{H})}$*, then* $\mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]} = \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g},}^{(\mathrm{H})}$.

Proof Let $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]} = \big(\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}\big)$ be a $\mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$-space and $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{(\mathrm{H})} = \big(\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}\big)$, a $\mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}$-space, and suppose $\mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]} \supseteq \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g},}^{(\mathrm{H})}$. Further, consider the $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}$-map $\pi_{\mathfrak{g}} : \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]} \longrightarrow \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{(\mathrm{H})}$ defined by $\pi_{\mathfrak{g}}(\xi) = \xi$. Since $\mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]} \supseteq \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g},}^{(\mathrm{H})}$, for every $\mathscr{O}_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g},}^{(\mathrm{H})}$, there exist $\mathscr{O}_{\mathfrak{g},\vartheta(\alpha)} \in \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$ such that $\pi_{\mathfrak{g}}^{-1}\big(\mathscr{O}_{\mathfrak{g},\vartheta(\alpha)}\big) = \mathscr{O}_{\mathfrak{g},\alpha} \subseteq \mathrm{op}_{\mathfrak{g}}(\mathscr{O}_{\mathfrak{g},\alpha})$. Consequently, $\pi_{\mathfrak{g}} : \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]} \longrightarrow \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Sigma}^{(\mathrm{H})}$ is a one-one and onto $\mathfrak{g}\text{-}\big(\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]}, \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Sigma}^{(\mathrm{H})}\big)$-continuous map from a $\mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$-space $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]}$ to a $\mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{(\mathrm{H})}$-space $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Sigma}^{(\mathrm{H})}$ and therefore, $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]} \cong \pi_{\mathfrak{g}}\big(\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g},\Omega}^{[\mathrm{A}]}\big)$. Hence, $\mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]} = \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g},}^{(\mathrm{H})}$. The proof of the proposition is complete. □

Theorem 3.11 *If* $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-}\mathrm{A}[\mathfrak{T}_{\mathfrak{g}}]$ *is a* $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}$*-compact set of a* $\mathscr{T}_{\mathfrak{g}}$*-space* $\mathfrak{T}_{\mathfrak{g}} = (\Omega, \mathscr{T}_{\mathfrak{g}})$*, then it is* *also countably* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-compact in* $\mathfrak{T}_{\mathfrak{g}}$.

Proof Let $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}]$ be a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact set of a $\mathscr{T}_{\mathfrak{g}}$-space $\mathfrak{T}_{\mathfrak{g}} = (\Omega, \mathscr{T}_{\mathfrak{g}})$ and suppose $\mathscr{R}_{\mathfrak{g}} \subset \mathscr{S}_{\mathfrak{g}}$ be any infinite $\mathfrak{T}_{\mathfrak{g}}$-subset of $\mathscr{S}_{\mathfrak{g}}$. Equivalently proved, it must be shown that, the assumption that $\mathscr{R}_{\mathfrak{g}}$ has no $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-accumulation point $\xi \in \mathscr{S}_{\mathfrak{g}}$ leads to a contradiction. Since $\mathscr{R}_{\mathfrak{g}} \subset \mathscr{S}_{\mathfrak{g}}$ is, by assumption, an infinite $\mathfrak{T}_{\mathfrak{g}}$-subset of $\mathscr{S}_{\mathfrak{g}}$ with no $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-accumulation point $\xi \in \mathscr{S}_{\mathfrak{g}}$, it follows that, for every $\xi \in \mathscr{S}_{\mathfrak{g}}$, there exists a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open set $\mathscr{U}_{\mathfrak{g},\xi} \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]$ which contains at most one point $\zeta \in \mathscr{R}_{\mathfrak{g}}$. It may be remarked, in passing, that $\langle \mathscr{U}_{\mathfrak{g},\xi} \rangle_{\xi \in \mathscr{S}_{\mathfrak{g}}}$ is a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open covering of the $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact set $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}]$ for $\mathscr{S}_{\mathfrak{g}} \subseteq \bigcup_{\xi \in \mathscr{S}_{\mathfrak{g}}} \mathscr{U}_{\mathfrak{g},\xi}$. Consequently, there exists a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open subcovering $\langle \mathscr{U}_{\mathfrak{g},\vartheta(\xi)} \rangle_{(\xi,\vartheta(\xi)) \in \mathscr{S}_{\mathfrak{g}} \times \hat{\mathscr{S}}_{\mathfrak{g}}} \prec \langle \mathscr{U}_{\mathfrak{g},\alpha} \rangle_{\alpha \in I_{\sigma}^{*}}$, where $\hat{\mathscr{S}}_{\mathfrak{g}} \subset \mathscr{S}_{\mathfrak{g}}$, such $\mathscr{R}_{\mathfrak{g}} \subseteq \mathscr{S}_{\mathfrak{g}} \subseteq \bigcup_{(\xi,\vartheta(\xi)) \in \mathscr{S}_{\mathfrak{g}} \times \hat{\mathscr{S}}_{\mathfrak{g}}} \mathscr{U}_{\mathfrak{g},\vartheta(\xi)}$. But, for every $(\xi, \vartheta(\xi)) \in \mathscr{S}_{\mathfrak{g}} \times \hat{\mathscr{S}}_{\mathfrak{g}}$, $\mathscr{U}_{\mathfrak{g},\vartheta(\xi)}$ contains at most one point $\zeta \in \mathscr{R}_{\mathfrak{g}}$. Therefore, the infinite $\mathfrak{T}_{\mathfrak{g}}$-subset $\mathscr{R}_{\mathfrak{g}}$ of $\mathscr{S}_{\mathfrak{g}}$, satisfying $\mathscr{R}_{\mathfrak{g}} \subseteq \bigcup_{(\xi,\vartheta(\xi)) \in \mathscr{S}_{\mathfrak{g}} \times \hat{\mathscr{S}}_{\mathfrak{g}}} \mathscr{U}_{\mathfrak{g},\vartheta(\xi)}$, can contain at most $\eta = \operatorname{card}(\hat{\mathscr{S}}_{\mathfrak{g}}) < \infty$ points. Accordingly, it follows that every infinite $\mathfrak{T}_{\mathfrak{g}}$-subset $\mathscr{R}_{\mathfrak{g}} \subset \mathscr{S}_{\mathfrak{g}}$ of $\mathscr{S}_{\mathfrak{g}}$ contains a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-accumulation point $\xi \in \mathscr{S}_{\mathfrak{g}}$. Hence, $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}]$ is also countably $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact in $\mathfrak{T}_{\mathfrak{g}}$. This completes the proof of the theorem. □

Corollary 3.12 *Every* $\mathscr{T}_{\mathfrak{g}}$*-space* $\mathfrak{T}_{\mathfrak{g}} = (\Omega, \mathscr{T}_{\mathfrak{g}})$ *having the property that every countable* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-open covering* $\langle \mathscr{U}_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}] \rangle_{\alpha \in I_{\sigma}^{*}}$ *of the* $\mathscr{T}_{\mathfrak{g}}$*-space* $\mathfrak{T}_{\mathfrak{g}}$ *contains a finite* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-open subcovering* $\langle \mathscr{U}_{\mathfrak{g},\vartheta(\alpha)} \rangle_{(\alpha,\vartheta(\alpha)) \in I_{\sigma}^{*} \times I_{\vartheta(\sigma)}^{*}} \prec \langle \mathscr{U}_{\mathfrak{g},\alpha} \rangle_{\alpha \in I_{\sigma}^{*}}$ *of* $\mathfrak{T}_{\mathfrak{g}}$ *is a countably* $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$*-space* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]} = (\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]})$.

Theorem 3.13 *Let* $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \longrightarrow \mathfrak{T}_{\mathfrak{g},\Sigma}$ *be a* $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$*-continuous map, where* $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathscr{T}_{\mathfrak{g},\Omega})$ *and* $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathscr{T}_{\mathfrak{g},\Omega})$ *are* $\mathscr{T}_{\mathfrak{g}}$*-spaces. If* $\mathscr{S}_{\mathfrak{g},\omega} \subset \mathfrak{T}_{\mathfrak{g},\Omega}$ *be a sequentially* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-compact set in* $\mathfrak{T}_{\mathfrak{g},\Omega}$*, then* $\operatorname{im}(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}}) \subset \mathfrak{T}_{\mathfrak{g},\Sigma}$ *is also a sequentially* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-compact set in* $\mathfrak{T}_{\mathfrak{g},\Sigma}$.

Proof Let $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-C}[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Sigma}]$, where $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathscr{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathscr{T}_{\mathfrak{g},\Omega})$ are $\mathscr{T}_{\mathfrak{g}}$-spaces, and suppose $\mathscr{S}_{\mathfrak{g},\omega} \subset \mathfrak{T}_{\mathfrak{g},\Omega}$ be a sequentially $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact set in $\mathfrak{T}_{\mathfrak{g},\Omega}$. If $\langle \zeta_{\alpha} \in \pi_{\mathfrak{g}}(\mathscr{S}_{\mathfrak{g},\omega}) \rangle_{\alpha \in I_{\infty}^{*}}$ be a sequence in $\operatorname{im}(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}}) \subset \mathfrak{T}_{\mathfrak{g},\Sigma}$, then there exists a sequence $\langle \xi_{\alpha} \in \mathscr{S}_{\mathfrak{g}} \rangle_{\alpha \in I_{\infty}^{*}}$ in $\mathscr{S}_{\mathfrak{g}}$ such $\pi_{\mathfrak{g}}(\xi_{\alpha}) = \zeta_{\alpha}$ that for every $\alpha \in I_{\infty}^{*}$. But, by hypothesis, $\mathscr{S}_{\mathfrak{g},\omega} \subset \mathfrak{T}_{\mathfrak{g},\Omega}$ is sequentially $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact in $\mathfrak{T}_{\mathfrak{g},\Omega}$. Therefore, there exists a subsequence $\langle \xi_{\alpha} \rangle_{\alpha \in I_{\infty}^{*}} \prec \langle \xi_{\vartheta(\alpha)} \rangle_{(\alpha,\vartheta(\alpha)) \in I_{\infty}^{*} \times I_{\infty}^{*}}$ which converges to a point $\xi \in \mathscr{S}_{\mathfrak{g}}$. On the other hand, $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-C}[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Sigma}]$ and therefore, $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \longrightarrow \mathfrak{T}_{\mathfrak{g},\Sigma}$ is sequentially $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-continuous. Consequently, it results that $\langle \pi_{\mathfrak{g}}(\xi_{\vartheta(\alpha)}) \rangle_{(\alpha,\vartheta(\alpha)) \in I_{\infty}^{*} \times I_{\infty}^{*}} = \langle \zeta_{\vartheta(\alpha)} \rangle_{(\alpha,\vartheta(\alpha)) \in I_{\infty}^{*} \times I_{\infty}^{*}}$ converges to $\pi_{\mathfrak{g}}(\xi) \in \operatorname{im}(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}})$. Hence,

$\mathrm{im}\big(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}}\big) \subset \mathfrak{T}_{\mathfrak{g},\Sigma}$ is sequentially $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact in $\mathfrak{T}_{\mathfrak{g},\Sigma}$. □

Proposition 3.14 *Let $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \longrightarrow \mathfrak{T}_{\mathfrak{g},\Sigma}$ be a $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-continuous map, where $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathscr{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathscr{T}_{\mathfrak{g},\Omega})$ are $\mathscr{T}_{\mathfrak{g}}$-spaces. If $\mathscr{S}_{\mathfrak{g},\omega} \in \mathfrak{g}\text{-}\mathrm{A}[\mathfrak{T}_{\mathfrak{g},\Omega}]$ is a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact set in $\mathfrak{T}_{\mathfrak{g},\Omega}$, then $\mathrm{im}\big(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}}\big) \in \mathrm{A}[\mathfrak{T}_{\mathfrak{g},\Sigma}]$ is also $\mathfrak{T}_{\mathfrak{g}}$-compact in $\mathfrak{T}_{\mathfrak{g},\Sigma}$.*

Proof Let $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-}\mathrm{C}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, where $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathscr{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathscr{T}_{\mathfrak{g},\Omega})$ are $\mathscr{T}_{\mathfrak{g}}$-spaces, and suppose $\langle \mathscr{U}_{\mathfrak{g},\alpha} \in \mathrm{O}[\mathfrak{T}_{\mathfrak{g},\Sigma}]\rangle_{\alpha\in I_{\eta}^{*}}$ be a $\mathfrak{T}_{\mathfrak{g}}$-open covering of $\mathscr{S}_{\mathfrak{g},\sigma} = \mathrm{im}\big(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}}\big) \subset \mathfrak{T}_{\mathfrak{g},\Sigma}$. Then, since the relation $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-}\mathrm{C}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$ holds, it results that $\langle \pi_{\mathfrak{g}}^{-1}(\mathscr{U}_{\mathfrak{g},\alpha})\rangle_{\alpha\in I_{\eta}^{*}}$ is a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open covering of $\mathscr{S}_{\mathfrak{g},\omega} = \pi_{\mathfrak{g}}^{-1}(\mathscr{S}_{\mathfrak{g},\sigma})$, because $\mathrm{O}[\mathfrak{T}_{\mathfrak{g},\Omega}] \subseteq \mathfrak{g}\text{-}\mathrm{O}[\mathfrak{T}_{\mathfrak{g},\Omega}]$. Since $\mathscr{S}_{\mathfrak{g},\omega} \in \mathfrak{g}\text{-}\mathrm{A}[\mathfrak{T}_{\mathfrak{g},\Omega}]$, a finite $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open subcovering $\langle \pi_{\mathfrak{g}}^{-1}(\mathscr{U}_{\mathfrak{g},\vartheta(\alpha)})\rangle_{(\alpha,\vartheta(\alpha))\in I_{\eta}^{*}\times I_{\vartheta(\eta)}^{*}} \prec \langle \pi_{\mathfrak{g}}^{-1}(\mathscr{U}_{\mathfrak{g},\alpha})\rangle_{\alpha\in I_{\eta}^{*}}$ exists, and such that, $\mathscr{S}_{\mathfrak{g},\omega} \subseteq \bigcup_{(\alpha,\vartheta(\alpha))\in I_{\eta}^{*}\times I_{\vartheta(\eta)}^{*}} \pi_{\mathfrak{g}}^{-1}(\mathscr{U}_{\mathfrak{g},\vartheta(\alpha)})$. Since $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-}\mathrm{C}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, it follows, consequently, that $\pi_{\mathfrak{g}}(\mathscr{S}_{\mathfrak{g},\omega}) \subseteq \bigcup_{(\alpha,\vartheta(\alpha))\in I_{\eta}^{*}\times I_{\vartheta(\eta)}^{*}} \mathscr{U}_{\mathfrak{g},\vartheta(\alpha)}$. Therefore, $\langle \pi_{\mathfrak{g}}^{-1}(\mathscr{U}_{\mathfrak{g},\alpha}) \in \mathrm{O}[\mathfrak{T}_{\mathfrak{g},\Sigma}]\rangle_{\alpha\in I_{\eta}^{*}}$ is a finite $\mathfrak{T}_{\mathfrak{g}}$-open subcovering of $\mathscr{S}_{\mathfrak{g},\sigma} \subset \mathfrak{T}_{\mathfrak{g},\Sigma}$. Hence, $\mathrm{im}\big(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}}\big) \in \mathrm{A}[\mathfrak{T}_{\mathfrak{g},\Sigma}]$ is also $\mathfrak{T}_{\mathfrak{g}}$-compact in $\mathfrak{T}_{\mathfrak{g},\Sigma}$. The proof of the proposition is complete. □

Theorem 3.15 *Let $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \longrightarrow \mathfrak{T}_{\mathfrak{g},\Sigma}$ be a $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$-continuous map, where $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathscr{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathscr{T}_{\mathfrak{g},\Omega})$ are $\mathscr{T}_{\mathfrak{g}}$-spaces. If $\mathscr{S}_{\mathfrak{g},\omega} \subset \mathfrak{T}_{\mathfrak{g},\Omega}$ is a countably $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact set in $\mathfrak{T}_{\mathfrak{g},\Omega}$, then $\mathrm{im}\big(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}}\big) \subset \mathfrak{T}_{\mathfrak{g},\Sigma}$ is also a countably $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact set in $\mathfrak{T}_{\mathfrak{g},\Sigma}$.*

Proof Let $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-}\mathrm{C}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$, where $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathscr{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathscr{T}_{\mathfrak{g},\Omega})$ are $\mathscr{T}_{\mathfrak{g}}$-spaces, and suppose $\mathscr{S}_{\mathfrak{g},\omega} \subset \mathfrak{T}_{\mathfrak{g},\Omega}$ be a countably $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact set in $\mathfrak{T}_{\mathfrak{g},\Omega}$. To prove that $\mathrm{im}\big(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}}\big) \subset \mathfrak{T}_{\mathfrak{g},\Sigma}$ is countably $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact in $\mathfrak{T}_{\mathfrak{g},\Sigma}$, let $\mathscr{S}_{\mathfrak{g},\sigma} \subseteq \pi_{\mathfrak{g}}(\mathscr{S}_{\mathfrak{g},\omega})$ be an infinite $\mathfrak{T}_{\mathfrak{g}}$-subset of $\mathrm{im}\big(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}}\big)$. Then, a denumerable $\mathfrak{T}_{\mathfrak{g}}$-subset $\mathscr{R}_{\mathfrak{g},\sigma} = \{\zeta_{\alpha} : \alpha \in I_{\infty}^{*}\} \subset \mathscr{S}_{\mathfrak{g},\sigma}$ exists. Since $\mathscr{R}_{\mathfrak{g},\sigma} \subset \mathscr{S}_{\mathfrak{g},\sigma} \subseteq \mathrm{im}\big(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}}\big) = \pi_{\mathfrak{g}}(\mathscr{S}_{\mathfrak{g},\omega})$, there exists a denumerable $\mathfrak{T}_{\mathfrak{g}}$-subset $\mathscr{R}_{\mathfrak{g},\omega} = \{\xi_{\alpha} : \alpha \in I_{\infty}^{*}\} \subset \mathscr{S}_{\mathfrak{g},\omega}$, with $\pi_{\mathfrak{g}}(\xi_{\alpha}) = \zeta_{\alpha}$ for every $\alpha \in I_{\infty}^{*}$. But, by hypothesis, $\mathscr{S}_{\mathfrak{g},\omega} \subset \mathfrak{T}_{\mathfrak{g},\Omega}$ is countably $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact in $\mathfrak{T}_{\mathfrak{g},\Omega}$, so $\mathscr{R}_{\mathfrak{g},\omega}$ contains a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-accumulation point $\xi \in \mathscr{S}_{\mathfrak{g},\omega}$. Thus, $\xi \in \mathscr{R}_{\mathfrak{g},\omega} \cup \mathrm{der}_{\mathfrak{g}}(\mathscr{R}_{\mathfrak{g},\omega}) \subseteq \mathscr{R}_{\mathfrak{g},\omega}$ and $\pi_{\mathfrak{g}}(\xi) \in \mathrm{im}\big(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}}\big) = \pi_{\mathfrak{g}}(\mathscr{S}_{\mathfrak{g},\omega})$; evidently, $\mathrm{der}_{\mathfrak{g}}(\mathscr{R}_{\mathfrak{g},\omega}) \in \mathfrak{g}\text{-}\mathrm{K}[\mathfrak{T}_{\mathfrak{g},\Omega}]$ and therefore, a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-closed set $\mathscr{V}_{\mathfrak{g},\omega} \in \mathfrak{g}\text{-}\mathrm{K}[\mathfrak{T}_{\mathfrak{g},\Omega}]$ exists such that, $\mathrm{der}_{\mathfrak{g}}(\mathscr{R}_{\mathfrak{g},\omega}) = \mathscr{V}_{\mathfrak{g},\omega}$. But, by hypothesis, $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-}\mathrm{C}[\mathfrak{T}_{\mathfrak{g},\Omega};\mathfrak{T}_{\mathfrak{g},\Sigma}]$. Consequently, $\pi_{\mathfrak{g}}\big(\mathscr{R}_{\mathfrak{g},\omega} \cup \mathrm{der}_{\mathfrak{g}}(\mathscr{R}_{\mathfrak{g},\omega})\big) \subseteq \pi_{\mathfrak{g}}(\mathscr{R}_{\mathfrak{g},\omega}) \cup \mathrm{der}_{\mathfrak{g}}\big(\pi_{\mathfrak{g}}(\mathscr{R}_{\mathfrak{g},\omega})\big) = \mathscr{R}_{\mathfrak{g},\sigma} \cup \mathrm{der}_{\mathfrak{g}}(\mathscr{R}_{\mathfrak{g},\sigma})$. But, $\xi \in \mathscr{R}_{\mathfrak{g},\omega} \cup \mathrm{der}_{\mathfrak{g}}(\mathscr{R}_{\mathfrak{g},\omega})$ and therefore, $\pi_{\mathfrak{g}}(\xi) \in \mathscr{R}_{\mathfrak{g},\sigma} \cup \mathrm{der}_{\mathfrak{g}}(\mathscr{R}_{\mathfrak{g},\sigma})$. Now, $\pi_{\mathfrak{g}}(\xi) \in \mathscr{R}_{\mathfrak{g},\sigma} \cup \mathrm{der}_{\mathfrak{g}}(\mathscr{R}_{\mathfrak{g},\sigma})$, so let it be claimed that

$\pi_{\mathfrak{g}}(\xi)$ is a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-accumulation point of $\mathscr{R}_{\mathfrak{g},\sigma}$. There are, then, two cases, namely, $\xi \notin \mathscr{R}_{\mathfrak{g},\omega}$ and $\xi \in \mathscr{R}_{\mathfrak{g},\omega}$.

I. *Case* $\xi \notin \mathscr{R}_{\mathfrak{g},\omega}$. If $\xi \notin \mathscr{R}_{\mathfrak{g},\omega}$, then $\pi_{\mathfrak{g}}(\xi) \notin (\mathscr{R}_{\mathfrak{g},\omega}) = \mathscr{R}_{\mathfrak{g},\sigma}$. But, $\pi_{\mathfrak{g}}(\xi) \in \mathscr{R}_{\mathfrak{g},\sigma} \cup \mathrm{der}_{\mathfrak{g}}(\mathscr{R}_{\mathfrak{g},\sigma})$ and consequently, $\pi_{\mathfrak{g}}(\xi)$ is a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-accumulation point of $\mathscr{R}_{\mathfrak{g},\sigma}$.

II. *Case* $\xi \in \mathscr{R}_{\mathfrak{g},\omega}$. If $\xi \in \mathscr{R}_{\mathfrak{g},\omega}$, choose a $\mu \in I_{\infty}^{*}$ such that $\xi = \xi_{\mu}$. Then, $\xi \notin \hat{\mathscr{R}}_{\mathfrak{g},\omega} = \{\xi_{\alpha} : \alpha \in I_{\infty}^{*} \setminus \{\mu\}\}$ and every $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open set $\mathscr{U}_{\mathfrak{g},\xi} \in \mathfrak{g}\text{-}\mathrm{O}[\mathfrak{T}_{\mathfrak{g}}]$ containing ξ contains at least a point $\hat{\xi} \in \hat{\mathscr{R}}_{\mathfrak{g},\omega} = \{\xi_{\alpha} : \alpha \in I_{\infty}^{*} \setminus \{\mu\}\}$ and therefore, ξ is a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-accumulation point of $\hat{\mathscr{R}}_{\mathfrak{g},\omega}$. But, $\pi_{\mathfrak{g}}(\hat{\mathscr{R}}_{\mathfrak{g},\omega}) = \{\zeta_{\alpha} : \alpha \in I_{\infty}^{*} \setminus \{\mu\}\}$ since, by hypothesis, $\pi_{\mathfrak{g}}(\xi_{\alpha}) = \zeta_{\alpha}$ for every $\alpha \in I_{\infty}^{*}$. Thus, $\pi_{\mathfrak{g}}(\xi)$ is a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-accumulation point of $\pi_{\mathfrak{g}}(\hat{\mathscr{R}}_{\mathfrak{g},\omega})$ where $\pi_{\mathfrak{g}}(\hat{\mathscr{R}}_{\mathfrak{g},\omega}) \subseteq \mathscr{R}_{\mathfrak{g},\sigma}$. Moreover, since $\pi_{\mathfrak{g}}(\hat{\mathscr{R}}_{\mathfrak{g},\omega} \cup \mathrm{der}_{\mathfrak{g}}(\hat{\mathscr{R}}_{\mathfrak{g},\omega})) \subseteq \pi_{\mathfrak{g}}(\hat{\mathscr{R}}_{\mathfrak{g},\omega}) \cup \mathrm{der}_{\mathfrak{g}}(\pi_{\mathfrak{g}}(\hat{\mathscr{R}}_{\mathfrak{g},\omega})) = \hat{\mathscr{R}}_{\mathfrak{g},\sigma} \cup \mathrm{der}_{\mathfrak{g}}(\hat{\mathscr{R}}_{\mathfrak{g},\sigma})$, it follows that, $\pi_{\mathfrak{g}}(\xi)$ is a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-accumulation point of $\hat{\mathscr{R}}_{\mathfrak{g},\sigma}$. Since $\mathscr{R}_{\mathfrak{g},\sigma} \subset \mathscr{S}_{\mathfrak{g},\sigma} \subseteq \mathrm{im}(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}}) = \pi_{\mathfrak{g}}(\mathscr{S}_{\mathfrak{g},\omega})$, $\pi_{\mathfrak{g}}(\xi)$ is also a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-accumulation point of $\mathscr{S}_{\mathfrak{g},\sigma}$ and $\pi_{\mathfrak{g}}(\xi) \in \mathrm{im}(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}}) = \pi_{\mathfrak{g}}(\mathscr{S}_{\mathfrak{g},\omega})$. Therefore, every infinite $\mathfrak{T}_{\mathfrak{g}}$-subset $\mathscr{S}_{\mathfrak{g},\sigma} \subseteq \mathrm{im}(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}})$ of $\pi_{\mathfrak{g}}(\mathscr{S}_{\mathfrak{g},\omega})$ contains a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-accumulation point in $\pi_{\mathfrak{g}}(\mathscr{S}_{\mathfrak{g},\omega})$ and hence, $\mathrm{im}(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}}) \subset \mathfrak{T}_{\mathfrak{g},\Sigma}$ is also a countably $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact set in $\mathfrak{T}_{\mathfrak{g},\Sigma}$. The proof of the theorem is complete. □

Proposition 3.16 *If $\mathscr{S}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g}}$ is a sequentially $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact set of a $\mathscr{T}_{\mathfrak{g}}$-space $\mathfrak{T}_{\mathfrak{g}} = (\Omega, \mathscr{T}_{\mathfrak{g}})$, then every countable $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open covering $\langle \mathscr{U}_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-}\mathrm{O}[\mathfrak{T}_{\mathfrak{g}}] \rangle_{\alpha \in I_{\sigma}^{*}}$ of the $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact set $\mathscr{S}_{\mathfrak{g}}$ is reducible to a finite $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open subcovering of the type $\langle \mathscr{U}_{\mathfrak{g},\vartheta(\alpha)} \rangle_{(\alpha,\vartheta(\alpha)) \in I_{\sigma}^{*} \times I_{\vartheta(\sigma)}^{*}} \prec \langle \mathscr{U}_{\mathfrak{g},\alpha} \rangle_{\alpha \in I_{\sigma}^{*}}$ of $\mathscr{S}_{\mathfrak{g}}$.*

Proof Let it be assumed that $\mathscr{S}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g}}$ is a sequentially $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact infinite set of a $\mathscr{T}_{\mathfrak{g}}$-space $\mathfrak{T}_{\mathfrak{g}} = (\Omega, \mathscr{T}_{\mathfrak{g}})$. Furthermore, assume that there exists a countable $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open covering $\langle \mathscr{U}_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-}\mathrm{O}[\mathfrak{T}_{\mathfrak{g}}] \rangle_{\alpha \in I_{\sigma}^{*}}$ of $\mathscr{S}_{\mathfrak{g}}$ with no finite $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open subcovering $\langle \mathscr{U}_{\mathfrak{g},\vartheta(\alpha)} \rangle_{(\alpha,\vartheta(\alpha)) \in I_{\sigma}^{*} \times I_{\vartheta(\sigma)}^{*}} \prec \langle \mathscr{U}_{\mathfrak{g},\alpha} \rangle_{\alpha \in I_{\sigma}^{*}}$ of $\mathscr{S}_{\mathfrak{g}}$. Finally, introduce the sequence $\langle \xi_{\alpha} \in \mathscr{S}_{\mathfrak{g}} \rangle_{\alpha \in I_{\infty}^{*}}$ and define its elements in the following manner. Let $\vartheta(1) \in I_{\vartheta(\sigma)}^{*} \subset I_{\sigma}^{*}$ be the smallest integer in $I_{\vartheta(\sigma)}^{*}$ such that $\mathscr{S}_{\mathfrak{g}} \cap \mathscr{U}_{\mathfrak{g},\vartheta(1)} \neq \emptyset$; choose $\xi_{1} \in \mathscr{S}_{\mathfrak{g}} \cap \mathscr{U}_{\mathfrak{g},\vartheta(1)}$. Let $\vartheta(2) \in I_{\vartheta(\sigma)}^{*} \subset I_{\sigma}^{*}$ be the least integer larger than $\vartheta(1)$ in $I_{\vartheta(\sigma)}^{*}$ such that $\mathscr{S}_{\mathfrak{g}} \cap \mathscr{U}_{\mathfrak{g},\vartheta(2)} \neq \emptyset$; choose $\xi_{2} \in (\mathscr{S}_{\mathfrak{g}} \cap \mathscr{U}_{\mathfrak{g},\vartheta(2)}) \setminus (\mathscr{S}_{\mathfrak{g}} \cap \mathscr{U}_{\mathfrak{g},\vartheta(1)})$. Note that, such a point ξ_{2} always exists, for otherwise $\mathscr{U}_{\mathfrak{g},\vartheta(1)}$ covers $\mathscr{S}_{\mathfrak{g}}$. Continuing in this way, the properties of $\langle \xi_{\alpha} \rangle_{\alpha \in I_{\infty}^{*}}$, for every $\alpha \in I_{\infty}^{*} \setminus \{1\}$, are

$$\xi_{\alpha} \in \mathscr{S}_{\mathfrak{g}} \cap \mathscr{U}_{\mathfrak{g},\vartheta(\alpha)}, \quad \xi_{\alpha} \notin \textstyle\bigcup_{\nu \in I_{\alpha-1}^{*}} (\mathscr{S}_{\mathfrak{g}} \cap \mathscr{U}_{\mathfrak{g},\vartheta(\nu)}), \quad \vartheta(\alpha) > \vartheta(\alpha - 1).$$

Let it be claimed that $\langle \xi_{\alpha} \rangle_{\alpha \in I_{\infty}^{*}}$ has no convergent subsequence $\langle \xi_{\vartheta(\alpha)} \rangle_{(\alpha,\vartheta(\alpha)) \in I_{\infty}^{*} \times I_{\infty}^{*}} \prec \langle \xi_{\alpha} \rangle_{\alpha \in I_{\infty}^{*}}$ in $\mathscr{S}_{\mathfrak{g}}$. Suppose $\xi \in \mathscr{S}_{\mathfrak{g}}$, then there exists a $\mu \in I_{\vartheta(\sigma)}^{*}$ such that $\xi \in \mathscr{U}_{\mathfrak{g},\vartheta(\mu)}$. Now, $\mathscr{S}_{\mathfrak{g}} \cap \mathscr{U}_{\mathfrak{g},\vartheta(\mu)} \neq \emptyset$ since, $\xi \in \mathscr{S}_{\mathfrak{g}} \cap \mathscr{U}_{\mathfrak{g},\vartheta(\mu)}$. Thus, there exists $\nu \in I_{\vartheta(\sigma)}^{*}$ such that, $\mathscr{U}_{\mathfrak{g},\vartheta(\nu)} = \mathscr{U}_{\mathfrak{g},\vartheta(\mu)}$. But, by the properties of the sequence $\langle \xi_{\alpha} \rangle_{\alpha \in I_{\infty}^{*}}$, $\alpha > \vartheta(\nu)$ implies $\xi_{\alpha} \notin \mathscr{U}_{\mathfrak{g},\vartheta(\mu)}$. Accordingly, since $\xi \in \mathscr{U}_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]$ no subsequence $\langle \xi_{\vartheta(\alpha)} \rangle_{(\alpha,\vartheta(\alpha)) \in I_{\infty}^{*} \times I_{\infty}^{*}} \prec \langle \xi_{\alpha} \rangle_{\alpha \in I_{\infty}^{*}}$ of $\langle \xi_{\alpha} \rangle_{\alpha \in I_{\infty}^{*}}$ converges to $\xi \in \mathscr{S}_{\mathfrak{g}}$. But, ξ was arbitrary and hence, $\mathscr{S}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g}}$ is not sequentially $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact in $\mathfrak{T}_{\mathfrak{g}}$. The proof of the proposition is complete. □

Theorem 3.17 *If $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}]$ is a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact set of a $\mathscr{T}_{\mathfrak{g}}$-space $\mathfrak{T}_{\mathfrak{g}} = (\Omega, \mathscr{T}_{\mathfrak{g}})$, then it is also locally $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact in $\mathfrak{T}_{\mathfrak{g}}$.*

Proof Let $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}]$ be a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact set of a $\mathscr{T}_{\mathfrak{g}}$-space $\mathfrak{T}_{\mathfrak{g}} = (\Omega, \mathscr{T}_{\mathfrak{g}})$. Since $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}]$, for every $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open covering $\langle \mathscr{U}_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}] \rangle_{\alpha \in I_{\sigma}^{*}}$, there exists a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open subcovering $\langle \mathscr{U}_{\mathfrak{g},\vartheta(\alpha)} \rangle_{(\alpha,\vartheta(\alpha)) \in I_{\sigma}^{*} \times I_{\vartheta(\sigma)}^{*}} \prec \langle \mathscr{U}_{\mathfrak{g},\alpha} \rangle_{\alpha \in I_{\sigma}^{*}}$ such that $\mathscr{S}_{\mathfrak{g}} \subseteq \bigcup_{(\alpha,\vartheta(\alpha)) \in I_{\sigma}^{*} \times I_{\vartheta(\sigma)}^{*}} \mathscr{U}_{\mathfrak{g},\vartheta(\alpha)}$. It is clear that, for every $\xi \in \mathscr{S}_{\mathfrak{g}}$, there exists $\mathscr{U}_{\mathfrak{g},\xi} \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]$ such that $\mathscr{S}_{\mathfrak{g}} \cap \mathscr{U}_{\mathfrak{g},\xi} = \mathscr{U}_{\mathfrak{g},\vartheta(\alpha)} \cap \mathscr{U}_{\mathfrak{g},\xi}$ for some $(\alpha, \vartheta(\alpha)) \in I_{\sigma}^{*} \times I_{\vartheta(\sigma)}^{*}$. For every $(\alpha, \xi, \vartheta(\alpha), \upsilon(\alpha,\xi)) \in I_{\sigma}^{*} \times \mathscr{S}_{\mathfrak{g}} \times I_{\vartheta(\sigma)}^{*} \times I_{\upsilon(\sigma)}^{*}$, set $\mathscr{U}_{\mathfrak{g},\upsilon(\alpha,\xi)} = \mathscr{U}_{\mathfrak{g},\vartheta(\alpha)} \cap \mathscr{U}_{\mathfrak{g},\xi}$. Then, since $(\mathscr{U}_{\mathfrak{g},\vartheta(\alpha)}, \mathscr{U}_{\mathfrak{g},\xi}) \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}] \times \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]$ for every $(\alpha, \xi, \vartheta(\alpha)) \in I_{\sigma}^{*} \times \mathscr{S}_{\mathfrak{g}} \times I_{\vartheta(\sigma)}^{*}$, there exists, for every $(\alpha, \xi, \vartheta(\alpha)) \in I_{\sigma}^{*} \times \mathscr{S}_{\mathfrak{g}} \times I_{\vartheta(\sigma)}^{*}$, a pair $(\mathscr{O}_{\mathfrak{g},\vartheta(\alpha)}, \mathscr{O}_{\mathfrak{g},\xi}) \in \mathscr{T}_{\mathfrak{g}} \times \mathscr{T}_{\mathfrak{g}}$ of $\mathscr{T}_{\mathfrak{g}}$-open sets such that, $(\mathscr{U}_{\mathfrak{g},\vartheta(\alpha)}, \mathscr{U}_{\mathfrak{g},\xi}) \subseteq (\mathrm{op}_{\mathfrak{g}}(\mathscr{O}_{\mathfrak{g},\vartheta(\alpha)}), \mathrm{op}_{\mathfrak{g}}(\mathscr{O}_{\mathfrak{g},\xi}))$. Consequently,

$$\mathscr{U}_{\mathfrak{g},\upsilon(\alpha,\xi)} = \mathscr{U}_{\mathfrak{g},\vartheta(\alpha)} \cap \mathscr{U}_{\mathfrak{g},\xi} \subseteq \mathrm{op}_{\mathfrak{g}}(\mathscr{O}_{\mathfrak{g},\vartheta(\alpha)}) \cap \mathrm{op}_{\mathfrak{g}}(\mathscr{O}_{\mathfrak{g},\xi}) \subseteq \mathrm{op}_{\mathfrak{g}}(\mathscr{O}_{\mathfrak{g},\vartheta(\alpha)} \cap \mathscr{O}_{\mathfrak{g},\xi}) = \mathrm{op}_{\mathfrak{g}}(\mathscr{O}_{\mathfrak{g},\upsilon(\alpha,\xi)}),$$

where $\mathscr{U}_{\mathfrak{g},\upsilon(\alpha,\xi)} = \mathscr{U}_{\mathfrak{g},\vartheta(\alpha)} \cap \mathscr{U}_{\mathfrak{g},\xi}$ for every $(\alpha, \xi, \vartheta(\alpha), \upsilon(\alpha,\xi)) \in I_{\sigma}^{*} \times \mathscr{S}_{\mathfrak{g}} \times I_{\vartheta(\sigma)}^{*} \times I_{\upsilon(\sigma)}^{*}$. Therefore, $\mathscr{U}_{\mathfrak{g},\upsilon(\alpha,\xi)} \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]$ for every $(\alpha, \xi, \vartheta(\alpha), \upsilon(\alpha,\xi)) \in I_{\sigma}^{*} \times \mathscr{S}_{\mathfrak{g}} \times I_{\vartheta(\sigma)}^{*} \times I_{\upsilon(\sigma)}^{*}$. But, since $\xi \in \mathscr{U}_{\mathfrak{g},\vartheta(\alpha,\xi)} \subseteq \mathscr{U}_{\mathfrak{g},\vartheta(\alpha,\xi)} \cup \mathrm{der}_{\mathfrak{g}}(\mathscr{U}_{\mathfrak{g},\vartheta(\alpha,\xi)})$ and $\mathscr{U}_{\mathfrak{g},\vartheta(\alpha)} \supset \mathscr{U}_{\mathfrak{g},\vartheta(\alpha,\xi)} \cup \mathrm{der}_{\mathfrak{g}}(\mathscr{U}_{\mathfrak{g},\vartheta(\alpha,\xi)}) \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}]$, it results that,

$$\xi \in \mathscr{U}_{\mathfrak{g},\vartheta(\alpha,\xi)} \subseteq \mathscr{U}_{\mathfrak{g},\vartheta(\alpha,\xi)} \cup \mathrm{der}_{\mathfrak{g}}(\mathscr{U}_{\mathfrak{g},\vartheta(\alpha,\xi)}) \subset \mathscr{U}_{\mathfrak{g},\vartheta(\alpha)}.$$

Thus, given any $(\xi, \mathscr{U}_{\mathfrak{g},\vartheta(\alpha)}) \in \mathscr{S}_{\mathfrak{g}} \times \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]$, there is a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open neighborhood $\mathscr{U}_{\mathfrak{g},\vartheta(\alpha,\xi)} \in \mathfrak{g}\text{-N}[\xi]$ of ξ such that $\mathscr{U}_{\mathfrak{g},\vartheta(\alpha,\xi)} \subset \mathscr{U}_{\mathfrak{g},\vartheta(\alpha)}$ and $\mathscr{U}_{\mathfrak{g},\vartheta(\alpha)} \cup \mathrm{der}_{\mathfrak{g}}(\mathscr{U}_{\mathfrak{g},\vartheta(\alpha)}) \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}]$. Hence, $\mathscr{S}_{\mathfrak{g}} \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g}}]$ implies that it is also locally $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact in $\mathfrak{T}_{\mathfrak{g}}$. The proof of the theorem is complete. □

Corollary 3.18 *Every $\mathscr{T}_{\mathfrak{g}}$-space $\mathfrak{T}_{\mathfrak{g}} = (\Omega, \mathscr{T}_{\mathfrak{g}})$ having the property that every local $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open* *covering* $\langle \mathscr{U}_{\mathfrak{g},\alpha} \in \mathfrak{g}\text{-O}[\mathfrak{T}_{\mathfrak{g}}]\rangle_{\alpha\in I_{\sigma}^{*}}$ *of the* $\mathscr{T}_{\mathfrak{g}}$*-space* $\mathfrak{T}_{\mathfrak{g}}$ *contains a finite* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-open subcovering* $\langle \mathscr{U}_{\mathfrak{g},\vartheta(\alpha)}\rangle_{(\alpha,\vartheta(\alpha))\in I_{\sigma}^{*}\times I_{\vartheta(\sigma)}^{*}} \prec \langle \mathscr{U}_{\mathfrak{g},\alpha}\rangle_{\alpha\in I_{\sigma}^{*}}$ *of* $\mathfrak{T}_{\mathfrak{g}}$ *is a locally* $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$*-space* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]} = (\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]})$.

Theorem 3.19 *Let* $\pi_{\mathfrak{g}} : \mathfrak{T}_{\mathfrak{g},\Omega} \longrightarrow \mathfrak{T}_{\mathfrak{g},\Sigma}$ *be a* $\mathfrak{g}$-$(\mathfrak{T}_{\mathfrak{g},\Omega}, \mathfrak{T}_{\mathfrak{g},\Sigma})$*-continuous map, where* $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathscr{T}_{\mathfrak{g},\Omega})$ *and* $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathscr{T}_{\mathfrak{g},\Omega})$ *are* $\mathscr{T}_{\mathfrak{g}}$*-spaces. If* $\mathscr{S}_{\mathfrak{g},\omega} \subset \mathfrak{T}_{\mathfrak{g},\Omega}$ *is a locally* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-compact set in* $\mathfrak{T}_{\mathfrak{g},\Omega}$*, then* $\mathrm{im}(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}}) \subset \mathfrak{T}_{\mathfrak{g},\Sigma}$ *is also a locally* $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$*-compact set in* $\mathfrak{T}_{\mathfrak{g},\Sigma}$.

Proof Let $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-C}[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Sigma}]$, where $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathscr{T}_{\mathfrak{g},\Omega})$ and $\mathfrak{T}_{\mathfrak{g},\Omega} = (\Omega, \mathscr{T}_{\mathfrak{g},\Omega})$ are $\mathscr{T}_{\mathfrak{g}}$-spaces, and suppose $\mathscr{S}_{\mathfrak{g},\omega} \subset \mathfrak{T}_{\mathfrak{g},\Omega}$ be locally $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact in $\mathfrak{T}_{\mathfrak{g},\Omega}$. Since $\mathscr{S}_{\mathfrak{g},\omega}$ is locally $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact, for any given $(\xi, \mathscr{N}_{\mathfrak{g},\xi}) \in \mathscr{S}_{\mathfrak{g},\omega} \times \mathfrak{g}\text{-N}[\xi]$, there is a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-neighborhood $\hat{\mathscr{N}}_{\mathfrak{g},\xi} \in \mathfrak{g}\text{-N}[\xi]$ of ξ such that $\hat{\mathscr{N}}_{\mathfrak{g},\xi} \subset \mathscr{N}_{\mathfrak{g},\xi}$ and $\hat{\mathscr{N}}_{\mathfrak{g},\xi} \cup \mathrm{der}_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi}) \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g},\Omega}]$. Consequently, $\xi \in \hat{\mathscr{N}}_{\mathfrak{g},\xi} \subseteq \hat{\mathscr{N}}_{\mathfrak{g},\xi} \cup \mathrm{der}_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi}) \subset \mathscr{N}_{\mathfrak{g},\xi}$ and thus, $\pi_{\mathfrak{g}}(\xi) \in \pi_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi}) \subseteq \pi_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi} \cup \mathrm{der}_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi})) \subset \pi_{\mathfrak{g}}(\mathscr{N}_{\mathfrak{g},\xi})$. But, $\pi_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi} \cup \mathrm{der}_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi})) \subseteq \pi_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi}) \cup \pi_{\mathfrak{g}}(\mathrm{der}_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi}))$ because, by hypothesis, $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-C}[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Sigma}]$. Therefore,

$$\pi_{\mathfrak{g}}(\xi) \in \pi_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi}) \subseteq \pi_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi} \cup \mathrm{der}_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi})) \subseteq \pi_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi}) \cup \mathrm{der}_{\mathfrak{g}}(\pi_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi})) \subset \pi_{\mathfrak{g}}(\mathscr{N}_{\mathfrak{g},\xi}).$$

Since $\hat{\mathscr{N}}_{\mathfrak{g},\xi} \subset \mathfrak{T}_{\mathfrak{g},\Omega}$ is a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-neighborhood in $\mathfrak{T}_{\mathfrak{g},\Omega}$ containing $\xi \in \mathscr{S}_{\mathfrak{g},\omega} \subset \mathfrak{T}_{\mathfrak{g},\Omega}$, $\pi_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi}) \subset \mathfrak{T}_{\mathfrak{g},\Sigma}$ is a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-neighborhood in $\mathfrak{T}_{\mathfrak{g},\Sigma}$ containing $\pi_{\mathfrak{g}}(\xi) \in \pi_{\mathfrak{g}}(\mathscr{S}_{\mathfrak{g},\omega}) \in \mathfrak{T}_{\mathfrak{g},\Sigma}$. Now $\pi_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi}) \cup \mathrm{der}_{\mathfrak{g}}(\pi_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi})) \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g},\Sigma}]$ by virtue of the statements $\hat{\mathscr{N}}_{\mathfrak{g},\xi} \cup \mathrm{der}_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi}) \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g},\Omega}]$ and $\pi_{\mathfrak{g}} \in \mathfrak{g}\text{-C}[\mathfrak{T}_{\mathfrak{g},\Omega}; \mathfrak{T}_{\mathfrak{g},\Sigma}]$. In other words, for any given $(\pi_{\mathfrak{g}}(\xi), \pi_{\mathfrak{g}}(\mathscr{N}_{\mathfrak{g},\zeta})) = (\zeta, \mathscr{N}_{\mathfrak{g},\zeta}) \in \mathscr{S}_{\mathfrak{g},\sigma} \times \mathfrak{g}\text{-N}[\zeta]$, there is a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-neighborhood $\pi_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi}) = \hat{\mathscr{N}}_{\mathfrak{g},\zeta} \in \mathfrak{g}\text{-N}[\zeta]$ of $\pi_{\mathfrak{g}}(\xi) = \zeta$ such that $\pi_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi}) = \hat{\mathscr{N}}_{\mathfrak{g},\zeta} \subseteq \mathscr{N}_{\mathfrak{g},\zeta} = \pi_{\mathfrak{g}}(\mathscr{N}_{\mathfrak{g},\xi})$ and $\pi_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi}) \cup \mathrm{der}_{\mathfrak{g}}(\pi_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\xi})) = \hat{\mathscr{N}}_{\mathfrak{g},\zeta} \cup \mathrm{der}_{\mathfrak{g}}(\hat{\mathscr{N}}_{\mathfrak{g},\zeta}) \in \mathfrak{g}\text{-A}[\mathfrak{T}_{\mathfrak{g},\Sigma}]$. Therefore, $\mathscr{S}_{\mathfrak{g},\sigma} \subset \mathfrak{T}_{\mathfrak{g},\Sigma}$ is locally $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact in $\mathfrak{T}_{\mathfrak{g},\Sigma}$. But, $\mathscr{S}_{\mathfrak{g},\sigma} = \pi_{\mathfrak{g}}(\mathscr{S}_{\mathfrak{g},\omega}) = \mathrm{im}(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}})$. Hence, $\mathrm{im}(\pi_{\mathfrak{g}|\mathscr{S}_{\mathfrak{g},\omega}}) \subset \mathfrak{T}_{\mathfrak{g},\Sigma}$ is locally $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact in $\mathfrak{T}_{\mathfrak{g},\Sigma}$. The proof of the theorem is complete. □

4. Discussion

4.1. Categorical Classifications

Having adopted a categorical approach in the classifications of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness in the $\mathscr{T}_{\mathfrak{g}}$-space $\mathfrak{T}_{\mathfrak{g}}$, the dual purposes of the this section are firstly, to establish the various relationships amongst the elements of the sequences $\langle \mathfrak{g}\text{-}\nu\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{E}]} = (\Omega, \mathfrak{g}\text{-}\nu\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{E}]})\rangle_{\nu\in I_{3}^{0}}$ and $\langle \mathfrak{g}\text{-}\nu\text{-}\mathfrak{T}^{[\mathrm{E}]} = (\Omega, \mathfrak{g}\text{-}\nu\text{-}\mathscr{T}^{[\mathrm{E}]})\rangle_{\nu\in I_{3}^{0}}$ of $\mathfrak{g}$-ν-$\mathscr{T}_{\mathfrak{g}}^{[\mathrm{E}]}$-spaces and $\mathfrak{g}$-ν-$\mathscr{T}^{[\mathrm{E}]}$-spaces, respectively, where $\mathrm{E} \in \{\mathrm{A}, \mathrm{CA}, \mathrm{SA}, \mathrm{LA}\}$, and secondly, to illustrate them through diagrams.

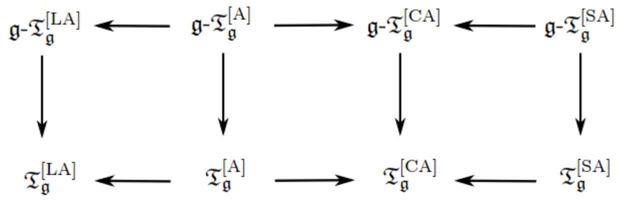

Figure 1: Relationships: $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact spaces and $\mathfrak{T}_{\mathfrak{g}}$-compact spaces

It is plain that $\mathfrak{T}_{\mathfrak{g}}$-compactness implies both countable $\mathfrak{T}_{\mathfrak{g}}$-compactness and local countable $\mathfrak{T}_{\mathfrak{g}}$-compactness; sequential $\mathfrak{T}_{\mathfrak{g}}$-compactness implies countable $\mathfrak{T}_{\mathfrak{g}}$-compactness. Moreover, the following implications also hold: $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{\mathrm{LA}} \longleftarrow \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{\mathrm{A}}$, $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{\mathrm{CA}} \longleftarrow \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{\mathrm{A}}$, and $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{\mathrm{CA}} \longleftarrow \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{\mathrm{SA}}$. Since the relation $\mathfrak{T}^{[\mathrm{E}]} \longleftarrow \mathfrak{g}\text{-}\mathfrak{T}^{[\mathrm{E}]}$ holds for every $\mathrm{E} \in \{\mathrm{A}, \mathrm{CA}, \mathrm{SA}, \mathrm{LA}\}$, taking this last statement together with those preceding it into account, the diagram presented in Figure 1 follows, in which are illustrated the various relationships amongst the elements of $\langle \mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{E}]} \rangle_{\mathrm{E}\in\Lambda}$ and $\langle \mathfrak{T}_{\mathfrak{g}}^{[\mathrm{E}]} \rangle_{\mathrm{E}\in\Lambda}$, where $\Lambda = \{\mathrm{A}, \mathrm{CA}, \mathrm{SA}, \mathrm{LA}\}$.

For each $\nu \in I_3^0$, these implications hold: $\mathfrak{g}\text{-}\nu\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{LA}]} \longleftarrow \mathfrak{g}\text{-}\nu\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}$, $\mathfrak{g}\text{-}\nu\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{CA}]} \longleftarrow \mathfrak{g}\text{-}\nu\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}$, and $\mathfrak{g}\text{-}\nu\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{CA}]} \longleftarrow \mathfrak{g}\text{-}\nu\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{SA}]}$. For each $\mathrm{E} \in \Lambda = \{\mathrm{A}, \mathrm{CA}, \mathrm{SA}, \mathrm{LA}\}$, these implications also hold: $\mathfrak{g}\text{-}0\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{E}]} \longleftarrow \mathfrak{g}\text{-}1\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{E}]}$, $\mathfrak{g}\text{-}1\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{E}]} \longleftarrow \mathfrak{g}\text{-}3\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{E}]}$, and $\mathfrak{g}\text{-}2\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{E}]} \longleftarrow \mathfrak{g}\text{-}3\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{E}]}$. When all these implications are taken into consideration, the resulting compactness diagram so obtained is that presented in Figure 2. It is reasonably correct to call them $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{E}]}$-spaces of type E and of category ν, where $(\nu, \mathrm{E}) \in I_3^0 \times \{\mathrm{A}, \mathrm{CA}, \mathrm{SA}, \mathrm{LA}\}$. As in the papers of [7] and [17], among others, the manner we have positioned the arrows is solely to stress that, in general, none of the implications in Figures 1 and 2 is reversible.

In order to exemplify the notion of $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{E}]}$-spaces of type E and of category ν, where $(\nu, \mathrm{E}) \in I_3^0 \times \{\mathrm{A}, \mathrm{CA}, \mathrm{SA}, \mathrm{LA}\}$, a nice application is presented in the following section.

4.2. A Nice Application

Focusing on basic concepts from the standpoint of the theory of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness, we shall now present a nice application.

Let $\mathscr{T}_{\mathfrak{g}} : \mathscr{P}(\Omega) \longrightarrow \mathscr{P}(\Omega)$ be the $\mathfrak{g}$-topology on $\Omega = \mathbb{N}$ (set of positive integers) generated by $\mathscr{T}_{\mathfrak{g}}$-open and $\mathscr{T}_{\mathfrak{g}}$-closed sets belonging to:

$$\mathscr{T}_{\mathfrak{g}} \stackrel{\text{def}}{=} \left\{\mathscr{O}_{\mathfrak{g},(2\mu-1,2\mu)} : \ (\forall \mu \in I_{\infty}^{*})\big(\big[\mathscr{O}_{\mathfrak{g},(2\mu-1,2\mu)} = \emptyset\big] \vee \big[\mathscr{O}_{\mathfrak{g},(2\mu-1,2\mu)} = \{2\mu-1, 2\mu\}\big]\big)\right\};$$

$$\neg\mathscr{T}_{\mathfrak{g}} \stackrel{\text{def}}{=} \left\{\mathscr{K}_{\mathfrak{g},(2\mu-1,2\mu)} : \ (\forall \mu \in I_{\infty}^{*})\big(\big[\mathscr{K}_{\mathfrak{g},(2\mu-1,2\mu)} = \mathbb{N}\big] \vee \big[\mathscr{K}_{\mathfrak{g},(2\mu-1,2\mu)} = \complement(\{2\mu-1, 2\mu\})\big]\big)\right\},$$

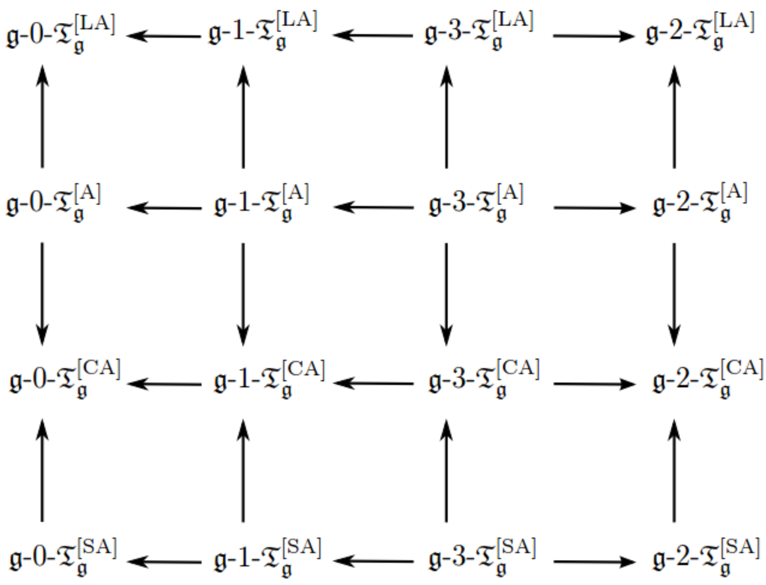

Figure 2: Relationships: $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact spaces

respectively. As in the above case, it results that $\mathscr{T}_{\mathfrak{g}} : \mathscr{P}(\Omega) \longrightarrow \mathscr{P}(\Omega)$ satisfies the relations $\mathscr{T}_{\mathfrak{g}}(\emptyset) = \emptyset$, $\mathscr{T}_{\mathfrak{g}}\big(\mathscr{O}_{\mathfrak{g},(2\mu-1,2\mu)}\big) \subseteq \{2\mu-1, 2\mu\} = \mathscr{O}_{\mathfrak{g},(2\mu-1,2\mu)}$ and, $\mathscr{T}_{\mathfrak{g}}\big(\bigcap_{\mu\in I_{\sigma}^{*}} \mathscr{O}_{\mathfrak{g},(2\mu-1,2\mu)}\big) = \bigcap_{\mu\in I_{\sigma}^{*}} \mathscr{T}_{\mathfrak{g}}\big(\mathscr{O}_{\mathfrak{g},(2\mu-1,2\mu)}\big)$ as well as $\mathscr{T}_{\mathfrak{g}}\big(\bigcup_{\mu\in I_{\infty}^{*}} \mathscr{O}_{\mathfrak{g},(2\mu-1,2\mu)}\big) = \bigcup_{\mu\in I_{\infty}^{*}} \mathscr{T}_{\mathfrak{g}}\big(\mathscr{O}_{\mathfrak{g},(2\mu-1,2\mu)}\big)$, since the two relations $\bigcap_{\mu\in I_{\sigma}^{*}} \mathscr{O}_{\mathfrak{g},(2\mu-1,2\mu)} = \emptyset \in \mathscr{T}_{\mathfrak{g}}$ and $\bigcup_{\mu\in I_{\infty}^{*}} \mathscr{O}_{\mathfrak{g},(2\mu-1,2\mu)} = \Omega \in \mathscr{T}_{\mathfrak{g}}$, respectively, hold. Therefore, $\mathfrak{T}_{\mathfrak{g}} = (\mathscr{T}_{\mathfrak{g}}, \Omega)$ is a $\mathscr{T}_{\mathfrak{g}}$-space and, moreover, since the relation $\mathfrak{T}_{\mathfrak{g}} = (\mathscr{T}_{\mathfrak{g}}, \Omega) = (\mathscr{T}, \Omega) = \mathfrak{T}$ holds, it is also a $\mathscr{T}$-space. Notice that $\big\langle \mathscr{O}_{\mathfrak{g},(2\alpha-1,2\alpha)} \big\rangle_{\alpha\in I_{\infty}^{*}}$ is a $\mathfrak{T}_{\mathfrak{g}}$-open covering of Ω, since $\mathscr{O}_{\mathfrak{g},(2\alpha-1,2\alpha)} \in \mathrm{O}[\mathfrak{T}_{\mathfrak{g}}]$ for every $\alpha \in I_{\infty}^{*}$ and furthermore, it is also a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open covering of Ω, since $\mathscr{O}_{\mathfrak{g},(2\alpha-1,2\alpha)} \subseteq \mathrm{op}_{\mathfrak{g}}\big(\mathscr{O}_{\mathfrak{g},(2\alpha-1,2\alpha)}\big) \in \mathfrak{g}\text{-}\mathrm{O}[\mathfrak{T}_{\mathfrak{g}}]$ for every $\alpha \in I_{\infty}^{*}$. However, $\mathfrak{T}_{\mathfrak{g}} = (\mathscr{T}_{\mathfrak{g}}, \Omega)$, where $\Omega = \mathbb{N}$, is not a $\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}$-space because $\big\langle \mathscr{O}_{\mathfrak{g},(2\alpha-1,2\alpha)} \big\rangle_{\alpha\in I_{\infty}^{*}}$ is a $\mathfrak{T}_{\mathfrak{g}}$-open covering of Ω with no finite $\mathfrak{T}_{\mathfrak{g}}$-open subcovering.

As stated above, since $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness implies $\mathfrak{T}_{\mathfrak{g}}$-compactness, it follows, obviously, that it is also not a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]}$-space. On the other hand, $\mathfrak{T}_{\mathfrak{g}} = (\mathscr{T}_{\mathfrak{g}}, \Omega)$, where $\Omega = \mathbb{N}$, is also not a sequentially $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compact $\mathscr{T}_{\mathfrak{g}}$-space for the simple reason that sequence $\langle \xi_{\alpha} = \alpha \in \Omega \rangle_{\alpha\in I_{\infty}^{*}}$ in $\mathfrak{T}_{\mathfrak{g}}$ contains no subsequence of the type $\big\langle \xi_{\vartheta(\alpha)} \big\rangle_{(\alpha,\vartheta(\alpha)\in\Omega)\in I_{\infty}^{*}\times I_{\infty}^{*}} \prec \langle \xi_{\alpha} \rangle_{\alpha\in I_{\infty}^{*}}$ which converges to a point $\xi \in \Omega$. Hence, $\mathfrak{T}_{\mathfrak{g}}$ is not a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{SA}]}$-space which, then, implies that it is also not a $\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{SA}]}$-space.

Let $\mathscr{S}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g}}$ be a non-empty $\mathfrak{T}_{\mathfrak{g}}$-set in $\mathfrak{T}_{\mathfrak{g}}$. Then, it is no error to express it in the form $\mathscr{S}_{\mathfrak{g}} = \mathscr{S}_{\mathfrak{g}}^{\mathrm{even}} \cup \mathscr{S}_{\mathfrak{g}}^{\mathrm{odd}}$, where $\mathscr{S}_{\mathfrak{g}}^{\mathrm{even}} = \big\{\mu : (\forall \alpha \in I_{\infty}^{*})\,[\mu = 2\alpha]\big\}$ and $\mathscr{S}_{\mathfrak{g}}^{\mathrm{odd}} = \big\{\mu : (\forall \alpha \in I_{\infty}^{*})\,[\mu = 2\alpha - 1]\big\}$. Since $\mathscr{S}_{\mathfrak{g}} \neq \emptyset$, consider an arbitrary point $\xi \in \mathscr{S}_{\mathfrak{g}}$. If $\xi \in \mathscr{S}_{\mathfrak{g}}^{\mathrm{even}}$

then, for every $\mathfrak{T}_{\mathfrak{g}}$-open set $\mathscr{U}_{\mathfrak{g},\xi} \in \mathrm{O}[\mathfrak{T}_{\mathfrak{g}}]$ containing ξ, $\mathscr{S}_{\mathfrak{g}}^{\mathrm{even}} \cap (\mathscr{U}_{\mathfrak{g},\xi} \setminus \{\xi\}) = \emptyset$ and $\mathscr{S}_{\mathfrak{g}}^{\mathrm{odd}} \cap (\mathscr{U}_{\mathfrak{g},\xi} \setminus \{\xi\}) \neq \emptyset$. But, if $\xi \in \mathscr{S}_{\mathfrak{g}}^{\mathrm{odd}}$ then, for every $\mathfrak{T}_{\mathfrak{g}}$-open set $\mathscr{U}_{\mathfrak{g},\xi} \in \mathrm{O}[\mathfrak{T}_{\mathfrak{g}}]$ containing ξ, $\mathscr{S}_{\mathfrak{g}}^{\mathrm{even}} \cap (\mathscr{U}_{\mathfrak{g},\xi} \setminus \{\xi\}) \neq \emptyset$ and $\mathscr{S}_{\mathfrak{g}}^{\mathrm{odd}} \cap (\mathscr{U}_{\mathfrak{g},\xi} \setminus \{\xi\}) = \emptyset$. In either case, it follows, then, that $\mathscr{S}_{\mathfrak{g}}$ have at least one $\mathfrak{T}_{\mathfrak{g}}$-accumulation point. Accordingly, $\mathfrak{T}_{\mathfrak{g}}$ is a $\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{CA}]}$-space. For every $\alpha \in I_{\infty}^{*}$, set $\mathscr{U}_{\mathfrak{g},2\alpha-1} = \{2\alpha-1\}$ and $\mathscr{U}_{\mathfrak{g},2\alpha} = \{2\alpha\}$. Accordingly, $\mathscr{U}_{\mathfrak{g},2\alpha-1}$, $\mathscr{U}_{\mathfrak{g},2\alpha} \in \mathfrak{g}\text{-}\mathrm{O}[\mathfrak{T}_{\mathfrak{g}}]$ since $\mathscr{U}_{\mathfrak{g},2\alpha-1}$, $\mathscr{U}_{\mathfrak{g},2\alpha} \subseteq \mathrm{op}_{\mathfrak{g}}(\mathscr{O}_{\mathfrak{g},(2\alpha-1,2\alpha)}) \in \mathfrak{g}\text{-}\mathrm{O}[\mathfrak{T}_{\mathfrak{g}}]$ for every $\alpha \in I_{\infty}^{*}$. Observe that, $\mathscr{S}_{\mathfrak{g}} \cap (\mathscr{U}_{\mathfrak{g},2\alpha-1} \setminus \{2\alpha-1\}) = \emptyset$ and $\mathscr{S}_{\mathfrak{g}} \cap (\mathscr{U}_{\mathfrak{g},2\alpha} \setminus \{2\alpha\}) = \emptyset$ for every $\alpha \in I_{\infty}^{*}$. This proves the existence of an infinite $\mathfrak{T}_{\mathfrak{g}}$-set $\mathscr{R}_{\mathfrak{g}} \subset \mathfrak{T}_{\mathfrak{g}}$ with no $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-accumulation point and hence, $\mathfrak{T}_{\mathfrak{g}}$ is not a $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{CA}]}$-space.

In relation to the above descriptions, further $\mathscr{T}_{\mathfrak{g}}$-properties amongst the $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$-spaces $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{A}]} = (\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]})$, $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{CA}]} = (\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{CA}]})$, $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{SA}]} = (\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{SA}]})$, and $\mathfrak{g}\text{-}\mathfrak{T}_{\mathfrak{g}}^{[\mathrm{LA}]} = (\Omega, \mathfrak{g}\text{-}\mathscr{T}_{\mathfrak{g}}^{[\mathrm{LA}]})$ called, respectively, $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$*-space*, *countably* $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$*-space*, *sequentially* $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$*-space*, and *locally* $\mathfrak{g}$-$\mathscr{T}_{\mathfrak{g}}^{[\mathrm{A}]}$*-space*, can be discussed in a similar way by slight modifications of some $\mathscr{T}_{\mathfrak{g}}$-properties found in those cases.

4.3. Concluding Remarks

In a recent paper [11] the study of a novel class of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness in $\mathscr{T}_{\mathfrak{g}}$-spaces was presented. In this paper, the concept is further studied and other derived concepts called countable, sequential, local $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness in $\mathscr{T}_{\mathfrak{g}}$-spaces have also been studied relatively. It was shown that $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness implies local $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness and countable $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness, sequential $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness implies countable $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness and $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness is a generalized topological property (briefly, $\mathscr{T}_{\mathfrak{g}}$*-property*).

For future research, it would be interesting to develop the theory of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness of mixed categories. More precisely, for some pair $(\nu,\mu) \in I_3^0 \times I_3^0$ such that $\nu \neq \mu$, to develop the theory of $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-compactness in terms of relatively $\mathfrak{g}$-$\mathfrak{T}_{\mathfrak{g}}$-open sets belonging to the class $\{\mathscr{U}_{\mathfrak{g}} = \mathscr{U}_{\mathfrak{g},\nu} \cup \mathscr{U}_{\mathfrak{g},\mu} : (\mathscr{U}_{\mathfrak{g},\nu}, \mathscr{U}_{\mathfrak{g},\mu}) \in \mathfrak{g}\text{-}\nu\text{-}\mathrm{O}[\mathfrak{T}_{\mathfrak{g}}] \times \mathfrak{g}\text{-}\mu\text{-}\mathrm{O}[\mathfrak{T}_{\mathfrak{g}}]\}$ in a $\mathscr{T}_{\mathfrak{g}}$-space $\mathfrak{T}_{\mathfrak{g}}$. Such a theory is what we thought would certainly be worth considering, and the discussion of this paper ends here.

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Declaration of Ethical Standards

The authors declare that the materials and methods used in their study do not require ethical committee and/or legal special permission.

Authors Contributions

Author [Mohammad Irshad Khodabocus]: Thought and designed the research/problem, collected the data, contribution to completing the research and solving the problem, wrote the manuscript (%80).

Author [Noor-Ul-Hacq Sookia]: Contributed to research method or evaluation of data (%20).

Conflicts of Interest

The authors declare no conflict of interest.

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