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RESEARCH ARTICLE

Deflection analysis of functionally graded equal strength beams

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Abstract: In this study, equal strength cantilever and simply supported beams made of functionally graded material (FGM) whose material properties vary along the thickness direction were investigated. Equal strength cantilever FGM beams were loaded with uniformly distributed load and a point load at the tip and simply supported FGM beams were loaded with uniformly distributed loads. They all have variable cross-sections and a straight axis. For calculating equivalent material properties of FGMs, power-law distribution and the Mori-Tanaka model were used. A computer program was developed for the analysis of the problem. The dimensionless deflection values for cantilever beams and simply supported beams were obtained for different materials with the help of the developed computer program. Obtained results are presented in tables and graphs which, may be helpful for the researchers.

Keywords: Functionally graded material (FGM); Mori-Tanaka model; equal strength beam; variable cross-section.

1. Introduction

Beams are mainly used for carrying vertical loads. In general, moment and shear forces are the main determinants of the deformations and stresses caused by the loading. The bending moment of the beam is variable along the beam axis. While the beam sections are designed, a constant cross-section can be selected according to the maximum bending moment value, or a variable cross-section can be used in accordance with the bending moment. If the beam is designed by keeping the maximum bending stress constant in all cross-sections along the axis, the cross-section will be reduced where the bending moment is small and will increase where the bending moment is large. These types of beams are called equal strength beams and materials are saved because of their variable cross-sections.

If equal strength beams are used, the cross-section area will be reduced. This will result in material savings and a reduction in the weight of the beam. The basic logic of the FGM is to make a composite material by changing the microstructure from one material to another with a particular gradient. This ensures that the new material created has the best properties of the constituent materials. Studies have shown that FGM's effectively increases systems' resistance to thermal effects, corrosion, fatigue, fracture, and stress cracking [1,2].

There are previous studies on equal strength beams. Many basic books about the strength of materials or mechanics of materials have sections about this subject. Inan, Bakioğlu covers the subject in their books [3,4].

Due to its useful features, research on FGM is continuing intensively. Özarslan found the equations of motion of a functionally graded plate made of zircon and aluminum materials by making the Navier solutions according to the classical plate theory and found the natural frequencies by making free vibration analyzes of plates in different combination ratios and different sizes in his graduate study [5]. Ersan investigated the behavior of functionally graded discs under thermal load by analytical and numerical analysis methods in her graduate study [6]. Kadoli et al. studied the static analysis of functionally graded beams using higher-order shear deformation theory [7]. Chauhan and Khan have published a review paper on the analysis of beam-type structures made of functionally graded material [8]. Alagöz et al. made a study about functionally graded materials and their usage areas [9]. Arslan et al. investigated the free vibration analysis of straight axis beams made of bidirectional functionally graded material (FGM) in frequency space [10]. In the study of Sınır and Çevik, linear and non-linear natural frequencies of the axially polynomial functionally graded beam by considering the non-linear Euler-Bernoulli beam theory were obtained [11]. Çalım studied the free vibration analysis of axially functionally graded beams with variable cross-sec-

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tions [12]. Rezaiee-Pajand et al. investigated the static behavior of non-prismatic sandwich beams composed of functionally graded materials [13]. Atmane et al. studied the free vibration behavior of exponential functionally graded beams with varying cross-sections [14].

In engineering fields such as aerospace, mechanical, and civil engineering, beams with variable cross-sections are commonly used. This study investigates the design of straight axis cantilever and simple beams with equal strength, made of functionally graded materials using different ceramic and metal materials. Equations of elastic curves of the beams are obtained.

A program is written for the analysis of beams examined using Matlab software. Results are obtained by using the developed codes. The obtained results were compared with the finite element program and the accuracy of the solution was demonstrated.

2. MATERIALS AND METHODS

The geometry of a functionally graded prismatic beam is shown in Figure 1. The *x*, *y,* and *z* axes are taken in width, height, and length directions, respectively. Since the geometry of the beam is symmetrical with respect to the vertical y-axis, the acting loads are in the yz plane.

In the study, it is assumed that the beam material behaves linearly elastic and small deformations occur.

As presented in Figure 1b, phase transitions of ceramic and metal materials functionally graded material along the cross-section height can be assumed as quasihomogeneous ceramic-metal layers and a continuous variation of the volume fraction of ceramic or metal materials.

2.1. Functionally Graded Materials

The functionally graded beam is typically made of ceramic and metal. It is assumed that with the thickness of the beam, the material properties vary. There is ceramic material on the upper surface of the beam and metal material on the lower surface.

The volume ratio distribution of functionally graded materials is as follows:

$$P = P_c V_c + P_m V_m \quad (1)$$

$$V_c + V_m = 1 \quad (2)$$

$$P = P_c + V_m \left(P_m - P_c \right) \quad (3A)$$

$$P = P_m + V_c \left(P_c - P_m \right) \quad (3B)$$

Material properties for ceramic and metal are shown with symbols, P_c and P_m, respectively. V_c and V_m are the material volume fractions of ceramic and metal [16].

There are several models developed to determine the material properties of FGM. Some of them are material distributions depending on various functions such as power-law distribution, sigmoid distribution, and exponential distribution [17]. The power-law distribution is given by

$$P(y) = P_c + \left(\frac{y}{h} + \frac{1}{2} \right)^n \left(P_m - P_c \right) \quad (4)$$

where *n* is a non-negative real number called the power-law index which identifies the material variation profile through the thickness of the beam.

In this study, material properties vary along the thickness dimension of the beam according to the power-law distribution.

The metal volume fraction is expressed as $V_m = \left((y/h) + (1/2) \right)^n$ according to the power-law distribution.

For the given fraction formula, metal distribution along the thickness direction for different power-law indices is given in Figure 2.

Many different models have been proposed to homogenize the mechanical properties of functionally graded materials [18,19]namely, Voigt (V. The Mori-Tanaka model was used in this study. Because this model is one of the most common homogenization techniques for modeling effective material properties. According to this model, the equivalent modulus of elasticity *E* is calculated as in equation (5):

$$E = \frac{9KG}{3K + G} \quad (5)$$

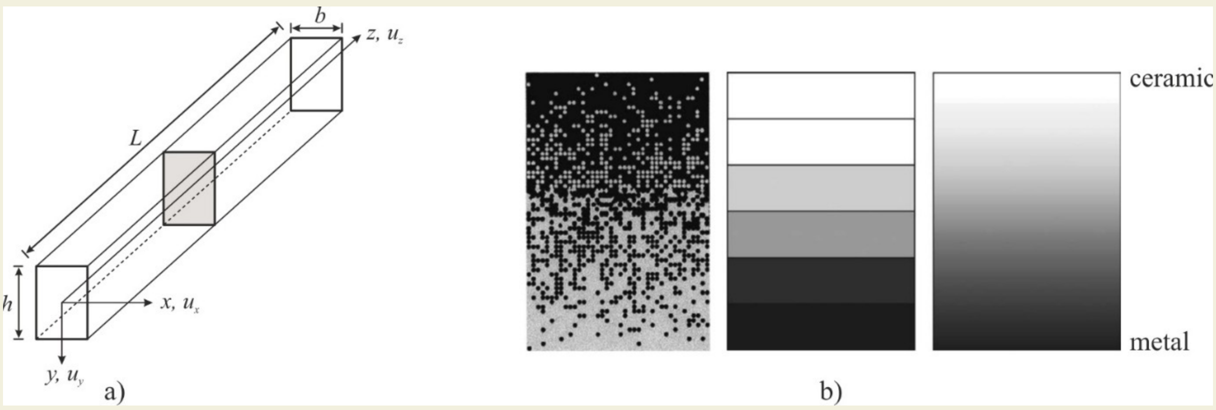


Figure 1. The geometry of a functionally graded beam and material distribution in cross-section [15,16].

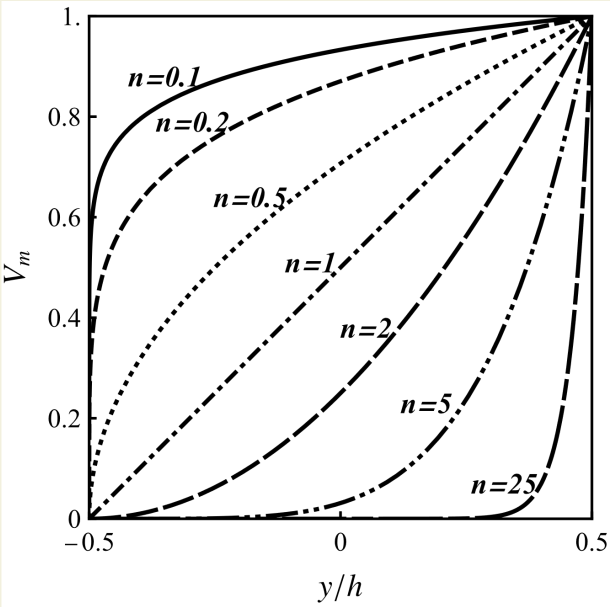


Figure 2. Value of metal volume fraction along the thickness direction depending on the index n.

In this expression, K denotes the bulk modulus and G denotes the shear modulus. According to the Mori-Tanaka model, these modules are expressed as follows [18,20].

$$\frac{K-K_c}{K_m-K_c}=\frac{V_m}{1+(1-V_m)\frac{3(K_m-K_c)}{3K_c+4G_c}}$$

$$\frac{G-G_c}{G_m-G_c}=\frac{V_m}{1+(1-V_m)\frac{G_m-G_c}{G_c+f}} \tag{6}$$

$$K_{c,m}=\frac{E_{c,m}}{3(1-2\nu_{c,m})} \qquad G_{c,m}=\frac{E_{c,m}}{2(1+\nu_{c,m})}$$

$$V_m=\left(\frac{y}{h}+\frac{1}{2}\right)^n \qquad f=G_c\frac{9K_c+8G_c}{6(K_c+2G_c)}$$

In this expression, K_m is the bulk modulus of the metal phase and K_c is the bulk modulus of the ceramic phase. G_c is the shear modulus of the ceramic phase.

V_c and V_m are the volume ratios calculated according to equation (2) depending on the thickness coordinate of the ceramic and metal material, respectively, and indicate the material distribution.

2.2. Equal Strength Beams

Internal forces in a general beam are one axial force, two shear forces, one torsion moment, and two bending moments. Due to these internal forces, stresses (σ, τ), strains (ε, γ), and elastic curve (vertical displacements u) occur. According to the Euler-Bernoulli beam theory, also called classical beam theory, bending moment M_x and shear force S_y are dominant in beams. Other internal forces are often not very effective. On the deformation of the beam, bending is more effective than the others.

The bending moment in the beam varies along the beam axis. While the beam sections are being designed, a constant section can be determined according to the maximum bending moment in the beam or a variable section can be designed with the bending moment variation. If the beam is designed by keeping the maximum absolute value of stress as constant in all cross-sections along the axis, the cross-section will decrease in places where the bending moment is small, and the cross-section will increase in places where it is large. Such beams are called equal strength beams.

For beams of equal strength, the ratio of bending moment to the first moment of area is constant as stated in equation (7) [3,4].

$$\sigma_{\max}=M_x(z)/W_x(z)=\sigma_{\text{allow}}=\text{constant} \tag{7}$$

In this expression, $W_x(z)$ is the first moment of area that varies along the beam axis z. It can be expressed mathematically by equation (8).

$$W_x(z)=M_x(z)/\sigma_{\text{allow}} \tag{8}$$

If the bending moment variation of the loaded beam is determined, the variation function of the first moment of area can be obtained from the condition that all sections are of equal stress. With this obtained function, variation of the cross-section that supply the condition was presented. The cross-section variation can be along with the height, width, or both.

First moment of area of a rectangular cross-section can be calculated in equation (9)

$$W_x(z)=\frac{I_x}{h_z/2}=\frac{b_z h_z^3/12}{h_z/2}=\frac{b_z h_z^2}{6} \tag{9}$$

The cantilever beam loaded with uniform load and a point load at the tip and simply supported beam loaded with the uniformly distributed load as shown in Figure 3 are chosen as example problems.

If the ratio of bending moments to required first moment of area is substituted as stated in equation (7), the expression becomes for the cantilever beam loaded with uniform load, a point load at the tip and simply supported beam loaded with uniformly distributed load respectively:

$$\text{constant}=\frac{M_x(z)}{W_x(z)}=\frac{-\frac{qL^2}{2}+\text{q}Lz-\frac{qz^2}{2}}{\frac{b_z h_z^2}{6}}=\frac{-3q(L-z)^2}{b_z h_z^2} \tag{10A}$$

$$\text{constant}=\frac{M_x(z)}{W_x(z)}=\frac{-P(L-z)}{\frac{b_z h_z^2}{6}}=\frac{-6P(L-z)}{b_z h_z^2} \tag{10B}$$

$$\text{constant}=\frac{M_x(z)}{W_x(z)}=\frac{-\frac{q}{2}(z^2-Lz)}{\frac{b_z h_z^2}{6}}=\frac{-3q(z^2-Lz)}{b_z h_z^2} \tag{10C}$$

If the section width $b_z=b_0$ is constant and the section

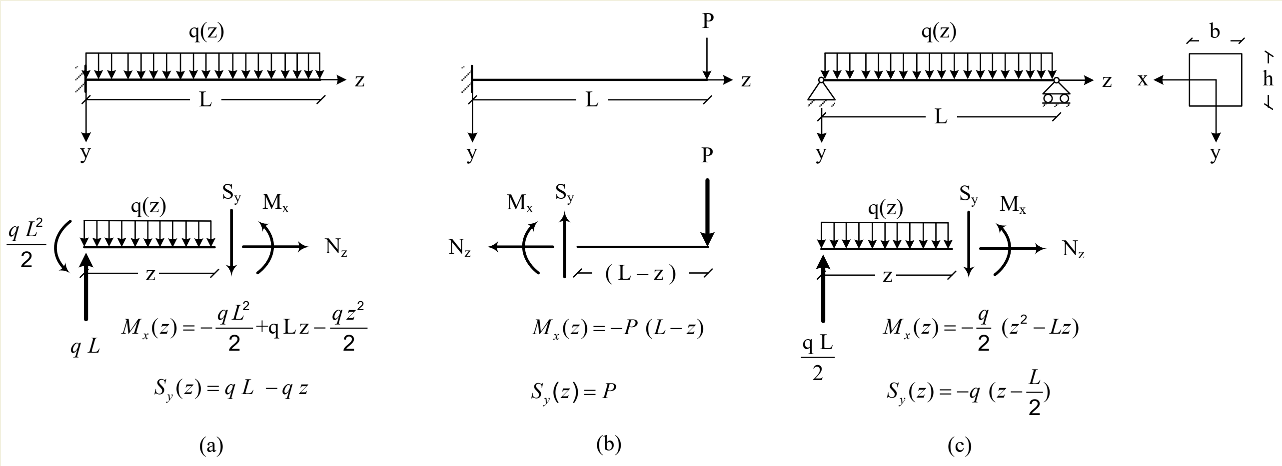
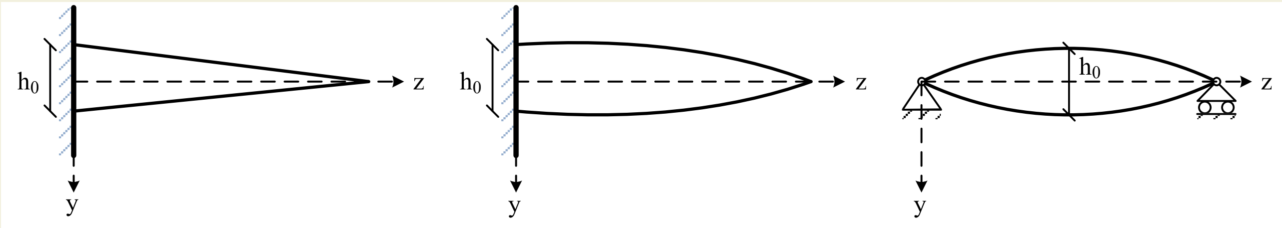


Figure 3. Beams with loads; (a) Uniform loaded cantilever beam (b) Point loaded cantilever beam (c) Uniform loaded simply supported beam [21].



(a) (b) (c)

Figure 4. The schematic representation of the cross-section height of beams. (a) Uniform loaded cantilever beam (b) Point loaded cantilever beam (c) Uniform loaded simply supported beam.

height h_z is chosen as a variable in the variable beam section, and if the section height for the cantilever beam $z=0$ at the fixed support and for the simply supported beam $z=L/2$ at the mid-point is defined as h_0, then the variable section height function will be for distributed loaded cantilever, point loaded cantilever and distributed loaded simple beams respectively as in equations below:

$$z=0 \Rightarrow \frac{-3q(L-z)^2}{b_0h_z^2}=\frac{-3q(L-0)^2}{b_0h_0^2} \quad \Rightarrow \quad h_z=\frac{h_0(L-z)}{L} \quad (11A)$$

$$z=0 \Rightarrow \frac{-6P(L-z)}{b_0h_z^2}=\frac{-6P(L-0)}{b_0h_0^2} \quad \Rightarrow \quad h_z=\sqrt{\frac{L-z}{L}}\,h_0 \quad (11B)$$

$$z=\frac{L}{2} \Rightarrow \frac{-3q(z^2-Lz)}{b_0h_z^2}=\frac{-3q\left(\left(\frac{L}{2}\right)^2-L\left(\frac{L}{2}\right)\right)}{b_0h_0^2} \quad \Rightarrow \quad h_z=\sqrt{\frac{4z(L-z)}{L^2}}\,h_0 \quad (11C)$$

The schematic representation of the cross-section height of beams, whose section width is uniform and height varies as in equations, is as in Figure 4.

With this calculated section height function of the beams, the moment of inertia will be for distributed loaded cantilever, point loaded cantilever, and distributed loaded simple beams respectively as in equations (12):

$$I_x=\frac{b_0h_z^3}{12}=\frac{b_0}{12}\left(\frac{h_0(L-z)}{L}\right)^3=\frac{b_0h_0^3}{12}\left(\frac{L-z}{L}\right)^3=I_0\left(\frac{L-z}{L}\right)^3 \quad (12A)$$

$$I_x=\frac{b_0h_z^3}{12}=\frac{b_0}{12}\left(\sqrt{\frac{L-z}{L}}\,h_0\right)^3=\frac{b_0h_0^3}{12}\left(\frac{L-z}{L}\right)^{3/2}=I_0\left(\frac{L-z}{L}\right)^{3/2} \quad (12B)$$

$$I_x=\frac{b_0h_z^3}{12}=\frac{b_0}{12}\left(\sqrt{\frac{4z(L-z)}{L^2}}\,h_0\right)^3=\frac{b_0h_0^3}{12}\left(\frac{4z(L-z)}{L^2}\right)^{3/2}=I_0\left(\frac{4z(L-z)}{L^2}\right)^{3/2} \quad (12C)$$

I_0 is the moment of inertia of the section at the fixed support point of the cantilever beams and at the mid-point of the simply supported beam in these expressions. Using the virtual work method, the critical displacements of the distributed loaded cantilever, point loaded cantilever, and distributed loaded simple beams are respectively as in equation (13).

$$\delta_{tip}=\frac{6qL^4}{Eb_0h_0^3} \quad (13A)$$

$$\delta_{tip}=\frac{8PL^3}{Eb_0h_0^3} \quad (13B)$$

$$\delta_{mid}=\frac{(\pi-2)3qL^4}{16Eb_0h_0^3} \quad (13C)$$

A computer program was developed with the help of MATLAB program within the scope of the study and the results were obtained using developed codes. In the first part of the computer program index n, the height of the cross-section h, Young's Modulus and Poisson's ratio of ceramic material and the metal material is taken from the user as input variables. With the help of the Mori-Tanaka model, the equivalent modulus of elasticity $E_{equivalent}$ was calculated with the volume modulus K and shear modulus G, according to this model. The developed computer program calculates the critical displacements using the virtual work method for three types of problem options; uniformly loaded cantilever beam, point loaded cantilever beam, and uniformly loaded simply supported beam. The flowchart of the program is shown in Figure 5.

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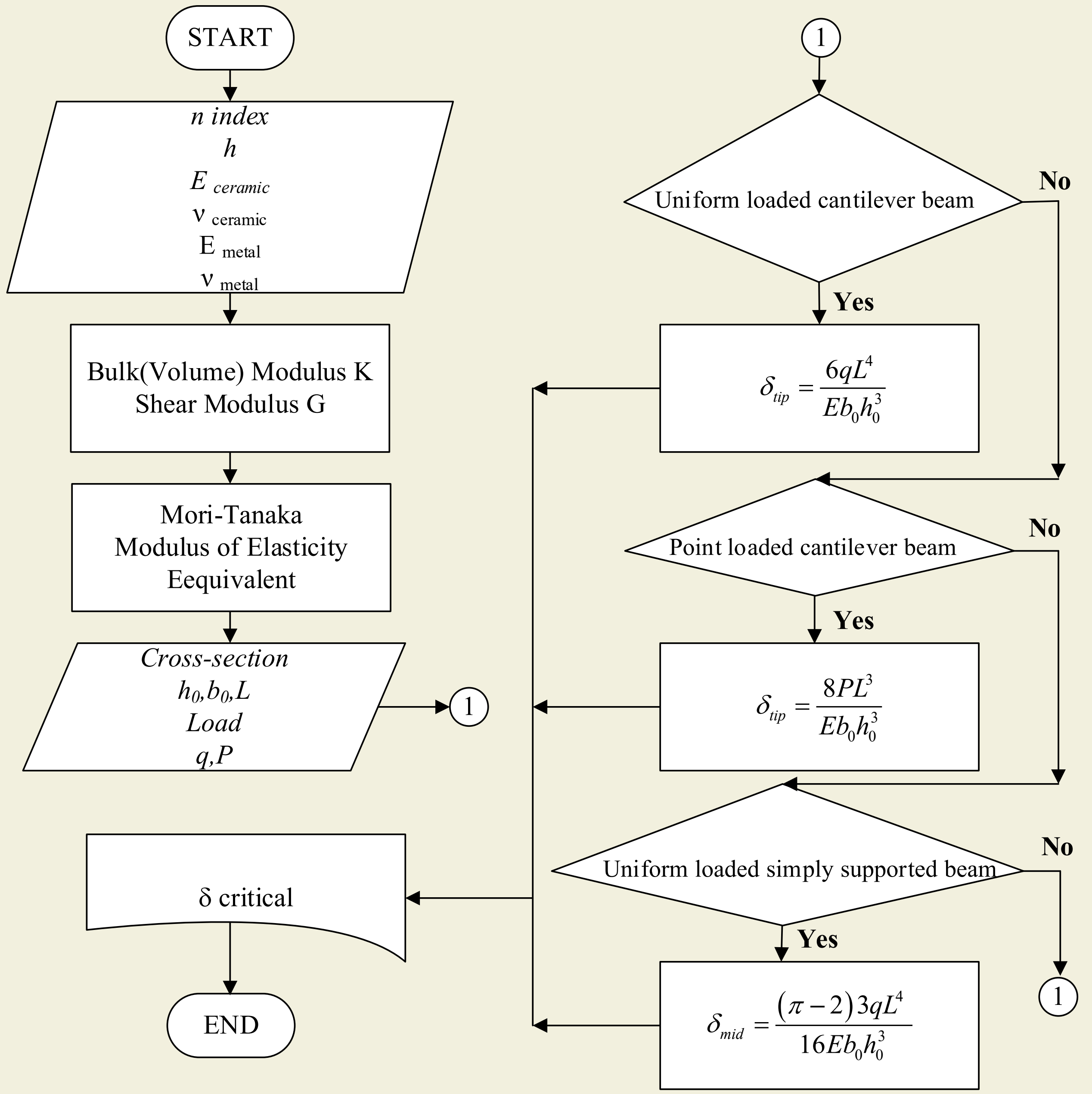

Figure 5. Flow chart of the developed computer program.

3. Results and Discussion

As an example, equal-strength cantilever functionally graded beam loaded with distributed load and point load and uniformly distributed loaded and equal-strength simply supported functionally graded beam loaded with uniformly distributed load are considered. The width of the beam, whose geometry is shown in Figure 1, is considered as b=0.1m and its height as h=0.1m. The length of the beam is L = 1m. In the problems, it is considered that point load P is 100 kN and uniformly distributed load q is 100 kN/m.

The calculated deflection values for point load and distributed load respectively are put in a dimensionless form by using equation (14).

$$\hat{u}_y = u_y \times \frac{E_m \times b \times h_0^3}{P \times L^3} \times 100$$
$$\hat{u}_y = u_y \times \frac{E_m \times b \times h_0^3}{q \times L^4} \times 100 \qquad (14)$$

In this expression, E_m is the modulus of elasticity of the metal material.

Dimensionlessalizing of deflection values with the constant dimensions of the problems as b, h_0, L, P, and q can be calculated by using equation (15)

$$\hat{u}_y = u_y \times \frac{E_m \times 0.1 \times 0.1^3}{100 \times 1^3} \times 100 \Rightarrow \hat{u}_y = \frac{1}{10\,000} \times u_y \times E_m \qquad (15)$$

In this expression, u_y is the deflection value as meter and

E_m is the Young's Modulus of metal material as kN/m^2.

Mechanical properties of ceramic and metal materials used for functionally graded materials are shown in Table 1 [20,22–26].

Table 1. Mechanical properties of ceramic and metal materials [20,22–26].

	Material	Young's Moduli (GPa)	Poisson's ratio
Ceramic	Zirconia (ZrO_2)	244.27	0.2882
	Titanium Carbide (TiC)	480.00	0.2000
	Aluminum Oxide (Al_2O_3)	349.55	0.2600
	Silicon Nitride (Si_3N_4)	348.43	0.2400
Metal	Stainless Steel (SUS304)	201.04	0.3262
	Nickel (Ni)	223.95	0.3100
	Aluminum (Al)	70.00	0.3100

In the functionally graded beam, ceramic and metal materials are used and the material properties vary with the beam's thicknes. Metal material is used at the bottom of the beam and ceramic material is used at the top.

The maximum dimensionless deflection values at the tip of the point loaded cantilever beam with variable cross-section depending on the variation in material distribution index *n* in the example whose material properties vary in the direction of beam height according to the power-law distribution are shown in Figure 6 b.

When the material distribution index *n* according to the power-law distribution is zero, the functionally graded beam is formed from completely ceramic (Figure 2). As can be seen from the calculations, the dimensionless deflection values are the maximum deflections in the all-ceramic beam models. As the index increases, the ceramic material will decrease in volume and the metal material will increase. When this value is 10, it can be considered that the cantilever beam is made of metal material. In the cantilever beam of equal strength and variable section, the maximum stress value in all sections along the beam axis is the same as the stress value at the fixed support point of the uniform section cantilever beam. As the index increases, the dimensionless deflection value at the tip point of the cantilever beam decreases (Figure 6). The behavior of the beam changes according to the mechanical properties of the ceramic and metal materials used in FGMs.

The dimensionless deflection functions for the material distribution index *n* of 0.2 in uniform cross-section and variable cross-section point loaded cantilever beams made of FGM using Aluminum Oxide (Al_2O_3) for ceramic material and Aluminum (Al) for metal material are shown in Figure7 b.

As can be seen in Figure7 b, the deflection of the point-loaded cantilever functionally graded (Al_2O_3 / Al) beam with the uniform section of height h_0 is less than the deflection of the equal strength beam with the same stress as this beam and a variable cross-section. While this variation is less in the fixed support, it is higher at the tip point of the cantilever beam where the point load is affected. A similar situation occurs for equal strength beams made of functionally graded materials using other ceramic and metal materials.

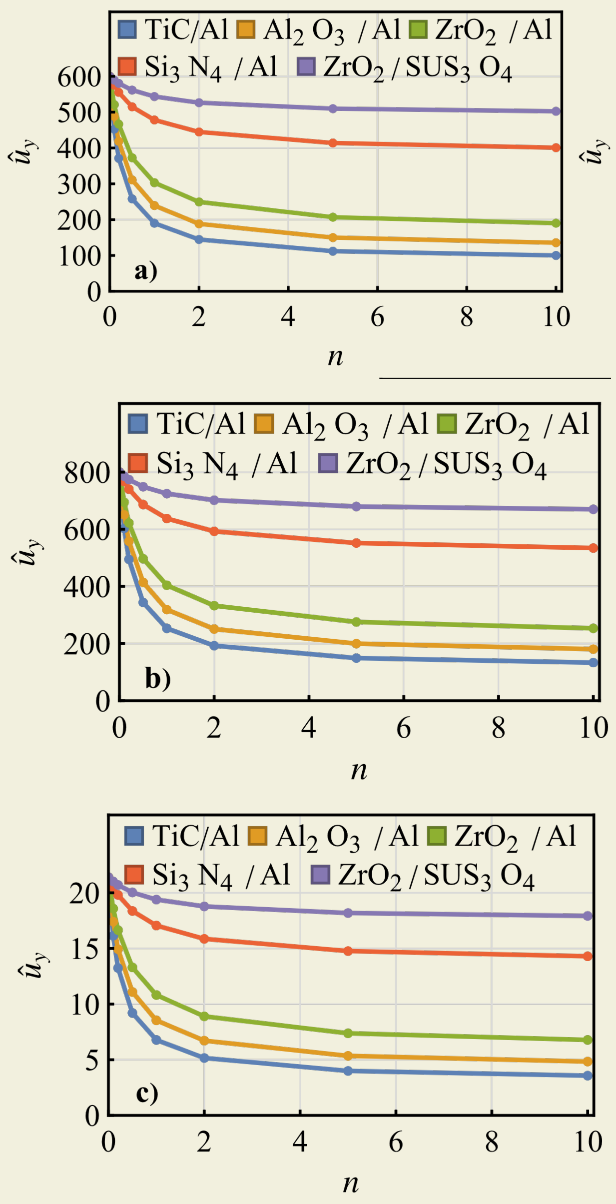

Figure 6. Distribution of dimensionless deflection values of equal strength functionally graded beams depending on the material distribution index *n* a) distributed loaded cantilever b) point loaded cantilever c) distributed loaded simply supported.

The maximum dimensionless deflection values at the tip of the distributed loaded cantilever beam with variable cross-section depending on the variation in material distribution index n in the example whose material properties vary in the direction of beam height according to the power-law distribution are shown in Figure 6 a.

As can be seen from the calculations, the dimensionless deflection value is the maximum for the all-ceramic beam models. As the index n increases, deflection values decrease.

The dimensionless deflection functions for the material distribution index *n* of 0.2 in uniform cross-section and variable cross-section cantilever beams made of FGM using Aluminum Oxide (Al_2O_3) for ceramic material and Aluminum (Al) for metal material are shown in Figure7 a.

As can be seen in Figure7 a, the deflection of the distributed loaded cantilever functionally graded (Al_2O_3 / Al) beam with the uniform section of height h_0 is less than the deflection of the equal strength beam with the same stress as this beam and a variable cross-section. While this variation is less in the fixed support, it is higher at the tip point of the cantilever beam where the point load is affected. A similar situation occurs for equal strength beams made of functionally graded materials using other ceramic and metal materials.

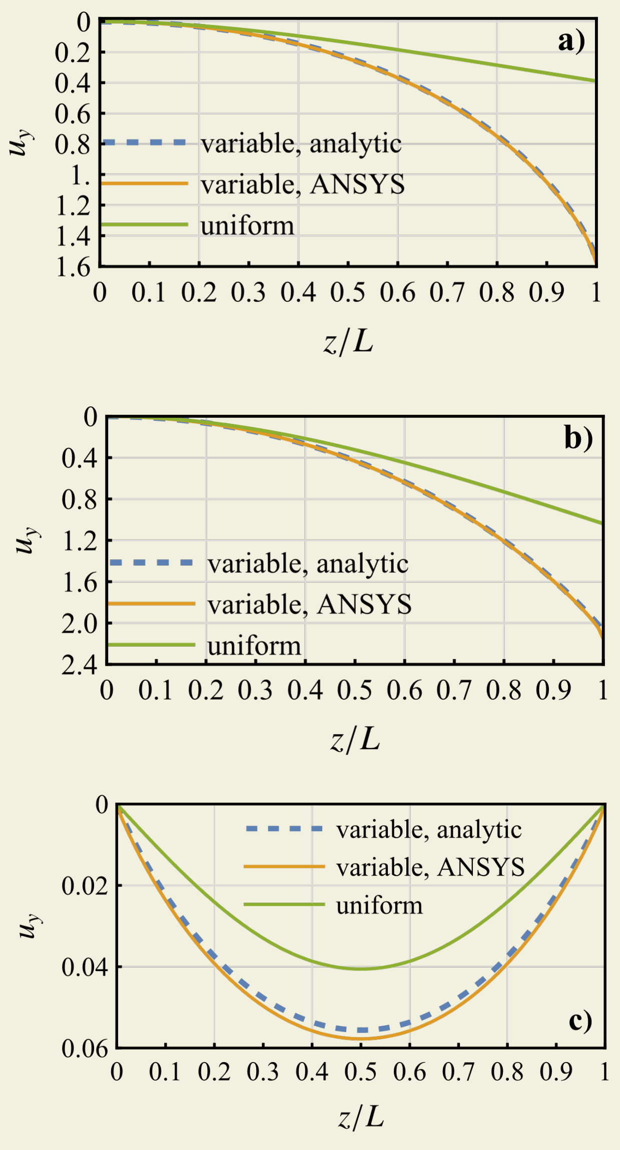

Figure7. Dimensionless deflection functions of functionally graded Al_2O_3/Al beams a) distributed loaded cantilever b) point loaded cantilever c) distributed loaded simply supported.

The variation with the dimensionless deflection values at the quarter-point of the simply supported beam with variable cross-section loaded with uniformly distributed load with the material distribution index n in the example whose material properties vary in the direction of beam height according to the power-law distribution are shown in Figure 6 c.

As can be seen from the calculations, the dimensionless deflection value is the maximum for the all-ceramic beam models as cantilever beams. As the index n increases, deflection values decrease.

The dimensionless deflection functions for the material distribution index *n* of 0.2 in uniform cross-section and variable cross-section simply supported beams made of FGM using Aluminum Oxide (Al_2O_3) for ceramic material and Aluminum (Al) for metal material are shown in Figure7 c.

As can be seen in Figure7 c, mid-point deflections are the same for variable and uniform cross-sections. Because they have the same section of height h_0 at the mid-point. While the deflection variation is less at the mid-point, it is higher at the supports of the simple beam where the height of the section decreases.

Table 2. Volume ratios of sample problems

Sample problem	Volume Ratio
Uniform loaded cantilever beam	0.500
Point loaded cantilever beam	0.667
Uniform loaded simply supported beam	0.785

The volumes of the equal strength beams are compared with the initial ones and the volume ratio is determined. A comparison of the volume savings of sample problems is shown in Table 2.

3.1. Verification with Finite Element Software ANSYS

Sample problems are modeled in the ANSYS program with the equivalent modulus of elasticity and equivalent Poisson's ratio. The dimensions of the example problems are taken the same as analytical studies.

The results obtained from the developed computer program were verified by the finite element software ANSYS solutions (Figure 8).

Table 3. Comparisons of the present solutions with ANSYS results of critical deflections.

Sample problem	Present	ANSYS	Difference
Uniform loaded cantilever beam	8.3045	8.4273	-1.46%
Point loaded cantilever beam	11.0726	11.4723	-3.48%
Uniform loaded simply supported beam	0.2963	0.3077	-3.72%

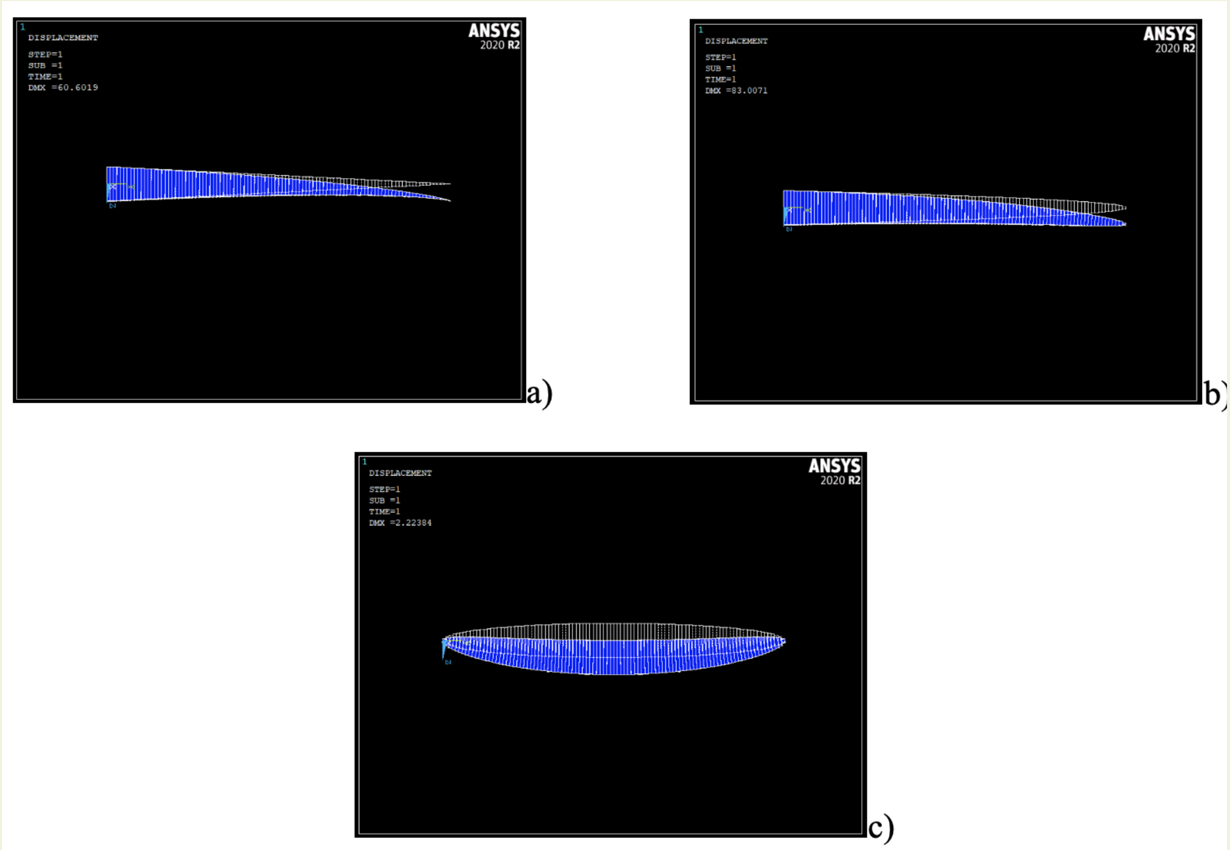

Figure 8. ANSYS solutions of discussed sample problems.

Dimensionless critical deflection values are given in Table 3 which are used for the verification of the sample problems.

There is a good agreement between the results of this study and ANSYS software results.

4. Conclusions

In this study, a computer program was developed. With the help of this program, the deflection functions of the equal-strength cantilever beam carrying a point load at the tip and equal-strength simply supported functionally graded beam loaded with uniformly distributed load made of FGM whose material properties vary with the height of the beams were obtained. The deflection values calculated for functionally graded beams with different ceramic and metal materials used in the literature have been made dimensionless and presented in the form of tables and graphics.

The material properties of FGMs have been modeled with different functions in the literature. In this study, the power-law distribution is considered. Mori-Tanaka model was used for the equivalent mechanical properties of the material.

The cross-sectional variation for the equivalent beam was calculated by keeping the stress constant calculated by the ratio of the bending moment to the required first moment of area. A function for beam height was obtained by keeping the beam width constant for the cross-section variation. Section height h_0 at the support of the cantilever beam and at the mid-point of the simple beam has decreased to zero along the beam axis according to this calculated function. Depending on this variation, the moment of inertia of the section and thus the bending stiffness were variable. The deflection function is obtained for equal strength beams made of FGMs. By giving some numerical values for the obtained function, the design of cantilever and simple beams of the variable cross-section with straight axis and equal strength made of different functionally graded materials is examined.

From the calculations, it is observed that when the index n increases the maximum deflection of the beam decreases. The maximum deflections of the problems of a cantilever beam with a point load at the tip and uniformly loaded simply supported beam with different ceramic and metal materials behave similarly. When the maximum deflection values for these problems are for the materials ZrO2/SUS304, for uniformly loaded cantilever beam problem it is for the materials of ZrO2/Al material.

As a result of this study, it is proved that it has been demonstrated that the flexibility of the FGM beam can be increased without reducing the strength by using equal strength beams made of functionally graded materials. In addition, analyses can be made for straight axis and equal strength variable cross-section beams made of FGM under different boundary conditions and different loadings.

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