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Balancing The Shirt Production Line Under Different Operational Constraints Using An Integer Programming Model

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ABSTRACT

Efficient use of capacity is significant to enable apparel businesses to work cost-effectively and provide timely service to their customers. The increase in assembly-line efficiency is associated with lower operating costs. Therefore, balancing assembly lines is mainly to manufacture products as profitable and quickly as possible. In this study, we consider a single-model assembly line balancing problem with workforce and machine constraints in the sewing department of an apparel company. We develop an integer programming (IP) model to optimally balance the shirt production line, considering parallel machines in each stage of the line and various operational constraints such as cycle time and precedence constraints, task machine eligibility, and the number of operators available. The IP model can either minimize the number of open workstations or both, minimize the number of open workstations and simultaneously assign tasks in subassembly parts close to each other. The model has been run under various scenarios using LINGO 15.0 optimization software. Additionally, we have balanced the shirt production line using the Ranked Positional Weight Method (RPWM) for comparison purposes. The IP model outperforms the RPWM results across all scenarios and finds 33 stations and 86.8% efficiency compared to 38 stations and 75.4% balance efficiency with the RPWM.

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1. INTRODUCTION

Assembly lines are one of the most popular production methods among flow-through manufacturing systems. They are extensively used in the production of high-quality standard products. Simultaneously, assembly lines have become increasingly important for producing small quantities of custom products [1]. Increasing product variability and shorter life cycles have shifted from traditional production methods to assembly lines. Assembly lines are expected to produce products fast, efficiently, cost-effectively, and with the necessary quality [2]. The assembly line balancing problem (ALBP) consists of assigning tasks to an orderly sequence of stations so that the precedence relationships between the tasks are satisfied and some performance measures are optimized (e.g., minimize the balance delay or minimize the number of workstations) [3].

The apparel industry is a very labor-intensive industry. The efficient use of capacity is of the utmost importance for apparel companies to operate cost-effectively and provide timely service to their clients. Delivering orders on time is essential to improve the relationship with customers. Reducing operational costs and delivering orders on time are closely linked to improving line efficiency. Even though the quantities ordered have declined over the last two decades, the variety of models has increased, making the rapid creation of a balanced line another crucial issue. Thus, studies about the ALBP have increased in the apparel industry. Many researchers have conducted studies using

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different heuristic methods and simulation modeling in the apparel industry. The real-case studies that solve the ALBP in the apparel industry are given below.

Kurşun and Kalaoğlu simulated a sweatshirt line and verified that the developed simulation model produced the same performance results as the existing system [4]. Kayar and Akalin examined the applicability of the Hoffman method to apparel assembly lines and compared it with the simulation model results [5], whereas Ünal et al. proposed a heuristic algorithm for line balancing and examined its effectiveness using simulation [6]. Ünal and Bilget created simulation models for three products using statistical task time distributions and implemented lean manufacturing principles. They developed a new algorithm to balance lines within a simulation application [7]. Ünal and Demirbas created an alternative production line to obtain more output with fewer operators using simulation [8]. Eryuruk et al.'s study compared the Ranked Positional Weight Method (RPWM) with the Probabilistic Line Balancing Technique (PLBT) and found that the RPWM's results were better [2]. Eryuruk et al. solved the ALBP via the PLBT to increase the line efficiency for a constant cycle time. They demonstrated that assigning tasks to stations with greater accuracy and obtaining reliable results is possible [9]. Güner et al. studied the applicability of five heuristic balancing methods and an improving method for a t-shirt production line. All the balancing methods achieved the same results, while the improving method increased the line's efficiency despite the increase in the number of stations [10]. Karabay examined two real practical line balancing techniques and compared their performance with the performance of the RPWM. The performance of these techniques was improved by using the precedence relations of tasks [11]. Ünal proposed a New Incremental Utilization Technique to address quality issues by grouping the same machinery and adjusting less circulating workflow for the ALBP [12]. Turkmen et al. developed a computer program that uses the Hoffman, Ranked Positional Weight, COMSOAL, and Kilbridge and Wester methods for the t-shirt and knitted pants ALBP [13]. Jirasirilerd et al. used a variable neighborhood adaptive search method to minimize cycle time for a simple ALBP in the garment industry, considering the number and types of machines used in each workstation [14].

Bongomi et al. improved a complex trouser assembly line efficiency using the RPWM and examined its applicability under two-line balancing scenarios (with and without resource constraints) [15]. The RPWM has recently drawn researchers' interest because of its capability of providing higher line efficiency than its other counterparts, such as the probabilistic line balancing technique, Hoffman method, and the Kilbridge and Wester method. The results indicated that the RPWM is appropriate if there is no constraint on the resource. However, it is ineffective for complicated clothing assembly lines with different machine types. Kayar and Akalın balanced the blouse manufacturing line using the RPWM, considering the operation durations obtained from the method study and the current operation times. They analyzed the effects of the method study on production volume and assembly-line efficiency to show the significance of the method study [16]. Ahmed et al. used the Largest Candidate Rule, the Kilbridge and Wester method, and the RPWM to reduce idle time, workstation number, and labor requirement. They found a new workflow to distribute the tasks across workstations and proposed an optimal layout to reduce idle time and workforce requirements [17]. Kayar and Akyalçın used the Ranked Positional Weight, Hoffman, COMSOAL, Moodie and Young, Kilbridge and Wester, Largest Candidate Rule, and Classical methods to balance the t-shirt production line [18]. A comparative analysis of these methods has been done, and the Classical method is evaluated as the most advantageous. In the study by Phan et al., five different heuristic assembly line balancing methods (RPWM, Probabilistic Line Balancing technique, Longest Task Time Method, Most Following Tasks Method, Organizing Synchronize the Work Stations Method) were used for t-shirt production in the Vietnam garment industry [19].

In most of the literature on the ALBP in the apparel industry, heuristic methods and simulation models have been used to balance single-model assembly lines to minimize the number of workstations. Mathematical models are used in only a few of them. Gürsoy initially created an IP model that minimizes the idle time per operator, then a new heuristic algorithm that reacts promptly to market demands and finds the minimum number of operators [20]. Gürsoy and Gürsoy found minimum idle time per worker for a given production rate using IP and catered to market demands using a genetic algorithm [21]. Xu et al. rearranged manufacturing tasks for apparel production to optimize one-piece flow assembly lines under certain conditions and minimize the number of workstations and the idle time of the assembly line. Their paper proposed a modified adaptive ant colony optimization method [22]. Ahmed and Ador reduced the cost, space, and cycle time for a mixed-model ALBP [23]. Their model ensures that the workstation time does not exceed the cycle time, precedence relations are satisfied, and only an allowed number of machines can be assigned to a workstation.

This study establishes a novel mathematical model that considers parallel workstations, manually performed and machine-requiring tasks, the available number of machine types and operators, and task assignment restrictions for the ALBP in the sewing department of a garment business. The mathematical model is developed to balance the shirt production line optimally, considering parallel machines in each stage of the line under the cycle time constraint. Seven different integer programming (IP) models were developed under various operating conditions. Helgeson and Birnie's RPWM has also been applied for comparison purposes [24]. We prefer RPWM in this study because when studies with heuristic line balancing methods are examined, this method is used in most studies, as seen in the literature above.

2. MATERIAL AND METHOD

2.1 Material

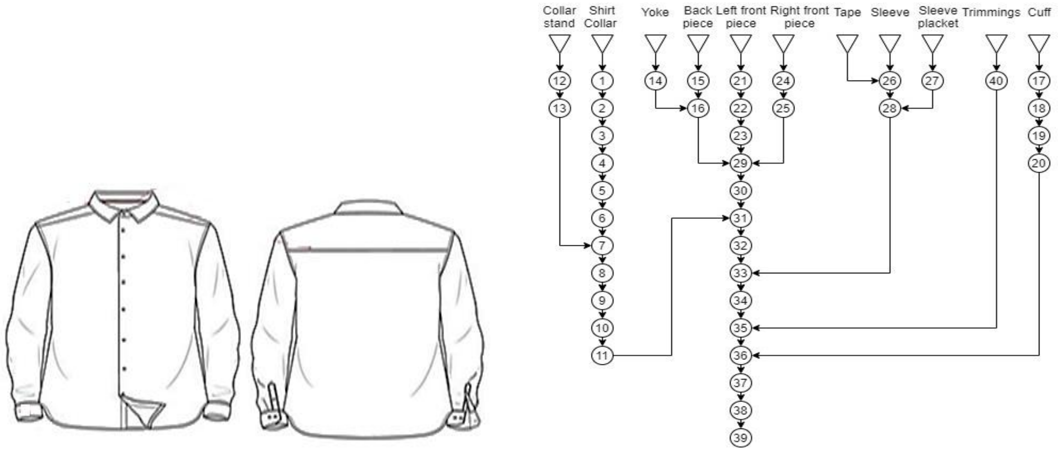
This study uses the proposed IP model and RPWM for the ALBP of an apparel company for shirt production. The computational results obtained from the two methods are compared with each other. The daily working time is 9 hours, and the targeted daily production rate is 750 pieces/day. The studied shirt model and its flowchart are shown in Figure 1, which has 20.617 min of assembly work, and the required cycle time is 0.72 min/piece.

2.2 Method- mathematical programming model

An IP model is developed to solve the ALBP of an apparel company. The model is generic in that it incorporates the assignment of workers and machinery necessary to perform tasks, accommodates parallel workstations, and minimizes the number of workers subject to a specified cycle time constraint. LINGO 15.0 Optimization software [25] was used to solve the proposed IP model optimally.

The basic assumptions considered when developing the IP model are as follows:

- The assembly line consists of a series of stages in which a workstation or parallel workstations are allowed.
- A workstation operates manually or requires a specific machine type to perform assigned tasks.
- Specific tasks are performed manually, while others can only be performed on a required machine type.



a) b)

Figure 1. a) Shirt model **b)** Flow diagram of the shirt

- Only one product model is produced on the assembly line.
- The precedence relationships between the tasks are known.
- Task times are deterministic.
- The workpiece is moved manually between workstations.
- A worker operates each type of machine.
- The number of workers available is limited.
- The number of machines available from each type is limited.

Indices, Sets, Parameters

i	The index for the tasks, $i = 1, \ldots, N$
s	The index for the potential stages on the assembly line, $s = 1, \ldots, S$
m	The index for the machine type assigned to a stage on the assembly line, $m = 1, \ldots, M$ ($m = 1$ indicates the manual workstation type)
P	The set of task pairs (i, j) in precedence relations
PR_i	The set of tasks preceding task i
S_i	The set of tasks succeeding task i
B	The set of tasks assigned to the first station
A_{im}	1, if machine type m is capable of performing task i and 0, otherwise
t_i	The processing time of task i
E_i	The earliest stage number that task i can be assigned
L_i	The latest stage number that task i can be assigned
C	The cycle time
K	The maximum number of parallel workstations allowed in each stage
TW	The total number of workers available
TM_m	The total number of type m machines available

Decision Variables

x_{is}	1, if task i is assigned to stage s and 0, otherwise
y_{sm}	1, if manual workstation type or machine type m is assigned to stage s and 0, otherwise
z_{sm}	number of manual workstations or type m machines assigned to stage s

Objective Function and Constraints

$$Minimize \sum_{s=1}^{S}\sum_{m=1}^{M} z_{sm} \qquad (1)$$

Subject to:

$$\sum_{s=E_i}^{L_i} x_{is} = 1 \qquad \forall i \qquad (2)$$

$$\sum_{m=1}^{M} y_{sm} \leq 1 \qquad \forall s \qquad (3)$$

$$\sum_{s=1}^{S} s \cdot x_{js} - \sum_{s=1}^{S} s \cdot x_{is} \geq 0 \qquad \forall\ (i,j) \in P \qquad (4)$$

$$\sum_{i=1}^{N} x_{is} \cdot A_{im} \cdot t_i \leq C \cdot z_{sm} \qquad \forall\ s,m \qquad (5)$$

$$z_{sm} \leq TM_m \cdot y_{sm} \qquad \forall\ s,m|m>1 \qquad (6)$$

$$z_{sm} \geq y_{sm} \qquad \forall\ s,m \qquad (7)$$

$$\sum_{s=1}^{S}\sum_{m=1}^{M} z_{sm} \leq TW \qquad (8)$$

$$z_{sm} \leq K \qquad \forall\ s,m \qquad (9)$$

$$\sum_{s=1}^{S} z_{sm} \leq TM_m \qquad \forall m|m>1 \qquad (10)$$

$$x_{i1} = 1 \qquad \forall\ i \in B \qquad (11)$$

$$\sum_{m=1}^{M} y_{sm} \leq \sum_{m=1}^{M} y_{s+1,m} \qquad \forall\ s|s<S \qquad (12)$$

$$x_{is}, y_{sm} \in \{0,1\} \qquad \forall\ i,s \qquad (13)$$

$$z_{sm} \geq 0 \text{ and integer} \qquad \forall\ s,m \qquad (14)$$

The objective function (1) minimizes the number of machines and manual workstations used, thus minimizing the overall number of workers on the assembly line. Constraint (2) ensures that each task is assigned to only one stage. Constraint (3) assigns at most one type of machine to a stage. All precedence relations among tasks are satisfied by Constraint (4), where task i is an immediate predecessor of task j. Constraint (5) ensures that the total duration of tasks assigned to a stage does not exceed the cycle time multiplied by the number of workstations. Under Constraint (6), the maximum number of type m machines assigned to a stage does not exceed the number available. This constraint also ensures that the variable z_{sm} is greater than zero only when the variable y_{sm} is set to 1. Constraint (7) imposes that z_{sm} is always positive when y_{sm} is set to 1. Constraints (6) and (7) together provide the necessary relation between the z_{sm} and y_{sm} variables. Constraint (8) ensures that the number of workers on the assembly line should not exceed the number of workers available, whereas Constraint (9) limits the number of parallel workstations in a stage. With Constraint (10), the number of machines from each type allocated to the assembly line should not exceed the available number. Constraint (11) assigns the specified tasks to the first stage of the assembly line. According to Constraint (12), the stages are opened in ascending order. Constraint (13) indicates the binary variables, while Constraint (14) indicates the integer variables.

The lower bound on the number of stages that should be opened, LB, can be estimated as follows:

$$LB = \left\lceil \frac{\sum_{i=1}^{N} t_i}{C.K} \right\rceil^{+} \qquad (15)$$

$$E_i = \left\lceil \frac{t_i + \sum_{j\in PR_i} t_j}{C.K} \right\rceil^{+} \qquad i = 1,\ldots,N \qquad (16)$$

$$L_i = LB + 1 - \left\lceil \frac{t_i + \sum_{j\in S_i} t_j}{C.K} \right\rceil^{+} \qquad i = 1,\ldots,N \qquad (17)$$

Ranked Positional Weight Method

The RPWM was developed by Helgeson and Birnie [24] and is commonly used in ALBPs. According to this method, each task has a positional weight calculated by summing its processing time and all processing times of the subsequent tasks. The steps of the RPWM are as follows:

1. The precedence diagram is created.
2. The positional weight value is calculated for each task.
3. The tasks are ranked in descending order of their positional weight.
4. The task with the greatest positional weight is selected as the next task to assign if its predecessor tasks are already assigned.
5. The selected task is assigned to the current open workstation. If the total workstation time exceeds the cycle time, the next task in the descending positional weight order is assigned as long as it does not violate the precedence relations. If no task can be assigned, a new station opens.
6. Steps 4 and 5 continue to be repeated until all tasks are assigned to stations.

3. RESULTS AND DISCUSSION

3.1 Balancing the line with the ranked positional weight method

The positional weights of all tasks required to sew the shirt model were calculated and ranked in ascending order. For each task, the task time, the type of machine required, all predecessor tasks, and the positional weight value (PWV) are given in Table 1. Some tasks are performed manually, referred to as "manual tasks," while some require specific machine types to process, called "machine tasks."

The daily working time of the company is 9 h. The required production rate for shirts is 750 pieces/day, resulting in a cycle time (C) of 0.72 minutes/piece. The tasks are allocated to the stations sequentially, starting with the highest positional weight and not exceeding the cycle time. The ordering of tasks depends on the type of machine needed and whether the preceding tasks are completed. Also, since some tasks have processing times exceeding the cycle time, it is necessary to open duplicate stations arranged in parallel to achieve the desired production quantity.

For example, tasks are assigned to Station 1 as follows: Task 1 has the highest positional weight at 14.256, and it is a preparatory operation performed on the fusing machine with a standard time of 0.106 min. Tasks 12, 17, and 21 are also preparatory tasks performed on the same machine, with no predecessor tasks required to be completed. Hence, although their positional weights are not the highest, they are also assigned to Station 1.

Table 1. Information on shirt production tasks and their positional weight values in descending order

Task no.	Task name	Machine type	Task time (min.)	Predecessor tasks	PWV
1	Fusing interlining to collar	Interlining Fusing press	0.106	-	14.25
2	Runstithcing collar	Lockstitch machine	0.7	1	14.15
3	Collar tip trimming and turning	Collar tip trimming and turning machine	0.3	2	13.45
4	Collar ironing	Collar press	0.25	3	13.15
5	Topstitching collar	Lockstitch machine	0.6	4	12.9
12	Fusing interlining to the collar stand	Interlining Fusing press	0.106	-	12.36
6	Collar edge trimming	Manual	0.22	5	12.3
13	Baste interlining at collar stand, fused	Lockstitch machine	0.156	12	12.25
7	Attaching collar stand and upper collar	Lockstitch machine	0.8	6,13	12.1
14	Attaching a label to yoke	Lockstitch machine	0.4	-	12.05
15	Attaching yoke to back and simultaneously lay two pleats manually	5 Thread Overlock	0.305	-	11.65
24	Sewing right placket	Lockstitch machine	0.31	-	11.46
21	Fusing interlining to left front placket	Interlining Fusing press	0.206	-	11.48
16	Topstitching back yoke	Lockstitch machine	0.35	14,15	11.35
8	Turning collar stand	Manual	0.35	7	11.3
22	Attaching placket to the left front	Lockstitch machine	0.13	21	11.28
23	Marking button hole positions	Manual	0.15	22	11.15
25	Marking button positions	Manual	0.15	24	11.15
29	Joining shoulders	5 Thread Overlock	0.5	16,23,25	11
9	Stitching through collar stand	Lockstitch machine	0.7	8	10.95
30	Topstitching shoulders	Lockstitch machine	0.6	29	10.5
10	Cutting of extensions of collar stand	Manual	0.2	9	10.25
11	Marking collar stand	Manual	0.15	10	10.05
31	Attaching collar	Lockstitch machine	0.85	11,30	9.9
26	Attaching sleeve tape	Lockstitch machine	0.4	-	9.1
32	Counterstitcing collar	Lockstitch machine	1.1	31	9.05
27	Sleeve placket pressing	Sleeve placket press	0.14	-	8.84
28	Attaching sleeve placket	Lockstitch machine	0.75	26,27	8.7
33	Attaching sleeves	5 Thread Overlock	1.05	32,28	7.95
34	Topstitching sleeves	Lockstitch machine	0.95	33	6.9
17	Fusing interlining to cuff	Interlining Fusing press	0.203	-	6.418
18	Runstitching two cuffs	Lockstitch machine	0.395	17	6.215
35	Side and sleeve close seaming and attaching trimming	5 Thread Overlock	1.2	34,40	5.95
40	Cutting trimmings	Manual	0.04	-	5.95
19	Cuff turning ironing	Cuff press	0.42	18	5.82
20	Topstitching cuffs	Lockstitch machine	0.65	19	5.4
36	Attaching cuffs	Lockstitch machine	1.15	35,20	4.75
37	Hemming	Lockstitch machine	0.9	36	3.6
38	Opening buttonholes	Buttonhole machine	1.5	37	2.7
39	Sewing buttons	Button sewing machine	1.3	38	1.3

Total task time of Station 1 = 0.106+0.106+0.206+0.203=0.621 min, Remaining time (Idle time) = 0.72-0.621=0.099 min

Table 2 shows the assignment of tasks to the stations according to the RPWM. Here, manual and machine tasks can be assigned to the same station. With an exception, tasks 3 and 4 can be performed consecutively by the same worker to prepare the collar, although they require different machine types, as tasks 27 and 28. The machine type abbreviations used in Table 2 are as follows: 5 Thread Overlock (5TO), Lockstitch Machine (LSM), Buttonhole Machine (BM), Button Sewing Machine (BSM), Interlining Fusing Press (IP), Cuff Press (CUP), Collar Press (COP), Collar Tip Cutting and Turning Machine (CTCM), Sleeve Placket Press (SPP), and Manual (MN).

Table 2. Assigning shirt operations to stations

Station no.	**Assigned Task no.**	**Machine type**	**PWV**	**Predecessor tasks**	**Task time (min.)**	**Cumulative time (min.)**	**Idle time (min.)**
Fusing							
1	1	IP	14.256	-	0.106	0.621	0.099
	12		12.362	-	0.106		
	21		11.486	-	0.206		
	17		6.418	-	0.203		
Collar preparation							
2	2	LSM	14.15	1	0.7	0.7	0.02
3	3	CTCM	13.45	2	0.3	0.55	0.17
	4	COP	13.15	3	0.25		
4 5	5	LSM	12.9	4	0.6	0.956	0.48
	6	MN	12.3	5	0.2		
	13	LSM	12.256	12	0.156		
6 7	7	LSM	12.1	6,13	0.8	1.15	0.29
	8	MN	11.3	7	0.35		
8 9	9	LSM	10.95	8	0.7	1.05	0.39
	10	MN	10.25	9	0.2		
	11	MN	10.05	10	0.15		
Front and back preparation							
10	14	LSM	12.055	-	0.4	0.4	0.32
11	15	5TO	11.655	14	0.305	0.305	0.415
12	16	LSM	11.35	14,15	0.35	0.48	0.24
	22	LSM	11.28	21	0.13		
13	23	MN	11.15	22	0.15	0.61	0.11
	24	LSM	11.46	23	0.31		
	25	MN	11.15	24	0.15		
Cuff preparation							
14	18	LSM	6.215	17	0.395	0.395	0.325
15	19	CUP	5.82	18	0.42	0.42	0.3
16	20	LSM	5.4	19	0.65	0.65	0.07
Sleeve preparation							
17	26	LSM	9.1	-	0.4	0.4	0.32
18 -19	27	SPP	8.84	-	0.14	0.89	0.55
	28	LSM	8.7	26,27	0.75		
Assembly							
20	29	5TO	11	16,23,25	0.5	0.5	0.22
21	30	LSM	10.5	29	0.6	0.6	0.12
22-23-24	31	LSM	9.9	11,30	0.85	1.95	0.21
	32	LSM	9.05	31	1.1		
25 - 26	33	5TO	7.95	32,28	1.05	1.05	0.39
27 - 28	34	LSM	6.9	33	0.95	0.99	0.45
	40	MN	5.95	-	0.04		
29 - 30	35	5TO	5.95	34,40	1.2	1.2	0.24
31 - 32	36	LSM	4.75	35,20	1.15	1.15	0.29
33 - 34	37	LSM	3.6	36	0.9	0.9	0.54
35 - 36	38	BM	2.7	37	1.4	1.4	0.04
37 - 38	39	BSM	1.3	38	1.3	1.3	0.14

The fact that there are tasks that cannot be assigned to the same station and that the unit times of the jobs are distributed over a wide range prevent the station times from being well balanced. Thirty-eight stations were opened to complete all task assignments using the RPWM under the determined conditions. Accordingly, the balance efficiency is 75.4%.

$$E\ (\%) = (sum\ of\ task\ times)/(cycle\ time \times number\ of\ workstations\ )$$

$$E\ (\%) = \frac{20.617\ min}{0.72\ min \times 38\ workstations} = 75.4\%$$

3.2 Balancing the line with the proposed mathematical model

The proposed IP model was modified, resulting in different versions to apply to the ALBP of shirt production under various operating conditions. The optimal results obtained using these models are presented in Tables 3 to 9.

The models have the following characteristics that differ from the original IP model in Section 2.2.

Model 1 (Original Model): Manual and machine tasks cannot be assigned to the same workstation. Also, there is no limit to the number of tasks assigned to a station, and the interlining operations with task numbers 1, 12, 17, and 21 are to be assigned to the first stage. Accordingly, the solution of Model 1 is given in Table 3.

Model 2: Manual tasks can be allocated to the same workstation with machine tasks. This operational flexibility is reflected in the proposed model by replacing Constraint (5) with Constraints (5a) -(5b) given below. Also, there is no limit to the number of tasks assigned to a station, as in Model 1. Accordingly, the solution of Model 2 is given in Table 4.

$$\sum_{i=1}^{N} x_{is} \cdot t_i \le C \cdot z_{sm} \qquad \forall\ s,m \qquad (5a)$$

$$A_{im} \cdot x_{is} \le y_{sm} \qquad \forall i,s,m \mid m > 1 \qquad (5b)$$

Model 3: It is the same as Model 2. Besides, tasks 3 and 4 and 27 and 28, although requiring different machine types, can be assigned to the same station. Equation (18) must be added to the model for these task pairs. The solution of Model 3 is given in Table 5.

$$x_{is} - x_{js} = 0 \qquad \forall\ (i,j) \in \{(3,4),(27,28)\} \qquad (18)$$

Model 4: It is the same as Model 3, except that a maximum of three tasks can be assigned to a station. This limitation does not apply to the interlining operations corresponding to task numbers 1,12,17, and 21, respectively, assigned to the first stage of the assembly line. The solution of Model 4 is given in Table 6.

Model 5: It is the same as Model 4, except that a maximum of two tasks can be assigned to a station instead of three. The solution of Model 5 is given in Table 7.

Model 6: It is the same as Model 4. The objective of this model is different from the other models, as given in Equation 19. Whereas the original model only minimizes the number of stations, i.e., the number of operators working on the assembly line, this model prioritizes the assignment of relevant tasks, such as tasks processed on the same piece of the shirt, to the same station where possible or nearby stations to minimize excessive transportation of such parts between workstations and the parameter w_2 indicates the importance weight of this objective. After then, it tries to minimize the number of stations for which the parameter w_1 specifies the importance weight of this objective. Here $w_1 = 1$ and $w_2 = 10$. The model requires Equation (20) as an additional constraint to determine whether task pairs that belong to the same piece of the shirt and in a precedence relation ($(i,j) \in Derived$) are assigned to a different stage. The variable dev_{ij} takes a value greater than zero when tasks in pair (i,j) is assigned to different stages and takes 0 when assigned to the same stage. The solution of Model 6 is given in Table 8.

$$Minimize\ w_1 \sum_{s=1}^{S} \sum_{m=1}^{M} z_{sm} + w_2 \sum_{(i,j) \in Derived} dev_{ij} \qquad (19)$$

$$dev_{ij} = \sum_{s=1}^{S} s.x_{js} - \sum_{s=1}^{S} s.x_{is} \qquad \forall\ (i,j) \in Derived \qquad (20)$$

Model 7: It is the same as Model 6. However, this time, the model's objective prioritizes minimizing the number of stations and then tries to assign relevant tasks closely. Here $w_1 = 10$ and $w_2 = 1$. This model consists of 1417 constraints and 1313 variables. The LINGO code for Model 7 is given in the Appendix, and the solution of Model 7 is given in Table 9.

In all models, interlining processes are gathered in a single station in accordance with the real case. Generally, workers perform manual tasks such as regulation, turning, and cutting in stations reserved for manual tasks only. As indicated, Model 1 uses separate stations for manual tasks, and the Model 1 solution consists of 36 stations, with three having only manual tasks performed. According to Model 2, manual tasks can be assigned to the same station together with machine tasks. With this flexibility, Model 2 reduces the number of stations needed to carry out the tasks from 36 to 34. Instead of assigning manual tasks to separate stations, assigning them to the same station with other machine tasks reduces remaining idle time at the stations and provides more efficient use of total station processing time.

In Model 3, binary tasks 3 and 4 and 27 and 28 are assigned to the same stations, although performed on different machine types. This assignment is allowed since they are already processed successively in the company. This reduces the number of stations needed to perform the tasks to 32, compared to 34 stations using Model 2. However, since there is no limit to the number of tasks assigned to a station, some stations have been assigned four tasks.

Although several different task allocations to a station help make the line more efficient, it can cause disruptions in the workflow and raise quality problems in practice.

Table 3. The solution of Model 1

Station no.	Machine type	Assigned task no.	Total task time	Avg. station time
1	LSM	24, 26	0.71	0.71
2	IP	1,12,17,21	0.62	0.62
3	LSM	2	0.7	0.7
4	LSM	13,18,22	0.68	0.68
5	LSM	14	0.4	0.4
6	CTCM	3	0.3	0.3
7	COP	4	0.25	0.25
8	LSM	5	0.6	0.6
9	MN	6,23,25	0.5	0.5
10	5TO	15	0.31	0.31
11,12	LSM	7,16	1.15	0.58
13	5TO	29	0.5	0.5
14	MN	8	0.35	0.35
15,16	LSM	9,30	1.3	0.65
17	MN	10,11,27,40	0.53	0.53
18	CUP	19	0.42	0.42
19,20,21,22	LSM	28,31,32	2.7	0.68
23,24	5TO	33	1.05	0.53
25,26	LSM	34	0.95	0.48
27,28	5TO	35	1.20	0.6
29	LSM	20	0.65	0.65
30,31,32	LSM	36,37	2.05	0.68
33,34	BM	38	1.40	0.7
35,36	BSM	39	1.30	0.65

Table 4. The solution of Model 2

Station No.	Machine type	Assigned task no.	Total task time	Avg. station time
1	IP	1,12, 17,21	0.62	0.62
2,3	LSM	2,13,22,26	1.39	0.69
4	LSM	14,24	0.71	0.71
5	5TO, MN	15,25	0.46	0.23
6	LSM	16	0.35	0.35
7	5TO, MN	23,29,40	0.69	0.69
8	CTCM	3	0.3	0.3
9	COP	4	0.25	0.25
10	LSM	5	0.6	0.6
11,12	LSM, MN	6,7,18	1.39	0.70
13	SPP, MN	8,27	0.49	0.49
14,15,16	LSM	9,28,30	2.05	0.68
17	CUP, MN	10,19	0.62	0.62
18,19,20	LSM, MN	11,31,32	2.1	0.7
21,22	5TO	33	1.05	0.53
23,24,25	LSM	20,34	1.6	0.53
26,27	5TO	35	1.2	0.6
28,29,30	LSM	36,37	2.05	0.68
31,32	BM	38	1.4	0.7
33,34	BSM	39	1.3	0.65

Table 5. The solution of Model 3

Station no.	Machine type	Assigned task no.	Total task time	Avg. station time
1	IP, MN	1,12,17,21,40	0.66	0.66
2,3	LSM, MN	2,22,24,25	1.29	0.65
4	CTCM, COP, MN	3,4,23	0.70	0.70
5,6,7	LSM	5,6,7,13,14	2.16	0.72
8	5TO	8,15	0.66	0.66
9,10	LSM	9,16	1.05	0.53
11	5TO	10,29	0.70	0.70
12,13,14	LSM, SPP	26,27,28,30	1.89	0.63
15,16,17	LSM	11,31,32	2.10	0.70
18,19	5TO	33	1.05	0.53
20,21	LSM	18,34	1.35	0.67
22	CUP	19	0.42	0.42
23	LSM	20	0.65	0.65
24,25	5TO	35	1.20	0.60
26,27,28	LSM	36,37	2.05	0.68
29,30	BM	38	1.40	0.70
31,32	BSM	39	1.30	0.65

In Model 4, the maximum number of tasks assigned to the stations is limited to three, thus increasing the number of stations required to perform the tasks from 32 stations found using Model 3 to 33. According to this model solution, successive tasks requiring the same machine type are mostly assigned to the same station. The sum of task times at the stations is quite well-balanced. Station 15, where only manual tasks are assigned, has the highest idle time among other stations. The stations involving manual and machine tasks are relatively better balanced.

In Model 5, when the maximum number of tasks assigned to a station is limited to two, the number of open workstations increases to 35 from 33 stations found using Model 4. The similarity between the average station times is distorted compared to Model 4 since the stations cannot be sufficiently balanced due to the task number limitation and the tasks' wide range of operation times.

Table 6. The solution of Model 4

Station no.	Machine type	Assigned task no.	Total task time	Avg. station time
1	IP	1,12,17,2	0.62	0.62
2	LSM	2	0.70	0.70
3	CTCM, COP	3,4	0.55	0.55
4	LSM	5	0.60	0.60
5	LSM	13,14,22	0.69	0.69
6,7	LSM, MN	6,7,18	1.40	0.70
8	5TO, MN	8,15	0.66	0.66
9	LSM	9	0.70	0.70
10	CUP, MN	10,19	0.62	0.62
11,12	LSM, SPP	26,27,28	1.29	0.65
13	LSM	20	0.65	0.65
14	LSM	16,24	0.66	0.66
15	MN	23,25,40	0.34	0.34
16	5TO, MN	11,29	0.65	0.65
17	LSM	30	0.60	0.60
18,19,20	LSM	31,32	1.95	0.65
21, 22	5TO	33	1.05	0.53
23,24	LSM	34	0.95	0.48
25,26	5TO	35	1.20	0.60
27,28,29	LSM	36,37	2.05	0.68
30,31	BM	38	1.40	0.70
32,33	BSM	39	1.30	0.65

Table 7. The solution of Model 5

Station No.	Machine type	Assigned task no.	Total task time	Avg. station time
1	IP	1,12,17,21	0.621	0.621
2	LSM	14,24	0.71	0.71
3	LSM	2	0.7	0.7
4	CTCM, COP	3,4	0.55	0.55
5	LSM	5	0.6	0.6
6	LSM, MN	6,13	0.356	0.356
7	LSM	18	0.395	0.395
8	LSM	22,26	0.53	0.53
9,10	LSM, MN	7,25	0.95	0.48
11	5TO, MN	8,15	0.66	0.66
12	LSM	9	0.70	0.70
13	LSM, MN	10,16	0.55	0.55
14	5TO, MN	23,29	0.65	0.65
15	LSM	30	0.60	0.60
16	CUP, MN	11,19	0.57	0.57
17,18	LSM, SPP	27,28	0.89	0.45
19, 20, 21	LSM	31,32	1.95	0.65
22,23	5TO, MN	33,40	1.09	0.55
24	LSM	20	0.65	0.65
25,26	LSM	34	0.95	0.48
27,28	5TO	35	1.20	0.60
29,30,31	LSM	36,37	2.05	0.68
32,33	BM	38	1.40	0.70
34,35	BSM	39	1.30	0.65

Models 6 and 7 were run with opposite priorities in fulfilling the objectives. In Model 6, the priority is to assign the jobs close to each other according to their precedence relations. Thirty-five stations have been used to allocate the tasks. The average processing times of stations 13, 14, and 15 are well below the cycle time. Since the tasks at these stations require different machines and must be performed in sequence, they have been assigned to separate stations consecutively. Thus, these stations are not working efficiently enough.

On the other hand, the movement of different workpieces of the shirt between stations has been reduced by successively assigning tasks to stations 3, 7, 9, 10, 11, 12, 16, and 17. Model 7 prioritizes the number of stations needed to complete the assembly work and assigns 33 stations. In this model, except for station 14, the average processing times of the stations are in a narrower range. Since the priority is to minimize the number of stations, only the appropriate consecutive tasks are assigned to the same station. Unlike Model 6, sequential manual tasks 8, 10, and 11 are assigned to different stations for more efficient balancing instead of being assigned to the same station.

Models 4 and 7 require a minimum of 33 stations to allocate all tasks. Since Model 7 tries assigning close tasks together as the second criterion, three consecutive tasks have been assigned one after the other to stations 3, 8, and 9. With this assignment, less work will have to be moved between stations than in Model 4. Therefore, Model 7 presents the most appropriate solution to this line-balancing problem.

Model 3 has the highest value of 89.5% in terms of efficiency. However, in this model, there are two stations with five tasks allocated and one station with four tasks. Model 7 is more appropriate in this regard since assigning many tasks to a station can disrupt the workflow. The assembly line layout for the solution of Model 7 is given in Figure 2. The RPWM has performed worse than all the models considering the balance efficiency, as illustrated in Table 10.

When considering all these models, Model 7 is thought to be more suitable regarding the layout of the machines, although it does not have the highest efficiency. In Model 7, the predecessors of tasks 13, 18, and 22 at station 2 are performed at station 1, where the interlining operation is performed. These tasks are not difficult concerning their level of practicality. In this respect, a similar interpretation can be made for tasks at station 3. As in many shirt businesses, tasks 3 and 4 of station 7 are carried out successively by the same worker. It is also observed that several manual tasks are assigned to stations, and tasks such as collar fitting (task no. 31) and sleeve fitting (task no. 33), which have a high degree of difficulty, are not assigned together.

According to these different scenarios considered by the models, assigning manual and machine tasks together contributes greatly to achieving workstation times close to the cycle time and ensuring a smooth workflow. Similar practices are also done in assigning and organizing tasks in modular production plants where the operators are initially assigned to perform tasks carried out with the sewing machine, and then they are assigned to manual tasks to fill the leisure time after these tasks are completed. It may be assumed that machine operators can also carry out manual tasks.

Table 8. The solution of Model 6

Station no.	Machine type	Assigned task no.	Total task time	Avg. station time
1	IP	1,12,17,21	0.621	0.621
2	LSM	13,18,22	0.681	0.681
3	LSM, MN	23,24,25	0.61	0.61
4	CUP	19	0.42	0.42
5,6	LSM	2,20	1.35	0.68
7	CTCM, COP	3,4	0.55	0.55
8	LSM	5	0.6	0.6
9,10	LSM, MN	6,7,8	1.35	0.675
11,12	LSM, MN	9,10,11	1.05	0.525
13	LSM	14	0.4	0.4
14	5TO	15	0.305	0.305
15	LSM	16	0.35	0.35
16,17	LSM, SPP	26,27,28	1.29	0.645
18	5TO, MN	29,40	0.54	0.54
19	LSM	30	0.6	0.6
20,21,22	LSM	31,32	1.95	0.65
23,24	LSM	33	1.05	0.525
25,26	LSM	34	0.95	0.475
27,28	5TO	35	1.2	0.6
29,30,31	LSM	36,37	2.05	0.68
32,33	BM	38	1.4	0.7
34,35	BSM	39	1.3	0.65

Table 9. The solution of Model 7

Station no.	Machine type	Assigned task no.	Total task time	Avg. station time
1	IP	1,12,17,21	0.621	0.621
2	LSM	13,18,22	0.681	0.681
3	LSM, MN	23,24,25	0.61	0.61
4	CUP	19	0.42	0.42
5	LSM	20	0.65	0.65
6	LSM	2	0.7	0.7
7	CTCM, COP	3,4	0.55	0.55
8,9	LSM, SPP	26,27,28	1.29	0.645
10	LSM	5	0.6	0.6
11,12	LSM, MN	6,7,14	1.35	0.675
13	5TO, MN	8,15	0.655	0.655
14	LSM	16	0.35	0.35
15	LSM	9	0.7	0.7
16	5TO, MN	10,29	0.7	0.7
17	LSM, MN	30,40	0.64	0.64
18,19,20	LSM, MN	11,31,32	2.1	0.7
21,22	5TO	33	1.05	0.525
23,24	LSM	34	0.95	0.475
25,26	5TO	35	1.2	0.6
27,28,29	LSM	36,37	2.05	0.68
30,31	BM	38	1.4	0.7
32,33	BSM	39	1.3	0.65

Table 10. Line efficiency values of all solutions

Models	Number of workstations	Efficiency %
Model 1	36	79.5
Model 2	34	84.2
Model 3	32	89.5
Model 4	33	86.8
Model 5	35	81.9
Model 6	35	81.9
Model 7	33	86.8
RPWM	38	75.4

Regarding the applicability of the proposed models in the factory environment, some other factors may need to be considered. In the business environment, task times may fluctuate within a given range, and workers may not be eligible to operate all machines and perform all tasks. On the other hand, it should be noted that, with the recent increase in model diversity, the changing competitive conditions have increased businesses' expectations for more workers to perform different tasks and use different machinery, and the companies have started training their workers subsequently.

4. CONCLUSION

In the literature on assembly line balancing, heuristic line balancing methods and simulation models have been widely used to balance single-model assembly lines. This paper has developed a unique balancing model for assembly lines that incorporates labor and machine constraints, parallel workstations, and task assignment restrictions to achieve the highest line efficiency using optimum labor and machinery for a fixed cycle time. In the first phase of the application, line balancing is performed using the RPWM. In the second phase, seven IP models are developed and implemented under various scenarios, and the results of their solutions are compared.

The line efficiency of the shirt sewing line is 75.4% for the RPWM, and the most appropriate IP model (Model 7) has resulted in 86.8% efficiency. Production speed is critical in the apparel industry. Setting up and balancing an assembly line takes time. With the developed IP model, establishing the line and assigning tasks can be found optimally quickly. Especially in multi-process models, the IP model with the given constraints can quickly create different line designs, and the most efficient design can be reached quickly.

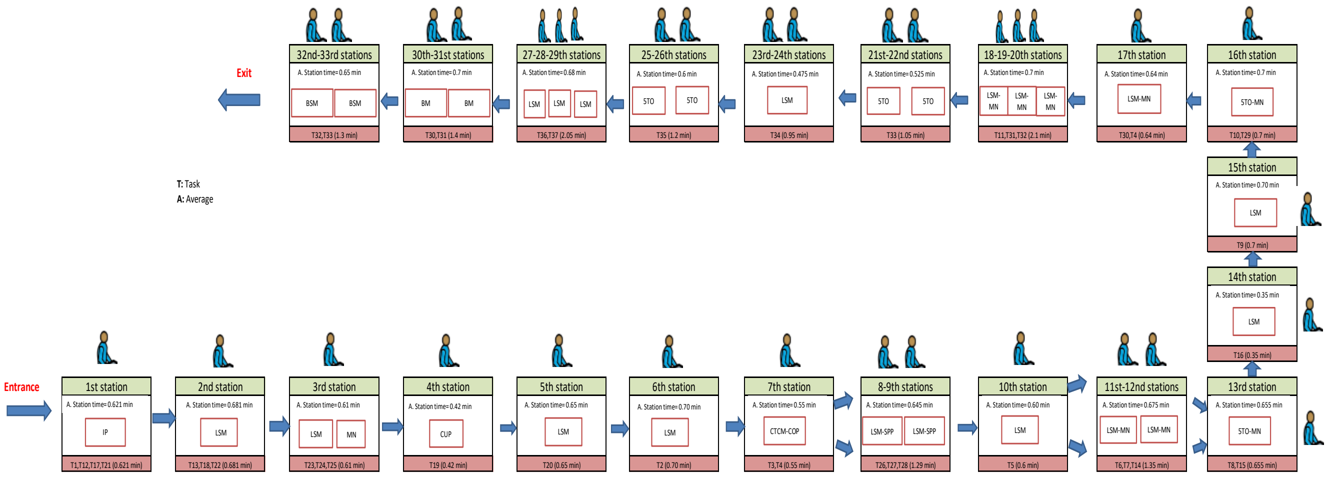


Figure 2. The layout of the assembly line for the Model 7 solution

In this research, different mathematical models were created; it has been seen that instead of doing the manual tasks at separate stations, their assignment with machine-operated tasks ensures that the station times are comparatively better balanced. Moreover, when there is no limitation on the number of tasks assigned to stations, the overall number of stations required to complete all tasks reduces, and the line efficiency increases; however, this way of the assignment of tasks is hard to implement within the enterprise since it increases the risk of poor product quality.

The proposed IP model can be run under different operating constraints. For this reason, companies can practically use the model to find the most suitable balancing solution. For future research, the model can be extended for mixed-model ALBPs. Also, it can be modified to include the limitation that the workers can only use certain machine types.

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APPENDIX

The LINGO program for the IP model of Model 7 is given below. The constraint numbers are the same as in the manuscript for an easy follow-up.

```
Sets:
Tasks/1..40/:t;
Stages/1..24/;
Precedences(Tasks,Tasks)/1 2, 2 3, 3 4, 4 5, 5 6, 6 7, 13 7, 7 8, 8 9, 9 10, 10 11, 12 13, 14 15, 14 16, 15 16, 17 18, 18 19, 19 20, 21 22, 22 23,
24 25, 26 28, 27 28,16 29, 23 29, 25 29, 29 30, 11 31, 30 31, 31 32, 32 33, 28 33, 33 34, 34 35, 40 35, 35 36, 20 36, 36 37, 37 38, 38 39/;
MachineType/Manuel,Pres,Duz, Yum, 5Ip, Mup, Im, Dm/:TM;
Derived(Tasks,Tasks)/1 2, 2 3, 3 4, 4 5, 5 6, 6 7, 7 8, 8 9, 9 10, 10 11, 12 13, 14 15, 14 16, 15 16, 17 18, 18 19, 19 20, 21 22, 22 23, 23
24, 24 25, 26 28, 27 28,29 30, 30 31, 31 32, 32 33, 33 34, 34 35, 35 36, 36 37, 37 38, 38 39/: dev;
Stages_MachineType(Stages,MachineType):z,y;
Tasks_Stages(Tasks,Stages):x;
Tasks_MachineType(Tasks,MachineType):A;
Together/1,12,17,21/;
Endsets

Data:
C=0.72;
t=0.106, 0.7, 0.3, 0.25, 0.6, 0.2, 0.8, 0.35, 0.7, 0.2, 0.15, 0.106, 0.156, 0.4, 0.305, 0.35, 0.203, 0.395, 0.42, 0.65, 0.206, 0.13, 0.15, 0.31,
0.15,  0.4, 0.14,
0.75, 0.5, 0.6, 0.85, 1.1, 1.05, 0.95, 1.2, 1.15, 0.9, 1.4, 1.3, 0.04;
TM=0,1,28,1,7,1,2,2; TW=38; K=3;
A=0 1 0 0  0 0 0 0
  0 0 1 0  0 0 0 0
  0 0 0 1  0 0 0 0
  0 0 0 1  0 0 0 0
  0 0 1 0  0 0 0 0
  1 0 0 0  0 0 0 0
  0 0 1 0  0 0 0 0
  1 0 0 0  0 0 0 0
  0 0 1 0  0 0 0 0
  1 0 0 0  0 0 0 0
  1 0 0 0  0 0 0 0
  0 1 0 0  0 0 0 0
  0 0 1 0  0 0 0 0
  0 0 1 0  0 0 0 0
  0 0 0 0  1 0 0 0
  0 0 1 0  0 0 0 0
  0 1 0 0  0 0 0 0
  0 0 1 0  0 0 0 0
  0 0 0 0  0 1 0 0
  0 0 1 0  0 0 0 0
  0 1 0 0  0 0 0 0
  0 0 1 0  0 0 0 0
  1 0 0 0  0 0 0 0
  0 0 1 0  0 0 0 0
  1 0 0 0  0 0 0 0
  0 0 1 0  0 0 0 0
  0 0 1 0  0 0 0 0
  0 0 1 0  0 0 0 0
  0 0 0 0  1 0 0 0
  0 0 1 0  0 0 0 0
  0 0 1 0  0 0 0 0
  0 0 1 0  0 0 0 0
  0 0 0 0  1 0 0 0
  0 0 1 0  0 0 0 0
  0 0 0 0  1 0 0 0
  0 0 1 0  0 0 0 0
  0 0 1 0  0 0 0 0
  0 0 0 0  0 0 1 0
  0 0 0 0  0 0 0 1
  1 0 0 0  0 0 0 0;
```

```
Enddata
!Objective function(1): Minimize the total number of open workstations;
!min=@sum(Stages_MachineType(s,m):z(s,m));

!Model 7 Objective Function (Equation 19);

min=10*@sum(Stages_MachineType(s,m):z(s,m)) + @sum(Derived(i,j):dev(i,j)) ;

!Constraint(2): Each task must be assigned to a stage;
@for(Tasks(i):@sum(Stages(s):x(i,s))=1);

!Constraint(3): At most one machine type can be assigned to a stage;
@for(Stages(s):@sum(MachineType(m):y(s,m))<=1);

!Constraint(4): Precedence relations among the tasks are provided;
@for(Precedences(i,j):@sum(Stages(s):s*x(j,s))-@sum(Stages(s):s*x(i,s))>= 0);

!Constraint(5): The sum of the processing times of the tasks assigned to a stage must not exceed the cycle time multiplied by the number
of open workstations;
!@for(Stages_MachineType(s,m):@sum(Tasks(i)|A(i,m) #EQ# 1 :x(i,s)*t(i))<=C*z(s,m));

!Constraint(5a);
@for(Stages(s):@sum(Tasks(i):x(i,s)*t(i))<= @sum(MachineType(m):C*z(s,m)));

!Constraint(5b);
@for(Stages_MachineType(s,m) | m #gt# 1 :@for(Tasks(i)|A(i,m) #EQ# 1 :x(i,s)<= y(s,m)));

!Constraint(6): The number of machines of a certain type assigned to a stage can only be positive when the same machine type is
assigned to the stage.
@for(Stages_MachineType(s,m)| m #GT#1: z(s,m)<=TM(m)*y(s,m));

!Constraint(7): The relation between z(s,m) and y(s,m) variables is provided;
@for(Stages_MachineType(s,m):z(s,m)>=y(s,m));

!Constraint(8): The number of workers assigned to work on the assembly line should not exceed the number of workers available;
@sum(Stages_MachineType(s,m): z(s,m))<= TW;

!Constraint(9): The number of parallel stations allowed in a stage should not exceed the specified number;
@for(Stages(s)| s #ne# 1: @sum(Tasks(i): x(i,s)) <=3;);
@for(Stages(s)| s #eq# 1: @sum(Tasks(i): x(i,s)) <=4);

!Constraint(10): The number of machines allocated to the assembly line of each type should not exceed the available number;
@for(MachineType(m) | m #GT# 1: @sum(Stages(s):z(s,m))<=TM(m));

!Constraint(11): The specified tasks are assigned to the first stage at the beginning;
x(1,1)=1;
x(12,1)=1;
x(17,1)=1;
x(21,1)=1;

!Constraint(12):;
@for(Stages(s) | s #LT# 24:@sum(MachineType(m):y(s,m))>=@sum(MachineType(m):y(s+1,m)));

!Constraint(13): Binary Constraints;
@for(Tasks_Stages(i,s):@bin(x(i,s)));
@for(Stages_MachineType(s,m):@bin(y(s,m)));

!Constraint(14): Integer Constraints;
@for(Stages_MachineType(s,m):@gin(z(s,m)));

!Constraint(18): The task pairs that must be assigned to the same workstation are specified;
@for(Stages(s): x(27,s)-x(28,s)=0);
@for(Stages(s): x(3,s)-x(4,s)=0);

!Equation(20);
@for(Derived(i,j): dev(i,j)=@sum(stages(s): s*x(j,s))- @sum(stages(s): s*x(i,s)));
```