Geometric Model Definition of Annular Type Tracheal Elements of Chard and Numerical Comparison

Ali ÖZDEMİR[1]*

[1]Celal Bayar University, Department of Mathematics, Faculty of Art and Science, Manisa
(orcid.org/0000-0001-9330-7084)
*e-mail: acaozdemir@gmail.com

Abstract: In this study, it has been given a geometric model description of the annular type tracheal elements of chard (*Beta vulgaris var. cicla* L.) Numerical comparisons have been also made by taking measurements from different parts of the leaf. It has been have shown that the tracheal elements which are taxonomical value of the plant can be considered as a surface of revolution or a tubular shape along a special curve.

Keywords: Geometric model, beta, tracheal elements, nümeric comparision

Pazı Bitkisinin Halkalı Tip Trake Elemanların Geometrik Modeli Tanımı ve Sayısal Karşılaştırması

Öz: Bu çalışmada, pazı bitkisinin (*Beta vulgaris var. cicla* L.) halka tipindeki (annular trake) iletim elemanlarının geometrik modelinin bir tanımı verilmiştir. Ayrıca yaprağın farklı kısımlarından bu elemanların çap ölçümleri alınarak nümerik karşılaştırmaları yapılmıştır. Çalışmada bitkinin taksonomik değeri olan iletim doku elemanlarının, özel bir eğri boyunca devinim yüzeyi veya tübüler bir şekil olarak düşünülebileceği gösterilmiştir.

Anahtar Kelimeler: Geometrik model, pazı, trake elemanları, sayısal karşılaştırma

1. Introduction

Tracheary elements are found in the xylem of all vascular plants. They are highly specialized cells. The cells of tracheary elements die at maturity but lignified cell walls remain and become tube with different geometric shapes as the conduits through which water is carried in the xylem vessel. So they serve for upward conduction of water and dissolved minerals in plants. Tracheary elements, which are the distinctive cells of the xylem are characterized by the formation of lignified cell wall with annular, spiral, scalariform and reticulate (Fukuda 1992; Höfte 2010; Devillard and Walter 2014). In this study, it was determined that the formation of lignified cell wall of tracheal elements are mostly annular types.

In addition, a geometric recognition of the tracheal elements which are taxonomical value of the plant is presented. The tracheal elements can be considered as a surface of revolution or a tubular shape along a special curve. Numerical comparisons were made by taking measurements from different parts of the annular tracheal elements.

2. Materials and Methods

Fresh samples of the plant were used. The surface sections which were stained photographed with a motorized Leica DM 3000 microscope. Measurements were taken using ocular micrometer of tracheal elements. Numerical comparisons were made by taking measurements of the diameters of annular type tracheal elements from 10 different parts of the five plant samples.

Geometric Works

Tubular surfaces and surface of revolution have an important role in 3D shape modeling.

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Definition

Let α:(a,b)→ ℝ2 be a regular planar curve given with the parametrization α(t)=(f(t),g(t)). When α revolved in ℝ3 about z-axis the resulting point set M is defined by

M: x(t,s)=(f(t)cos(s),f(t)sin(s),g(t))

is called the surface of revolution generated by the profile curve α. The z-axis is called axis of revolution (Gray 1993).

In the present study it has been shown that the types tracheal elements are considered a certain part of surface of revolution about a plane curve α(t)=(f(t),g(t)). To draw these kind of surfaces we type into the Maple plotting command;

Plot 3d([f(t), g(t)*cos(s), g(t)*sin(s)], t=a..b, s=c..d);.

3. Results and Discussion

In this study it was determined that the formation of lignified cell wall in surface sections taken from different parts of Beta vulgaris var. cicla L. are mostly annular types in the surface sections. The sizes of these cells vary. The sizes vary between 150 μm and 10 μm. The results of the experimental analyzes and measurements have been shown by photographs and Tables (Figure 1,2 Table 1,2).

Table 1. Numerical features of the tracheal elements
Çizelge 1. *Trake elemanlarının sayısal değerleri*

	A	B	C	D	E
			Trache diameter (µm)		
	Mean±SD	Mean±SD	Mean±SD	Mean±SD	Mean±SD
1	35.3±1.50	18.9±1.20	15.3±1.50	43.0±10.2	22.0±0.18
2	32.5±4.00	44.7±3.44	36.3±3.45	54.0±9.06	43.0±6.26
3	86.0±3.77	62.9±3.01	76.6±13.5	55.3±3.10	32.3±6.53
4	112.9±1.22	95.0±6.88	67.1±4.18	37.9±2.78	122.2±13.2
5	15.0±3.29	20.0±5.14	10.5±1.51	20.0±4.20	12.0±0.18
6	30.6±3.09	64.5±4.14	28.5±3.21	60.0±17.2	57.90±13.8
7	76.2±7.51	17.50±1.86	87.1±12.3	45.0±11.9	30.0±0.12
8	57.3±4.98	70.2±3.00	49.0±2.67	34.4±4.15	28.0±3.19
9	120.0±9.90	62.7±3.15	80.0±5.28	150.0±15.0	57.0±7.11
10	29.0±3.16	14.3±2.45	31.1±1.21	30.0±11.0	15.0±0.13

SD:Standart deviation A-E:The plant samples,1-10: Different parts of the plant samples.

Geometric work

In the present study it has been given a geometric recognition the annular types of tracheal elements and has been made nümerical comparisions of them. Firstly, we describe the annular type tracheal element of the plant as a collection of tubular surfaces about the space curves

β(t)=(cos(t),sin(t),b)

where b is a real constant.

Table 2. Correlation between diameter measurements (Regression Analysis)
Çizelge 2. *Çap ölçümleri arasında karşılaştırma (Regresyon Analizi)*

	MS	F-value	Probility	Significant
1-2	6776.3	8.71	0.018	*
1-3	789.4	5.52	0.043	*
1-4	258.0	0.16	0.698	NS
1-5	1768.3	7.34	0.002	**
2-3	1142.7	0.66	0.213	NS
2-4	188.9	0.23	0.641	NS
2-5	4015.8	12.23	0.048	*
3-4	5326.9	10.43	0.047	*
3-5	4920.3	13.30	0.028	*
4-5	26.60	0.17	0.694	**

MS: mean square; NS: not significant; *P<.05; **P<.01. 1-5: the plant samples
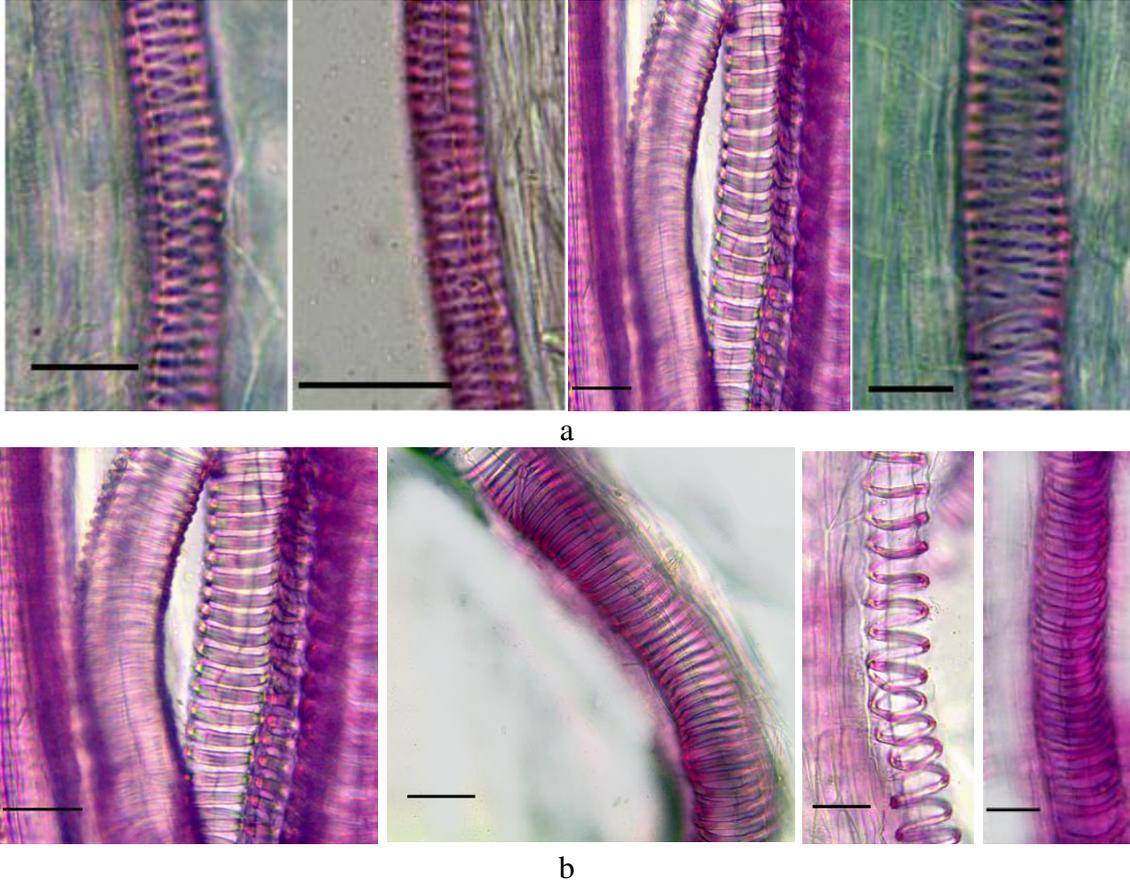
Figure 1. The types of tracheal elements in surface sections taken from different parts of the leaf; (**a**) spiral, scalariform and reticulate type (**b**) annular type (Scala Bar:50 μm)

Şekil 1. Yaprağın farklı kısımlarından alınan yüzeysel kesitlerde trake eleman tipleri; (a) sarmal, merdivensi ve ağsı tip (b) halkalı tip (Ölçek:50 μm)

To draw this surface we type into the Maple plotting command (Figure 2).

```
> K:=tubeplot([cos(t),sin(t),-3],t=0..2*Pi,radius=1/4):
> A:=tubeplot([cos(t),sin(t),-2],t=0..2*Pi,radius=1/4):
> B:=tubeplot([cos(t),sin(t),-1],t=0..2*Pi,radius=1/4):
> C:=tubeplot([cos(t),sin(t),0],t=0..2*Pi,radius=1/4):
> E:=tubeplot([cos(t),sin(t),1],t=0..2*Pi,radius=1/4):
> F:=tubeplot([cos(t),sin(t),2],t=0..2*Pi,radius=1/4):
> G:=tubeplot([cos(t),sin(t),3],t=0..2*Pi,radius=1/4):
> H:=plot3d([1.27*cos(s), 1.27*sin(s), t], s=0..Pi,
  t=-Pi..Pi,color=pink):
> display({K, A,B,C,E,F,G,H});
```

Finally, a geometric model for the annular type of tracheal elements was generated. This type of shapes is described as a collection of tubular surfaces about the space curves

$$\beta(t)=(\cos(t),\sin(t),b).$$

Tracheary elements are highly specialized cells. These non-living cells are elongated with lignified cell walls. The tracheal elements form geometric shapes such as spiral, annular, scaliform by accumulation of lignin in cell walls (Figure 1,2).
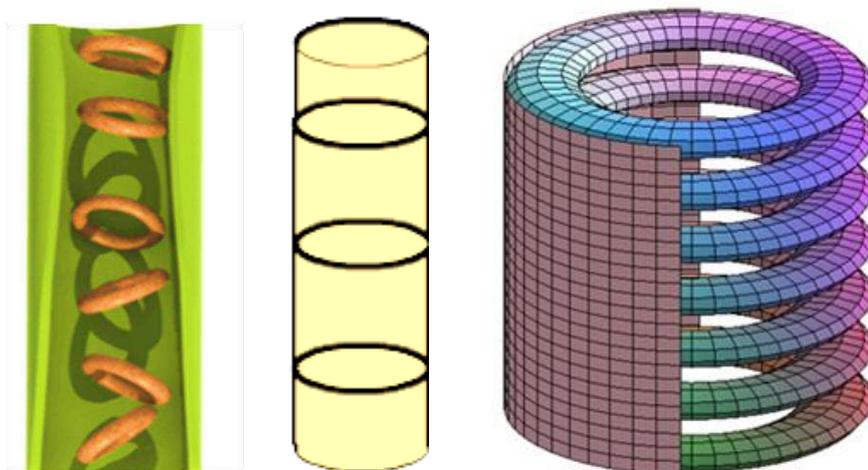
Figure 2. The general drawing and geometric model surfaces of annular type
Şekil 2. Halkalı tip yüzeyinin genel çizimi ve geometrik modeli

In this study, we aimed to demonstrate the characteristics of tracheal elements of the plant by geometrical and numerical method. Non-numeric characters provide evidence concerning the interrelationships of larger groups of plant. But in many cases, it is not sufficient to examine in detail. Therefore, it is of special importance that the numeric characters are examined in a comparative and scientific discipline (Özörgücü et al. 1991). The shape of the anatomical structure are very important for taxonomic studies (Metcalfe and Chalk, 1983).

In this study It has been tried to find evidence which can be use in addition to non- nümerical characters to distinguish plant using geometric modeling of the tracheal elements. This numerical features can be used as taxon-distinguishing feature. Thus, the geometric model of the tracheal elements can be viewed as a taxonomic character for classification of plants. There are several studies on the geometric description of some other live parts in the literature (Arslan et al.2009; Coşkun et al. 2007; Ezentaş et al. 2005; Yıldız et al. 2001). We think that this work will bring a different perspective to future researchers working on similar issues therefore will provide a new comparing opportunity for the future researches on the related subjects.

As shown in Table 2 there are important correlations between 1-2,3,5; 2-5 ; 3-4,5 and 4-5 at levels of 0.01 and 0.05. It can be said that diameter measurements of annular trachea are the nümeric characters which represents the variations in them. It has been also found that the results from nümerical analysis can provide distinct evidences of plants.

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