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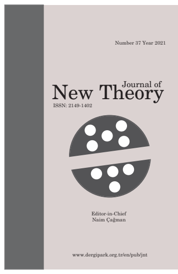

Bianchi VI_0 Universe with Magnetized Strange Quark Matter in $f(R,T)$ Theory

Sinem Kalkan[1], **Can Aktaş**[2]

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Abstract — This study discusses $f(R,T)$ theory, one of the alternative theories. It then studies magnetized strange quark matter in the universe model Bianchi VI_0, homogeneous and anisotropic. Afterwards, it has determined whether the energy conditions are provided by using the deceleration parameter while obtaining the solutions. Moreover, the evolution of the cosmic universe is examined with the help of graphics and tables.

Keywords — *$f(R,T)$ theory, Bianchi VI_0, deceleration parameter*

Mathematics Subject Classification (2020) — 83C05, 83C15

1. Introduction

Recent observations have shown that the universe is accelerating and expanding [1–3]. The reasons for this acceleration and expansion still remain a mystery. Einstein's General Relativity theory has tried to explain the universe in general. However, it has fallen short of explaining acceleration and expansion. Therefore, alternative theories to this theory have been put forward by many scientists. Among these alternative theories, there are theories such as $f(R)$ theory [4], $f(G)$ theory [5], and Lyra theory [6]. In this study, we have discussed the $f(R,T)$ theory put forward by Harko et al. in 2011 [4]. $f(R,T)$ theory has also been studied by many scientists [7–16]. Although it is known that the magnetic field played an important role in the formation of structures in the early universe, its cause has still not fully understood. The magnetic field is thought to affect the formation of galaxies [17]. Strange quark matter (SQM) with a magnetic field provides information about the accelerating expansion of the universe according to Supernova-type Ia observations [18]. One of the reasons we have studied quark matter with magnetic field is because quark-gluon matter must be stable and it must ensure charge neutrality [19]. Moreover, Singh and Beesham have studied SQM with the cosmological term in $f(R,T)$ theory [20]. Besides, Katore et al. have studied in Bianchi VI_0 universe model [21]. In addition, Aktaş has investigated magnetized strange quark matter (MSQM) in $f(R,T)$ theory [22]. Further, Nagpal et al. have studied MSQM and SQM distribution in $f(R,T)$ theory [17]. Furthermore, Pradhan and Bali have studied magnetized Bianchi VI_0 universe model [23]. Additionally, Aktaş et al. have studied MSQM distribution for the Marder universe model in $f(R,T)$ theory [24]. Moreover, Sahoo et al. have investigated MSQM in $f(R,T)$ theory [18]. Besides, some scientists have investigated MSQM and SQM [25–30].

[1]sinemkalkan@stu.comu.edu.tr (Corresponding Author); [2]canaktas@comu.edu.tr
[1]Institute of Graduate Education, Çanakkale Onsekiz Mart University, Çanakkale, Turkey
[2]Department of Mathematics, Faculty of Arts and Sciences, Çanakkale Onsekiz Mart University, Çanakkale, Turkey

The Section 2 of this study presents basic equations in the $f(R,T)$ theory with cosmological term by considering MSQM. The Section 3 obtains modified field equations by using an equation of state and a decelaration parameter. Finally, the graphs of the obtained solutions are interpreted and the energy states of the MSQM in this universe are examined. This study is a part of the first author's master's thesis.

2. Field Equations in $f(R,T)$ Modified Gravition Theory

In 2011, Harko et al. suggested a new theory as a function $f(R,T)$ connected to R and T instead of Ricci scalar R in the Einstein-Hilbert type action function [4]. The action in the modified gravitational theories is as follows.

$$S = \frac{1}{16\pi}\int (f(R,T) + 2\Lambda)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x \tag{1}$$

where T is the trace of $T_{ik} = -2\frac{\delta(\sqrt{-g}L_m)}{\sqrt{-g}\delta g^{ik}}$, R is Ricci scaler, g is the determinant of the metric tensor g_{ik}, and L_m is matter Lagrangian. Here, assume that the Lagrangian L_m depends only on the metric tensor component g_{ik} rather than derivatives [4]. By changing the action S in the Equation (1) regarding g_{ik}, the $f(R,T)$ gravitational field equations are obtained as follows [4,31]:

$$f_R(R,T)R_{ik} - \frac{1}{2}f(R,T)g_{ik} + (g_{ik}\Box - \nabla_i\nabla_k)f_R(R,T) = 8\pi T_{ik} - f_T(R,T)T_{ik} - f_T(R,T)\Theta_{ik} + \Lambda g_{ik} \tag{2}$$

here Θ_{ik} is defined by [4]

$$\Theta_{ik} = -2T_{ik} + g_{ik}L_m - 2\frac{\partial^2 L_m}{\partial g^{ik}\partial^{lm}}g^{lm}$$

where $f_T(R,T)$ and $f_R(R,T)$ indicate partial derivatives of $f(R,T)$ in regard to T and R, respectively, ∇_i is the covariant derivative, and $\Box = \nabla_i\nabla^i$ [4].

Contracting the Equation (2), we get

$$3\Box f_R(R,T) + f_R(R,T)R - 2f(R,T) = \big(8\pi - f_T(R,T)\big)T - f_T(R,T)\Theta + 4\Lambda \tag{3}$$

where $\Theta = g^{ik}\Theta_{ik}$. From Equations (2) and (3), we have

$$\begin{aligned} f_R(R,T)(R_{ik} - \frac{1}{3}Rg_{ik}) + \frac{1}{6}f(R,T)g_{ik} &= 8\pi\big(T_{ik} - \frac{1}{3}Tg_{ik}\big) - f_T(R,T)(T_{ik} - \frac{1}{3}Tg_{ik}) \\ &\quad - f_T(R,T)\big(\Theta_{ik} - \frac{1}{3}\Theta g_{ik}\big) + \nabla_i\nabla_k f_R(R,T) + \Lambda g_{ik} \end{aligned}$$

In this study, we consider $f(R,T) = R+2f(T)$. The gravitational field equation is as follows [4,31]:

$$R_{ik} - \frac{1}{2}Rg_{ik} = 8\pi T_{ik} - 2f'(T)T_{ik} - 2f'(T)\Theta_{ik} + (f(T) + \Lambda)g_{ik}$$

In addition, in this study, $f(T) = \mu T$ such that μ is an arbitrary constant [4]. Moreover, in this theory, the field equation (3) with cosmological term Λ is given by

$$G_{ik} = (2\mu + 8\pi)T_{ik} + \big(\mu(\rho - p) + \Lambda\big)g_{ik} \tag{4}$$

such that ρ and p are the energy density and pressure, respectively.

The energy momentum tensor in MSQM is considered as

$$T_{ik} = h^2(u_iu_k - \frac{1}{2}g_{ik}) + u_iu_k(\rho + p) - pg_{ik} - h_ih_k \tag{5}$$

where h^2 is magnetic field [32,33]. Further, $u^i = \delta^4_i$ that satisfies the condition $u_iu^i = 1$.

3. Modified Einstein Field Equations and Solutions in Bianchi VI_0 Universe

The Bianchi universe VI_0, a homogeneous-anisotropic metric, is

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2m^2x}dy^2 - C^2e^{2m^2x}dz^2 \tag{6}$$

where A, B, and C are functions of t and m is constant. Besides, the coordinates: (x, y, z, t). From Equations (4)-(6), the collection of modified field equations for the Bianchi universe VI_0 are acquired as follows:

$$\frac{m^4}{A^2} + \frac{\ddot{C}}{C} + \frac{\ddot{B}}{B} + \frac{\dot{C}\dot{B}}{CB} = 4h^2\pi + h^2\mu - 3p\mu + \mu\rho - 8p\pi + \Lambda \tag{7}$$

$$-\frac{m^4}{A^2} + \frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -4h^2\pi - h^2\mu - 3p\mu + \mu\rho - 8p\pi + \Lambda \tag{8}$$

$$-\frac{m^4}{A^2} + \frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{B}\dot{A}}{BA} = -4h^2\pi - h^2\mu - 3p\mu + \mu\rho - 8p\pi + \Lambda \tag{9}$$

$$-\frac{m^4}{A^2} + \frac{\dot{C}\dot{B}}{CB} + \frac{\dot{C}\dot{A}}{CA} + \frac{\dot{A}\dot{B}}{AB} = 8\pi\rho + 3\mu\rho + 4h^2\pi + h^2\mu - p\mu + \Lambda \tag{10}$$

$$-\frac{m^2\dot{C}}{C} + \frac{m^2\dot{B}}{B} = 0 \tag{11}$$

here "$\cdot$" indicates the derivative with respect to t.

In this study, there are five $f(R,T)$ field equations and seven unknowns. The unknowns are p, Λ, h, A, B, C, and ρ. To solve this system of the equations, auxiliary equations must be considered. First, the following MSQM's equation of state is used to completely solve the system.

$$p = \frac{\rho - 4B_c}{3} \tag{12}$$

where B_c is a bag constant [19,34]. Furthermore, deceleration parameter can be taken as follows [35]:

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) = -1 + \frac{1}{2\beta\sqrt{t+\alpha}} \tag{13}$$

where α and β are constants and H is Hubble parameter. Using Equation (11),

$$B = c_1C \tag{14}$$

such that c_1 is a constant. Later on, if we use Equation (13), then we obtain the equation

$$C = \frac{c_2e^{2\beta\sqrt{t+\alpha}}}{\sqrt{c_3}} \tag{15}$$

where c_2 and c_3 are constants. From Equations (7)-(15), the metric potential A, magnetic field h^2, energy density ρ, pressure p, and cosmological term Λ are obtained as follows:

$$A = c_3e^{2\beta\sqrt{t+\alpha}}$$

$$h^2 = \frac{m^4}{(4\pi+\mu)c_3^2(e^{2\beta\sqrt{\alpha+t}})^2}$$

$$\rho = \frac{3}{8(4\pi+\mu)}\left(\frac{\beta}{(\alpha+t)^{\frac{3}{2}}} - \frac{2m^4}{c_3^2e^{4\beta\sqrt{\alpha+t}}}\right) + B_c$$

$$p = \frac{1}{8(4\pi+\mu)}\left(\frac{\beta}{(\alpha+t)^{\frac{3}{2}}} - \frac{2m^4}{c_3^2 e^{4\beta\sqrt{\alpha+t}}}\right) - B_c$$

and

$$\Lambda = -\frac{2\pi m^4}{c_3^2(4\pi+\mu)e^{4\beta\sqrt{\alpha+t}}} - \frac{\beta(3\pi+\mu)}{(4\pi+\mu)(\alpha+t)^{\frac{3}{2}}} + \frac{3\beta^2}{\alpha+t} - 4(2\pi+\mu)B_c$$

Phase transition have realized while $q(t_{tr}) = 0$. Here,

$$t_{tr} = \frac{1}{4\beta^2}\left(1 - 4\beta^2\alpha\right)$$

As the phase transition in our universe happens after the Big Bang, $t_{tr} > 0 \Rightarrow \beta\sqrt{\alpha} < 0.5$ [35]. Moreover, because $q > 0$ for, $t = 0$, the cosmic universe passes from deceleration to acceleration [35]. Given the fact that $\beta\sqrt{\alpha} < 0.5$, different values q_0 are given in Table 1 for different constants.

Table 1. Values of the deceleration parameter q_0 corresponding to β values for $\alpha = 0.05$

β	$\beta\sqrt{\alpha}$	$q_0 = q(t_0)$
0.15	$0.03354101966 < 0.5$	-0.62
0.25	$0.05590169942 < 0.5$	-0.46
0.35	$0.0782623792 < 0.5$	-0.104

t_0 is the age of the universe and is taken as 13.8 billion years.

4. Conclusion

This section provides the graphs A, B, C, h^2, p, Λ, ρ, and q corresponding to different q_0 values. Table 2 shows that the examined universe model satisfies the energy conditions (EC), that is, the energy conditions preserve the properties of the substance.

Table 2. Analyze of energy conditions

q_0	Null EC	Weak EC	Dominant EC	Strong EC
	$\{p+\rho \geq 0\}$	$\{ p+\rho \geq 0 \ \& \ 0 \leq \rho\}$	$\{ -p+\rho \geq 0 \ \& \ -\lvert p\rvert+\rho \geq 0\}$	$\{p+\rho \geq 0 \ \& \ 3p+\rho \geq 0\}$
-0.62	$\bigoplus$	$\bigoplus$	$\bigoplus$	$\bigoplus$
-0.46	$\bigoplus$	$\bigoplus$	$\bigoplus$	$\bigoplus$
-0.104	$\bigoplus$	$\bigoplus$	$\bigoplus$	$\bigoplus$

$\bigoplus$: Energy condition is provided.

Figure 1 provides that as accepted in today's models, the transition of the considered universe model is from deceleration to acceleration. Further, the evolution of the metric potential A, B, and C are presented in Figures 2-4, respectively.

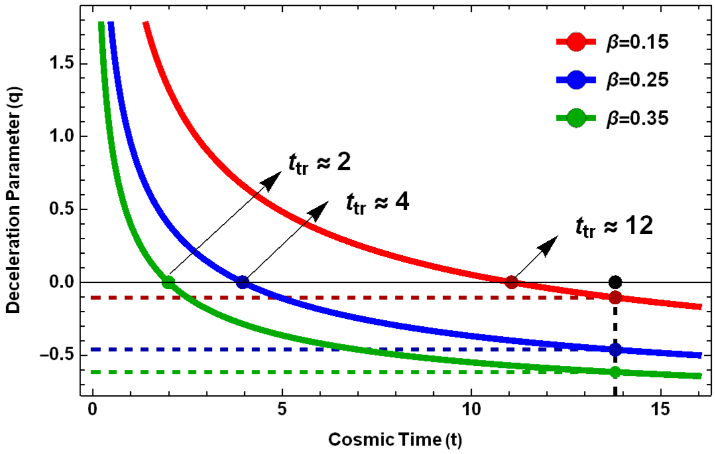

Fig. 1. The evolution of the deceleration parameter with respect to cosmic time for $\alpha = 0.05$

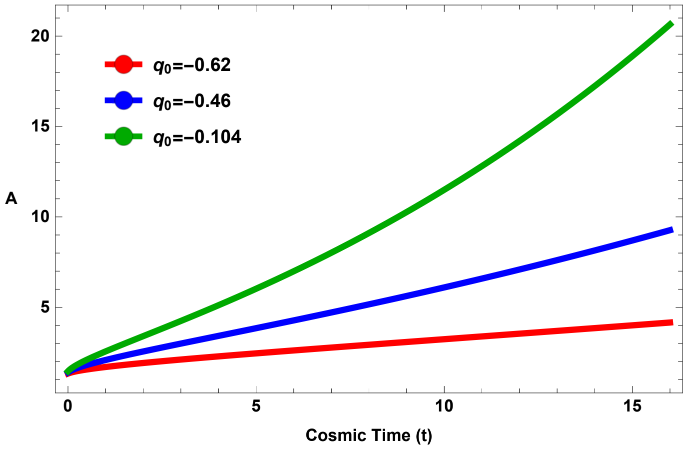

Fig. 2. The evolution of the metric potential A with respect to cosmic time for $m = 0.4$, $c_3 = 1.25$, and $\alpha = 0.05$

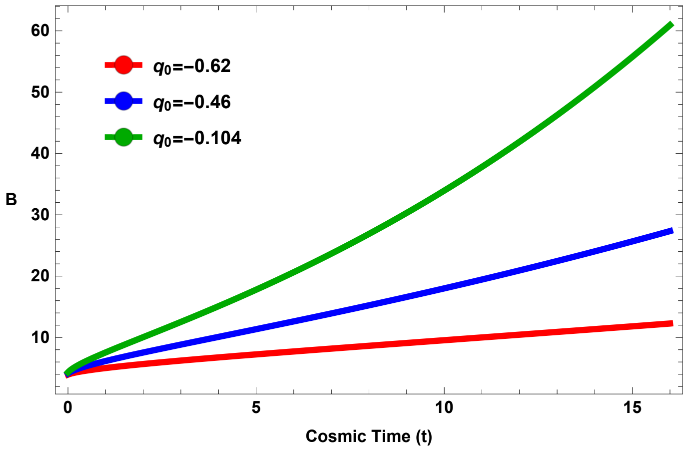

Fig. 3. The evolution of the metric potential B with respect to cosmic time for $c_1 = 1.5$, $c_2 = 2.75$, $c_3 = 1.25$, and $\alpha = 0.05$

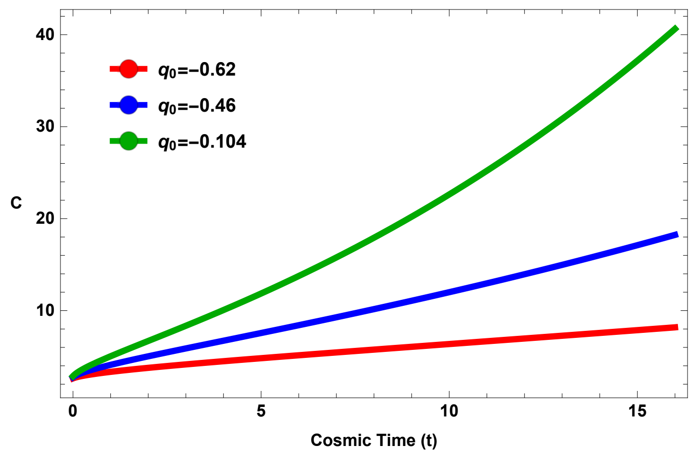

Fig. 4. The evolution of the metric potential C with respect to cosmic time for $c_2 = 2.75$, $c_3 = 1.25$, and $\alpha = 0.05$

Figures 2-4 show that the metric potentials increases over time. Moreover, for $q_0 = -0.62$, smaller than the other q_0 values, the increase is less. Similarly, for $q_0 = -0.104$, larger than the other q_0 values, the increase is more.

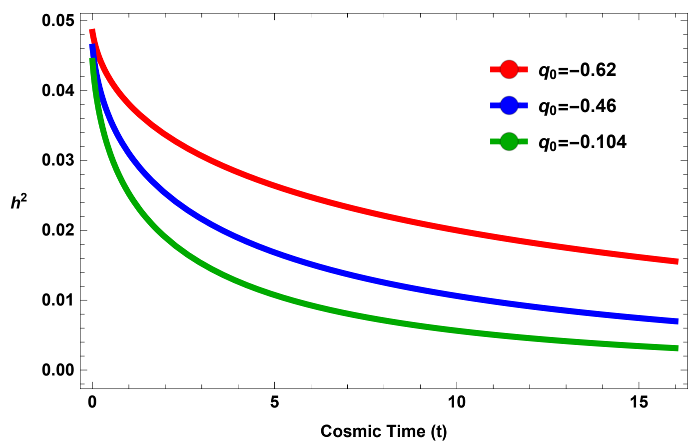

Fig. 5. The evolution of the magnetic field with respect to cosmic time for $m = 0.4$, $c_3 = 1.25$, $\alpha = 0.05$, and $\mu = -6.2734$

In Figure 5, the variation of the magnetic field relative to cosmic time decreases over time. Further, because $h^2 = 0$, for $m = 0$, the universe model turns into a non-magnetic universe model.

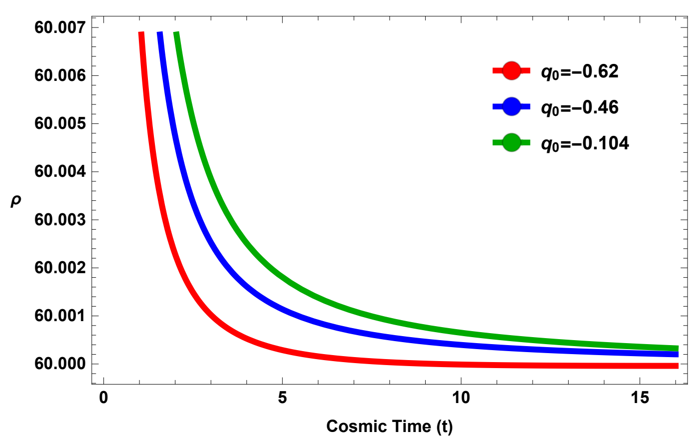

Fig. 6. The evolution of the energy density with respect to cosmic time for $m = 0.4$, $c_3 = 1.25$, $\alpha = 0.05$, $\mu = -6.2734$, and $B_c = 60$

Figure 6 shows $\rho > 0$. As $\rho > 0$, the universe model does not collapse on itself. In addition, it also approaches the bag constant over time.

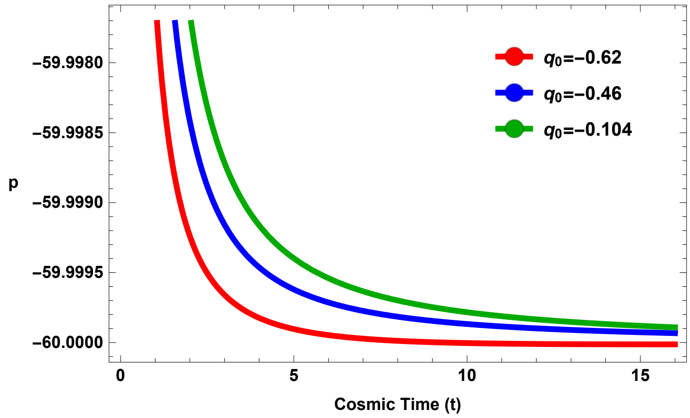

Fig. 7. The evolution of the pressure with respect to cosmic time for $m = 0.4$, $c_3 = 1.25$, $\alpha = 0.05$, $\mu = -6.2734$, and $B_c = 60$

Figure 7 shows that in the universe model, pressure decreases and approaches $-B_c$ over time.

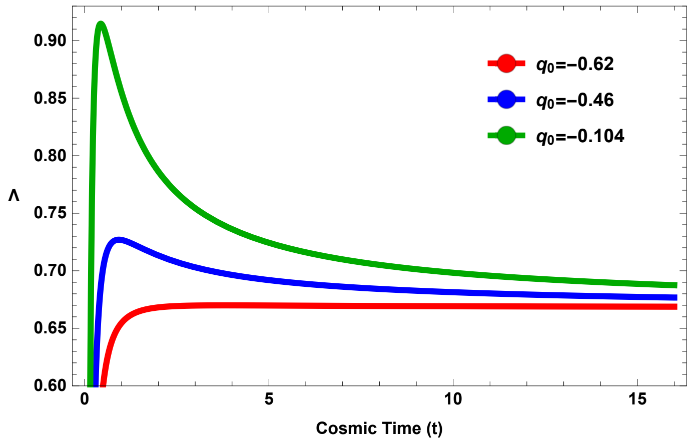

Fig. 8. The evolution of the cosmological term with respect to cosmic time for $m = 0.4$, $c_3 = 1.25$, $\alpha = 0.05$, $\mu = -6.2734$, and $B_c = 60$

Figure 8 shows that the cosmological term is positive. Therefore, it contributes to the accelerating expansion of the universe.

Author Contributions

All the authors contributed equally to this work. They all read and approved the last version of the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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