Research Article GU J Sci 36(4): 1811-1832 (2023) DOI: 10.35378/gujs.1038212

Gazi University
Journal of Science
http://dergipark.gov.tr/gujs

Bootstrap Approach for Testing More Than Two Population Means with Ranked Set Sampling

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Highlights

- This paper focuses on bootstrap method of ranked set sampling.
- Bootstrap sample selection methods are adapted for ANOVA in ranked set sampling.
- Simulation study results were obtained.

Article Info

Received: 18 Dec 2021
Accepted: 12 Oct 2022

Keywords

Ranked set sampling
Bootstrap
ANOVA
Hypothesis testing

Abstract

In this study, hypothesis test is investigated based on Bootstrap sample selection methods to compare more than two population means under Ranked Set Sampling. Bootstrap sample selection methods are obtained by adapting Hui's sample selection methods for confidence interval. We also compare these adapted methods with bootstrap simple random sampling and bootstrap ranked set sampling methods using simulation study. Simulation study shows that adapted methods which proposed in this paper perform quite well.

1. INTRODUCTION

Ranked set sampling (RSS) is used where the actual measurement of sample units is difficult but ranking the sample units is easy without actual quantification. RSS was proposed by McIntyre [1] for estimating mean pasture yield. RSS has many applications in medical, ecological and environmental studies. Takahasi and Wakimoto [2] suggested the mathematical theory of concerning technique. Dell and Clutter [3] showed that regardless of ranking error, the RSS estimator of the population mean is unbiased, and it is at least as efficient as the simple random sampling (SRS) estimator. The RSS technique is also used in parameter estimation, confidence interval and hypothesis testing. Muttlak [4] examined the parameter estimation of one-way ANalysis Of VAriance (ANOVA) under RSS. Albatineh et al. [5] conducted a confidence interval study for the population coefficient of variation using RSS. Mahdizadeh and Zamanzade [6] studied an asymptotic interval estimation for the stress resistance model based on the method of the RSS. In addition to this, Mahdizadeh and Zamanzade [7] obtained the confidence interval for the population quantiles based on the RSS and carried out an application study on the real medical data set. Shen [8] proposed a new test statistic for hypothesis test of population mean under normal distribution using RSS. Abu-Dayyeh and Muttlak [9] studied hypothesis tests for the parameters about exponential and uniform distributions. Özdemir and Gökpınar [10], studied hypothesis test of population mean under different RSS designs and obtained power values using SRS and different RSS designs. Özdemir et al. [11] investigated the hypothesis testing for the difference of means of two populations under RSS for normal distributions with unknown variances. Besides, Özdemir et al. [12], conducted hypothesis test study for two population means difference under median RSS for homogenous and heterogenous variance cases. Also, Karadağ and Bacanlı [13] considered the hypothesis test for the population mean of inverse Gaussian distribution using ranked set sampling is considered when the scale parameter is both known and unknown.

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In statistical inferences such as confidence interval or hypothesis testing, the distribution of statistic is needed. However, this distribution cannot be obtained in most cases. In such cases, resampling techniques such as bootstrap are used to obtain distribution of statistic. Bootstrap also offers an alternative approach to estimate standard error of the statistic. Bootstrap method was firstly used by Efron [14]. Chernick [15], Davison and Hinkley [16] and Manly [17] present different studies related to Bootstrap. If sample selection process in RSS is performed visually ranking or personal judgement to minimize the ranking error, it is not preferred to have set size more than 5. In addition to this, it may be difficult to obtain asymptotic distribution of the statistic for small sample sizes cases. The use of bootstrap under RSS was given firstly by Hui et al. [18]. Hui et al. [18] considered bootstrapping as a way to construct confidence interval for estimation of the population mean for linear regression under RSS and proposed different bootstrap sample selection methods. In addition, Yeniay et al. [19] adapted the bootstrap sample selection methods given by Hui et al. [18] for testing population mean under RSS and also, they obtained Type I error rates and powers of tests for this case.

In the literature, there is no study about equality of means for more than two groups by using ranked set sampling as far as we investigated. However, the distribution of the test statistics based on ranked set sampling is very hard to obtain. For this reason, in this study, we proposed a new test by adapting bootstrap selection methods which were proposed by Hui et al. [18] to compare more than two population means. We give testing algorithm for more than two groups by using ranked set sampling with bootstrap. We also perform a simulation study for obtain type I error and power of test for this method and its alternatives.

The article is arranged as follows. Sample selection procedure in RSS and bootstrap sample selection methods by Hui et.al [18] in RSS are given in section 2. section 3 describes the Bootstrap sample selection methods adapted for ANOVA in RSS, simulation study is conducted in section 4 and concluding remarks are summarized in section 5.

2. SAMPLE SELECTION PROCEDURE AND BOOTSTRAP METHODS BY HUI ET AL. [18] FOR RSS

In this section, RSS sample selection procedure was introduced. Afterward, Bootstrap sample selection methods which were proposed Hui et al. [18] were explained under RSS.

Ranked set sampling has a two-stage selection process. In the first step, *m* sets of size *m* are selected from the population using SRS. The *m* units of each sample are ordered using auxiliary information or visual ranking methods. Then the smallest unit from the first set, the second smallest unit from the second set, and then the largest unit from the m^{th} set are selected and measured. This process given by Table 1.

Table 1. *Sample selection procedure in RSS with sample size m*

Set	Selected sample units from population				Ranked sample units				Sample units			
1	X_{11}	X_{12}	$\cdots$	X_{1m}	$X_{[1]1}$	$X_{[2]1}$	$\cdots$	$X_{[m]1}$	$X_{(1)1}$	$*$	$\cdots$	$*$
2	X_{21}	X_{22}	$\cdots$	X_{2m}	$X_{[1]2}$	$X_{[2]2}$	$\cdots$	$X_{[m]2}$	$*$	$X_{(2)2}$	$\cdots$	$*$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
m	X_{m1}	X_{m2}		X_{mm}	$X_{[1]m}$	$X_{[2]m}$		$X_{[m]m}$	$*$	$*$	$\cdots$	$X_{(m)m}$

This cycle may be repeated *r* times to obtain a RSS sample of size *mr*. It was illustrated in Table 2.

Table 2. *Ranked set sample with r cycle*

Cycle *1*				Cycle *2*				$\cdots$	Cycle *r*			
$X_{(1)1}$	$*$	$\cdots$	$*$	$X_{(1)2}$	$*$	$\cdots$	$*$		$X_{(1)r}$	$*$	$\cdots$	$*$
$*$	$X_{(2)1}$	$\cdots$	$*$	$*$	$X_{(2)2}$	$\cdots$	$*$	$\cdots$	$*$	$X_{(2)r}$	$\cdots$	$*$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$		$\vdots$	$\vdots$	$\ddots$	$\vdots$
$*$	$*$	$\cdots$	$X_{(m)1}$	$*$	$*$		$X_{(m)2}$		$*$	$*$	$\cdots$	$X_{(m)r}$

Let $X_1, X_2, \ldots, X_m$ be a random sample with probability function *f(x)* with mean μ and variance σ^2. Let $X_{11}, X_{12}, \ldots, X_{1m}; X_{21}, X_{22}, \ldots, X_{2m}; \ldots; X_{m1}, X_{m2}, \ldots, X_{mm}$ be independent random variables all with the same cumulative distribution function *F(x)*. Let assume that the cycle is repeated *r* times, $X_{(i)j}$ represents i^{th} order statistics of i^{th} set in j^{th} cycle (*i=1.2,…,m; j=1.2,…,r).*

The unbiased estimator of the population mean using RSS is defined as:

$$\bar{X}_{RSS} = \frac{1}{mr}\sum_{i=1}^{m}\sum_{j=1}^{r} X_{(i)j} \tag{1}$$

$\bar{X}_{RSS}$ is unbiased estimator of population mean.

Due to the visual ranking in RSS, it is recommended to study with small sample size to minimize ranking error. In cases where small sample sizes are studied, resampling techniques such as bootstrap are used to obtain the distribution information of statistics. The Bootstrap technique is one of the most popular methods of resampling methods if the distributions of the statistic could not be obtained analytically, this method would be preferred in practice. Hui et al. [18] used to Bootstrap methods in RSS and obtained confidence interval for the population mean. Bootstrap RSS by rows (Method 1) and Bootstrap RSS (Method 2) given by Hui et al. [18] as below.

Method 1

1. Assign to each element of the i^{th} row in Table 2, a probability of $\frac{1}{r}$ and select *r* elements randomly with replacement to obtain $X^*_{(i)1}, \ldots, X^*_{(i)r}$.
2. Perform step 1. for *i=1.2,…,m*, to obtain a bootstrap ranked set sample $\{X^*_{(i)j}\}$.

Method 2

1. Assign to each element of the ranked set sample a probability of $\frac{1}{mr}$.
2. Randomly draw *m* elements in Table 2, sort them in ascending order $y_1, y_2, \ldots y_m \sim F_{n.}$ $y_{(1)} \leq y_{(2)} \leq \cdots \leq y_{(m)}$ and retain $X^*_{(i)1} = y_{(1)}$. Where $y_{(1)} \leq y_{(2)} \leq \cdots \leq y_{(m)}$ denote ordered statistics.
3. Perform step 2 for *i=1.2,…,m*.
4. Repeat step 2 and step 3 *r* times to obtain $\{X^*_{(i)j}\}$.

3. ADAPTED BOOTSTRAP METHODS FOR ANOVA IN RSS

In this section, we adapt the bootstrap sample selection methods which were proposed by Hui et al. [18] to compare more than two population means**.** The one-way ANOVA is used to determine whether there are any statistically significant differences between the means of more than two independent groups. In practice, it can occur in situations where the measurement in dependent variable is difficult in terms of cost, labor and time. In this case, cost-effective measurements can be made using RSS. Sample selection process for *a* group under RSS is as follows: m^2 units are randomly selected from *k*. group *(k=1.2,…,a)* by SRS and the units are then randomly divided into *m* sets of *m* sizes. Units in each *m* sets are ordered in terms of dependent variable using auxiliary information or visual ranking methods. The first unit is taken from 1^{st} set and second unit is taken from 2^{nd} set and then the finally m^{th} unit is selected from m^{th}set. This process repeats r_k times for k^{th} group. These process repeats for each group and the obtained ranked set samples for *a* groups are given as Table 3.

Table 3. *Ranked set samples for a groups in ANOVA*

Groups												
1				2					a			
$X_{1(1)1}$	$X_{1(1)2}$	$\cdots$	$X_{1(1)r_1}$	$X_{2(1)1}$	$X_{2(1)2}$	$\cdots$	$X_{2(1)r_2}$		$X_{a(1)1}$	$X_{a(1)2}$	$\cdots$	$X_{a(1)r_a}$
$X_{1(2)1}$	$X_{1(2)2}$	$\cdots$	$X_{1(2)r_1}$	$X_{2(2)1}$	$X_{2(2)2}$	$\cdots$	$X_{2(2)r_2}$	$\cdots$	$X_{a(2)1}$	$X_{a(2)2}$	$\cdots$	$X_{a(2)r_a}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$	$\vdots$
$X_{1(m)1}$	$X_{1(m)2}$	$\cdots$	$X_{1(m)r_1}$	$X_{2(m)1}$	$X_{2(m)2}$	$\cdots$	$X_{2(m)r_2}$		$X_{a(m)1}$	$X_{a(m)2}$	$\cdots$	$X_{a(m)r_a}$

In Table 3, $X_{k(i)j}$ denotes i^{th} order statistic in j^{th} cycle in k^{th}group *(k=1.2,…,a; i=1.2,…,m; j=1.2,…,*r_k*)*. Here, the sample size is $n = \sum_{k=1}^{a} n_k, n_k = mr_k$.

Here, we are interested in testing the null hypothesis. $H_0: \mu_1 = \mu_2 = \cdots = \mu_a = \mu$ against the alternative hypothesis

$H_1: \exists \mu_k \neq \mu_{k'}, \exists k \neq k' = 1.2, \dots, a.$

The test statistic for testing H_0 against H_1 can be defined as follows

$$F_{RSS} = \frac{\sum_{k=1}^{a} n_k \left(\bar{X}_{k(.).} - \bar{X}_{.(.).}\right)^2 / (a-1)}{\sum_{k=1}^{a} \sum_{i=1}^{m} \sum_{j}^{r_k} \left(X_{k(i)j} - \bar{X}_{k(.).}\right)^2 / (n-a)} \tag{2}$$

where $\bar{X}_{k(.).} = \sum_{i=1}^{m} \sum_{j=1}^{r_k} X_{k(i)j}$, $\bar{X}_{.(.).} = \frac{X_{.(.).}}{n}$, $X_{.(.).} = \sum_{i=1}^{m} \sum_{j=1}^{r_k} \sum_{k=1}^{a} X_{k(i)j}$, $(i = 1.2, \dots, m; j = 1.2, \dots, r_k; k = 1.2, \dots, a)$.

Since small sample sizes are used to minimize the ranking error in RSS, it is not possible to obtain distribution of the statistic. Then, bootstrap technique which is one of the resampling techniques can be used. In the bootstrap technique, B bootstrap sample are generated from original data set. Each bootstrap sample has *n* elements, generated by sampling with replacement. The value of statistic computed for each individual resample. The bootstrap is a very convenient and practical tool for statistical analysis because it is not requiring any theoretical and empirical assumption (Efron, 1979).

In the rest of this section, the sample selection methods which were proposed by Hui et al. [18] for the confidence interval for population mean are adapted for to compare more than two population means which are given as Method 1 and Method 2 below.

The bootstrap sample selection methods which were given by Hui et al. [18] are adapted to ANOVA testing procedure as below. Let T matrix denotes combined ranked set samples for each group to test ANOVA. For this, the sample units in Table 3 are combined as given in Equation (3). This matrix is defined as T matrix and given as follows:

$$T = \begin{bmatrix} X_{1(1)1}X_{1(1)2} \dots X_{1(1)r_1}; X_{2(1)1}X_{2(1)2}, \dots, X_{2(1)r_2}; \dots X_{a(1)1}X_{a(1)2} \dots X_{a(1)r_a} \\ X_{1(2)1}X_{1(2)2} \dots X_{1(2)r_1}; X_{2(2)1}X_{2(2)2}, \dots, X_{2(2)r_2}; \dots X_{a(2)1}X_{a(2)2} \dots X_{a(2)r_a} \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ X_{1(m)1}X_{1(m)2} \dots X_{1(m)r_1}; X_{2(m)1}X_{2(m)2}, \dots, X_{2(m)r_2}; \dots X_{a(m)1}X_{a(m)2} \dots X_{a(m)r_a} \end{bmatrix} \tag{3}$$

Selected bootstrap samples according to methods 1 and 2 from the T matrix will be denoted by superscript ‘*’.

Method 1

1. Compute F_{RSS} statistic using Equation (2) for ranked set sample.

2. r_1 unit for *1st* group, r_2 unit for *2nd* group and r_a unit for *ath* group are selected from *ith* row of T in Equation (3) by randomly with replacement.
3. Perform step 2 for *i=1.2,…,m* to obtain bootstrapped ranked set sample T^*.

4. Compute F^*_{RSS} from bootstrap ranked set sample T^*as follows:

$$F^*_{RSS} = \frac{\sum_{k=1}^{a} n_k\left(\bar{X}^*_{k(.).}-\bar{X}^*_{.(.).}\right)^2/(a-1)}{\sum_{k=1}^{a}\sum_{i=1}^{m}\sum_{j}^{r_k}\left(X^*_{k(i)j}-\bar{X}^*_{k(.).}\right)^2/(n-a)} \ ,$$

$$\bar{X}^*_{k(.).}=\sum_{i=1}^{m}\sum_{j=1}^{r_k} X^*_{k(i)j} \ , \bar{X}^*_{.(.).} = \frac{X^*_{.(.).}}{n}, X^*_{.(.).} = \sum_{i=1}^{m}\sum_{j=1}^{r_k}\sum_{k=1}^{a} X^*_{k(i)j}.$$

5. If B is assumed as the number of bootstrap samples generated from the original sample, $F^*_{1(RSS)}, F^*_{2(RSS)}, \dots, F^*_{B(RSS)}$ are calculated by repeating the steps 3-5.

6. p value is estimated by comparing the $F^*_{b(RSS)}$ test statistic value calculated from each bootstrap sample with the F_{RSS} test statistic calculated in step 1 as follows:

$$\hat{p} = \frac{\#(F^*_{b(RSS)} > F_{RSS})}{B}, \ b=1.2,\dots,B$$

7. If $\hat{p} < \alpha$ then H_0 is rejected.

Method 2

1. Compute F_{RSS} statistic using Equation (2) for ranked set sample.

2. Randomly *m* elements are selected from T in Equation (3), sort them ascending order and smallest unit is taken, and this unit is denoted as $X^*_{1(1)1}$. Similarly, *m* units are randomly selected from T and sorted, second unit is taken, and this unit is denoted as $X^*_{1(2)1}$. This process continues until *mth* unit is obtained as $X^*_{1(m)1}$. This sample selection process repeats r_1 times for the first group.

3. Perform step 2 for k^{th} group r_k *(k=1.2,..,a)* times and then T^*bootstrap ranked set samples are obtained for *a* groups.

4. Compute F^*_{RSS} from bootstrap ranked set sample T^*as follows:

$$F^*_{RSS} = \frac{\sum_{k=1}^{a} n_k\left(\bar{X}^*_{k(.).}-\bar{X}^*_{.(.).}\right)^2/(a-1)}{\sum_{k=1}^{a}\sum_{i=1}^{m}\sum_{j}^{r_k}\left(X^*_{k(i)j}-\bar{X}^*_{k(.).}\right)^2/(n-a)} \ ,$$

$$\bar{X}^*_{k(.).}=\sum_{i=1}^{m}\sum_{j=1}^{r_k} X^*_{k(i)j} \ , \bar{X}^*_{.(.).} = \frac{X^*_{.(.).}}{n}, X^*_{.(.).} = \sum_{i=1}^{m}\sum_{j=1}^{r_k}\sum_{k=1}^{a} X^*_{k(i)j}.$$

5. B bootstrap samples are generated and $F^*_{1(RSS)}, F^*_{2(RSS)}, \dots, F^*_{B(RSS)}$ are calculated by repeating step 3-5.

6. p value is estimated by comparing the $F^*_{b(RSS)}$ test statistic value calculated from each bootstrap sample with the F_{RSS} test statistic calculated in step 1 as follows:

$$\hat{p} = \frac{\#(F^*_{b(RSS)} > F_{RSS})}{B}, \ b=1.2,\dots,B$$

7. If $\hat{p} < \alpha$ then H_0 is rejected.

Simulation study was carried out in different situations to examine *p* value.

3.1. Classical Bootstrap for ANOVA Using SRS and RSS

In the simulation study, classical bootstrap is used to compare more than two group means apart from the methods given above using SRS (SRSboot) and RSS (RSSboot) methods. These methods are described below.

SRSboot Method

1. F_{SRS} statistic is computed using the following formula for the selected simple random sample.

$$F_{SRS} = \frac{\sum_{k=1}^{a} n_k(\bar{X}_{k.}-\bar{X}_{..})^2/(a-1)}{\sum_{k=1}^{a}\sum_{i=1}^{n_k}(X_{ki}-\bar{X}_{k.})^2/(n-a)}$$

Where $\bar{X}_{k.} = \sum_{k=1}^{a}\sum_{i=1}^{n_k} X_{ki}$, $\bar{X}_{..} = \frac{X_{..}}{n}$, $X_{..} = \sum_{k=1}^{a}\sum_{i=1}^{n_k} X_{ki}$, $(k = 1.2, \dots, a; i = 1.2, \dots, n_k)$.

2. Samples which are selected for the SRS technique is combined just like T matrix for the bootstrapping. The classical bootstrap method is applied to this matrix to obtain bootstrap samples.

3. F_{SRS}^* from bootstrap sample is computed as follows:

$$F_{SRS}^* = \frac{\sum_{k=1}^{a} n_k\left(\bar{X}_{k.}^*-\bar{X}_{..}^*\right)^2/(a-1)}{\sum_{k=1}^{a}\sum_{i=1}^{n_k}\left(X_{ki}^*-\bar{X}_{k.}^*\right)^2/(n-a)} ,$$

$\bar{X}_{k.}^* = \sum_{k=1}^{a}\sum_{i=1}^{n_k} X_{ki}^*$, $\bar{X}_{..}^* = \frac{X_{..}^*}{n}$, $X_{..}^* = \sum_{k=1}^{a}\sum_{i=1}^{n_k} X_{ki}^*$

4. B bootstrap samples are generated and $F_{1(SRS)}^*, F_{2(SRS)}^*, \dots, F_{B(SRS)}^*$ are calculated.

5. p value is estimated as follows

$$\hat{p} = \frac{\#(F_{b(SRS)}^* > F_{SRS})}{B},\ b=1.2,\dots,B$$

6. If $\hat{p} < \alpha$ then H_0 is rejected.

RSSboot Method

1. F_{RSS} statistic is computed using Equation (2) for ranked set sample.
2. The classical bootstrap method is applied to T matrix to obtain T^*.

3. F_{RSS}^* from bootstrap ranked set sample $T^*$is computed as follows:

$$F_{RSS}^* = \frac{\sum_{k=1}^{a} n_k\left(\bar{X}_{k(.).}^*-\bar{X}_{.(.).}^*\right)^2/(a-1)}{\sum_{k=1}^{a}\sum_{i=1}^{m}\sum_{j}^{r_k}\left(X_{k(i)j}^*-\bar{X}_{k(.).}^*\right)^2/(n-a)} ,$$

$\bar{X}_{k(.).}^* = \sum_{i=1}^{m}\sum_{j=1}^{r_k} X_{k(i)j}^*$, $\bar{X}_{.(.).}^* = \frac{X_{.(.).}^*}{n}$, $X_{.(.).}^* = \sum_{i=1}^{m}\sum_{j=1}^{r_k}\sum_{k=1}^{a} X_{k(i)j}^*$.

4. B bootstrap samples are generated and $F_{1(RSS)}^*, F_{2(RSS)}^*, \dots, F_{B(RSS)}^*$ are calculated.

5. p value is estimated as follows:

$$\hat{p} = \frac{\#(F_{b(RSS)}^* > F_{RSS})}{B},\ b=1.2,\dots,B$$

6. If $\hat{p} < \alpha$, then H_0 is rejected.

4. SIMULATION STUDY

In this section, the hypothesis given above $H_0: \mu_1 = \mu_2 = \cdots = \mu_a = \mu$ was considered based on RSS with bootstrap methods. Original data sets are generated from Standard Normal distribution under H_0. For calculating the powers of tests, data sets are generated from normal distribution. The variances of normal distribution are taken as σ_I^2= 1. (i=1.2,…,k) for all groups. Also, the means of normal distribution , μ= $[\mu_1, \mu_2, \dots, \mu_a]$ are taken as follows: for a=3, μ=[-d 0 d] ; for a=4, μ=[-d 0 0 d] and for a =, μ=[-d –d 0 d d] . Thus, type I error rates can be calculated when d values are zero and Powers of tests are also obtained for different *d* values *(0.125,0.250…0.750)*. To compare the type I error rates and powers of tests, *m* was taken as 3,4,5. *r* values used in this simulation study for different *a* values are given in Table 4.

Table 4. $r = [r_1, r_2, \dots, r_a]$ *values for a=3,4,5*

a=3	**a=4**	**a=5**
[2 2 2]	[2 2 2 2]	[2 2 2 2 2]
[3 3 3]	[3 3 3 3]	[3 3 3 3 3]
[4 4 4]	[4 4 4 4]	[4 4 4 4 4]
[5 5 5]	[5 5 5 5]	[5 5 5 5 5]
[10 10 10]	[10 10 10 10]	[10 10 10 10 10]
[2 3 4]	[2 3 3 4]	[2 2 3 4 4]
[4 5 6]	[4 5 5 6]	[4 4 5 6 6]
[2 5 8]	[2 5 5 8]	[2 2 5 8 8]

We use the nominal level 0.05. Number of bootstrap repetitions is 2000 and number of Monte Carlo iteration is 2000. Simulation study was conducted by using MATLAB R2007b. Figures are given to visually support the results of the simulation study. Figures only show the case of *d*=0.25.

Table 5. *Type I error rates and powers of tests when m=3 and a=3*

$[r_1\ \ r_2\ \ r_3]$	d	1.method	2.method	RSSboot	SRSboot
[2 2 2]	0.000	0.0520	0.0445	0.0030	0.0565
	0.125	0.0965	0.0740	0.0095	0.0690
	0.250	0.1760	0.1480	0.0195	0.1095
	0.375	0.3205	0.2840	0.0505	0.1625
	0.500	0.5125	0.4780	0.1320	0.2690
	0.750	0.052	0.0445	0.003	0.0565
[3 3 3]	0.000	0.0570	0.0500	0.0040	0.0440
	0.125	0.1060	0.0900	0.0065	0.0680
	0.250	0.2275	0.2180	0.0390	0.1370
	0.375	0.4685	0.4470	0.1190	0.2330
	0.500	0.7250	0.7210	0.2995	0.4100
	0.750	0.9740	0.9715	0.7705	0.7750
[4 4 4]	0.000	0.0555	0.0530	0.0050	0.0540
	0.125	0.1085	0.1025	0.0145	0.0810
	0.250	0.3135	0.2930	0.0585	0.1820
	0.375	0.5820	0.5690	0.1990	0.3455
	0.500	0.8460	0.8400	0.4970	0.5510
	0.750	0.9955	0.9945	0.9280	0.8960
[5 5 5]	0.000	0.0505	0.0550	0.0030	0.0530
	0.125	0.1305	0.1270	0.0095	0.0885
	0.250	0.3855	0.3700	0.0885	0.2020
	0.375	0.7165	0.6935	0.3095	0.4070
	0.500	0.9210	0.9235	0.6425	0.6410
	0.750	1.0000	1.0000	0.9845	0.9390
[10 10 10]	0.000	0.0480	0.0460	0.0050	0.0520
	0.125	0.2090	0.1990	0.0415	0.1320
	0.250	0.6455	0.6495	0.2795	0.3760
	0.375	0.9615	0.9585	0.7515	0.7185
	0.500	0.9990	0.9985	0.9725	0.9350
	0.750	1.0000	1.0000	1.0000	1.0000
[2 3 4]	0.000	0.0460	0.0405	0.0025	0.0510
	0.125	0.0905	0.0770	0.0060	0.0715
	0.250	0.2105	0.2030	0.0345	0.1210
	0.375	0.4405	0.4150	0.1075	0.2130
	0.500	0.6905	0.6740	0.2695	0.3940
	0.750	0.9600	0.9500	0.7155	0.7230
[4 5 6]	0.000	0.0505	0.0440	0.0045	0.0545
	0.125	0.1315	0.1195	0.0105	0.0900
	0.250	0.3600	0.3470	0.0830	0.1905
	0.375	0.6975	0.6925	0.2930	0.4005
	0.500	0.9075	0.9030	0.6150	0.6440
	0.750	0.9995	1.0000	0.9830	0.9460
[2 5 8]	0.000	0.0535	0.0500	0.0035	0.0450
	0.125	0.1120	0.1050	0.0070	0.0785
	0.250	0.3065	0.2935	0.0630	0.1640
	0.375	0.5765	0.5690	0.1970	0.5150
	0.500	0.9375	0.8305	0.4665	0.5330
	0.750	0.9940	0.9935	0.9265	0.8710

Table 6. *Type I error rates and powers of test when m=4 and a=3*

$[r_1\ r_2\ r_3]$	d	1.method	2.method	RSSboot	SRSboot
[2 2 2]	0.000	0.0550	0.0465	0.0003	0.0565
	0.125	0.1165	0.0905	0.0020	0.0575
	0.250	0.2970	0.2520	0.0215	0.1135
	0.375	0.5125	0.4650	0.0710	0.2330
	0.500	0.7585	0.7420	0.1950	0.3580
	0.750	0.9805	0.9790	0.6740	0.7055
[3 3 3]	0.000	0.0595	0.0530	0.0015	0.0520
	0.125	0.1320	0.1140	0.0045	0.0775
	0.250	0.3710	0.3525	0.0400	0.1660
	0.375	0.6920	0.6775	0.1705	0.3530
	0.500	0.9180	0.9085	0.4595	0.5440
	0.750	0.9985	0.9985	0.9440	0.8890
[4 4 4]	0.000	0.0575	0.0475	0.0015	0.0515
	0.125	0.1630	0.1445	0.0075	0.0905
	0.250	0.4535	0.4420	0.0580	0.2235
	0.375	0.8100	0.7995	0.2955	0.4340
	0.500	0.9780	0.9760	0.7000	0.6770
	0.750	0.9995	0.9995	0.9940	0.9625
[5 5 5]	0.000	0.0600	0.0515	0.0000	0.0495
	0.125	0.1920	0.1765	0.0105	0.1010
	0.250	0.5940	0.5770	0.1215	0.2605
	0.375	0.8980	0.8980	0.4555	0.5380
	0.500	0.9975	0.9960	0.8435	0.7900
	0.750	1.0000	1.0000	0.9995	0.9930
[10 10 10]	0.000	0.0535	0.0480	0.0000	0.0430
	0.125	0.3245	0.3090	0.0255	0.1495
	0.250	0.8765	0.8685	0.4030	0.5075
	0.375	0.9970	0.9960	0.9100	0.8670
	0.500	1.0000	1.0000	0.9995	0.9855
	0.750	1.0000	1.0000	1.0000	1.0000
[2 3 4]	0.000	0.0565	0.0535	0.0000	0.0530
	0.125	0.1260	0.1085	0.0030	0.0765
	0.250	0.3370	0.3075	0.0375	0.1555
	0.375	0.6415	0.6340	0.1520	0.3215
	0.500	0.9020	0.8910	0.4305	0.5165
	0.750	0.9960	0.9975	0.9160	0.8760
[4 5 6]	0.000	0.0565	0.0540	0.0020	0.0445
	0.125	0.1780	0.1685	0.0105	0.1110
	0.250	0.5750	0.5550	0.1120	0.2555
	0.375	0.8885	0.8795	0.4440	0.5380
	0.500	0.9930	0.9925	0.8320	0.7640
	0.750	1.0000	1.0000	0.9995	0.9840
[2 5 8]	0.000	0.0540	0.0460	0.0005	0.0535
	0.125	0.1420	0.1285	0.0065	0.0910
	0.250	0.4665	0.4540	0.0695	0.2060
	0.375	0.7980	0.7975	0.3030	0.4035
	0.500	0.9720	0.9715	0.6770	0.6875
	0.750	1.0000	1.0000	0.9935	0.9545

Table 7. *Type I error rates and powers of test when m=5 and a=3*

$[r_1\ r_2\ r_3]$	d	1.method	2.method	RSSboot	SRSboot
[2 2 2]	0.000	0.0575	0.0435	0.0000	0.0555
	0.125	0.1535	0.1145	0.0000	0.0650
	0.250	0.3945	0.3305	0.0180	0.1385
	0.375	0.6960	0.6590	0.0770	0.2835
	0.500	0.9265	0.9080	0.3150	0.4605
	0.750	0.9990	0.9980	0.8655	0.8075
[3 3 3]	0.000	0.0600	0.0535	0.0000	0.0535
	0.125	0.1780	0.1555	0.0020	0.0840
	0.250	0.5165	0.4855	0.0420	0.1930
	0.375	0.8755	0.8630	0.2620	0.4115
	0.500	0.9875	0.9855	0.6425	0.6535
	0.750	1.0000	1.0000	0.9935	0.9465
[4 4 4]	0.000	0.0585	0.0465	0.0000	0.0475
	0.125	0.2250	0.2065	0.0095	0.0945
	0.250	0.6505	0.6285	0.0880	0.2750
	0.375	0.9395	0.9360	0.4265	0.5280
	0.500	0.9990	0.9995	0.8580	0.8040
	0.750	1.0000	1.0000	1.0000	0.9930
[5 5 5]	0.000	0.0530	0.0500	0.0000	0.0490
	0.125	0.2495	0.2330	0.0100	0.1145
	0.250	0.7300	0.7195	0.1370	0.3470
	0.375	0.9815	0.9795	0.6385	0.6365
	0.500	1.0000	1.0000	0.9510	0.8840
	0.750	1.0000	1.0000	1.0000	0.9980
[10 10 10]	0.000	0.0545	0.0545	0.0000	0.0445
	0.125	0.4525	0.4510	0.0355	0.1870
	0.250	0.9625	0.9620	0.5430	0.5790
	0.375	1.0000	1.0000	0.9870	0.9250
	0.500	1.0000	1.0000	1.0000	0.9965
	0.750	1.0000	1.0000	1.0000	1.0000
[2 3 4]	0.000	0.0545	0.0505	0.0000	0.0570
	0.125	0.1730	0.1460	0.0000	0.0785
	0.250	0.5085	0.4690	0.0375	0.1795
	0.375	0.8450	0.8295	0.2125	0.3945
	0.500	0.9835	0.9790	0.5845	0.6220
	0.750	1.0000	1.0000	0.9875	0.9340
[4 5 6]	0.000	0.0585	0.0520	0.0000	0.0525
	0.125	0.2485	0.2275	0.0075	0.1095
	0.250	0.7175	0.7135	0.1290	0.3220
	0.375	0.9790	0.9785	0.6240	0.6265
	0.500	1.0000	0.9995	0.9490	0.8760
	0.750	1.0000	1.0000	1.0000	0.9975
[2 5 8]	0.000	0.0500	0.0480	0.0000	0.0435
	0.125	0.2245	0.2045	0.0070	0.0870
	0.250	0.6175	0.6105	0.0830	0.2350
	0.375	0.9450	0.9420	0.4305	0.5060
	0.500	0.9995	0.9990	0.8360	0.7660
	0.750	1.0000	1.0000	1.0000	0.9860

The results in Table 5 give the type I error rates and powers of tests when *m=3* and *a=3*. It is seen that the type I error rates obtained by the method 1. 2 and SRSboot are at the nominal level 0.05. Type I error rates obtained by the RSSboot are considerably lower than nominal alpha 0.05. In addition, powers of tests are obtained by method 1 is higher than others. Powers of test increases as the sample size increases. Moreover, considering the cases [3 3 3] and [2 3 4] where total sample size is same the case of [3 3 3] have higher power values than [2 3 4] for method 1. Similar situation is true for [5 5 5] and [2 5 8]. For example, the cases of [3 3 3] and [2 3 4] for when *d=0.375*, the power values are obtained for method 1 in [3 3 3] are higher than the power values in [2 3 4]. In addition to this, considering [2 5 8] with [4 5 6] cases where the numbers of cycles are different but the total sample sizes are the same, power values are obtained in [4 5 6] are higher than in [2 5 8].

Table 6 presents Type I error rates and powers of tests when *m=4* and *a=3*. According to Table 6, Type I error rates which are obtained based on method 1. 2 and SRSboot are at its nominal level 0.05. All the same, Type I error rates obtained by the RSSboot are considerably lower than nominal alpha 0.05. Powers of tests obtained by method 1 are higher than the results are obtained by method 2 and SRSboot in Table 6. The cases of [3 3 3] with [2 3 4] and [5 5 5] with [2 5 8] where the total sample sizes are the same are considered, power values obtained are higher when the numbers of cycles are equal. Also, [2 5 8] and [4 5 6] cases where the numbers of cycles are different but the total sample sizes are the same, power values are obtained in [4 5 6] are higher than in [2 5 8]. For example, when d=0.5000. the power value is 0.9720 for r_k=[2 5 8] while the power value is 0.9930 for r_k = [4 5 6].

Table 7 gives Type I error rates and powers of tests when *m=5* and *a=3*. The results obtained are in line with the previous results. Figures 1-3 for a=3, m=3,4,5 as follows.

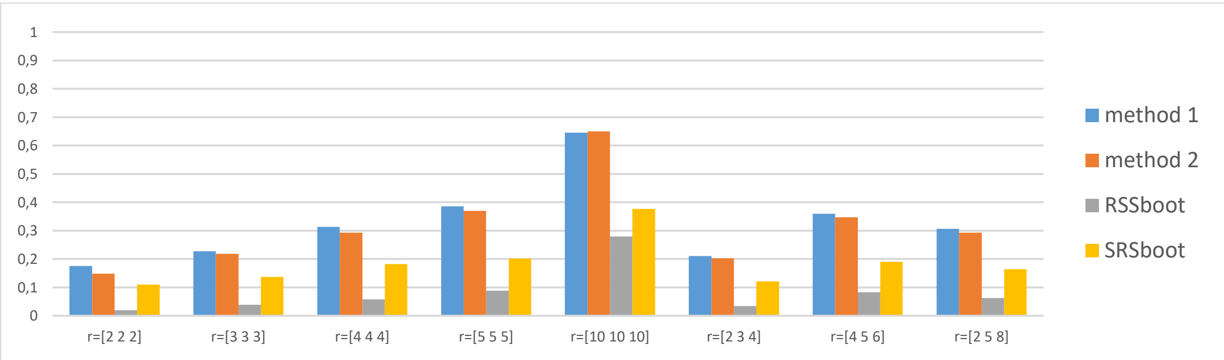

Figure 1. *Powers of tests for m=3, a=3 and d=0.25*

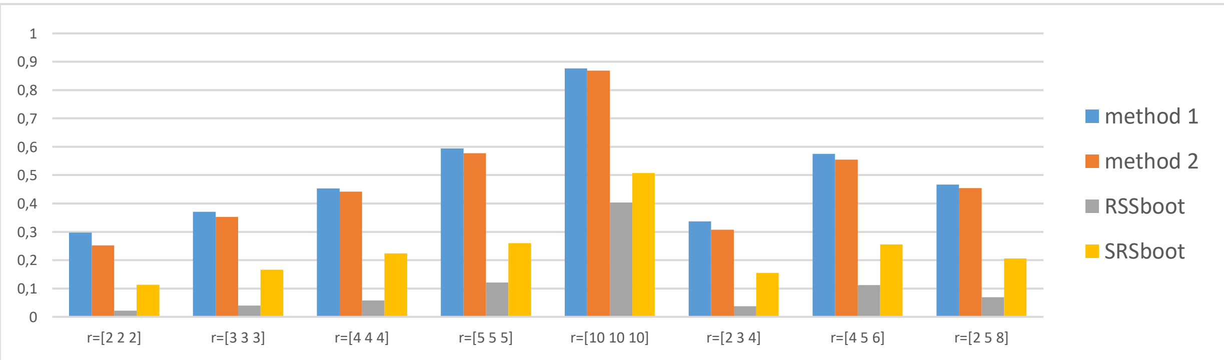

Figure 2. *Powers of tests for m=4, a=3 and d=0.25*

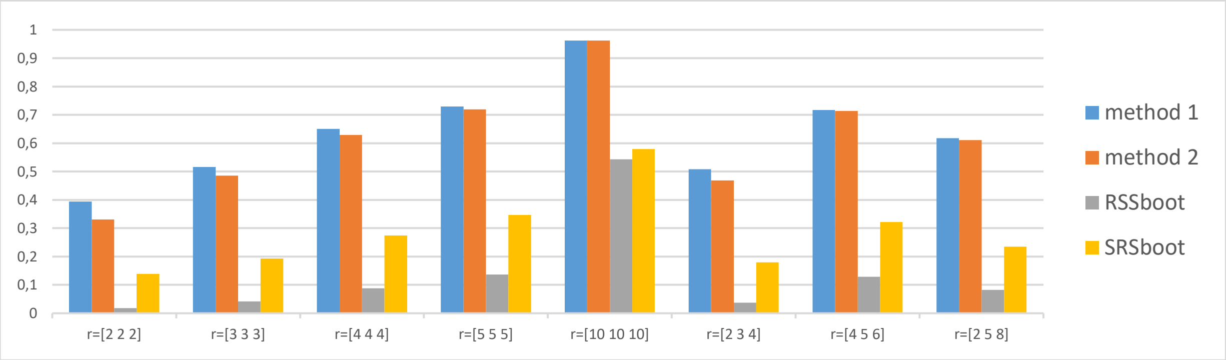

Figure 3. *Powers of tests for m=5, a=3 and d=0.25*

Figure 1-3 give the powers of tests values when *a*=3; *m*=3,4,5; *d*=0.25 with method 1.2, SRSboot and RSSboot methods. It is seen that the powers of tests values obtained with method 1 are higher than the powers of tests values obtained with other methods for all considered cases. Also, considering the cases [3 3 3] and [2 3 4] where the total cycle size is 9 and [5 5 5] and [2 5 8] where the total cycle size is 15, the cases of [3 3 3] and [5 5 5] have higher power values than [2 3 4] and [2 5 8] respectively. Thus, we can say that groups containing equal number of units will give higher power values than other situations that give the same sample size. In addition to this, powers of test values are increase as set size increase.

Table 8. *Type I error rates and powers of test when m=3 and a=4*

$[r_1\ \ r_2\ \ r_3\ \ r_4]$	*d*	1.method	2.method	RSSboot	SRSboot
[2 2 2 2]	0.000	0.0560	0.0490	0.0000	0.0440
	0.125	0.0755	0.0625	0.0015	0.0640
	0.250	0.1440	0.1240	0.0080	0.0840
	0.375	0.2705	0.2615	0.0385	0.1290
	0.500	0.4305	0.4265	0.0810	0.2335
	0.750	0.8025	0.7945	0.3400	0.4885
[3 3 3 3]	0.000	0.0585	0.0475	0.0000	0.0420
	0.125	0.0855	0.0720	0.0050	0.0630
	0.250	0.2025	0.1895	0.0240	0.1085
	0.375	0.3885	0.3720	0.0575	0.1945
	0.500	0.6430	0.6440	0.1860	0.3535
	0.750	0.9595	0.9590	0.6615	0.7035
[4 4 4 4]	0.000	0.0535	0.0450	0.0030	0.0510
	0.125	0.1055	0.0985	0.0045	0.0705
	0.250	0.2645	0.2565	0.0345	0.1340
	0.375	0.5255	0.5225	0.1285	0.2520
	0.500	0.7970	0.7930	0.3565	0.4650
	0.750	0.9925	0.9915	0.8700	0.8665
[5 5 5 5]	0.000	0.0570	0.0440	0.0015	0.0450
	0.125	0.1015	0.0975	0.0060	0.0750
	0.250	0.3035	0.2980	0.0380	0.1770
	0.375	0.6375	0.6370	0.1955	0.3525
	0.500	0.8755	0.8755	0.5130	0.5655
	0.750	0.9985	0.9980	0.9505	0.9235
[10 10 10 10]	0.000	0.0585	0.0530	0.0040	0.0480
	0.125	0.1715	0.1675	0.0215	0.1090
	0.250	0.5825	0.5680	0.1720	0.3395
	0.375	0.9305	0.9275	0.6130	0.6785
	0.500	0.9965	0.9960	0.9360	0.8965
	0.750	1.0000	1.0000	1.0000	0.9990
[2 3 3 4]	0.000	0.0565	0.0460	0.0020	0.0515
	0.125	0.0825	0.0725	0.0045	0.0665
	0.250	0.1780	0.1795	0.0185	0.1185
	0.375	0.3860	0.3670	0.0565	0.1815
	0.500	0.6180	0.6060	0.1665	0.3270
	0.750	0.9415	0.9380	0.6135	0.6825
[4 5 5 6]	0.000	0.0580	0.0540	0.0015	0.0470
	0.125	0.1055	0.0955	0.0080	0.0785
	0.250	0.3165	0.3230	0.0470	0.1635
	0.375	0.6225	0.6130	0.1825	0.3535
	0.500	0.8860	0.8830	0.5060	0.5660
	0.750	0.9975	0.9975	0.9570	0.9240
[2 5 5 8]	0.000	0.0445	0.0420	0.0020	0.0565
	0.125	0.1030	0.1030	0.0045	0.0880
	0.250	0.2635	0.2415	0.0265	0.1480
	0.375	0.5590	0.5460	0.1430	0.3120
	0.500	0.8185	0.8075	0.3840	0.4745
	0.750	0.9895	0.9940	0.8905	0.8740

Table 9. *Type I error rates and powers of test when m=4 and a=4*

$[r_1\ r_2\ r_3\ r_4]$	*d*	1.method	2.method	RSSboot	SRSboot
[2 2 2 2]	0.000	0.0555	0.0420	0.0000	0.0495
	0.125	0.1100	0.0745	0.0000	0.0660
	0.250	0.2345	0.2065	0.0075	0.0995
	0.375	0.4465	0.4140	0.0345	0.1930
	0.500	0.7040	0.6910	0.1160	0.3230
	0.750	0.9690	0.9665	0.5570	0.6405
[3 3 3 3]	0.000	0.0580	0.0550	0.0000	0.0510
	0.125	0.1145	0.0980	0.0015	0.0660
	0.250	0.3015	0.2800	0.0115	0.1340
	0.375	0.6170	0.6085	0.0960	0.2755
	0.500	0.8810	0.8715	0.3045	0.4650
	0.750	0.9975	0.9990	0.8710	0.8665
[4 4 4 4]	0.000	0.0550	0.0435	0.0000	0.0550
	0.125	0.1395	0.1230	0.0030	0.0770
	0.250	0.4035	0.3875	0.0300	0.1845
	0.375	0.7680	0.7600	0.1810	0.3600
	0.500	0.9570	0.9555	0.5210	0.6230
	0.750	1.0000	1.0000	0.9790	0.9510
[5 5 5 5]	0.000	0.0500	0.0490	0.0000	0.0460
	0.125	0.1600	0.1480	0.0035	0.0935
	0.250	0.5075	0.4865	0.0550	0.2390
	0.375	0.8590	0.8565	0.3030	0.4770
	0.500	0.9890	0.9880	0.7315	0.7360
	0.750	1.0000	1.0000	0.9980	0.9815
[10 10 10 10]	0.000	0.0435	0.0440	0.0000	0.0540
	0.125	0.2765	0.2725	0.0120	0.1305
	0.250	0.8265	0.8185	0.2480	0.4285
	0.375	0.9950	0.9935	0.8365	0.8170
	0.500	1.0000	1.0000	0.9965	0.9735
	0.750	1.0000	1.0000	1.0000	1.0000
[2 3 3 4]	0.000	0.0575	0.0460	0.0000	0.0515
	0.125	0.1125	0.0990	0.0000	0.0720
	0.250	0.3225	0.3000	0.0150	0.1315
	0.375	0.6150	0.6000	0.0860	0.2825
	0.500	0.8620	0.8500	0.2745	0.4660
	0.750	0.9955	0.9950	0.8420	0.8345
[4 5 5 6]	0.000	0.0515	0.0500	0.0000	0.0545
	0.125	0.1570	0.1420	0.0004	0.0905
	0.250	0.4940	0.4865	0.0530	0.2125
	0.375	0.8580	0.8575	0.2880	0.4590
	0.500	0.9850	0.9850	0.7150	0.7305
	0.750	1.0000	1.0000	0.9990	0.9805
[2 5 5 8]	0.000	0.0460	0.0425	0.0000	0.0605
	0.125	0.1350	0.1245	0.0025	0.0890
	0.250	0.4510	0.4320	0.0415	0.2045
	0.375	0.8500	0.7755	0.1985	0.4135
	0.500	0.9705	0.9690	0.5650	0.6590
	0.750	1.0000	1.0000	0.9865	0.9530

Table 10. *Type I error rates and powers of test when m=5 and a=4*

$[r_1\ \ r_2\ \ r_3\ \ r_4]$	*d*	1.method	2.method	RSSboot	SRSboot
[2 2 2 2]	0.000	0.0575	0.0460	0.0000	0.0540
	0.125	0.1245	0.1015	0.0000	0.0680
	0.250	0.3355	0.2960	0.0070	0.1290
	0.375	0.6370	0.6050	0.0355	0.2425
	0.500	0.8710	0.8630	0.1710	0.4080
	0.750	0.9960	0.9980	0.7290	0.7855
[3 3 3 3]	0.000	0.0545	0.0435	0.0000	0.0500
	0.125	0.1535	0.1145	0.0000	0.0845
	0.250	0.3945	0.3305	0.0180	0.1790
	0.375	0.6960	0.6590	0.0770	0.5760
	0.500	0.9265	0.9080	0.3150	0.9385
	0.750	1.0000	1.0000	0.9750	0.9820
[4 4 4 4]	0.000	0.0515	0.0415	0.0000	0.0495
	0.125	0.1790	0.1655	0.0000	0.0830
	0.250	0.5815	0.5575	0.0300	0.2320
	0.375	0.9195	0.9160	0.2665	0.4735
	0.500	0.9940	0.9930	0.7255	0.7465
	0.750	1.0000	1.0000	1.0000	0.9810
[5 5 5 5]	0.000	0.0555	0.0490	0.0000	0.0505
	0.125	0.2085	0.1970	0.0020	0.0865
	0.250	0.6865	0.6825	0.0570	0.2660
	0.375	0.9720	0.9950	0.4395	0.5765
	0.500	0.9980	0.9985	0.8818	0.8540
	0.750	1.0000	1.0000	1.0000	0.9960
[10 10 10 10]	0.000	0.0505	0.0450	0.0000	0.0450
	0.125	0.4025	0.3900	0.0090	0.1580
	0.250	0.9480	0.9490	0.3505	0.5240
	0.375	1.0000	1.0000	0.9545	0.9085
	0.500	1.0000	1.0000	0.9995	0.9930
	0.750	1.0000	1.0000	1.0000	1.0000
[2 3 3 4]	0.000	0.0530	0.0415	0.0000	0.0495
	0.125	0.1350	0.1180	0.0000	0.0760
	0.250	0.4385	0.4190	0.0150	0.1735
	0.375	0.8065	0.7825	0.0985	0.3260
	0.500	0.9655	0.9590	0.4000	0.5655
	0.750	1.0000	1.0000	1.0000	0.9160
[4 5 5 6]	0.000	0.0485	0.0425	0.0000	0.0505
	0.125	0.2065	0.1885	0.0015	0.1020
	0.250	0.6810	0.6765	0.0685	0.2570
	0.375	0.9660	0.9645	0.4130	0.5795
	0.500	1.0000	0.9995	0.8795	0.8485
	0.750	1.0000	1.0000	0.9995	0.9985
[2 5 5 8]	0.000	0.0555	0.0440	0.0000	0.0515
	0.125	0.1840	0.1640	0.0004	0.0825
	0.250	0.5905	0.5830	0.0375	0.2375
	0.375	0.9235	0.9205	0.2880	0.4885
	0.500	0.9965	0.9970	0.7570	0.7500
	0.750	1.0000	1.0000	0.9995	0.9925

Tables 8-10 present Type I error rates and powers of tests when *a=4*, *m=3,4,5* respectively. According to these tables, it is possible to say Type I error rates which are obtained based on method 1. 2 and SRSboot are at their nominal level 0.05 and Type I error rates obtained by the RSSboot are considerably lower than nominal alpha 0.05. Additionally, considering the cases [3 3 3 3] and [2 3 3 4], where total sample sizes are same, the case of [3 3 3 3] have higher power values than [2 3 3 4] for method 1. Similar situation is true for [5 5 5 5] and [2 5 5 8]. For example, for *d=0.375*, *m*=3, powers of test values which are obtained from method 1 for the case of [3 3 3 3] higher than the case of [2 3 3 4]. In addition to this, [2 5 5 8] with [4 5 5 6] cases where the numbers of cycles are different but the total sample sizes are the same, power values are obtained in [4 5 5 6] are higher than in [2 5 5 8].

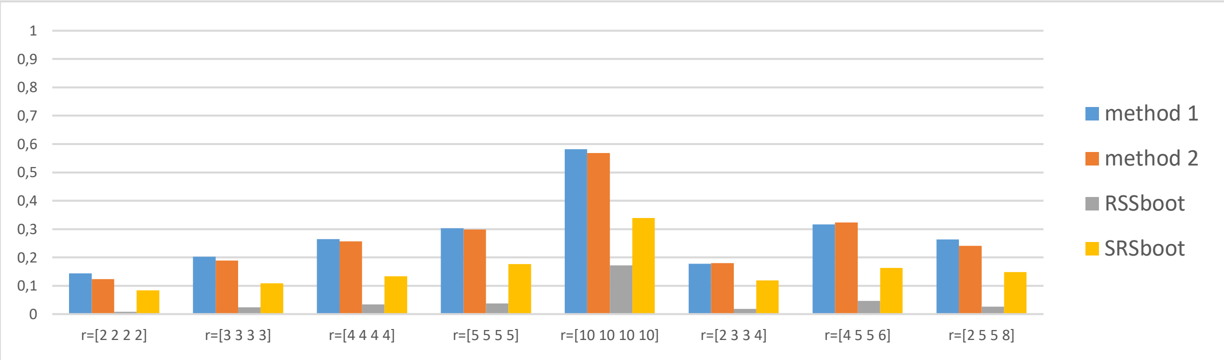

Figure 4. *Powers of tests for m=3, a=4 and d=0.25*

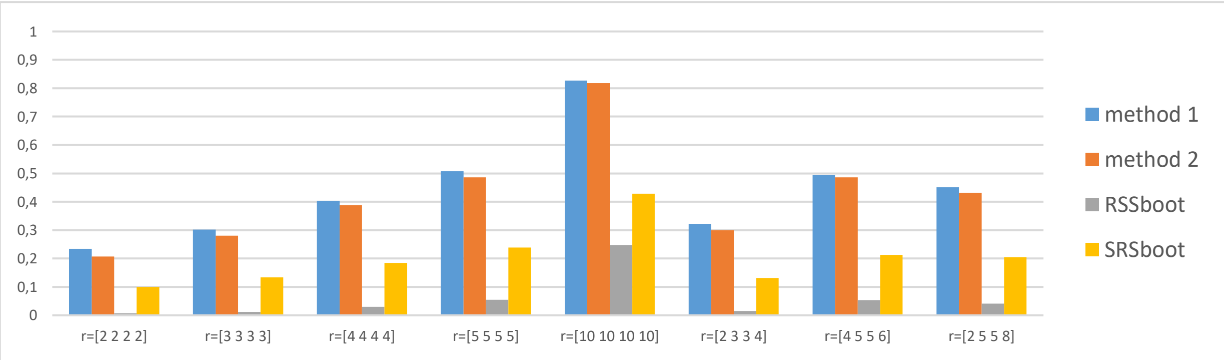

Figure 5. *Powers of tests for m=4, a=4 and d=0.25*

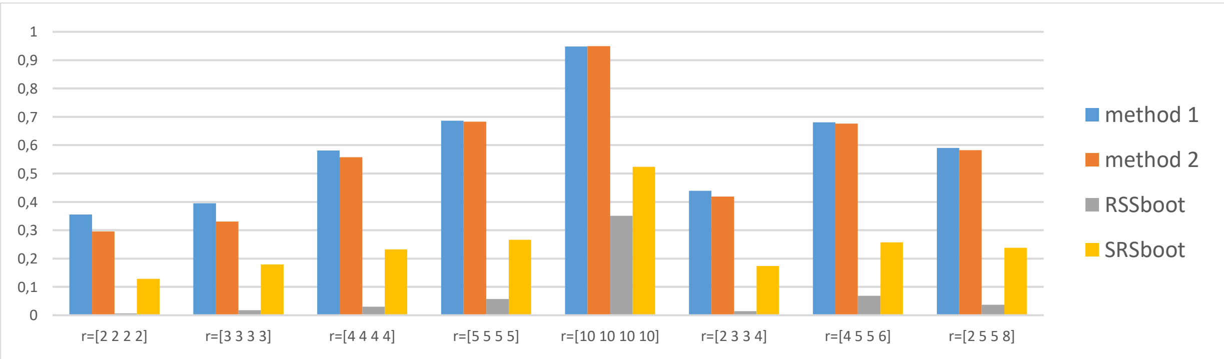

Figure 6. *Powers of tests for m=5, a=4 and d=0.25*

Figures 4-6 show the powers of tests when *a*=4, *m*=3,4,5 and d=0.25. As seen from the Figures 4-6, the highest powers of test values are obtained with method 1. Similarly the case *a*=3, powers of test values are increase as set size increase.

Table 11. *Type I error rates and powers of test when m=3 and a=5*

$[r_1\ r_2\ r_3\ r_4\ r_5]$	d	1.method	2.method	RSSboot	SRSboot
[2 2 2 2 2]	0.000	0.0515	0.0510	0.0000	0.0455
	0.125	0.0910	0.0840	0.0003	0.0645
	0.250	0.1750	0.1665	0.0150	0.1290
	0.375	0.4595	0.4445	0.0605	0.2320
	0.500	0.7105	0.7190	0.2030	0.3930
	0.750	0.9765	0.9775	0.6790	0.7585
[3 3 3 3 3]	0.000	0.0570	0.0560	0.0000	0.0505
	0.125	0.1065	0.0975	0.0030	0.0760
	0.250	0.3240	0.3205	0.0320	0.1740
	0.375	0.6605	0.6595	0.1700	0.3675
	0.500	0.8945	0.8955	0.4920	0.6105
	0.750	1.0000	1.0000	0.9580	0.9395
[4 4 4 4 4]	0.000	0.0510	0.0515	0.0000	0.0575
	0.125	0.1365	0.1345	0.0065	0.0910
	0.250	0.4370	0.4290	0.0710	0.2310
	0.375	0.7970	0.7950	0.3340	0.5030
	0.500	0.9720	0.9735	0.7385	0.7520
	0.750	1.0000	1.0000	0.9955	0.9840
[5 5 5 5 5]	0.000	0.0545	0.0535	0.0020	0.0490
	0.125	0.1580	0.1475	0.0075	0.1080
	0.250	0.5195	0.5275	0.1005	0.2880
	0.375	0.8850	0.8950	0.4780	0.6005
	0.500	0.9945	0.9940	0.8680	0.8800
	0.750	1.0000	1.0000	0.9990	0.9990
[10 10 10 10 10]	0.000	0.0565	0.0545	0.0020	0.0520
	0.125	0.2650	0.2605	0.0245	0.1585
	0.250	0.8445	0.8520	0.4375	0.5580
	0.375	0.9985	0.9980	0.9405	0.9135
	0.500	1.0000	1.0000	0.9995	1.0000
	0.750	1.0000	1.0000	1.0000	1.0000
[2 2 3 4 4]	0.000	0.0570	0.0525	0.0015	0.0600
	0.125	0.1100	0.0985	0.0025	0.0745
	0.250	0.2960	0.2895	0.0320	0.1740
	0.375	0.5985	0.6095	0.1320	0.3300
	0.500	0.8695	0.8700	0.4250	0.5660
	0.750	0.9975	0.9990	0.9340	0.9190
[4 4 5 6 6]	0.000	0.0585	0.0550	0.0020	0.0480
	0.125	0.1735	0.1580	0.0105	0.0910
	0.250	0.5140	0.5070	0.0970	0.2805
	0.375	0.8895	0.8815	0.4650	0.5740
	0.500	0.9920	0.9930	0.8520	0.8295
	0.750	1.0000	1.0000	1.0000	0.9975
[2 2 5 8 8]	0.000	0.0555	0.0545	0.0035	0.0505
	0.125	0.1270	0.1240	0.0060	0.0855
	0.250	0.3785	0.3795	0.0605	0.2430
	0.375	0.7585	0.7635	0.2765	0.4505
	0.500	0.9550	0.9545	0.6540	0.7065
	0.750	1.0000	1.0000	0.9960	0.9865

Table 12. *Type I error rates and powers of test when m=4 and a=5*

$[r_1\ r_2\ r_3\ r_4\ r_5]$	d	1.method	2.method	RSSboot	SRSboot
[2 2 2 2 2]	0.000	0.0570	0.0475	0.0000	0.0465
	0.125	0.1225	0.1095	0.0020	0.0835
	0.250	0.3045	0.2705	0.0145	0.1465
	0.375	0.6955	0.6955	0.0770	0.3295
	0.500	0.9260	0.9250	0.3380	0.5065
	0.750	0.9995	1.0000	0.9110	0.9075
[3 3 3 3 3]	0.000	0.0575	0.0470	0.0000	0.0575
	0.125	0.1555	0.1420	0.0015	0.0885
	0.250	0.5350	0.5240	0.0415	0.2310
	0.375	0.8985	0.9000	0.2605	0.4775
	0.500	0.9940	0.9955	0.7210	0.7520
	0.750	1.0000	1.0000	0.9970	0.9840
[4 4 4 4 4]	0.000	0.0505	0.0460	0.0000	0.0440
	0.125	0.2050	0.1935	0.0040	0.1020
	0.250	0.6775	0.6775	0.0855	0.3100
	0.375	0.9675	0.9685	0.4835	0.6285
	0.500	0.9995	0.9990	0.9195	0.8840
	0.750	1.0000	1.0000	1.0000	0.9985
[5 5 5 5 5]	0.000	0.0530	0.0450	0.0000	0.0465
	0.125	0.2345	0.2185	0.0065	0.1175
	0.250	0.7660	0.7690	0.1525	0.3800
	0.375	0.9905	0.9905	0.7025	0.7485
	0.500	1.0000	1.0000	0.9835	0.9490
	0.750	1.0000	1.0000	1.0000	1.0000
[10 10 10 10 10]	0.000	0.0460	0.0450	0.0000	0.0510
	0.125	0.4505	0.4530	0.0285	0.2080
	0.250	0.9860	0.9835	0.6350	0.7170
	0.375	1.0000	1.0000	0.9970	0.9735
	0.500	1.0000	1.0000	1.0000	1.0000
	0.750	1.0000	1.0000	1.0000	1.0000
[2 2 3 4 4]	0.000	0.0500	0.0475	0.0000	0.0485
	0.125	0.1560	0.1420	0.0000	0.0790
	0.250	0.4735	0.4665	0.0260	0.2240
	0.375	0.8520	0.8535	0.2265	0.4535
	0.500	0.9920	0.9905	0.6290	0.7055
	0.750	1.0000	1.0000	0.9960	0.9785
[4 4 5 6 6]	0.000	0.0570	0.0530	0.0000	0.0510
	0.125	0.2375	0.2290	0.0065	0.1180
	0.250	0.7545	0.7545	0.1345	0.3775
	0.375	0.9900	0.9885	0.6885	0.7340
	0.500	1.0000	1.0000	0.9775	0.9530
	0.750	1.0000	1.0000	1.0000	1.0000
[2 2 5 8 8]	0.000	0.0545	0.0505	0.0000	0.0565
	0.125	0.1905	0.1775	0.0035	0.1010
	0.250	0.6040	0.5945	0.0665	0.2730
	0.375	0.9450	0.9390	0.4390	0.5755
	0.500	0.9995	0.9990	0.8685	0.8535
	0.750	1.0000	1.0000	1.0000	0.9980

Table 13. *Type I error rates and powers of test when m=5 and a=5*

$[r_1\ r_2\ r_3\ r_4\ r_5]$	*d*	1.method	2.method	RSSboot	SRSboot
[2 2 2 2 2]	0.000	0.0565	0.0405	0.0000	0.0515
	0.125	0.1600	0.1420	0.0000	0.0920
	0.250	0.5175	0.4885	0.0120	0.1985
	0.375	0.8785	0.8790	0.1280	0.3765
	0.500	0.9900	0.9905	0.5295	0.6650
	0.750	1.0000	1.0000	0.9850	0.9630
[3 3 3 3 3]	0.000	0.0590	0.0500	0.0000	0.0530
	0.125	0.2270	0.2080	0.0015	0.0910
	0.250	0.7265	0.7180	0.0550	0.2925
	0.375	0.9815	0.9810	0.3950	0.5780
	0.500	1.0000	1.0000	0.8935	0.8680
	0.750	1.0000	1.0000	1.0000	0.9985
[4 4 4 4 4]	0.000	0.0535	0.0455	0.0000	0.0535
	0.125	0.2870	0.2790	0.0015	0.1130
	0.250	0.8455	0.8400	0.1285	0.3795
	0.375	0.9945	0.9930	0.6915	0.1130
	0.500	1.0000	1.0000	0.9855	0.9525
	0.750	1.0000	1.0000	1.0000	1.0000
[5 5 5 5 5]	0.000	0.0510	0.0455	0.0000	0.0435
	0.125	0.3505	0.3355	0.0035	0.1385
	0.250	0.9280	0.9270	0.2250	0.4925
	0.375	0.9990	0.9990	0.8820	0.8480
	0.500	1.0000	1.0000	0.9985	0.9850
	0.750	1.0000	1.0000	1.0000	1.0000
[10 10 10 10 10]	0.000	0.0495	0.0450	0.0000	0.0440
	0.125	0.6585	0.6545	0.0335	0.2580
	0.250	0.9985	0.9985	0.8230	0.8110
	0.375	1.0000	1.0000	1.0000	0.9960
	0.500	1.0000	1.0000	1.0000	1.0000
	0.750	1.0000	1.0000	1.0000	1.0000
[2 2 3 4 4]	0.000	0.0570	0.0485	0.0000	0.0490
	0.125	0.2095	0.1985	0.0000	0.0885
	0.250	0.6680	0.6670	0.0355	0.2635
	0.375	0.9675	0.9655	0.3380	0.5735
	0.500	1.0000	1.0000	0.8340	0.8155
	0.750	1.0000	1.0000	1.0000	0.9945
[4 4 5 6 6]	0.000	0.0575	0.0560	0.0000	0.0505
	0.125	0.3315	0.3240	0.0060	0.1340
	0.250	0.9280	0.9200	0.1935	0.4590
	0.375	1.0000	1.0000	0.8680	0.8400
	0.500	1.0000	1.0000	0.9975	0.9870
	0.750	1.0000	1.0000	1.0000	1.0000
[2 2 5 8 8]	0.000	0.0480	0.0455	0.0000	0.0485
	0.125	0.2530	0.2475	0.0015	0.1145
	0.250	0.8130	0.8040	0.0915	0.3660
	0.375	0.9965	0.9945	0.6070	0.6905
	0.500	1.0000	1.0000	0.9695	0.9515
	0.750	1.0000	1.0000	1.0000	1.0000

Type I error rates and powers of tests when *m=3,4,5* and *a=5* are presented in Tables 11-13, respectively.

It can be said that Type I error rates which are obtained based on method 1. 2 and SRSboot are at their nominal level 0.05 and Type I error rates obtained by the RSSboot are considerably lower than nominal alpha 0.05. Similar to *a*=3 and *a*=4, since the total sample sizes are same, the case of [3 3 3 3 3] have higher power values than [2 2 3 4 4] for method 1. For example, for *d*=0.375, *m*=3, powers of test values which are obtained method 1 for the case of [3 3 3 3 3] higher than the case of [2 2 3 4 4]. Similar results are valid in the case of [5 5 5 5 5] with [2 2 5 8 8].

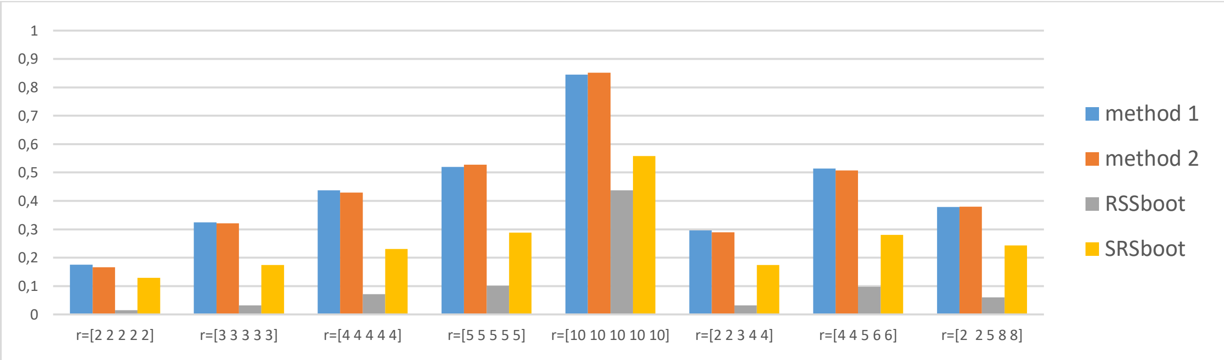

Figure 7. *Powers of tests for m=3, a=5 and d=0.25*

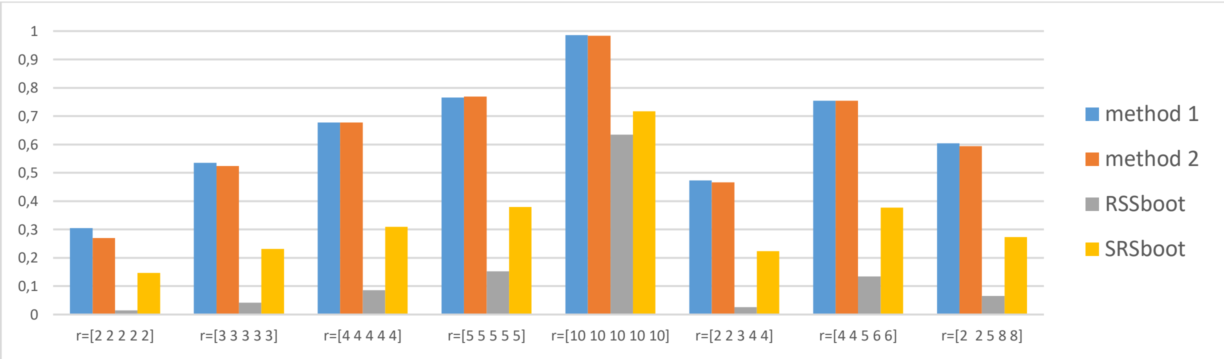

Figure 8. *Powers of tests for m=4, a=5 and d=0.25*

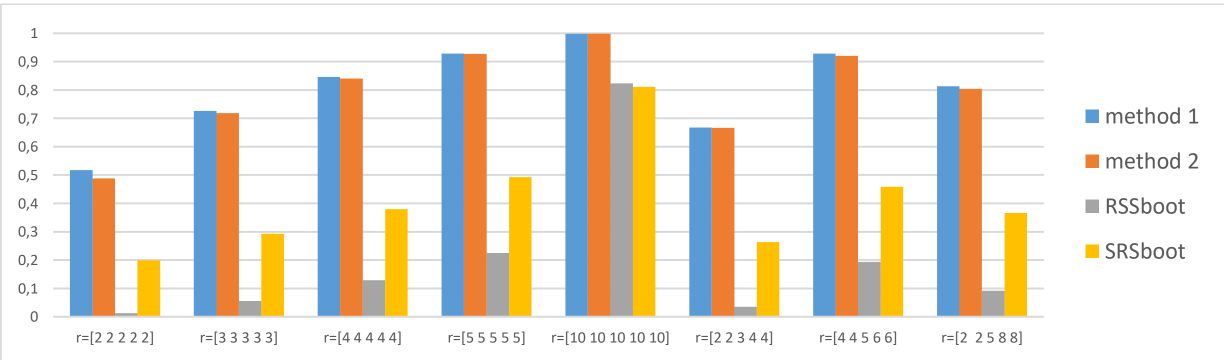

Figure 9. *Powers of tests for m=5, a=5 and d=0.25*

Figures 7-9 reported powers of tests values when *a*=8, *m*=3,4,5 and *d*=0.25. Power values in line with other power values.

5. CONCLUSION

In this study, the Bootstrap sample selection methods in RSS for ANOVA is examined. Bootstrap sample selection methods for confidence interval of population mean were proposed by Hui [18]. In this study, firstly we give these bootstrap sample selection methods in RSS for hypothesis testing. Then, we adapt these sample selection methods to test the hypothesis of equality of more than two population means. We give algorithms of hypothesis testing for proposed bootstrap methods. To compare with classical bootstrap methods in SRS and RSS, we also give the algorithms for these methods. The performance of the new bootstrap tests called method 1 and method 2 using RSS are analyzed with a simulation study. According to simulation study results, Type I error rates of method 1, 2 and classical bootstrap method in SRS are close to nominal level 0.05 in all of the considered cases. However, Type I error rates of classical bootstrap method based on RSS are notably lower than 0.05. Moreover, the powers of method 1 are greater than other methods in all considered cases.The power of this method are getting greater, especially when the sample sizes of groups are the same, according to the other situations. For this reason, Method 1 may be preferred in almost all cases.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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