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Fuzzy linear programming approach for the capacitated vehicle routing problem

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Abstract

In recent years, with both technological advances and the effect of globalization, businesses have entered a very intense competition. In this harsh competitive environment, customer satisfaction has become a critical concept. Firms implement costs reducing strategies in order to increase profitability levels and gain competitive advantage in the market. Researches show that a significant ratio of the cost of a product is derived from the transport activities. Therefore, it has become important for companies to find the optimal route during transportation. In this study, it is addressed as capacity constrained vehicle routing. First, a deterministic model is proposed for the addressed problem. Second, in accordance with real life conditions, a fuzzy linear programming model has been developed in case of the vehicle capacities are uncertain. The Verdegay approach has been adopted for the fuzzy linear programming model. The proposed model was implemented to a real life problem in the food sector. The results are compared with the results of a deterministic model and they show that the fuzzy linear programming model proposed in this study gives cost effective results in uncertain environment.

1. Introduction

In recent years, companies have entered a very competitive environment with increasing globalization. The companies in the markets where competition is experienced have adopted to increase their profits by decreasing the costs instead of increasing the product sales price in order to increase the operating profit without damaging the competitive advantage in the market. Today, it is known that the share of distribution costs in the unit product costs is approximately 15-20% of the total cost (Güvez et al., 2012). Therefore, a cost-effective distribution planning provides firms with both cost advantage and significant competitive (Öztürk, 2014).

Vehicle routing problem (VRP) represents the issue of identifying the smallest cost routes for a vehicle fleet to meet customer needs while aiming to find the optimum route starting from a warehouse to provide all customers with services or products in the shortest time and with minimum cost (Toth & Vigo, 2002). Logistical planning in real life includes high level of uncertainty and dynamic in nature (Öztürk & Kaya, 2020). It is not always possible to obtain all the required information for a decision making process (Öztürk, 2021). Therefore, usually these kinds of processes are performed without complete or precise information (Ayvaz et al., 2018; Öztürk et al., 2020). This is where the fuzzy sets become appropriate to apply since they deal with optimization or decision making under uncertainty (Brito et al., 2009). The cases for VRP where the demand is uncertain (Werners & Drawe, 2003) or the time windows are imprecise (Brito et al., 2008); (Tang et al., 2009) are examined in the literature with details. Unlike the previous studies, our aim is to deal with the uncertainty in capacity since it is not always possible to measure the loading capacity of a vehicle perfectly in terms of how many boxes or how much load it can carry.

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In this study, two models are proposed for multi-vehicle and capacity restricted vehicle routing problem. Initially, a deterministic model was developed with the assumption that the customer capacity of the vehicle is known. Moreover, capacity constrained fuzzy linear programming model is proposed assuming that vehicle capacities are uncertain in accordance with real life conditions. For the fuzzy capacity limited fuzzy linear programming model, different parameter values were analyzed and the model with the best results were chosen and compared with deterministic model results. The proposed models were adapted to the problem of product distribution to a bakery in Hatay province and the optimal route was obtained. The results show that the capacity limited fuzzy linear programming model proposed in this study gives effective results.

In the first part of the study, literature review was provided. In the second part, the methodology and the problem are discussed. In the third part, real life usage of the built model has been made. In the last section, the results were reported.

2. Literature Review

In this part of the study, some studies about fuzzy linear programming with the literature studies related to vehicle routing are included. A recent study where an optimization model with two objectives to minimize cost and time for municipal waste collection was conducted by Aliahmadi et al. (2021). The model aims to deal with the uncertainty of waste generation amounts by using fuzzy credibility theory. In more recent years, fuzzy green VRP has begun to catch more attention and the number of publications considering fuzzy green VRP follows and increasing trend (Yaşar Boz & Aras, 2021). Giallanza and Li Puma discussed a fuzzy green VRP for an agricultural supply chain with uncertain demands (2020). A VRP model for electric vehicles with fuzzy service times, travel times and battery energy consumption is developed by Zhang et al. (2020). Men et al. focused on VRP for hazardous material transportation (2019). They introduced the variables in the objective function in trapezoidal interval type-2 fuzzy form to deal with the uncertainty which causes transportation risks. Radojičić et al. proposed a fuzzy greedy randomized adaptive search procedure method coordinated with path relinking to solve a VRP with additional risk constraints (2018). Bahri et al. introduced an approach to deal with a multi-objective VRP by ranking generated solutions according to fuzzy pareto dominance and applying generic evolutionary algorithms with fuzzy extensions (2018). Nadizadeh and Kafash have discussed a routing and cost problem of a simultaneous pickup and delivery demands (2019). Arab et al. have worked on a product inventory-routing problem with multiple periods (2020). A two-level model which involves several retailers and a distributor is proposed with the aim of minimization of two objectives, namely, the system total cost (which includes start-up, maintenance, and distribution costs) and transportation costs based on risk factors. Eryavuz and Gencer discussed the problem of vehicle routing to make the total distance minimum for the personnel service vehicles of Balıkesir Ordnance School and Training Center (2001). Savings and stochastic saving algorithms intuitions were used to solve the proposed model. Başkaya and Öztürk focused on the vehicle routing problem for the distribution of bread to the five sales branches of a bread factory (2005). For the solution of the model, branch-cutting method was proposed and the shortest routes for the vehicles were determined. Szeto et al. has studied the artificial bee colony intuition to implement on the vehicle routing problem (2011). The performance of the suggested solution method was analyzed using experimental data. Yücenur and Demirel developed a hybrid metadata intuitive structure consisting of ant colony optimization and genetic algorithm for multi-port vehicle routing problem (2011). Their objective was to minimize the total distance traveled. Güvez et al. focuses on the vehicle routing problem of a collector firm in the waste sector, which enables to minimize the total cost of collecting medical waste from health institutions (2012). In the solution of the problem, an integer linear programming model was used. Kesen discussed the vehicle routing problem in the distribution decisions (2012). The study also proposed a new model that solves the problems of production and distribution simultaneously. Yu and Dong discussed the routing problem of a distribution vehicle under time and weight constraints to maximize the delivery profit (2013). A genetic algorithm approach incorporating CPLEX into the model has been developed for the solution of the model.

A fuzzy set framework approach is proposed to model imprecision and flexibility for uncertain information which may be faced in a VRP by El-Sherbeny (2011). Taş et al. discussed the ARP with flexible time window (2014). Özkök and Kurul addressed the capacity restricted vehicle routing problem of a distribution company operating in the food sector to determine the optimum route for the vehicles used in the product distribution to the customer group (2014). Şahin and Eroğlu have developed genetic algorithm-based methods that can solve order picking and vehicle routing problems hierarchically in a structure that is interconnected in both classical and crossover warehouse systems (2015). Customer and order groups were determined using genetic algorithms, vehicle routes, savings and the nearest neighbor intuition. Kovacs et al. proposed multi-purpose generalized restricted ARP model (2015). In addition to general ARP constraints, improvement of driver consistency, consistency at time of reaching and route cost minimization are discussed. Zhang et al. proposed an ARP model taking into account fuel consumption and carbon emissions (2015). For the solution of the model, a new method

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with tabular search intuitive basis has been developed. Atmaca et al. deals with the vehicle routing problem with time windows to determine the routes of a white goods authorized service vehicles distributing the products (2015). Lin et al. addressed the problem of electric vehicle routing to find minimum travel time and energy cost (2016). It is assumed that the vehicle charging points are at service points. Ultaş et al. used the method of saving algorithm in order to solve the vehicle routing problem of a bakery in Sivas (2017). Shao et al. handled electric vehicle routing problem with charge time and variable travel time (2017). Genetic algorithm heuristic is used in the solution of the developed model.

Examples of some literature studies using fuzzy linear programming method are as follows: Güngör and Ergülen introduced a vehicle routing linear programming model with fuzzy demand parameters and applied on the case of a food manufacturer (2006). Ergülen and Kazan have developed a fuzzy integer linear programming model for the route evaluation of freight transport systems in the transport sector (2007). In the study, the demand parameter is explained with fuzzy numbers. Bilgen proposed a fuzzy integer linear programming model for the production and distribution problem in the supply chain (2010). In the proposed model, the capacity parameter is considered to be fuzzy. The model is applied in the consumer goods sector. Fazlollahtabar et al. has developed a fuzzy mathematical model focusing on supply chain network design with multi-ware, multi-vehicle, multi-product, multi-customer under different time periods (2013). decision variables are assumed to be fuzzy, besides the demand and cost parameters. Mousavi et al. studied a fuzzy probabilistic-stochastic programming model for multiple cross-shipment and vehicle routing problems (2014). In the study, parameters such as distance, product availability cost, facility opening cost, processing cost, vehicle operating cost, transportation cost, vehicle capacity, product volume, collection and delivery times, maximum working time of collection and delivery of vehicles are stated as fuzzy numbers. Baykaşoğlu and Subulan developed a mathematical model for reverse logistic network design where the decision variables are fuzzy (2015). In the study, the amount of waste, product rate to be disposed, weight of waste, recycling rate, transportation costs and plant capacities are expressed in fuzzy numbers. In addition to these parameters, the decision variables expressing the amount of waste to be transported between plants were also taken as fuzzy. Dai and Zheng proposed a model for chance-constrained programming and fuzzy programming approaches for multi-stage, multi-cycle, multi-stage closed-loop supply chain network design under uncertainties (2015). In the proposed model, capacity, demand, product ratio parameters to be disposed are uncertain and expressed in fuzzy numbers. Mohammed and Wang proposed a fuzzy multipurpose linear programming model for green meat supply chain design (2017). The objectives covered are minimization of transport and processing costs, minimization of CO_2 emissions during transportation, minimization of delivery time and maximization of delivered product rate. In the study, fuzzy numbers were used for the cost of transportation and processing, demand and capacity parameters.

According to the results of the literature research, it is still clear that some parameters are uncertain in the vehicle routing problem in accordance with real life conditions. In this study, the capacity constrained multi-vehicle vehicle routing problem has been addressed by considering vehicle capacity uncertainties.

3. Methodology

3.1 Vehicle Routing Problem

VRP basically aims to transport products from one or more production centers with multiple vehicles to the relevant demand points / customers at minimum distance or minimum cost. In this type of problem, the distributor has a fleet of vehicles with certain capacities (Başkaya & Öztürk, 2005). In the problem, each customer has a special demand, the vehicle to meet customer orders should create many tours starting from the warehouse and ending in the warehouse (Karagül & Güngör, 2014). Starting from this definition, the assumptions of the problem are determined as a vehicle goes to each customer, each route begins from the warehouse and finishes in the warehouse, and meeting the known customer demand and total demand of the customer does not exceed the vehicle capacity (Alağaş et al., 2016).

Although many studies have been carried out in the field of this problem for many years, no exact formula has been found for the optimum route for VRP. Therefore, intuitive and meta-intuitive methods have been used in the solution of vehicle routing problems according to the constraints and characteristics of the problem type, and with their results, they have been used in this field (Yücenur & Demirel, 2011).

The assumptions of VRP are as follows (Karagül & Güngör, 2014):

• Each route begins and end at the warehouse and there is a single warehouse in each city.
• Every customer is visited by definitely one vehicle and once.
• The total demand of any route cannot be more than the vehicle capacity.

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• Vehicles used in the fleet are identical in terms of specifications and capacity.
• Total route cost is minimized.

The mathematical model of the vehicle routing problem is set up as follows: n denotes the total number of cities or warehouses, Q denotes the vehicle capacities, c_{ij} denotes the distance between cities or warehouses, K denotes the number of vehicles and the d_i denotes the demand amounts (Caccetta & Hill, 2001):

The integer linear programming model of VRP is shown below:

$$min \sum_{i=0}^{n} \sum_{i \neq j, j=0}^{n} c_{ij}.x_{ij} \quad (1)$$

$$\sum_{i=1, i \neq j}^{n} x_{ij} = 1 \qquad (\forall j) \quad (2)$$

$$\sum_{j=1, j \neq i}^{n} x_{ij} = 1 \qquad (\forall i) \quad (3)$$

$$\sum_{j=1}^{n} x_{0j} = K \quad (4)$$

$$\sum_{i=1}^{n} x_{i0} = K \quad (5)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} d_i.x_{ij} \leq Q \qquad (i \neq j) \quad (6)$$

$$\mathrm{u_i - u_j + n.x_{ij} \leq n - 1} \qquad (1 \leq \mathrm{i} \neq \mathrm{j} \leq \mathrm{n}) \quad (7)$$

$$(0 \leq x_{ij} \leq 1) \quad (8)$$

The objective function (1) minimizes the total travel distances of the vehicles. (2) and (3) indicate that one point can only be reached from one point and can go to only one point. (4) indicates that the number of vehicles starting from the starting point is m, (5) indicates that the n vehicles completing the round must return to the point where they started. (6) ensures that the request on node i does not exceed Q, the capacity of the vehicle. (7) is sub-tour elimination constraint. (8) gives the variable indicating when a vehicle should go to another point must be taken as {0,1}.

3.2 Linear Programming with Constraints and Verdegay Approach

The general mathematical structure of linear programming problems with fuzzy right-side constants are as follows:

$$maks\ Z = c^T.x \quad (9)$$

$$Ax \leq \tilde{b} \quad (10)$$

$$x \geq 0 \quad (11)$$

There are two approaches, namely Verdegay and Werners, to solve linear programming models with fuzzy right-side constants. Verdegay has proposed a model only in cases where the right side constants are fuzzy (Verdegay, 1984). Werners has developed a model that considers the objective function to be fuzzy because of the right-hand side constants (Werners, 1987).

3.3 Verdegay Approach

It is a non-symmetric approach developed by Verdegay for the solution of fuzzy linear programming models, with only right-side constants being fuzzy. In non-symmetrical models, the idea is derived from that there are differences between the objective functions and the constraints (Başkaya, 2011).

$$maks\ Z = c_j.x_j \quad (12)$$

$$\sum_{j=1}^{n} A_{ij}.x_j \leq \tilde{b}_i \quad (13)$$

$$x_j \geq 0 \quad (14)$$

The membership function of fuzzy constraints is expressed as follows.

$$\mu_i(x) = \begin{cases} 1, A_{ij}.x_j \leq b_i \\ 1 - \left[\dfrac{A_{ij}.x_j - b_i}{P_i}\right], b_i \leq A_{ij}.x_j \leq b_i + P_i \\ 0, A_{ij}.x_j \geq b_i + P_i \end{cases} \quad (15)$$

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In non-symmetrical models, fuzzy linear programming problems are solved by transforming them into parametric linear programming problems. The parametric linear programming model is as follows (Paksoy et al., 2013):

$$maks\ Z = c_j . x_j \tag{16}$$

$$(A_{ij} . x_j \leq b_i + P_i . (1 - \alpha) \qquad \alpha \in [0,1] \tag{17}$$

$$x_j \geq 0 \tag{18}$$

In the Verdegay approach, the possible tolerance limits (P_i) corresponding to the constant values of the fuzzy variables are requested from the decision maker. As $\theta \in [0,1]$, θ is written instead of (1-α) to obtain the parametric solution. In this solution, $\alpha = 1$is for $\theta = 0$. When the θ value goes from zero to one, the degree of satisfaction moves from 100% to zero. There is no deviation for $\theta = 0$ and the satisfaction rating is $1. \theta = 1$ shows the highest tolerance and takes the highest value in the maximization problem (Paksoy et al., 2013).

4. Problem Identification and Model Formulation

In this study, fuzzy vehicle routing problem is taken into consideration in order to meet the demand of each demand point where vehicle capacities are taken into consideration. Two models have been proposed in the study. The first model is a generic deterministic model with vehicle capacities, nearly identical as the one described in the previous section, with only a few constraints are changed. The difference is that our deterministic model forces the solution to be where a path between two nodes be traveled by only one vehicle and it is formed to provide benchmark solutions for our second model. The second model is a fuzzy integer linear programming model where vehicle capacities are uncertain. Here, the capacity constrained fuzzy vehicle routing model adapted for the problem addressed is as follows. *n* denotes the total customer point, *Q* denotes the vehicle capacities, c_{ij} denotes the distance between customers, *k* denotes the number of vehicles and d_i denotes the amount of demand at customer points.

4.1 Model 1: Deterministic Model

$$min \sum_{i=0}^{n} \sum_{i \neq j, j=0}^{n} c_{ij} . x_{ijk} \tag{19}$$

$$\sum_{i=0, i \neq n}^{n} x_{ink} = \sum_{j=1, j \neq n}^{n} x_{njk} \qquad \forall\ \text{k}, \forall\ (1 \leq i \leq n) \tag{20}$$

$$\sum_{k=1}^{K} \sum_{j=1}^{n} x_{ijk} = 1 \qquad \forall\ (2 \leq i \leq n) \tag{21}$$

$$\sum_{j=1}^{n} x_{0jk} = 1 \qquad \forall\ k \tag{22}$$

$$\sum_{i=1}^{n} x_{ijk} = 1 \qquad \forall\ k \tag{23}$$

$$\sum_{i=0}^{n} \sum_{j=0, j \neq i}^{n} d_i . x_{ijk} \leq Q_k \qquad \forall\ k \tag{24}$$

$$u_i - u_j + n . x_{ij} \leq n - 1 \qquad (1 \leq i \neq j \leq n) \tag{25}$$

$$(0 \leq x_{ij} \leq 1) \tag{26}$$

The objective function (19) minimizes the total travel distances of the vehicles. (20) and (21) indicate that one point can only be reached from one point and can go to only one point. (22) means that the number of vehicles starting from the start point is *m*, (23) indicates that the *m* vehicles which has completed the round must return to the point where they started. (24) ensures that the request on node *i* does not exceed the capacity of the vehicle. The vehicle capacity, which is the right-side constant of this constraint, is taken as indefinite and it is expressed as fuzzy number. (25) is sub-tour elimination constraint. (26) enables the decision variable indicating when a vehicle should go to another point to take the {0,1} value.

4.2 Model 2: Fuzzy integer linear programming model

In the fuzzy integer linear programming model where the vehicle capacities are unclear, only the constraint (24) is defined as in (27).

$$\sum_{i=0}^{n} \sum_{j=0, j \neq i}^{n} d_i . x_{ijk} \leq \widetilde{Q_k} \qquad \forall\ k \tag{27}$$

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In this study, the tolerance limit (P_k) for the fuzzy capacity constraint was accepted as 10% for the right-side constant. This tolerance limit is the upper limit for the vehicle capacities and it is based on the experience of the bakery owner or the decision maker. According to the decision maker, the vehicles usually exceed their default capacities around 10% and for the sake of cost-efficiency, it is assumed as the strict upper limit for the vehicle capacities. Since the cost associated with the travelled distance is the objective function of our problem, the hidden costs of manipulation on the right-hand side of the constraints are disregarded.

The linear programming model, which is fuzzy with the help of the membership function proposed by Verdegay, has been transformed into a parametric linear programming model. The constraint (27) has been turned into a new form (28).

$$\sum_{i=0}^{n} \sum_{j=0, j\neq i}^{n} d_i . x_{ijk} \leq Q_k + P_k . (1-\alpha) \qquad \forall\, k \tag{28}$$

The parametrical linear programming model was analyzed by using GAMS 23.5 package program for different α = (0.1,0.2,0.3, α, 1) values and optimum values were obtained.

5. Application and Results

In this study, the proposed capacity limited fuzzy vehicle routing model was applied to the problem of distance minimization of bread deliveries of round-trips from İskenderun district of Hatay province to 57 distribution points with four vehicles. The distribution points are given in Table 1 with their geographic locations. Additionally in Figure 1, positions of the bakery and distribution points on a map is shown.

The assumptions of the model are as follows:

- The vehicles exit the center and return to the center.
- A vehicle goes to each distribution point.
- The amount of cargo to be distributed on the route cannot exceed the vehicle capacity.
- All distribution points must be covered.
- Demands are known in advance.

Table 1. The coordinates of the bakery and distribution points

Location	X coordinate	Y coordinate	Location	X coordinate	Y coordinate
BAKERY	36.578361	36.167617	**29**	36.587286	36.170316
1	36.578231	36.157889	**30**	36.589667	36.154611
2	36.577267	36.167295	**31**	36.591250	36.158806
3	36.576658	36.179000	**32**	36.525835	36.179017
4	36.582536	36.163167	**33**	36.539139	36.165049
5	36.570778	36.161455	**34**	36.541795	36.162979
6	36.579538	36.178883	**35**	36.541502	36.157985
7	36.592739	36.163034	**36**	36.546023	36.163761
8	36.563278	36.135017	**37**	36.555382	36.155230
9	36.576000	36.169517	**38**	36.551665	36.154549
10	36.576028	36.169239	**39**	36.558649	36.158566
11	36.576056	36.169045	**40**	36.566438	36.149912
12	36.564750	36.158639	**41**	36.562423	36.149694
13	36.578656	36.161278	**42**	36.582912	36.164151
14	36.585500	36.160250	**43**	36.573987	36.164145
15	36.580917	36.165944	**44**	36.571188	36.174729
16	36.576761	36.168284	**45**	36.573957	36.170056
17	36.577833	36.170034	**46**	36.559186	36.142639
18	36.571351	36.161722	**47**	36.544759	36.152173
19	36.574703	36.162767	**48**	36.569988	36.157611
20	36.574669	36.162692	**49**	36.547500	36.155528
21	36.571944	36.159500	**50**	36.541715	36.086867
22	36.571953	36.159511	**51**	36.580206	36.159545
23	36.581656	36.194573	**52**	36.558380	36.158528
24	36.592436	36.167066	**53**	36.541333	36.159111
25	36.589000	36.156545	**54**	36.579222	36.189455

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26	36.587417	36.163705	**55**	36.582739	36.191333
27	36.590139	36.164778	**56**	36.586472	36.171639
28	36.588017	36.172778	**57**	36.587306	36.168611

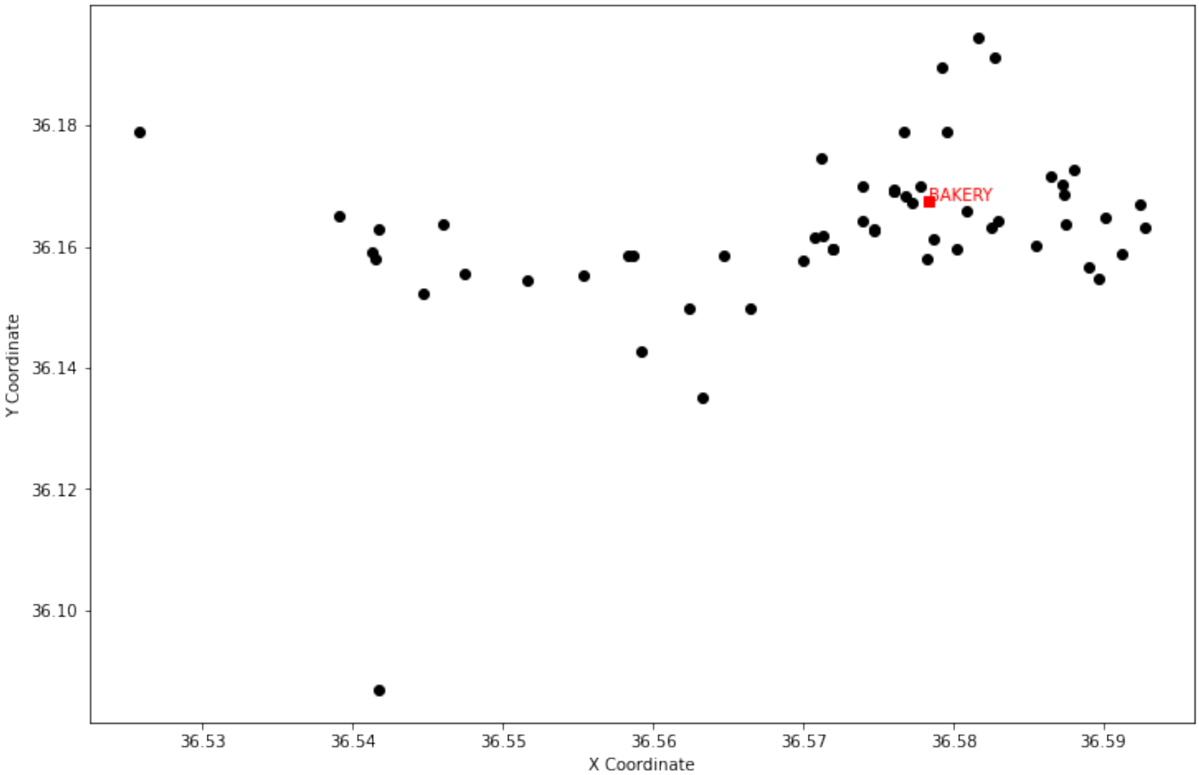

Figure 1. Map of the bakery and distribution points.

Demand amounts of distribution points / customers are as in Table 2. Vehicle capacities are 900, 900, 2500 and 2500 respectively.

Table 2. Demand quantities for demand points (Qty)

Location	Demand (qty)	Location	Demand (qty)	Location	Demand (qty)
1	30	**20**	45	**39**	75
2	30	**21**	60	**40**	75
3	45	**22**	30	**41**	75
4	45	**23**	75	**42**	75
5	45	**24**	60	**43**	45
6	60	**25**	75	**44**	60
7	60	**26**	75	**45**	60
8	60	**27**	60	**46**	30
9	30	**28**	90	**47**	60
10	45	**29**	90	**48**	120
11	45	**30**	75	**49**	75
12	45	**31**	75	**50**	210
13	45	**32**	60	**51**	75
14	30	**33**	75	**52**	60
15	45	**34**	75	**53**	60
16	45	**35**	75	**54**	600
17	45	**36**	75	**55**	450
18	45	**37**	90	**56**	750
19	60	**38**	75	**57**	900

GAMS 23.5 package program was used to solve the developed model. The results obtained are as follows:

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5.1 Results for model 1

Minimal value of total travel distances of vehicles with objective function by solving deterministic model was found as 49,972 km. Vehicle routes obtained for optimal results are given in Table 3. The optimal routes are displayed on map in Figure 2.

Table 3. Deterministic model results

Vehicle	Route
Vehicle #1	Bakery-10-9-17-4-42- Bakery
Vehicle #2	Bakery -18-21-22-48-41-40-8-50-46-37-52- Bakery
Vehicle #3	Bakery -11-3-54-6-28-56-15-12-39-38-49-47-35-53-34-33-32-36-5-43- Bakery
Vehicle #41	Bakery -2-19-20-1-51-13-14-25-30-31-7-26-27-24-57-29-55-23-44-45-16- Bakery

5.2 Results for model 2

When the fuzzy linear programming model is solved for the different α values by the Verdegay Approach, the results in Table 4 are obtained.

Table 4. Fuzzy VRP model results

Alpha	Objective function
0,0	48,473
0,1	50,509
0,2	51,385
0,3	54,870
0,4	57,090
0,5	50,940
0,6	52,127
0,7	55,610
0,8	47,062
0,9	53,509
1,0	49,972

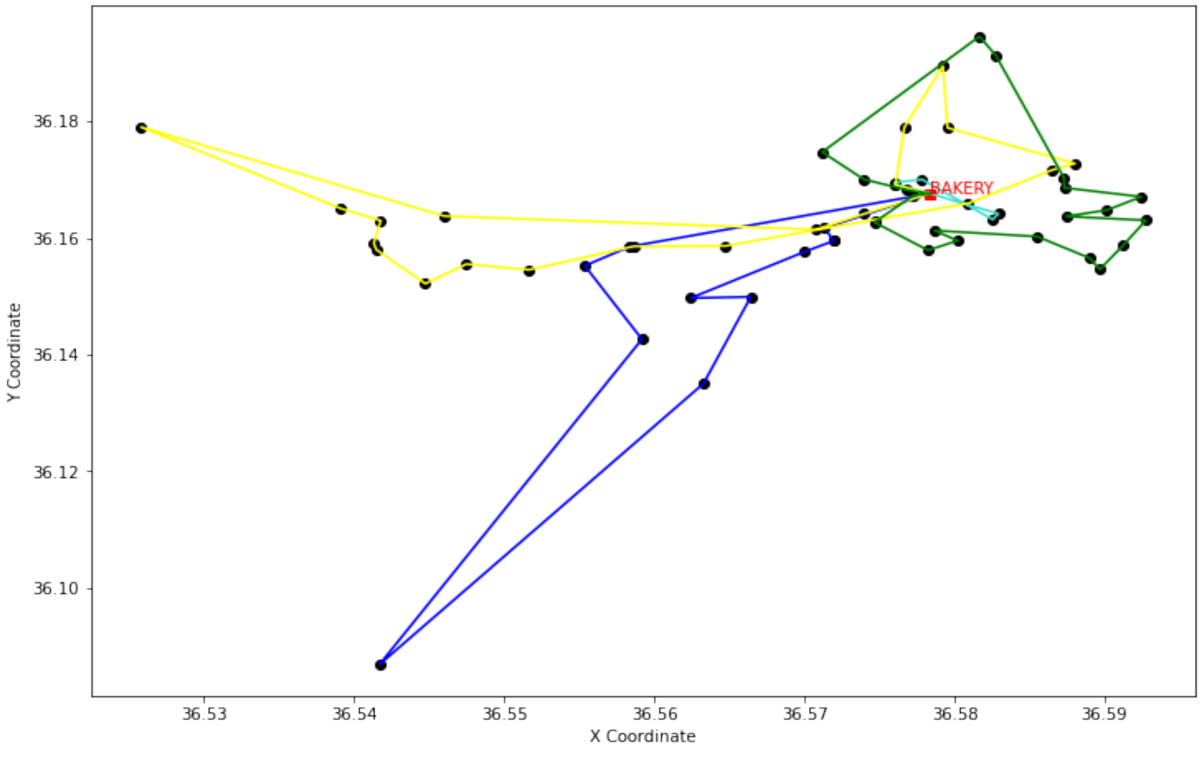

Figure 2. Optimal routes for the vehicles in deterministic model.

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Accordingly, the minimum distance is 47,062 km for α = 0,8 (Table 4; Figure 3) and the routes obtained for this solution are as follows. The vehicle routes obtained for the α = 0.8 value which gives the best solution are given in Table 5.

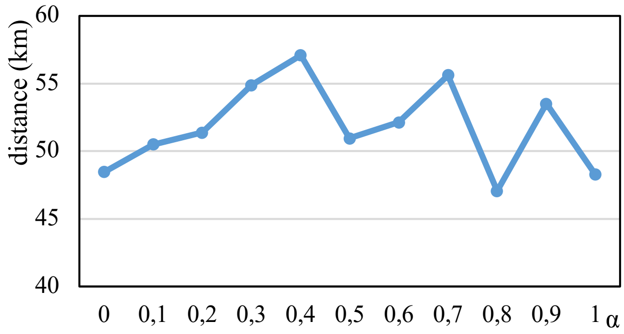

Figure 3. Change in objective function according to alpha value.

Table 5. Vehicle routes that provide the best solution

Vehicle	Route
Vehicle #1	Bakery -45-44-32-33-34-36-53-35-47-49-38-37-5- Bakery
Vehicle #2	Bakery -2-19-20-43-13-51-4-42- Bakery
Vehicle #3	Bakery -17-3-6-55-23-54-28-56-29- Bakery
Vehicle #4	Bakery -15-26-57-24-27-7-31-30-25-14-1-21-22-12-39-52-41-46-50-8-40-48-18-11-9-10-16- Bakery

The improvements in vehicle routes by applying fuzzy linear programing model are apparent and are shown in Figure 4.

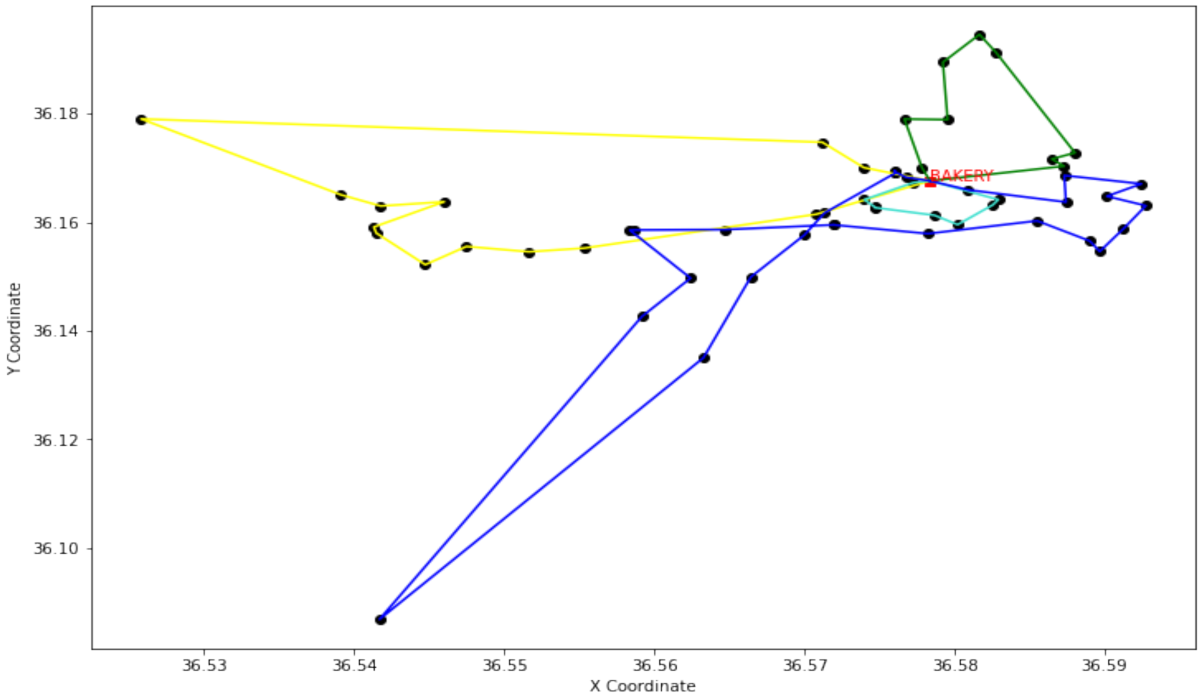

Figure 4. Optimal routes for the vehicles in fuzzy linear programming model.

In Table 6, the solution obtained for α = 0.8, which gives the best solution value, was compared with the deterministic model solution.

Accordingly, the fuzzy proposed VRP model is better than the deterministic model. When the run-times are examined, compare to the deterministic model run-time is longer by 34 seconds due to fuzzy vehicle capacities and Verdegay method. In deterministic method, solution has been reached with 2,208,776 iterations, while in fuzzy VRP model it has been reached with 1,955,204 iterations. Although the fuzzy VRP model gives a better objective function, it can be concluded that the deterministic model is faster in achieving an optimal solution.

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Table 6. The effectiveness of calculations

	Deterministic model	Fuzzy VRP model
Objective function (km)	49.972	47,062
Elapsed time (sec)	943,26	977,88
The iteration	2208776	1955204
Variables	13,286	13,286
Non-zero elements	84,821	84,821
Discrete variables	13,228	13,228

6. Conclusions

According to the researches, in today's business world, where distribution costs constitute 15-20% of the unit product costs, determining an efficient distribution route in terms of cost and distribution time gives enterprises both cost advantage and significant competitive advantage. Problem of identifying the smallest cost routes for a vehicle fleet to meet customer needs is known in the literature as vehicle routing problem. In this study, capacity constrained vehicle routing problem were handled by expressing customer demands as fuzzy numbers. The proposed model has been applied to a real problem in order to find optimal routes in the distribution of products to the customers of a bakery operating in Hatay province. The results showed the effectiveness of the fuzzy linear programming method. The proposed fuzzy mathematical model is more effective than the deterministic model in real life conditions where some critical parameters are indeterminate.

In subsequent studies, the distances between the customer and the source point using the geographic information system can be expressed as real distance based on the geographic information system instead of the Euclidean relation. Stochastic programming model can be developed by showing with uncertainty probability distributions in customer demands. Likewise, it is not only the minimization of the path taken, but also the multipurpose models which can minimize the carbon dioxide emissions as well.

Contribution of Researchers

Fatih Öztürk carried out model calibrations and data analysis. Seçkin Ünver reviewed the literature and contributed to computational efforts.

Conflicts of Interest

The authors declared that there is no conflict of interest.

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