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ON THE PHASE AND GROUP VELOCITY OF THERMOELASTIC RAYLEIGH WAVES IN TRANSVERSELY ISOTROPIC MATERIALS

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Abstract

Rayleigh wave speed in a heat conducting transversely isotropic material with thermal relaxation is studied. Phase and group velocity for the first four modes have been computed for aluminum alloy plate at different thermal relaxation times. It is observed that if modal phase velocity decrease with increasing frequency normally dispersive profiles, phase velocity is greater than group velocity and consequently carrier travels faster than the envelope. Thus in such cases if a phase disturbance appears at the beginning of the pulse, then it overtakes and finally it disappears in the front. Rayleigh wave speed is computed the medium and compared. It is observed that thermal relaxation time effect plays a significant role thermoelastic speed of Rayleigh waves at the low values of wave number limits.

Keywords: Rayleigh waves, dispersion, phase velocity, group velocity, transversely isotropic material

1. Introduction

Waves propagate in a medium are largely body waves and surface waves. Surface waves are generated only in presence of a free boundary and they can be essentially Love waves and Rayleigh waves. In [17] Rayleigh waves are elastic surface waves which propagate along the traction-free surface with the phase velocity in the subsonic range, with their amplitude decays exponentially with depth below that surface. Such waves serve as useful tool in nondestructive characterization of materials and have particular importance in seismology, acoustic, geophysics, material sciences, telecommunication industry and the other fields of physics. In view of the fact that velocity of the Rayleigh wave is the fundamental quantity which used in wide range of applications, thus it is significant to study their analytical behavior and derive accurate expressions in the simple forms.

Rayleigh waves firstly introduced as solution of the free vibration problem for an elastic half-space in [17]. It is anticipated the significance that such kind of wave could have in earthquake tremor transmission in this study in thermal environment. In fact the introduction of surface waves was preceded by some seismic interpretation that couldn’t be explained using only body wave theory, which was well known at that time. First of all the nature of the major tremor was not clear, because the first arrivals were a couple of minor tremors corresponding to P and S waves respectively. The greater amount of energy associated to this late tremor if compared to that of body wave was a strong evidence of less attenuation passing through the same medium and this could be explained only assuming that this kind of wave was essentially confined to the surface.

For isotropic elasticity, such waves are well known [2-7] and for anisotropic elasticity surface waves studied by many researchers [8-12]. In a transversely isotropic material elastic waves were considered by [9] and the same problem was investigated in the rotating materials by [12]. Waves that propagated along the plane surface of elastic solid, therefore, surface waves firstly introduced by Rayleigh are known by his name. Later number of researches studied the Rayleigh wave speed by using different techniques in different kind of materials. Recently [13] discussed the Rayleigh waves speed in orthotropic elastic solids, in this article rotational effects on Rayleigh waves speed on a transversely isotropic medium are studied.

Theory of coupled thermoelasticity [14, 15] is of the diffusion type heat equation that predicts an infinite speed of propagation of the heat wave which is physically unacceptable. To get liberate of this paradox, theories of generalized thermoelasticity were developed. Among the various such theories the approach proposed by [16], is based on a modified Fourier's law and [17] allows second sound without violating the classical Fourier's law are most accepted theories. These theories are structurally unlike, and one cannot be obtained as a particular case of the other have been developed by introducing one or two relaxation times in the basic equation of motions and heat equations with intend to remove the paradox of an infinite speed for the propagation of thermal signals. The two theories both ensure finite speeds of propagation for thermal wave. Lord and Shulman theory of generalized thermoelasticity extended for anisotropic media by [18]. Various problems characterizing these two theories have been investigated by [20, 21] and [22-24] and have revealed some interesting phenomena. A detailed review is reported in [19].

In this paper Rayleigh wave speed in a heat conducting transversely isotropic material with thermal relaxation is studied. Phase and group velocity for the first four modes have been computed for aluminum alloy plate at different thermal relaxation times. It is observed that if modal phase velocity decrease with increasing frequency normally dispersive profiles, phase velocity is greater than group velocity and consequently carrier travels faster than the envelope. Thus in such cases if a phase disturbance appears at the beginning of the pulse, then it overtakes and finally it disappears in the front. Rayleigh wave speed is computed the medium and compared. It is observed that thermal relaxation time effect plays a significant role thermoelastic speed of Rayleigh waves at the low values of wave number limits.

2. Boundary Value Problem and Secular Equation

Consider a semi-infinite stress-free surface of a homogeneous heat-conducting elastic material which is transversely isotropic in both elastic and thermal response and we choose a system of rectangular Cartesian co-ordinate system.

$$\sum_{j=1}^{3}\left(\frac{\partial \tau_{ij}(\mathbf{u})}{\partial x_j}\right)+f_i=\rho\frac{\partial^2 u_i}{\partial t^2}\qquad i=1,2,3 \tag{1}$$

$$K_{ij}T_{,ij}-\rho C_e\left(\dot{T}+\tau_0\ddot{T}\right)=T_0\beta_{ij}\left(\dot{u}_{i,j}+\tau_0\ddot{u}_{i,j}\right) \tag{2}$$

where

$$\tau_{ij}=C_{ijkl}e_{kl}-\beta_{ij}T,\quad \beta_{ij}=C_{ijkl}\alpha_{kl} \tag{3}$$

the summation convention is implied; ρ is the density, t is the time, u_i is the displacement in he x_i direction, K_{ij} are the thermal conductivities, C_e and τ_0 are respectively the specific heat at constant strain, and thermal relaxation time, σ_{ij} and e_{ij} are the stress and strain tensor respectively; β_{ij} are thermal moduli; α_{ij} is the thermal expansion tensor; T is temperature; and the fourth order tensor of the elasticity C_{ijkl} satisfies the (Green) symmetry conditions:

$$c_{ijkl} = c_{klij} = c_{ijlk} = c_{jikl}\ \text{, and}\ \alpha_{ij} = \alpha_{ji},\ \ \beta_{ij} = \beta_{ji},\ K_{ij} = K_{ji}\ . \tag{4}$$

For a transversely isotropic medium the matrix may be simplified to a 6 x 6 array of constants using symmetry arguments and the standard index substitution. For a transversely isotropic material in the symmetric coordinate system can be expressed by

$$\begin{bmatrix}\tau_{11}\\ \tau_{22}\\ \tau_{33}\\ \tau_{23}\\ \tau_{13}\\ \tau_{12}\end{bmatrix}=\begin{bmatrix}c_{11} & c_{12} & c_{13} & 0 & 0 & 0\\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0\\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & c_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & c_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & \dfrac{c_{11}-c_{12}}{2}\end{bmatrix}\begin{bmatrix}e_{11}\\ e_{22}\\ e_{33}\\ 2e_{23}\\ 2e_{13}\\ 2e_{12}\end{bmatrix}-T\begin{bmatrix}\beta_1\\ \beta_1\\ \beta_3\\ 0\\ 0\\ 0\end{bmatrix} \tag{5}$$

$$\left[K_1\left(T_{,11}+T_{,22}\right)+K_3T_{,33}\right]-\rho C_e\left(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2}\right)$$
$$=T_0\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\left[\beta_1\left(\tilde{u}_{1,1}+\tilde{u}_{2,2}\right)+\beta_3\tilde{u}_{3,3}\right] \tag{6}$$

where the z-axis is assumed to be the symmetry axis (fiber axis) as shown in Figure 1. In this case we may write the equations of motion in operator notation as

$$\tau_{ij,j} = \rho\ddot{u}_i\ , \tag{7}$$

$$\tau_{ij,j} = C_{ijkl}u_{k,jl} - \beta_{ij}T_{,j}\ , \tag{8}$$

$$\left[c_{11}\frac{\partial^2}{\partial x_1^2}+c_{44}\frac{\partial^2}{\partial x_3^2}-\rho\frac{\partial^2}{\partial t^2}+\right]u_1+\left[(c_{13}+c_{44})\frac{\partial^2}{\partial x_1\partial x_3}\right]u_3+\beta_1\frac{\partial}{\partial x_1}T=0, \tag{9}$$

$$\left[(c_{13}+c_{44})\frac{\partial^2}{\partial x_1\partial x_3}\right]u_1+\left[c_{44}\frac{\partial^2}{\partial x_1^2}+c_{33}\frac{\partial^2}{\partial x_3^2}-\rho\frac{\partial^2}{\partial t^2}\right]u_3-\beta_1\frac{\partial}{\partial x_3}T=0, \tag{10}$$

$$K_1T_{,11}+K_3T_{,33}-\rho C_e(\dot{T}+\tau_0\ddot{T})=T_0\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\left[\beta_1u_{1,1}+\beta_3u_{3,3}\right]. \tag{11}$$

The boundary conditions of zero traction are, which satisfies the boundary conditions for the problem, which are given by

$$\tau_{31}\big|_{x_3=0} = \tau_{33}\big|_{x_3=0} = \frac{\partial T}{\partial x_3}\bigg|_{x_3=0} = 0, \tag{12a}$$

where

$$\begin{aligned}\tau_{33} &= c_{11}e_{11}+c_{13}e_{22}+c_{33}e_{33}-\beta_3 T\\ \tau_{13} &= 2c_{44}e_{13}\end{aligned} \tag{12b}$$

Using the definition of strains, we have

$$\tau_{33} = c_{11} jk\phi_1 + kc_{33}\phi_3' - \beta_3\chi$$
$$\tau_{13} = c_{44}\left(k\phi_1' + ik\phi_3\right)$$
$$T' = \frac{\partial T}{\partial x_3} = k\chi'$$

This implies

$$k\ c_{11}\ \sqrt{-1}\ \phi_1 + c_{33}k\phi_3' - \beta_3\chi = 0$$
$$\phi_1' + i\phi_3 = 0$$
$$\chi' = 0$$
$$\tag{13}$$

Usual requirements that the displacements and the stress components decay away from the boundary implies $u_i \to 0, \quad \sigma_{ij} \to 0 \quad (i,\ j = 1,3) \quad \text{as} \quad x_3 \to -\infty.$

(14)

Following the approach for isotropic media, for wave propagation, considering the harmonic waves propagating in x_1 -direction,

$$(u_j, T) = (\phi_j, \chi)(z)e^{i(kx_1 - ct)}, i = \sqrt{-1} \tag{15}$$

where k is the wave number, c is the wave speed and $z = kx_3; \phi_j,\ j = 1,3$ are the functions to be determined. Substituting (15) into (9) to (10) implies

$$c_{44}k^2\phi_1'' + ik^2(c_{13} + c_{44})\phi_3' + k^2\left(\rho c^2 - c_{11}\right)\phi_1 + ik\beta_1\chi = 0 \tag{16}$$

$$c_{33}k^2\phi_3'' + ik^2(c_{13} + c_{44})\phi_1' + k^2\left(\rho c^2 - c_{11}\right)\phi_3 + k\beta_3\chi' = 0 \tag{17}$$

$$K_3k^2\chi'' + k^3c^3\tau^*\beta_3T_0\phi_3' + ik^3c^3\tau^*\beta_1T_0\phi_1' + k^2\left(\rho c^2\tau^*C_e - K_1\right)\chi = 0 \tag{18}$$

On applying Laplace transform on equations Eqs. (16)-(18) and by using (13), it is obtained

$$\begin{aligned}\left[c_{44}s^2 + \rho c^2 - c_{11}\right]k^2\overline{\phi}_1(s) + ik^2(c_{13} + c_{44})s\overline{\phi}_3(s) + ik\beta_1\overline{\chi}(s) \\ = c_{44}k^2\left\{s\phi_1(0) + \phi'_1(0)\right\} + ik^2(c_{13} + c_{44})\phi_3(0)\end{aligned} \tag{19a}$$

$$\begin{aligned}ik^2(c_{13} + c_{44})s\overline{\phi_1}(s) + k^2\left(c_{33}s^2 - c_{44} + \rho c^2\right)\overline{\phi_3}(s) + k\beta_3 s\overline{\chi}(s) \\ = i(c_{13} + c_{44})\phi_1(0) + c_{33}\left\{\phi_3(0)s + \phi_3'(0)\right\} + k\beta_3\chi(0)\end{aligned} \tag{19b}$$

$$\begin{aligned}ik^3c^3\tau^*\beta_1T_0s\overline{\phi_1}(s) + k^3c^3\tau^*\beta_3T_0s\overline{\phi_3}(s) + \left\{k^2\left(K_3s^2 + \left(\rho c^2\tau^*C_e - K_1\right)\right)\right\}\overline{\chi}(s) \\ = ik^3c^3\tau^*\beta_1T_0\phi_1(0) + k^3c^3\tau^*\beta_3T_0\phi_3(0) + K_3\left\{\chi(0)s + \chi'(0)\right\}\end{aligned} \tag{19c}$$

$$M_{11}\overline{\phi}_1(s) + M_{12}\overline{\phi}_3(s) + M_{13}\overline{\chi}(s) = M_{14} \tag{20a}$$

$$M_{21}\overline{\phi_1}(s) + M_{22}\overline{\phi_3}(s) + M_{23}\overline{\chi}(s) = M_{24} \tag{20b}$$

$$M_{31}\overline{\phi_1}(s) + M_{32}\overline{\phi_3}(s) + M_{33}\overline{\chi}(s) = M_{34} \tag{20c}$$

where

$$
\begin{aligned}
&M_{11}=k^2\left(c_{44}s^2+\rho c^2-c_{11}\right),\ \ M_{12}=ik^2(c_{13}+c_{44})s,\ \ M_{13}=ik\beta_1\\
&M_{14}=c_{44}k^2\left\{s\phi_1(0)+\phi'_1(0)\right\}+ik^2(c_{13}+c_{44})\phi_3(0)\\
&M_{21}=i(c_{13}+c_{44})k^2s,\ M_{22}=\left(c_{33}s^2-c_{44}+\rho c^2\right)k^2,\ M_{23}=k\beta_3 s\\
&M_{24}=i(c_{13}+c_{44})\phi_1(0)+c_{33}\left\{\phi_3(0)s+\phi'_3(0)\right\}+k\beta_3\chi(0)\\
&M_{31}=ik^3c^3\tau^*\beta_1T_0s,\ M_{32}=k^3c^3\tau^*\beta_3T_0s,\ \ M_{33}=\left\{k^2\left(K_3s^2+\left(\rho c^2\tau^*C_e-K_1\right)\right)\right\}\\
&M_{34}=ik^3c^3\tau^*\beta_1T_0\phi_1(0)+k^3c^3\tau^*\beta_3T_0\phi_3(0)+K_3\left\{\chi(0)s+\chi'(0)\right\}
\end{aligned}
\tag{20d}
$$

On solving above equations, we have

$$\bar{\phi}_1(s)=\frac{\Delta_1}{\Delta},\bar{\phi}_3(s)=\frac{\Delta_2}{\Delta},\bar{\chi}(s)=\frac{\Delta_3}{\Delta}, \tag{20e}$$

where

$$
\Delta_1=\begin{bmatrix} M_{14} & M_{12} & M_{13}\\ M_{24} & M_{22} & M_{23}\\ M_{34} & M_{32} & M_{33}\end{bmatrix},\ \Delta_2=\begin{bmatrix} M_{11} & M_{14} & M_{13}\\ M_{21} & M_{24} & M_{23}\\ M_{31} & M_{34} & M_{33}\end{bmatrix},
$$

$$
\Delta_3=\begin{bmatrix} M_{11} & M_{12} & M_{14}\\ M_{21} & M_{22} & M_{24}\\ M_{31} & M_{32} & M_{34}\end{bmatrix},\ \Delta=\begin{bmatrix} M_{11} & M_{12} & M_{13}\\ M_{21} & M_{22} & M_{23}\\ M_{31} & M_{32} & M_{33}\end{bmatrix}
$$

Let s_1^2, s_2^2 and s_3^2 are the roots (having real parts positive) of determinant Δ

$$A_0s^6+A_1s^4+A_2s^2+A_3=0, \tag{21}$$

where

$$
\begin{aligned}
A_0&=c_{11}c_{33}k_{33}k^2\\
A_1&=\left(c_{44}c_{33}\rho c^2C_e-c_{44}T_0c^3\beta_3^2\right)\tau_1k^2-c_{44}c_{33}k_{11}k^2\\
&\quad+\left[\left(c_{44}+c_{33}\right)\rho c^2-c_{11}c_{33}+c_{13}^2+2c_{13}c_{44}\right]k^2k_{33}\\
A_2&=\Big[\left(c_{11}-\rho c^2\right)\tau_1T_0\beta_3^2c^3-2\left(c_{13}-c_{44}\right)\tau_1T_0\beta_1\beta_3c^3+\left(c_{11}c_{33}-2c_{13}c_{44}-c_{13}^2\right)k_{11}\\
&\quad+\left[\left(c_{33}+c_{44}\right)\rho^2c^4-\left(c_{11}c_{33}-2c_{13}c_{44}-c_{13}^2\right)\rho c^2\right]\tau_1C_e\\
&\quad-\left[\left(c_{33}+c_{44}\right)k_{11}+\left(c_{11}+c_{44}\right)k_{33}\right]\rho c^2+c_{11}c_{44}k_{33}+k_{33}\rho^2c^4+c_{33}T_0\beta_1^2c^3\Big]k^2\\
A_3&=\left(\rho c^2-c_{44}\right)k^2\left[\left\{\left(c_{11}-\rho c^2\right)k_{11}+\left(\rho^2c^4C_e+T_0c^3\beta_1^2-\rho c^2c_{11}C_e\right)\tau_1\right\}\right]
\end{aligned}
$$

This implies

$$\bar{\phi}_1(s)=\frac{A_1}{s-s_1}+\frac{A_2}{s-s_2}+\frac{A_3}{s-s_3}+\frac{A_4}{s+s_1}+\frac{A_5}{s+s_2}+\frac{A_6}{s+s_3} \tag{22}$$

where A_j, $j=1,2,3...6$ are the constants to be determined. Taking inverse Laplace transform and applying (14), implies

$$\bar{\phi}_1(y)=\sum_{j=1}^{3}A_j\exp(s_jy) \tag{23}$$

In view of (16)-(18), (14) and (23), we have

$$\bar{\phi}_3(y) = \sum_{j=1}^{3} A_j p_j \exp(s_j y) \tag{24}$$

$$\bar{\chi}(y) = \sum_{j=1}^{3} A_j q_j \exp(s_j y) \tag{25}$$

Where

$$p_j = \frac{\left[\left(c_{44}s_j^2 + \rho c^2 - c_{11}\right)\right] i s_j + \left[i\left(c_{13}+c_{44}\right)s_j + \rho c^2 - c_{44}\right]\dfrac{\beta_1}{\beta_3}}{\left[\left(c_{13}+c_{44}\right) - c_{33}\dfrac{\beta_1}{\beta_3}\right] s_j^2} \tag{26}$$

$$q_j = \frac{\left[c_{33}c_{44}s_j^3 + \left[(c_{13}+c_{44})^2 - c_{33}\left(c_{11}-\rho c^2\right)\right]s_j + i\left(c_{13}c_{44} + c_{44}{}^2 - \left(c_{13}+c_{44}\right)\rho c^2\right)\right]k}{\left[\left(c_{13}+c_{44}\right) - c_{33}\dfrac{\beta_1}{\beta_3}\right] s_j} \tag{27}$$

Putting the values of ϕ_1, ϕ_3 and χ in (13), we obtain

$$\sum_{j=1}^{3}\left(ic_{11} + c_{33}p_j s_j - \frac{\beta_3}{k}q_j\right)A_j = 0$$

$$\sum_{j=1}^{3}(s_j + ip_j)A_j = 0 \qquad , \quad i = \sqrt{-1} \tag{28}$$

$$\sum_{j=1}^{3} s_j q_j A_j = 0$$

$$(c_{11}i + c_{33}p_1 s_1 - \frac{\beta_3}{k}q_1)A_1 + (c_{11}i + c_{33}p_2 s_2 - \frac{\beta_3}{k}q_2)A_2 + (c_{11}i + c_{33}p_3 s_3 - \frac{\beta_3}{k}q_3)A_3 = 0$$

$$(s_1 + ip_1)A_1 + (s_2 + ip_2)A_2 + (s_3 + ip_3)A_3 = 0 \tag{29}$$

$$q_1 s_1 A_1 + q_2 s_2 A_2 + q_3 s_3 A_3 = 0$$

$$(c_{11}i + c_{33}p_1 s_1 - \frac{\beta_3}{k}q_1)A_1 + (c_{11}i + c_{33}p_2 s_2 - \frac{\beta_3}{k}q_2)A_2 + (c_{11}i + c_{33}p_3 s_3 - \frac{\beta_3}{k}q_3)A_3 = 0$$

$$(s_1 + ip_1)A_1 + (s_2 + ip_2)A_2 + (s_3 + ip_3)A_3 = 0 \tag{30}$$

$$q_1 s_1 A_1 + q_2 s_2 A_2 + q_3 s_3 A_3 = 0$$

For non trivial solution of above linear homogeneous system of equations, the determinant must vanishes

$$\begin{vmatrix} c_{11}i + c_{33}p_1 s_1 - \frac{\beta_3}{k}q_1 & c_{11}i + c_{33}p_2 s_2 - \frac{\beta_3}{k}q_2 & c_{11}i + c_{33}p_3 s_3 - \frac{\beta_3}{k}q_3 \\ s_1 + ip_1 & s_2 + ip_2 & s_3 + ip_3 \\ q_1 s_1 & q_2 s_2 & q_3 s_3 \end{vmatrix} = 0 \tag{31}$$

2.1 Uncoupled Thermoelasticity

This case corresponds to the situation when the strain and temperature fields are not coupled with each other i.e. $\beta_1 = \beta_3 = 0$. In this case the equation (21) decoupled, and the analysis reduced to uncoupled generalized thermoelasticity, thus purely elastic and thermal waves decoupled.

2.2 Coupled Thermoelasticity

This case corresponds to no thermal relaxation time, i.e. τ_0 =0, then the analysis reduced to the definition classical coupled thermoelasticity.

3. Numerical results and discussion

As the frequency equations (31) are complex equation, so that these transcendental equations enable us to evaluate not only the phase velocity, but also the thermoelastic energy dissipation for the propagation of thermoelastic waves in an infinite plate. In general, the waves are dispersive and dissipate energy according to the LS theory of generalized thermoelasticity. The manner in which the long and short wavelength limits are connected requires numerical solution of Eqs. (14). Moreover, for values of c which make s_1, s_2, and s_3 imaginary, the hyperbolic functions become periodic and so an infinite number of higher modes exists. Computations for the symmetric and antisymmetric modes have been carried out for aluminum alloy plate whose physical data are given in Table I.

Table 1 Physical Properties of an Aluminum Alloy

Young's modulus E	72.6 GPa
Poisson ratio ν	0.33
Density ρ	2800 kg.m^{-3}
Specific heat C_e	960 J.kg^{-1}.K^{-1}
Thermal diffusivity k_1	7×10^{-5} m^2 .s^{-1}
Expansion coefficient α_t	2.35×10^{-5} K^{-1}
Initial temperature T_0	293 ^{0}K

An important consequence of surface wave dispersive behavior is the existence of a group velocity. When we talk about velocity of propagation of surface waves, we use the term phase velocity, that is the velocity of a wave front (locus of constant phase points), such as a peak or a trough. In a dispersive medium, this is not the same as the velocity of a pulse of energy, indeed the latter can be seen (Fourier analysis) as composed of several single frequency signals, each one travelling with its own velocity because of dispersion. In Figure 1 and Figure 2 dispersion curves are depicted at the thermal relaxation times $\tau_0 = 1.10^{-6}s$ and $\tau_0 = 1.10^{-7}s$ which clarifies this concept. The velocity of the wave train, i.e. the velocity of the envelope is indicated as group velocity, in contrast with that of the carrier that is the phase velocity. Obviously for a non-dispersive medium group velocity and phase velocity coincide.

The group velocity U is computed using the following expressions, which involve the derivative of phase velocity with respect to frequency f or to wavelength λ

$$U = \frac{d\omega}{dk(\omega)} \approx c + f\frac{dc}{df} = c - \lambda\frac{dc}{d\lambda}$$

where all the values are the average ones over the dominant frequency range. From the above expression it is clear that if modal phase velocity decrease with increasing frequency (normally dispersive profiles), c is greater than U and hence the carrier travels faster than the envelope. Thus in such cases if a phase disturbance appears at the beginning of the pulse, then it overtakes and finally it disappears in the front (as shown in Figure 1, Figure 2 and Figure 3).

Also from Figures 1 to Figure 3 it is also observed that with changes in thermal relaxation times lower modes have more influenced whereas small variation is noticed in the high modes. At low wave number limits, although wave speed modes are dispersive, but are different from coupled case. Thus in generalized thermoelasticity, at low values of the wave number, only the lower modes get affected and the little change is seen at relatively high values of wave number. The low value region of the wave number is found to be of more physical interest in generalized thermoelasticity. Further as at high wave number limits has no effect in generalized thermoelasticity, and so the second sound effects are short lived. Phase Velocity Vs wave number dispersion curves for uncoupled and coupled thermoelasticity are shown in Figure 4 and Figure 5.

5. Conclusion

Thermoelastic Rayleigh waves are generated by the presence of a free surface in solids and they travel in a confined zone along the free surface. In general for a given frequency several free wave modes exist, each one characterized by a given wave number and hence a given phase velocity. The effective phase velocity is a combination of modal values and it is spatially dependent. Because of the difference between phase velocity and group velocity, mode separation takes place going away from the source and hence the pulse changes shape. For a normally dispersive profile the fundamental mode is strongly predominant on the higher modes at every frequency. Consequently, the effective phase velocity is practically coincident with the fundamental mode one. The wave propagation interests a portion of the medium nearly equal to a wavelength. Further separation among mode types is somewhat an artificial, when we learn thermoelastic waves in anisotropic plates, as the equation for thermoelastic wave modes i.e. quasi-longitudinal and quasi-transverse and shear horizontal modes are being coupled with quasi-thermal modes. Also wave propagation in the higher symmetry materials as in isotropic case, some wave types revert to pure modes, leading simple characteristic equation of lower order, and consequence of thermoelastic anisotropy in media is the loss of pure wave modes for general propagation direction. It is also observed that at low wave number limits, modes are found to highly influence and it vary with the thermal relaxation times. At relatively low values of the wave number, little change is observed in these values. As wave number increases others high modes appear, one of the modes seems to be associated with quick change in the slope of the mode. For Coupled and uncoupled thermoelasticity, each of figure display coupled wave speeds corresponding to quasi-longitudinal, two transverse and quasi-thermal at zero wave number limits, for the higher value wave numbers higher modes appear with wave number increases.

Legends in the figures -35 are:

Group Velocity Mode 1
Group Velocity Mode 2
Group Velocity Mode 3
Group Velocity Mode 4
Phase Velocity Mode 1
Phase Velocity Mode 2
Phase Velocity Mode 3
Phase Velocity Mode 4

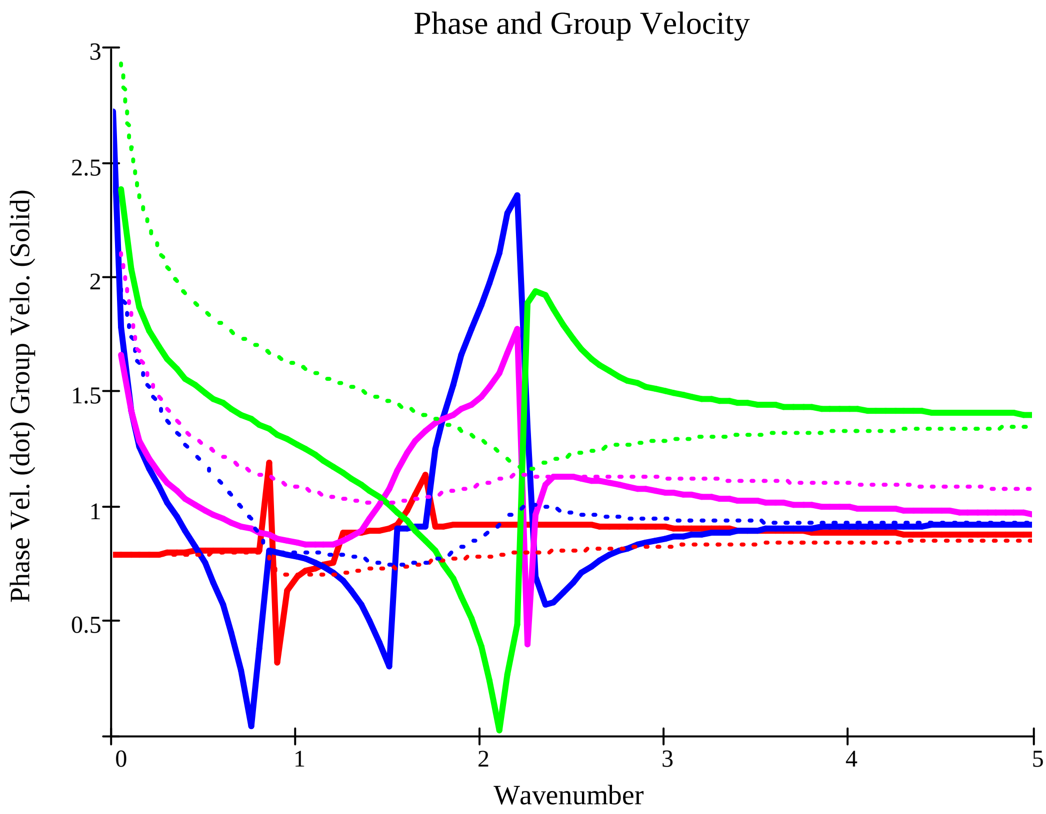


Figure 1. Group and Phase Velocity aluminum alloy plate when thermal relaxation time $\tau_0 = 1.10^{-6} s$

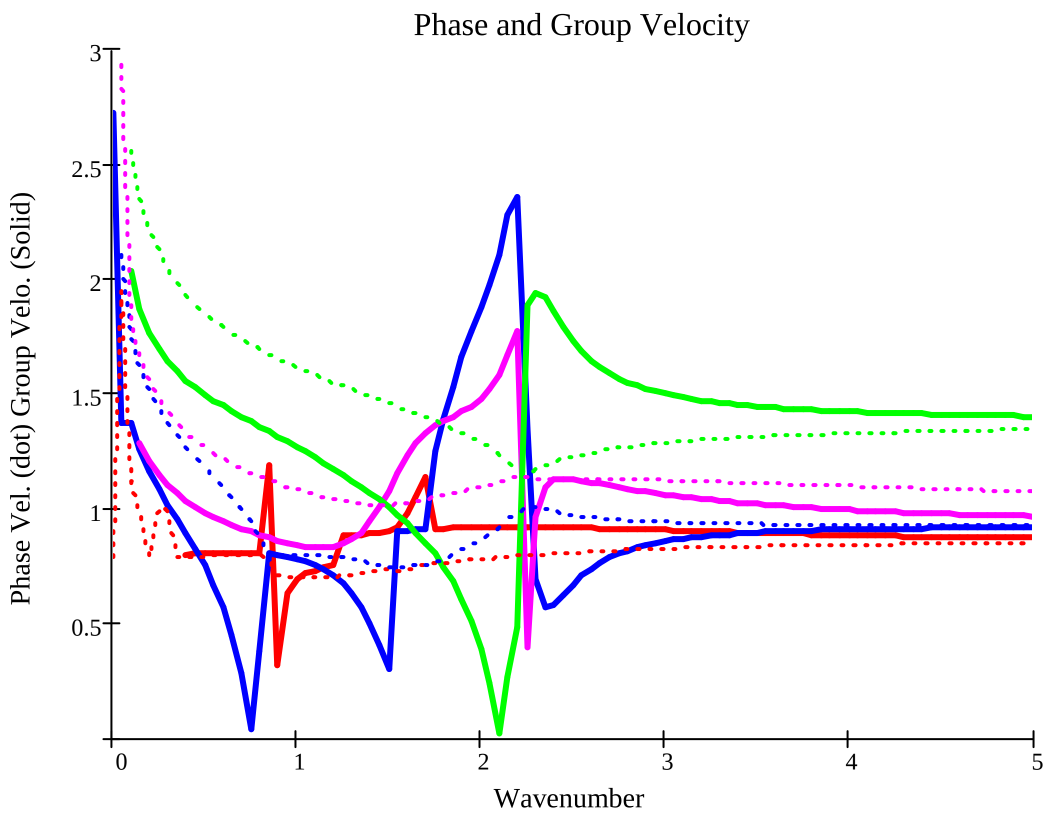


Figure 2. Group and Phase Velocity aluminum alloy plate when thermal relaxation time $\tau_0 = 1.10^{-7} s$

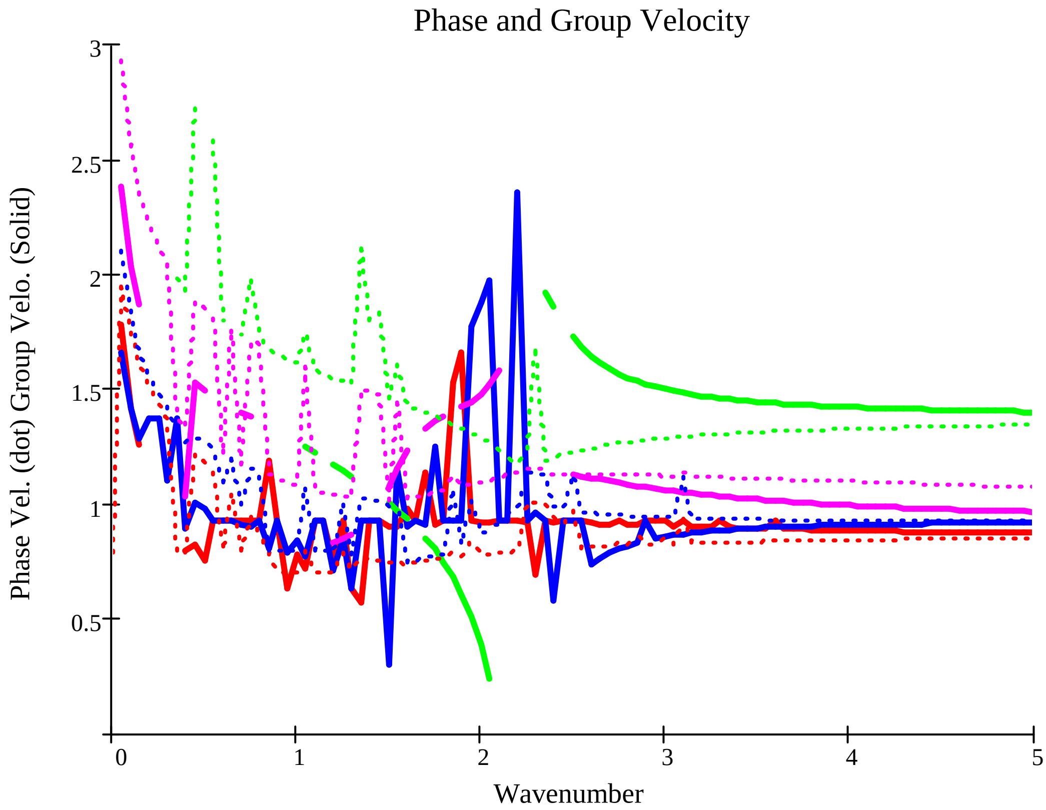


Figure 3. Group and Phase Velocity aluminum alloy plate when thermal relaxation time $\tau_0 = 1.10^{-8}\, s$

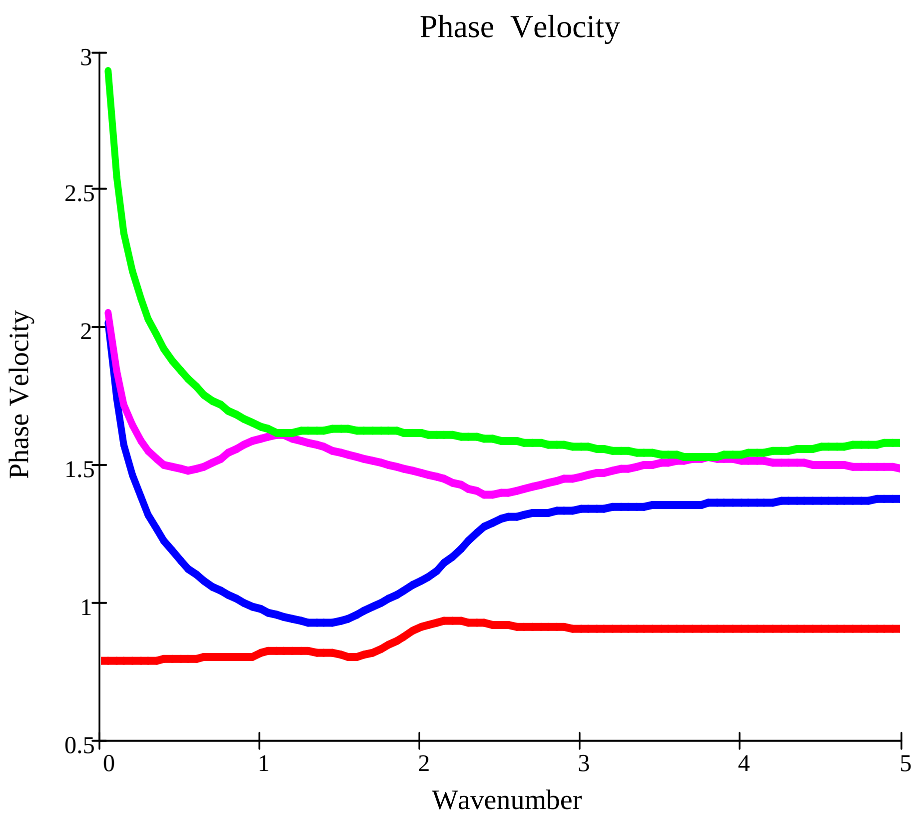


Figure 4. Phase Velocity aluminum alloy plate in case of Uncoupled Thermoelasticity

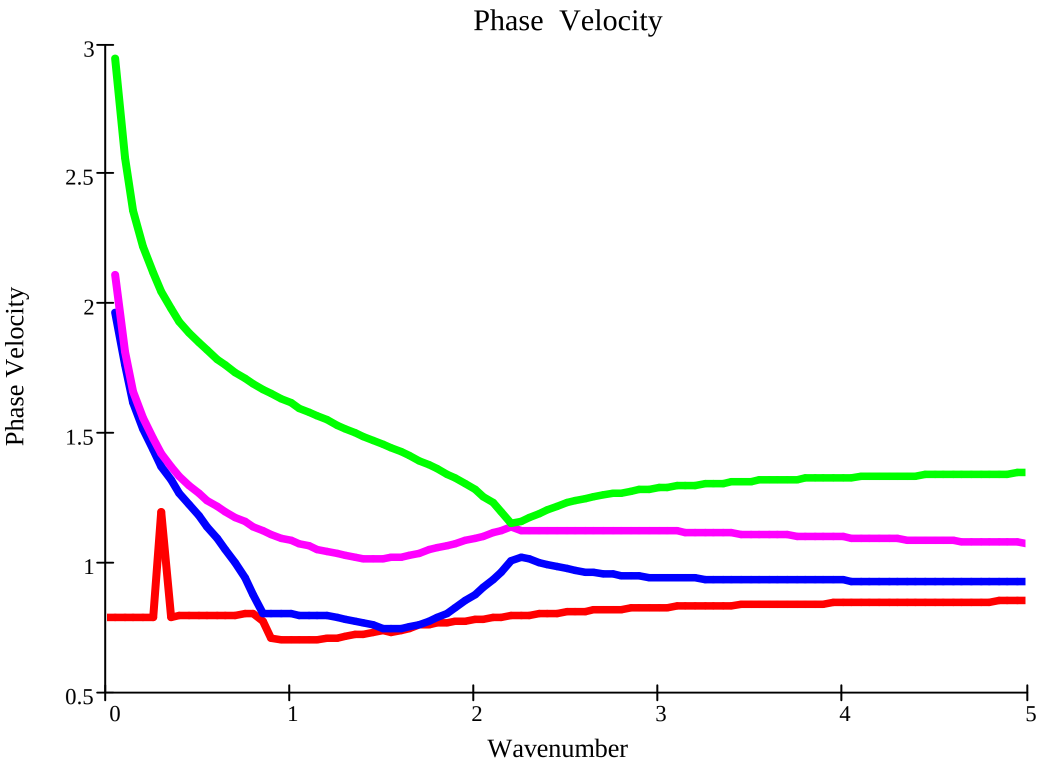
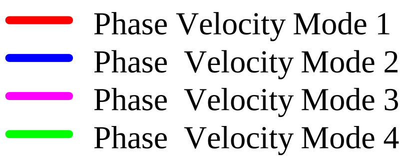


Figure 5. Phase Velocity aluminum alloy plate in case of Coupled Thermoelasticity

Legends in the figures 4 and 5 are:



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