International Journal of Engineering & Applied Sciences (IJEAS)

Vol.6, Issue 2(2014)32-51

EFFECT OF TWO - PARAMETER FOUNDATION ON FREE TRANSVERSE VIBRATION OF NON- HOMOGENEOUS ORTHOTROPIC RECTANGULAR PLATE OF LINEARLY VARYING THICKNESS

U.S.Gupta[a], Seema Sharma[b] and Prag Singhal [c*]

[a]Department of Mathematics, Indian Institute of Technology Roorkee, Roorkee, India
[b]Department of Mathematics, Gurukul Kangri University, Hardwar, India.
[c]Department of Applied Sciences and Humanities, ABES Engineering College, Ghaziabad, India.
[*]Email: prag.singhal@gmail.com

Abstract

Differential Quadrature Method (DQM) is employed to obtain natural frequencies and mode shapes of non-homogeneous rectangular orthotropic plates of linearly varying thickness resting on two -parameter foundation (Pasternak). The analysis is based on classical plate theory. Numerical results are presented for various values of plate parameters for different boundary conditions. Convergence studies have been made to ensure accuracy of the results. A comparison of our results with those available in the literature shows the versatility and accuracy of DQM.

Keywords: DQM, varying thickness, non-homogeneity, two-parameter foundation.

1. Introduction

An extensive review of transverse vibrations of plates of uniform thickness has been given by Leissa[1] in his monograph followed by a series of review articles [2-7]. Due to the desirability of light weight, high strength and stiffness of structural components in various engineering applications such as aeronautical, naval, mechanical, space vehicles and printed circuit boards led to the study of vibrations of orthotropic plates. The increasing use of fiber reinforced materials and plates fabricated out of modern composites such as boron-epoxy, glass-epoxy and Kevlar in various modern structures demand that the effect of non-homogeneity together with anisotropy of the material should be taken into account to predict their vibration behavior with a fair amount of accuracy reported in the references [8-16]. The consideration of thickness variation provides the added advantage of reduction in weight, size and stiffness enhancing and meeting the architectural requirements.

Furthermore, the problem of plate resting on an elastic foundation has achieved great importance in modern technological and foundation engineering such as reinforced concrete pavements of high runways, foundation of deep wells, machine basis, building footing. In order to avoid the mathematical complexities and to investigate the interaction between plates and foundations, many idealized elastic foundations have been proposed by Selvadurai [17], Kerr [18] and Hetenyi [19]. Most of the studies on the vibration behavior of beams and rectangular plates resting on elastic foundation are devoted to Winkler foundation and reported in the references [20-22]. Winkler foundation is the simplest model in which the foundation is assumed to be replaced by a series of unconnected closely spaced vertical

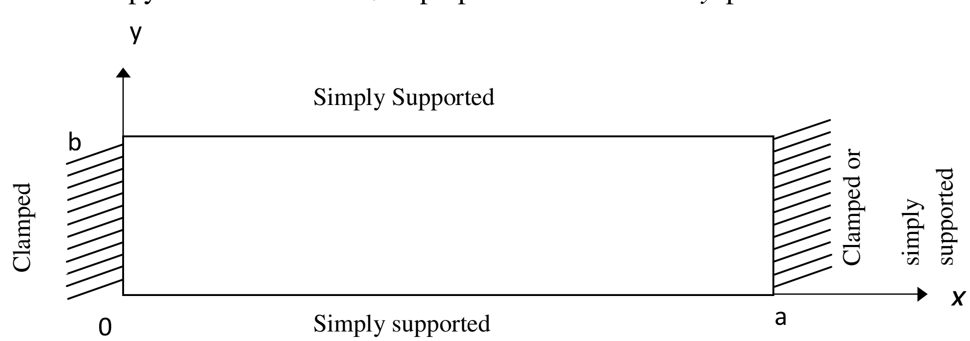
[3]Corresponding author. Tel.: +919999056894 (Residence).
E-mail address: prag.singhal@gmail.com (Prag Singhal)

elastic springs. In view of the important deficiency of discontinuity of displacement in Winkler model, various other models have been proposed in literature by different researchers. Kerr [23] has given an excellent discussion about various foundation models. Out of these, Pasternak model provides a better approximation to foundation reaction as it takes into account not only its transverse reaction but also a shear interaction between the spring elements. A survey of literature shows that most of the earlier studies on Pasternak foundation are devoted to homogeneous rectangular plates of uniform/ non-uniform thickness [24-32]. Out of these, Omurtag and Kadioglu [30] have presented finite element method solution on free vibration analysis of orthotropic Kirchhoff rectangular plates resting on Pasternak foundation. Malekzadeh and Karami [31] discussed the free vibration analysis of isotropic non-uniform thick plates by DQM. Leung and Zhu [32] analyzed the transverse vibration of midlin plates of various geometric on two parameter foundation using finite element method with trapezoidal elements. Recently, Lal and Dhanpati [33] analysed the effects of Pasternak foundation on the vibration of non-homogeneous orthotropic rectangular plates using Chebyshev collocation method.

In this paper, a study dealing with transverse vibrations of non-homogeneous orthotropic rectangular plates of linearly varying thickness along with one direction and resting on a Pasternak type elastic foundation has been presented by employing classical plate theory. For non-homogeneity of the plate material, it is assumed that Young's moduli and density vary exponentially along with one direction. The governing differential equation for such plates with two opposite edges simply supported while other two may be simply supported or clamped reduces to fourth order differential equation with variable coefficients whose analytical solution is not feasible. Differential quadrature method has been employed to obtain the natural frequencies for different combinations of clamped and simply supported boundary conditions at the other two edges. The effects of various plate parameters have been studied on the natural frequencies for the first three modes of vibration. A Huber type orthotropic material 'ORTHO1' Biancolini et al. [34] has been taken as an example of rectangular orthotropic material.

2. Mathematical Formulation

Referred to Cartesian co-ordinate system (x, y, z), the configuration of rectangular orthotropic plate of side dimension $a \times b$, thickness $h(x, y)$, density $\rho(x, y)$ and resting on two parameter elastic foundation with foundation moduli k_f and G_f. The middle surface being $z = 0$ and the origin is at one of the corners of the plate. The x and y axes are taken alongwith the principal directions of orthotropy and the axis of z is perpendicular to the x y plane as shown in the Figure (1).



Boundary Conditions.

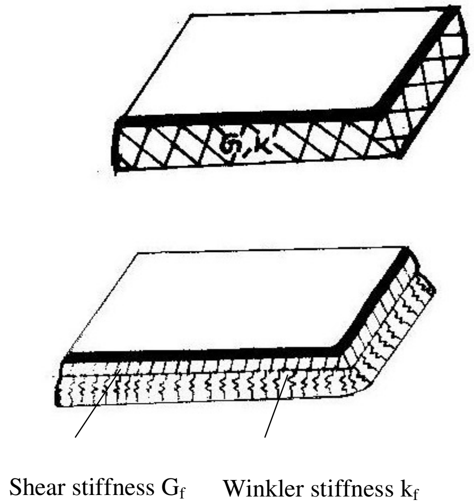


Figure 1: Rectangular plate resting on two-parameter foundation.

The differential equation governing the transverse vibration of such plates is given by [15]:

$$\begin{aligned}
&D_x \frac{\partial^4 w}{\partial x^4} + D_y \frac{\partial^4 w}{\partial y^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + 2\frac{\partial H}{\partial x}\frac{\partial^3 w}{\partial x \partial y^2} + 2\frac{\partial H}{\partial y}\frac{\partial^3 w}{\partial y \partial x^2} + 2\frac{\partial D_x}{\partial x}\frac{\partial^3 w}{\partial x^3} + 2\frac{\partial D_y}{\partial y}\frac{\partial^3 w}{\partial y^3} \\
&+ \frac{\partial^2 D_x}{\partial x^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 D_y}{\partial y^2}\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 D_1}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 D_1}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 4\frac{\partial^2 D_{xy}}{\partial x \partial y}\frac{\partial^2 w}{\partial y \partial x} + \rho h \frac{\partial^2 w}{\partial t^2} \\
&- G_f\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + k_f w = 0,
\end{aligned} \tag{1}$$

Here,

$$D_x = E_x^* h^3/12,\ D_y = E_y^* h^3/12,\ D_{xy} = G_{xy} h^3/12,\ D_1 = E^* h^3/12,\ H = D_1 + 2D_{xy},$$
$$(E_x^*, E_y^*) = (E_x, E_y)/(1-\nu_x \nu_y),\ E^* = \nu_y E_x^* = \nu_x E_y^*,$$

and *w(x,y,t)* is the transverse deflection, *t* the time, ρ the mass density, E_x, E_y, ν_x, ν_y and G_{xy} are material constants in proper directions defined by an orthotropic stress-strain law.

Let us assume that the thickness of the plate varies in the *x*-direction only i.e. $h = h(x)$ and the two opposite edges $y = 0$ and $y = b$ are simply supported. For a harmonic solution, the deflection w (Lévy approach) is assumed to be:

$$w(x, y, t) = \bar{w}(x)\sin(p\pi y/b)e^{i\omega t}\ , \tag{2}$$

Here, p is a positive integer and ω is the frequency in radians/sec.

Substituting equation (2) in (1), we get:

$$
\begin{aligned}
&D_x \bar{w}^{iv} + \left(2\frac{\partial D_x}{\partial x}\right)\bar{w}''' + \left(-2\bar{\lambda}^2 H + 2\bar{\lambda}\frac{\partial H}{\partial y} + \frac{\partial^2 D_x}{\partial x^2} + \frac{\partial^2 D_1}{\partial y^2} - G_f\right)\bar{w}'' \\
&+\left(-2\bar{\lambda}^2\frac{\partial H}{\partial x} + 4\bar{\lambda}\frac{\partial^2 D_{xy}}{\partial x \partial y}\right)\bar{w}' + \left\{\bar{\lambda}^4 D_y - 2\bar{\lambda}^3\frac{\partial D_y}{\partial y} - \bar{\lambda}^2\left(\frac{\partial^2 D_y}{\partial y^2} - \frac{\partial^2 D_1}{\partial x^2} + G_f\right) + k_f - h\rho\omega^2\right\}\bar{w} = 0,
\end{aligned} \tag{3}
$$

For thickness variation of the plate and non-homogeneity in material it is assumed that h, E_x, E_y and density ρ all are functions of space variable x only and following Lekhnitskii [35] and Panc [36] and the shear modulus is $G_{xy} = \sqrt{E_x E_y} / 2(1+\sqrt{\nu_x \nu_y})$.

By introducing the non-dimensional variables $X = x/a$, $Y = y/b$, $\bar{h} = h/a$, $W = \bar{w}/a$ and by considering above assumptions, equation (3) reduces to

$$
\begin{aligned}
&\bar{h}^3 E_x W^{iv} + [2(\bar{h}^3 E_x' + 3\bar{h}^2 \bar{h}' E_x)]W''' + [(6\bar{h}\bar{h}'^2 + 3\bar{h}^2 \bar{h}'')E_x + 6\bar{h}^2 \bar{h}' E_x' + \bar{h}^3 E_x'' \\
&-2(1-\nu_x\nu_y)\{\lambda^2 \bar{h}^3 (E^* + 2G_{xy}) + 6(G_f/a)\}]W'' - [2\lambda^2\{3\bar{h}^2 \bar{h}'(\nu_y E_x + 2(1-\nu_x\nu_y)G_{xy}) \\
&+\bar{h}^3(\nu_y E_x' + 2(1-\nu_x\nu_y)G_{xy}')\}]W' + [\lambda^4 \bar{h}^3 E_y - \lambda^2\nu_y\{\bar{h}^3 E_x'' + 6\bar{h}^2 \bar{h}' E_x' + (6\bar{h}\bar{h}'^2 + 3\bar{h}^2 \bar{h}'')E_x\} \\
&+12(1-\nu_x\nu_y)(a k_f + \lambda^2 (G_f/a) - \rho \bar{h} a^2 \omega^2)]W = 0\,,
\end{aligned} \tag{4}
$$

Here $\lambda^2 = p^2 a^2 \pi^2 / b^2$ and primes denote differentiation with respect to X.

Assuming linear variation in thickness, [10] i.e. $\bar{h} = h_0(1+ \alpha X)$ and following references, [13, 14] for non-homogeneity of the plate material in X direction as follows:

$$E_x = E_1 e^{\mu X}, \quad E_y = E_2 e^{\mu X}, \quad \rho = \rho_0 e^{\beta X}\,, \tag{5}$$

where $(h_0, \rho_0) = (h, \rho)\big|_{X = X_0}$, μ is the non-homogeneity parameter, α is the taper parameter, β is the density parameter and E_1, E_2 are Young's moduli in proper directions at $X = 0$, equation (4) now reduces to:

$$A_0 W^{iv} + A_1 W''' + A_2 W'' + A_3 W' + A_4 W = 0 \tag{6}$$

Here,

$$A_0=1, A_1=2(\mu+\frac{3\alpha}{(1+\alpha X)}),$$

$$A_2=\frac{6\alpha^2}{(1+\alpha X)^2}+\frac{6\mu\alpha}{(1+\alpha X)}+\mu^2-2\{\lambda^2(\nu_y+\frac{\sqrt{E_2/E_1}}{1+\sqrt{\nu_x\nu_y}}(1-\nu_x\nu_y)+\frac{6G}{h_0^3(1+\alpha X)^3}e^{-\mu X}\}$$

$$A_3=-2\lambda^2(\frac{3\alpha}{(1+\alpha X)}+\mu)(\nu_y+\frac{\sqrt{E_2/E_1}}{1+\sqrt{\nu_x\nu_y}}(1-\nu_x\nu_y),$$

$$A_4=\lambda^4E_2/E_1-\lambda^2\nu_y(\mu^2+\frac{6\mu\alpha}{(1+\alpha X)}+\frac{6\alpha^2}{(1+\alpha X)^2})$$

$$-\frac{\Omega^2}{(1+\alpha X)^2}e^{(\beta-\mu)X}+\frac{12\mathrm{K}}{h_o^3(1+\alpha X)^3}e^{-\mu X}+\frac{12\lambda^2G}{h_0^3(1+\alpha X)^3}e^{-\mu X}$$

$\Omega^2 = 12\,\rho_0\,(1- \nu_x\,\nu_y)\,\mathrm{a}^2\omega^2/E_1\,h_0^{\;2}$, K= a $k_f(1- \nu_x\,\nu_y)/E_1$ and $G = G_f(1- \nu_x\,\nu_y)/\,\mathrm{a}\,E_1$

Equation (6) in conjunction with boundary conditions at the edges $X=0$ and $X=1$ gives rise to a two-point boundary value problem with variable coefficients whose closed form solution is not possible. An approximate solution is obtained by employing Differential Quadrature Method (DQM).

3. Method of Solution: Differential Quadrature Method

Let X_1, X_2 , … ., X_m be the m grid points in the applicability range [0, 1] of the plate. According to the DQM, the n^{th} order derivative of $W(X)$ with respect to X can be expressed discretely at the point X_i as:

$$\frac{d^nW(X_i)}{dX^n}=\sum_{j=1}^{m}c_{ij}^{(n)}W(X_j)\ ,\qquad n=1,2,3,4 \text{ and } i=1,2,\ldots,m \tag{7}$$

Here $c_{ij}^{(n)}$ are the weighting coefficients associated with the nth order derivative of W(X) with respect to X at discrete point X_i.

Following Shu [37], the weighting coefficients in equation (7) are given by are given by

$$c_{ij}^{(1)}=\frac{M^{(1)}(X_i)}{(X_i-X_j)M^{(1)}(X_j)}\quad ,\quad i,\ j=1,2,\ldots,m \text{ and } i\neq j \tag{8}$$

$$M^{(1)}(X_i)=\prod_{\substack{j=1\\ j\neq i}}^{m}(X_i-X_j)\,, \tag{9}$$

$$c_{ij}^{(n)}=n\left(c_{ii}^{(n-1)}c_{ij}^{(1)}-\frac{c_{ij}^{(n-1)}}{x_i-x_j}\right)\quad for\ \ i,j=1,2,\ldots,m,\ j\neq i\quad and\ n=2,3,4 \tag{10}$$

$$c_{ii}^{(n)} = -\sum_{\substack{j=1 \\ j\neq i}}^{m} c_{ij}^{(n)} \qquad for \;\; i = 1, 2, \ldots, m \qquad and \;\; n = 1, 2, 3, 4 . \tag{11}$$

Discretizing equation (6) at grid points X_i where $i = 3, 4, \ldots, m-2.$, it reduces to,

$$A_0 W^{iv}(X_i) + A_{1,i} W'''(X_i) + A_{2,i} W''(X_i) + A_{3,i} W'(X_i) + A_{4,i}\, W(X_i) = 0, \tag{12}$$

Substituting the values of W and its derivatives at the i^{th} grid point in the equation (12) using relations (7) to (8), equation (12) becomes:

$$\sum_{j=1}^{m} (A_0 c_{ij}^{(4)} + A_{1,i} c_{ij}^{(3)} + A_{2,i} c_{ij}^{(2)} + A_{3,i} c_{ij}^{(1)})\, W(X_j) + A_{4,i} W(X_i) = 0. \quad i = 3, 4, \ldots, m-2 \tag{13}$$

Equation (13) is a set of (m-4) equations in terms of unknowns $W_j (\equiv W(X_j)),\; j = 1, 2, \ldots, m$ which can be written in the matrix form as:

$$[B][W^*] = [0], \tag{14}$$

Here B and W^* are matrices of order (m-4) x m and (m x 1) respectively.
Here, the (m-2) internal grid points chosen for collocation, are the zeros of shifted Chebyshev polynomial of order (m-2) with orthogonality range [0, 1] given by:

$$X_{k+1} = \frac{1}{2}[1 + \cos(\frac{2k-1}{m-2}\frac{\pi}{2})] \;, \qquad k = 1, 2, \ldots, m-2. \tag{15}$$

4. Boundary Conditions and Frequency Equations

The two different combinations of boundary conditions namely, C-C and C-S have been considered here, where C and S stand for clamped and simply supported boundary respectively. First symbol denotes the condition at the edge $X = 0$ and second symbol at the edge $X = 1$. By satisfying the relations,

$$W = \frac{dW}{dX} = 0 \;; W = \frac{d^2W}{dX^2} - (E^* / E_x^*)\lambda^2 W = 0 \;;$$

For clamped and simply supported conditions respectively, a set of four homogeneous equations in terms of unknown W_j are obtained. These equations together with field equation (14) give a complete set of m homogeneous equations in m unknowns. For C-C plate this set of equations can be written as:

$$\begin{bmatrix} B \\ B^{CC} \end{bmatrix} [W^*] = [0] \tag{16}$$

Here B^{CC} is a matrix of order $4\times m$.

For a non-trivial solution of equation (16), the frequency determinant must vanish and hence,

$$\begin{vmatrix} B \\ B^{CC} \end{vmatrix} = 0 \;. \tag{17}$$

Similarly for C-S plate, the frequency determinants can be written as,

$$\begin{vmatrix} B \\ B^{CS} \end{vmatrix} = 0 \tag{18}$$

5. Numerical Results and Discussion

The frequency equations (17 & 18) provide the values of the frequency parameter Ω for various values of plate parameters. Numerical computation has been carried out to investigate the effects of foundation together with orthotropy, non-homogeneity, thickness variation and aspect ratio on the frequency parameter Ω for p = 1. The work presented in this paper, the values of various parameters are taken as follows: foundation parameters K = 0.0 (0.02) 0.1, G = 0.0 (0.002) 0.01, non-homogeneity parameter μ = -0.5 (0.1) 1.0, density parameter β = -0.5 (0.1) 1.0, taper parameter α = -0.5 (0.1) 1.0 and aspect ratio a/b = 0.5 (0.5) 2.0 for C-C and C-S boundary conditions. The elastic constants for the plate material 'ORTHO1' Biancolini [34] are taken as $E_1 = 1\times10^{10}\ MPa$, $E_2 = 5\times10^{9}\ MPa$, $\nu_x = 0.2$, $\nu_y = 0.1$. The value of h_0 which is thickness of plate at the edge X=0 has been taken as 0.1.

To choose the appropriate value of the number of collocation points m, convergence studies have been carried out for different sets of parameters. For a specified plate, graphs are shown in Figures 2(a, b) for μ = 0.5, α = 0.5, β = -0.5, K = 0.02, G = 0.001 and a/b = 1 for C-C and C-S plates respectively. For this data, the maximum deviations were observed. In all, the computation m = 20 has been fixed because further increase in m does not improve the results even in the fourth place of decimal in the third mode of vibration for all the plates.

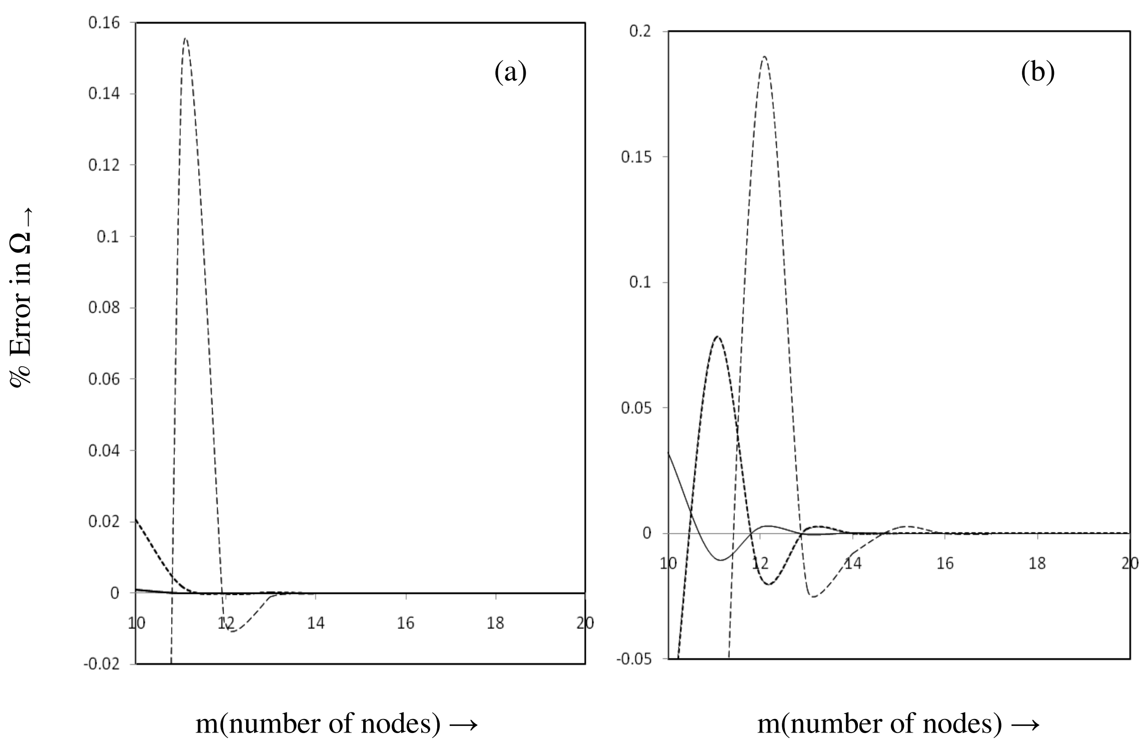
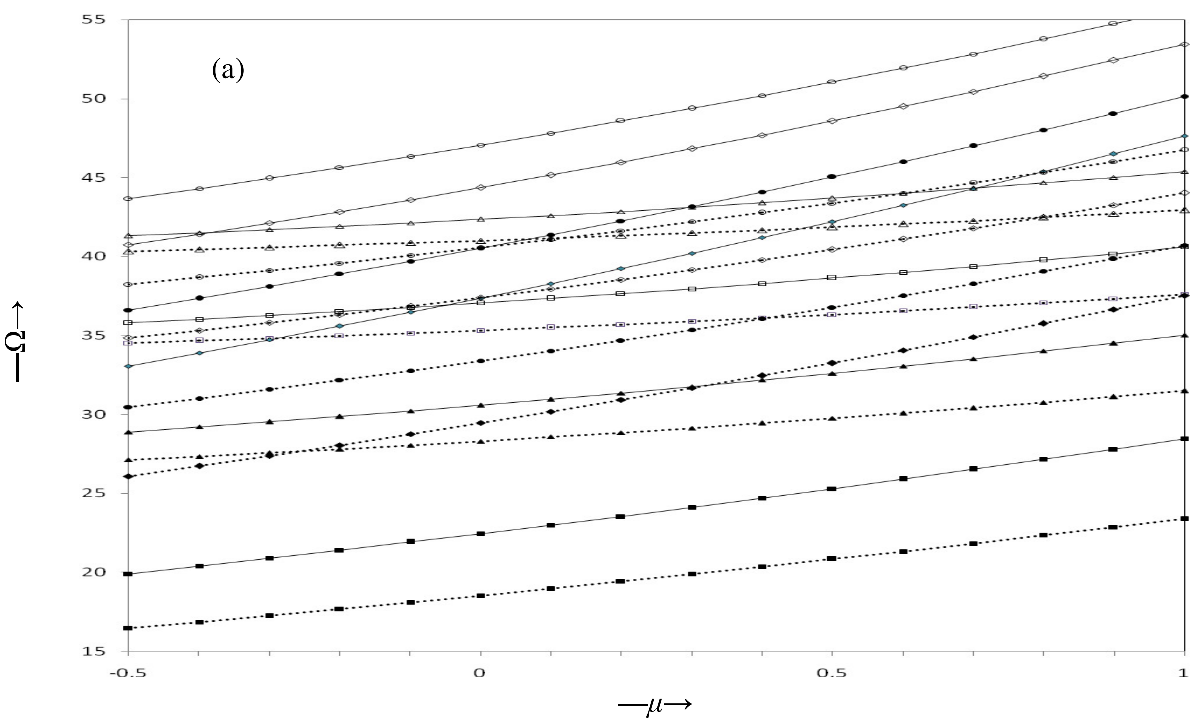


Figure 2: Percentage error in frequency parameter Ω; (a) C-C plate, and (b) C-S plate, for a/b= 1.0, K= 0.02, g = 0.001, μ= 0.5, β= -0.5, α = 0.5, ———, first mode, ---------, second mode, – – – – –, third mode. % error = $[(\Omega_m - \Omega_{20})/\ \Omega_{20}] \times 100$.

The numerical results for specified plate parameters are given in Figures (3 - 9). It is found that the value of frequency parameter Ω for a C-C plate is higher than that for a C-S plate. Frequency parameter Ω increases with the increasing values of non-homogeneity parameter μ, taper parameter α, aspect ratio *a*/*b*, Winkler foundation parameter *K*, Pasternak foundation parameter *G* and decreases with the increasing values of density parameter β.

Figures 3(a-c) show the behavior of frequency parameter Ω versus non-homogeneity parameter μ for density parameter β = -0.5, aspect ratio *a*/*b* = 1, two different values of taper parameter α = -0.5, 0.5, Winkler foundation parameter *K* = 0.0, 0.02 and Pasternak foundation parameter *G* = 0.0, 0.002, for C-C and C-S plates vibrating in first, second and third mode respectively. It is observed that the frequency parameter Ω is found to increase linearly with the increasing values of non-homogeneity parameter μ keeping all other parameters fixed. The rate of increase of Ω with μ is higher for a C-C plate than that for a C-S plate. This rate increases with the increase in the value of taper parameter α but decreases with increasing values of both the foundation parameters *K* and *G*. In case of second and third modes of vibration the behavior of frequency parameter Ω is same as in case of first mode except that the rate of increase of Ω with μ increases with the number of modes.



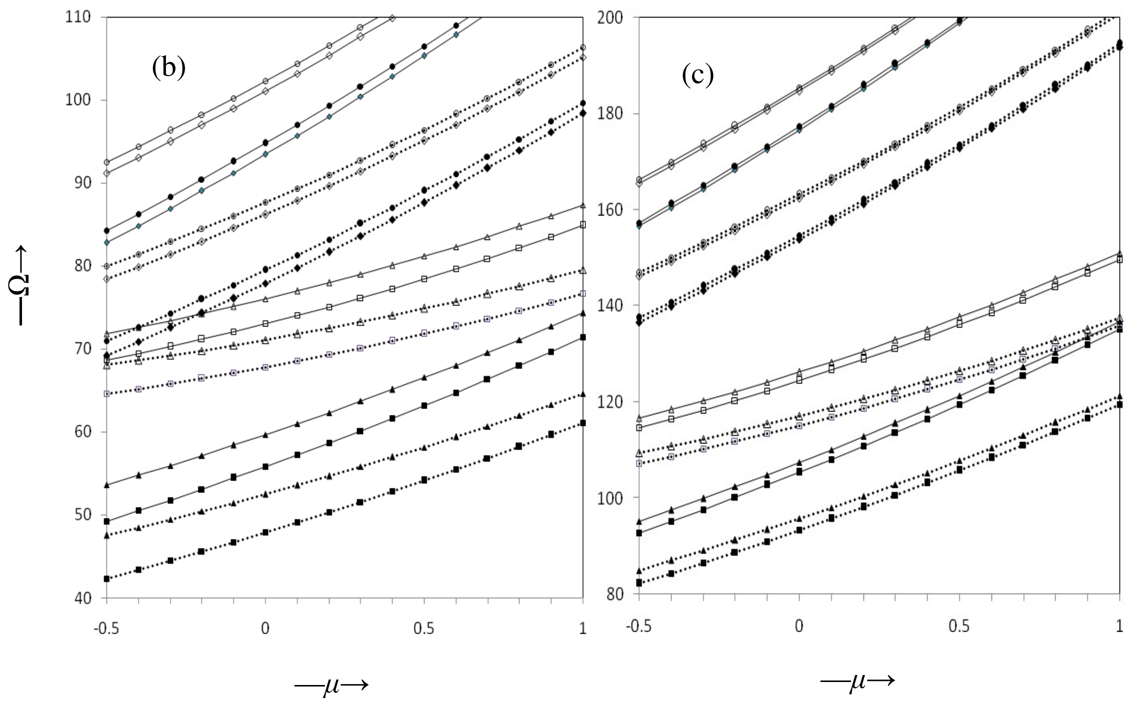


Figure 3: Frequency parameter for C-C and C-S plates vibrating in (a) first mode (b) second mode and (c) third mode, for β = -0.5, *a/b* = 1. ———, C-C; -----------, C-S;
■, □, α = -0.5, *K*=0.0; ♦,◊, α = 0.5, *K*=0.0; ▲, △, α = -0.5, *K*=0.02; •,o, α = 0.5, *K*=0.02.
■, ♦, ▲,•, *G*=0.0; □,◊, △,o, *G*=0.002.

Figure 4(a) shows the behavior of frequency parameter Ω with aspect ratio of plate *a*/*b* for two different values of taper parameter α = -0.5, 0.5, foundation parameters (*K* = 0.0, 0.02 & *G* = 0.0, 0.002), non-homogeneity parameter μ = 0.5 and density parameter β = 0.5 for C-C and C-S plates vibrating in first mode. It is found that the frequency parameter Ω increases with the increasing values of aspect ratio *a*/*b* irrespective of other plate parameters. The rate of increase of frequency parameter Ω with aspect ratio *a*/*b* increases with the increasing values of taper parameter α and *G* but decreases with the foundation parameter *K* for both the plates. The rate of increase of Ω with *a*/*b* is more prominent for *a*/*b*>1 as compare to *a*/*b*<1. The behavior is almost same in case of second and third mode of vibration as that of first mode. The rate of increase of Ω with *a*/*b* increases with the increasing number of modes, Figures 4(b) and (c).

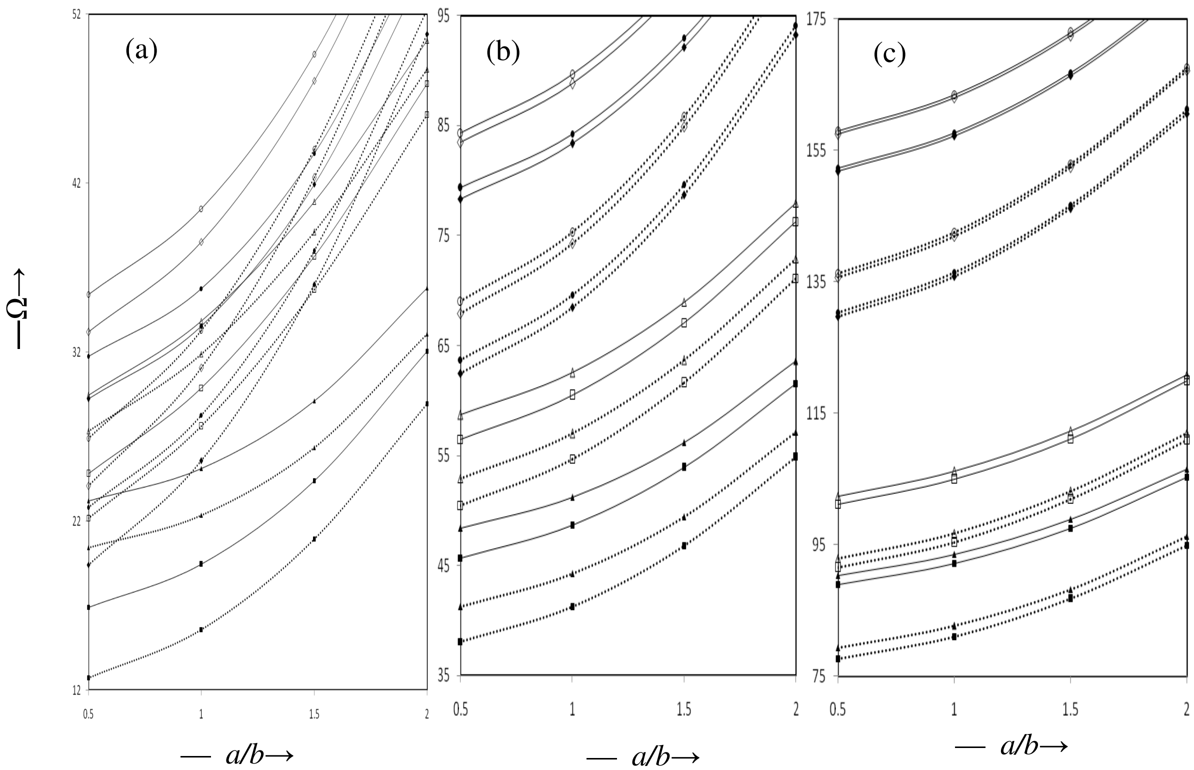


Figure 4: Frequency parameter for C-C and C-S plates vibrating in (a) first mode (b) second mode and (c) third mode, for $\beta = 0.5$, $\mu = 0.5$. ——, C-C; ----------, C-S; ■, □, $\alpha = -0.5$, K=0.0 ; ♦,◊, $\alpha = 0.5$, K=0.0; ▲, △, $\alpha = -0.5$, K=0.02; •,o, $\alpha = 0.5$, K=0.02; ■, ♦, ▲, •, G=0.0; □,◊, △, o, G=0.002.

The Figure 5(a) depicts the variation of frequency parameter Ω with density parameter β for non-homogeneity parameter μ = 0.5, aspect ratio a/b = 1, two different values of taper parameter α = -0.5, 0.5, foundation parameters K = 0.0, 0.02 and G = 0.0, 0.002 for both the C-C and C-S plates vibrating in the first mode. The frequency parameter Ω is found to decrease with the increasing values of density parameter β keeping other parameters fixed. The rate of decrease of frequency parameter Ω with β increases with the increase in the values of taper parameter α, foundation parameters K and G. This rate of decrease is smaller for a C-S plate as compared to a C-C plate. Figures 5(b) and (c) show the similar behavior for the second and third modes of vibration for both the plates. However, the rate of decrease in frequency parameter Ω increases with the increase in the number of modes.

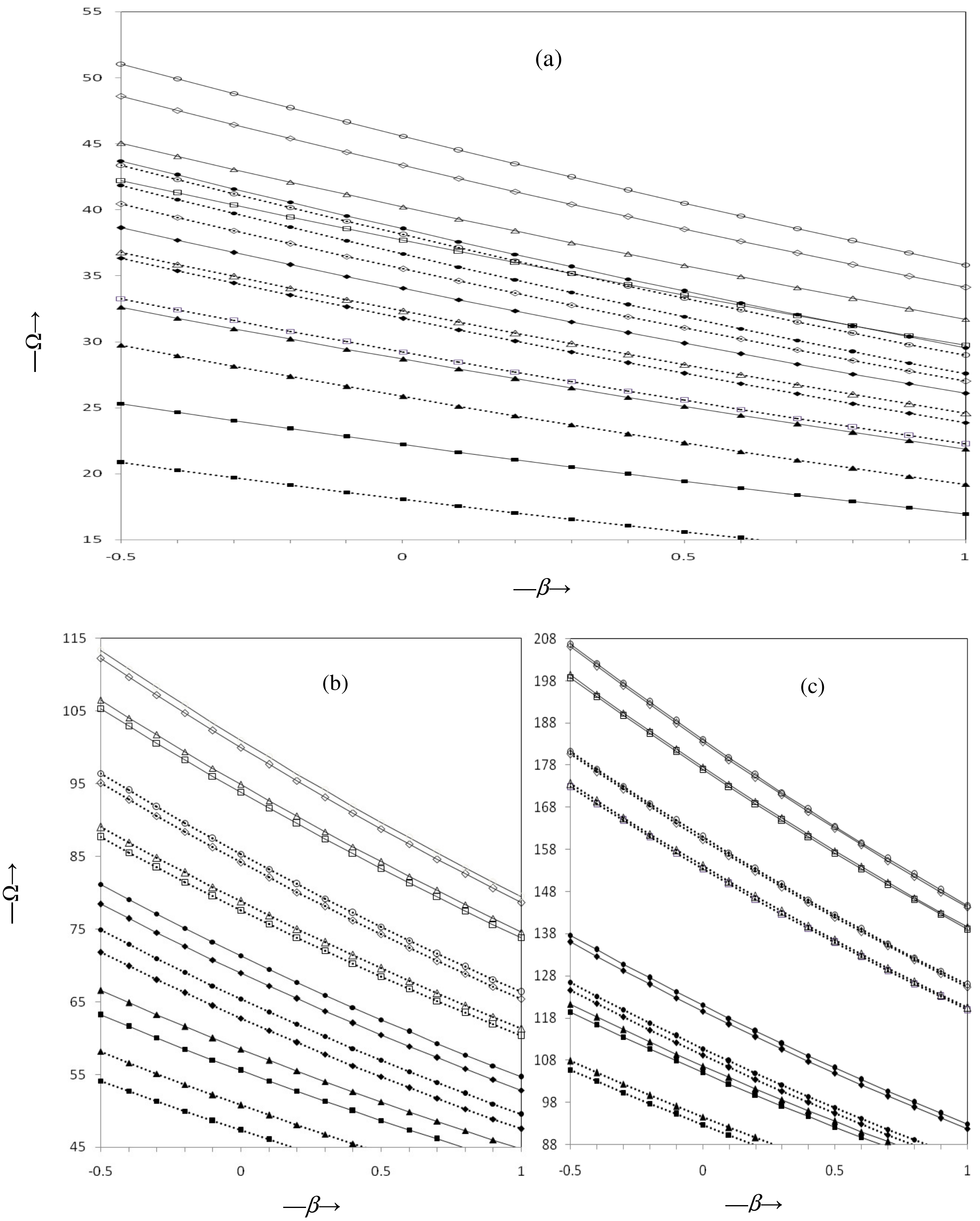
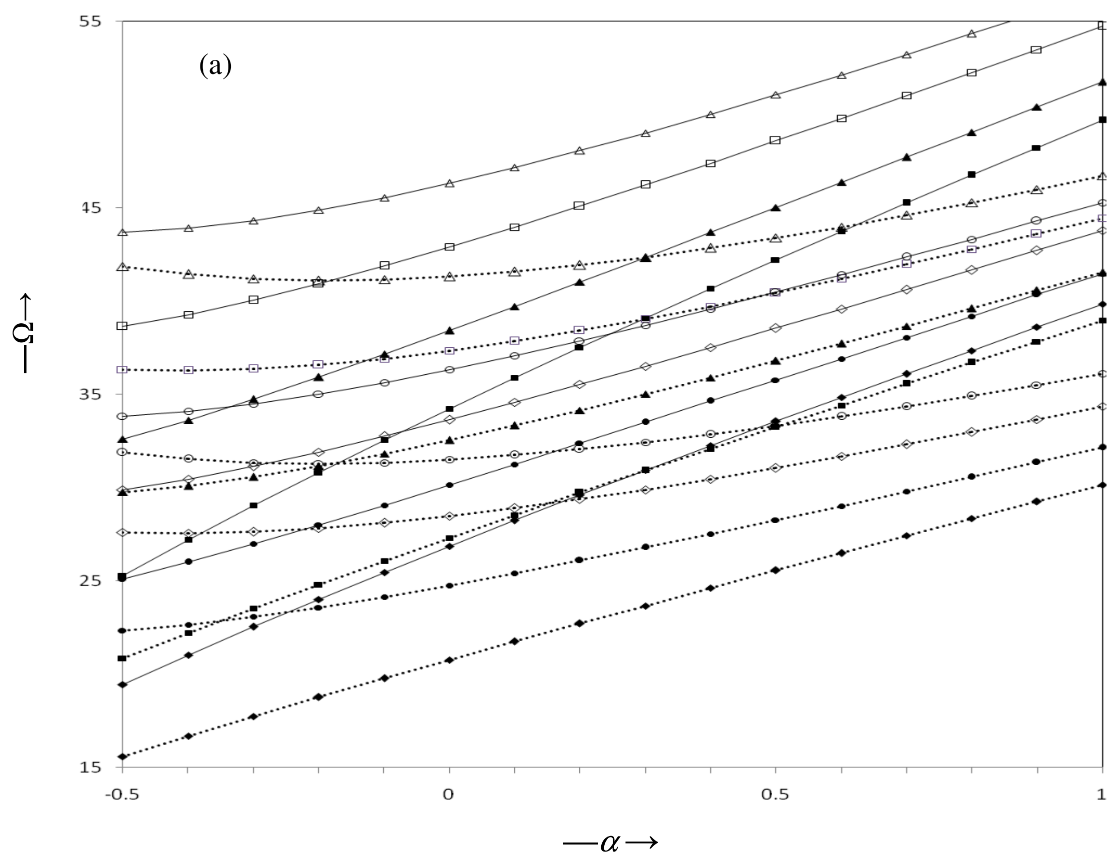


Figure 5: Frequency parameter for C-C and C-S plates vibrating in (a) first mode (b) second mode and (c) third mode, for $\mu = 0.5$, $a/b = 1$. ————, C-C; ------------, C-S; ■, □, $\alpha = -0.5$, K=0.0 ; ♦,◊, $\alpha = 0.5$, K=0.0; ▲, △, $\alpha = -0.5$, K=0.02; •,o, $\alpha = 0.5$, K=0.02. ■, ♦, ▲, •, G=0.0; □,◊, △, o, G=0.002.

The effect of taper parameter α on the frequency parameter Ω has been shown in Figure 6 (a) for two different values of density parameter β = -0.5, 0.5, Winkler stiffness K = 0.0, 0.02, shear stiffness G = 0.0, 0.002, μ = 0.5 and a/b = 1 for C-C and C-S plates vibrating in first mode. It is found that in the absence of foundation the frequency parameter Ω increases with the increasing values of taper parameter α for both the plates. However, in presence of foundation and β = -0.5,0.5, the frequency parameter Ω first decreases and then increases with the increasing values of taper parameter α giving rise to a case of local minima. In particular, for β = -0.5, K = 0.02, G = 0.002 there is a local minima in the vicinity of α = -0.2 for C-S plate. A case of local minima is also observed in the vicinity of α = -0.1 for β = 0.5, K= 0.02 and G = 0.002 in case of C-S plate. In case of second mode of vibration Figure 6 (b) shows that the frequency parameter Ω decreases with the increasing values of density parameter β for both the plates irrespective of the other plate parameters. The rate of increase of frequency parameter Ω increases with the increasing values of density parameter β but it decreases with the increase in the values of foundation parameters. In case of third mode of vibration Figure 6(c), the behavior is similar as that of second mode of vibration. Also, the rate of increase of Ω with α increases with the increase in number of modes.



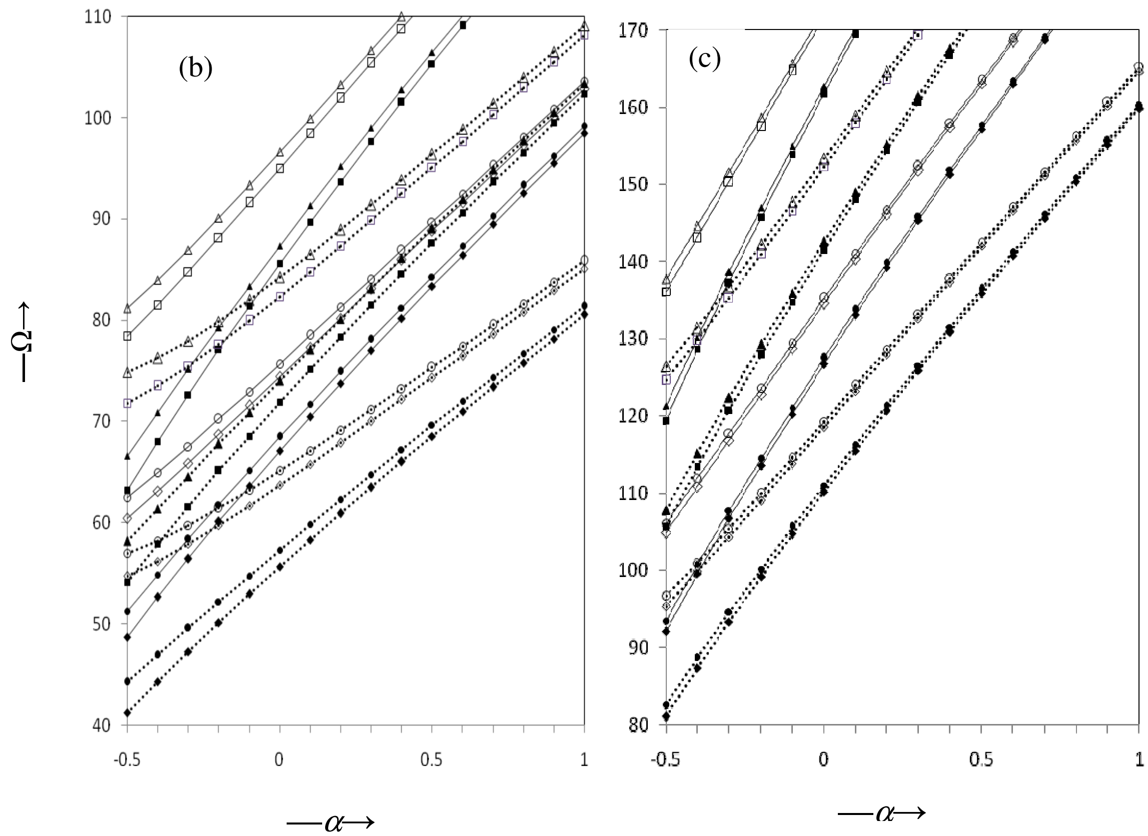


Figure 6: Frequency parameter for C-C and C-S plates vibrating in (a) first mode (b) second mode and (c) third mode for μ = 0.5, a/b = 1. ————, C-C; ----------, C-S; ■, □, β = -0.5, K=0.0; ♦,◊, β = 0.5, K=0.0; ▲, △, β = -0.5, K=0.02; •,o, β = 0.5, K=0.02. ■, ♦, ▲, •, G=0.0; □,◊, △, o, G=0.002.

Figure 7(a) shows the plots of the frequency parameter Ω versus Winkler foundation parameter K for two different values of taper parameter α = -0.5, 0.5, non-homogeneity parameter μ = -0.5, 0.5, Pasternak foundation parameter G = 0.00, 0.002, β = 0.5 and a/b=1 for C-C and C-S plates vibrating in first mode. It is found that the frequency parameter Ω increases with the increasing values of K. The rate of increase of frequency parameter Ω with K is smaller for a C-C plate as compared to a C-S plate keeping other parameters fixed. For second and third modes of vibration, Figures 7(b) and (c) show the similar inferences for both the plates. The rate of increase of Ω with K decreases with the increasing number of modes.

Figures 8(a-c) depict the behavior of frequency parameter Ω versus Pasternak foundation parameter G for two different values of taper parameter α = -0.5, 0.5 non-homogeneity parameter μ = -0.5, 0.5, Winkler foundation parameter K = 0.0, 0.02, density parameter β = 0.5 and a/b = 1 for C-C and C-S plates vibrating in first, second and third mode respectively. The frequency parameter Ω increases with the increasing values of foundation parameter G. The rate of increase of frequency parameter Ω with G is higher for a C-C plate than that for a C-S plate. Also, the rate of increase in Ω with Pasternak parameter G increases with the increase in number of modes.

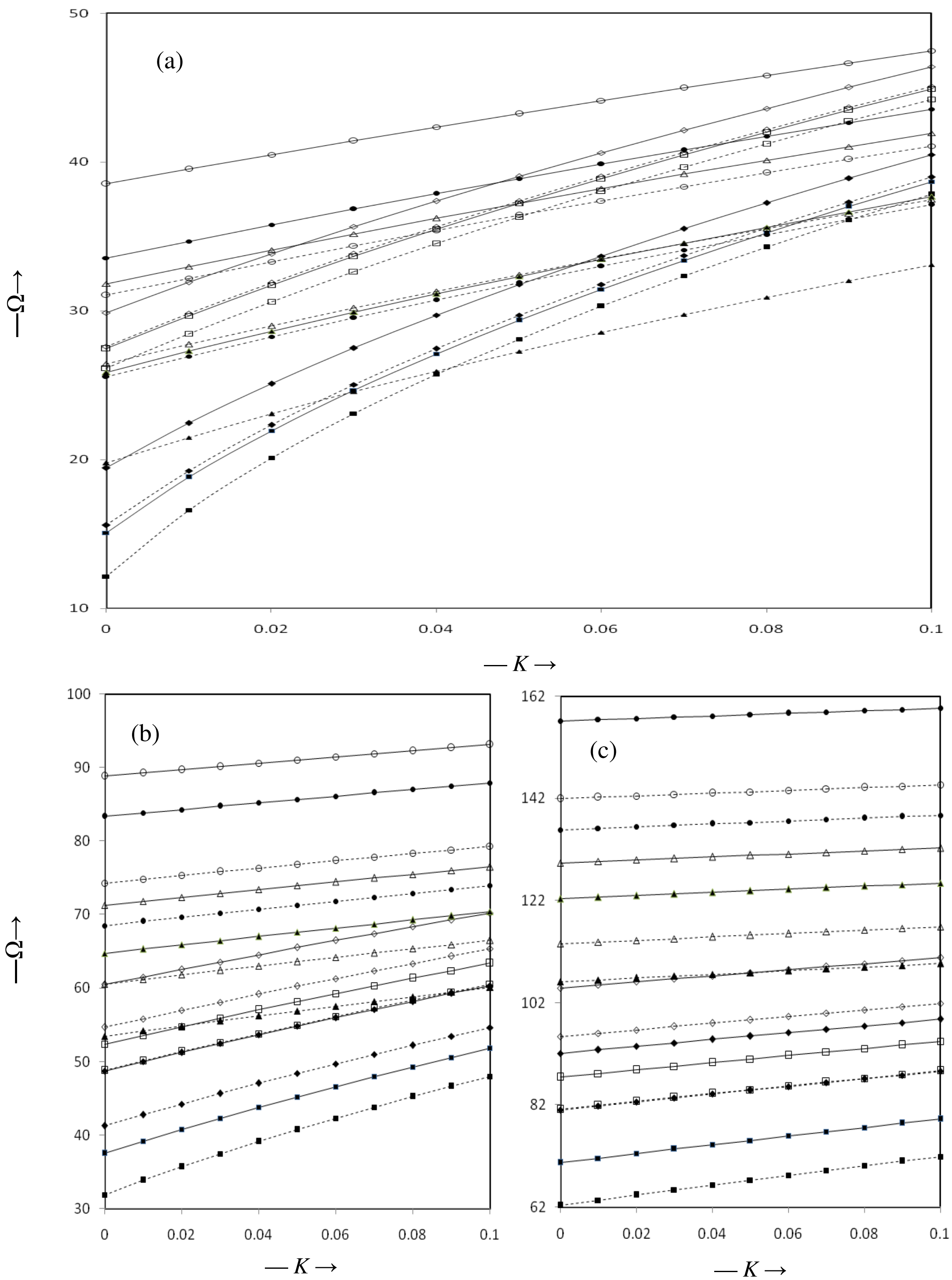


Figure 7: Frequency parameter for C-C and C-S plates vibrating in (a) first mode (b) second mode and (c) third mode, for $\beta = 0.5$, $a/b = 1$. ————, C-C; ----------, C-S; ■, □, $\alpha = -0.5$, $\mu = -0.5$; ♦,◊, $\alpha = -0.5$, $\mu = 0.5$; ▲, △ , $\alpha = 0.5$, $\mu = -0.5$; •,o, $\alpha = 0.5$,

μ = 0.5. ■, ♦, ▲, •, G=0.0; □, ◊, △, o, G=0.002.

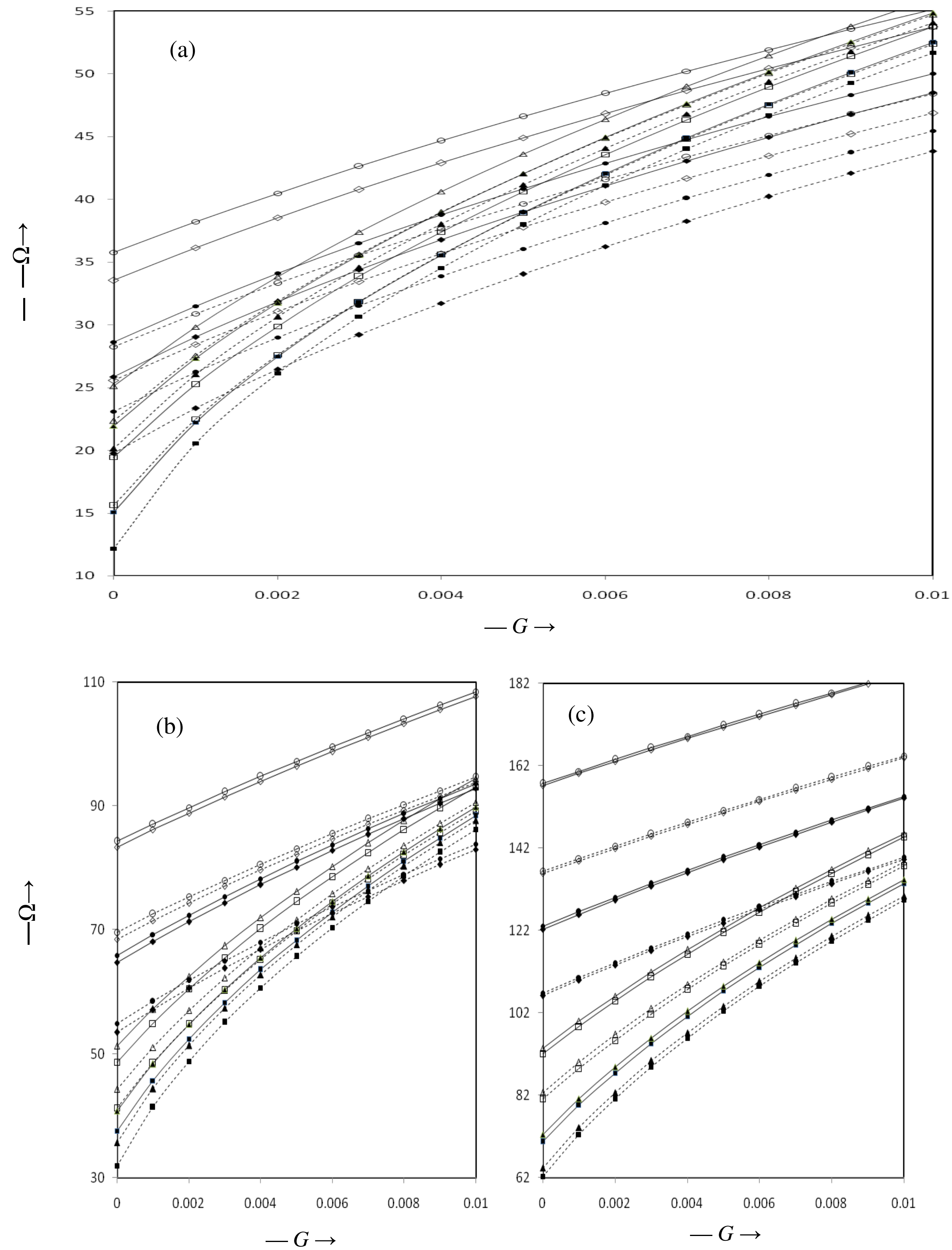


Figure 8: Frequency parameter for C-C and C-S plates vibrating in (a) first mode (b) second

mode and (c) third mode, for β = 0.5,*a/b* = 1. ————, C-C; ----------, C-S; ■, □, α = -0.5, *K*=0.00; ♦,◊, α = 0.5, *K*=0.00; ▲, △, α = -0.5, *K*=0.02; •,o, α = 0.5, *K*=0.02. ■, ♦, ▲, •, μ = -0.5; □,◊, △, o, μ = 0.5.

Figures 9(a, b) show the plots for normalised transverse displacement for a square plate i.e. *a*/*b* = 1 have been computed for two values of non-homogeneity parameter μ = -0.5, 0.5, density parameter β = -0.5, 0.5, α = 0.5, *K*=0.02 and *G* = 0.001 for the first three modes of vibration for C-C and C-S plates, respectively. The nodal lines are seen to shift towards the edge *X* = 0, as β increases from -0.5 to 0.5. The radii of nodal circle decrease as μ increases from -0.5 to 0.5.

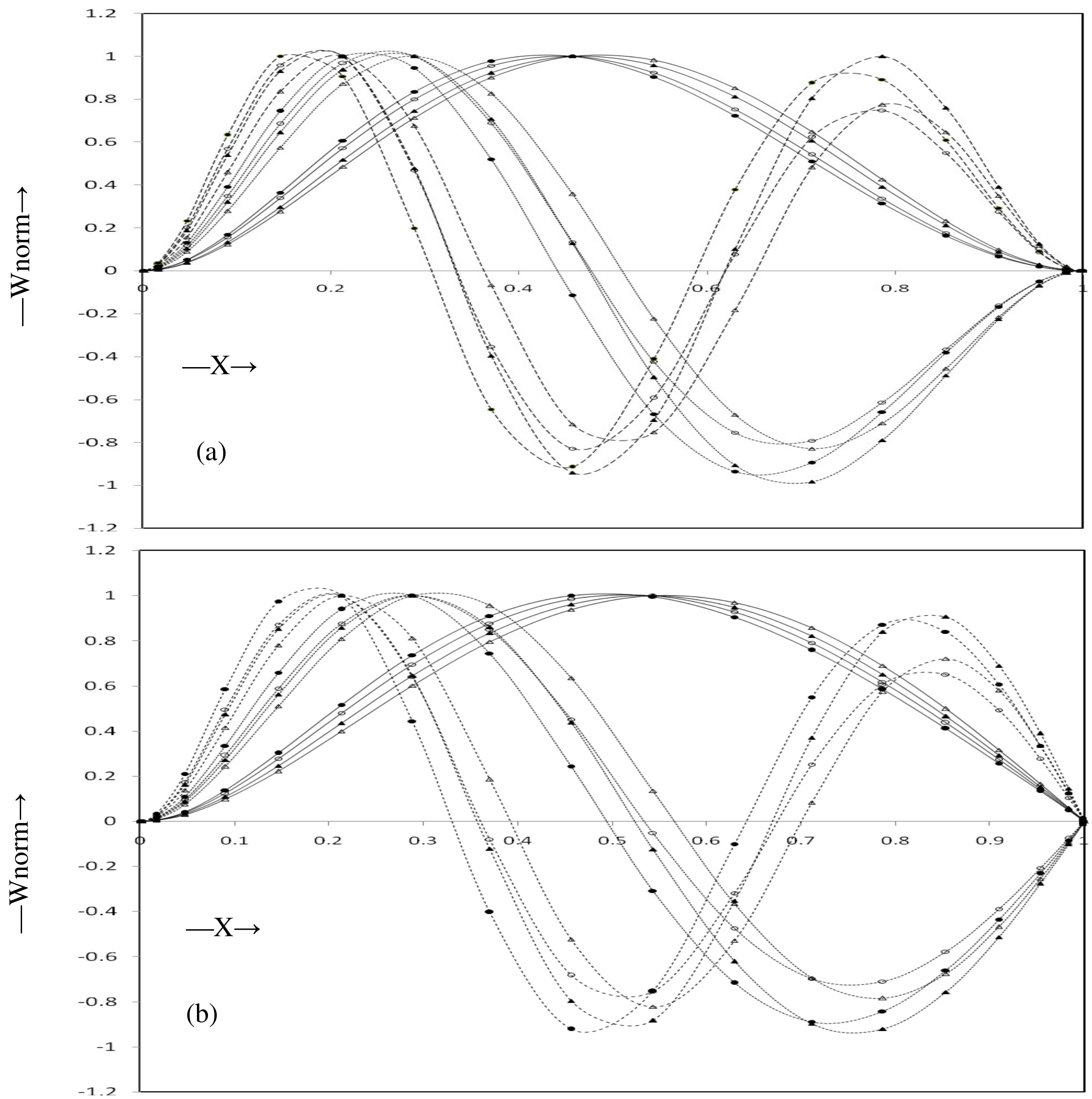


Figure.9: Normal displacements for the first three modes of vibration for (a) C-C plate and (b) C-S plate, for *a/b*=1.0, α =0.5, *K* = 0.02, *G* =0.001. ——— , first mode;

…….., second mode; ----------, third mode; ▲, △, μ = -0.5; ●,○, μ = 0.5;
▲, ●, β = -0.5; △,○, β = 0.5.

In the absence of foundation ($K = G = 0$), a comparison of results for homogeneous ($\mu = \beta = 0$), isotropic (E_2/E_1=1) plates with other methods has been presented in Tables 1 and 2 for υ = 0.3. Table 1 shows a comparison of results for plates of uniform (α = 0.0) thickness with exact solutions [1] and with those obtained by differential quadrature method [10], quintic splines [14] and Frobènius method [38] for two values of aspect ratio a/b = 0.5 and 1.0. As a special case for p = 2 the results have been computed and compared with those obtained by quintic splines [14] and Frobènius method [38] for a/b = 0.5, 1 and given in Table 2. The comparison shows a very good agreement of the results.

Table 1: Comparison of frequency parameter Ω for isotropic (E_2/E_1=1), homogeneous ($\mu = \beta = 0$), and uniform (α = 0) C-C and C-S plates for υ= 0.3.

Boundary Condition	**Mode**	***K* = 0.0** *a/b* = 0.5		***K* = 0.0** *a/b* = 1	***K* = 0.01** *a/b* = 0.5		***K* = 0.01** *a/b* = 1
	I	**23.815**		**28.950**	**26.2142**		**30.954**
C-C		23.820[a]		28.950[a]	26.219[a]		30.953[a]
		—		28.946[b]	—		—
		23.816[c]		28.951[c]	—		—
		23.820[d]		28.960[d]	—		—
	II	**63.5345**		**69.3270**	**64.4720**		**70.1872**
		63.603[a]		69.380[a]	64.479[a]		70.189[a]
		—		69.320[b]	—		—
		63.535[c]		69.327[c]	—		—
C-S	**I**	**17.3318**		**23.6363**	**20.5034**		**26.0605**
		17.335[a]		23.647[a]	20.506[a]		26.061[a]
		—		23.646[b]	—		—
		17.332[c]		23.646[c]	—		—
	II	**52.0979**		**58.6464**	**53.2372**		**59.6607**
		52.150[a]		58.688[a]	53.237[a]		59.702[a]
		—		58.641[b]	—		—
		52.097[c]		58.646[c]	—		—

a. Values by quintic splines technique from Lal and Dhanpati [14].
c. Exact values from Liessa [1].
d. b.Values by Frobénius method from Jain and Soni [38].
e. Values by differential quadrature method from Gutierrez and Laura [10].

Table 2: Comparison of frequency parameter Ω for homogeneous (μ=β=0) and isotropic (E_2/E_1) and uniform ($\alpha = 0$) C-C and C-S Plates for $p = 2$, and $\nu = 0.3$.

Mode	C-C			C-S	
	Values of *a/b*			Values of *a/b*	
	0.5	1.0		0.5	1.0
I	**28.9502** 28.9499[a] 28.9508[b]	**54.7431** 54.7312[a] 54.7430[b]		**23.6463** 23.6468[a] 23.6463[b]	**51.6743** 51.6700[a] 51.6742[b]
II	**69.3270** 69.3796[a] 69.3270[b]	**94.5853** 94.5927[a] 94.5852[b]		**58.6464** 58.6880[a] 58.6463[b]	**86.1345** 86.1493[a] 86.1344[b]
III	**129.0956** 129.3675[a] 129.0956[b]	**154.7757** 154.9509[a] 154.7757[b]		**113.8281** 113.4541[a] 113.2281[b]	**140.8456** 141.0035[a] 140.8455[b]

a. Values by quintic splines Lal and Dhanpati (2007).
b. Values taken from Jain and Soni (1973).

6. Conclusion

The effects of non-homogeneity which is presumed to arise due to variation in Young's moduli and density on natural frequencies of rectangular orthotropic plates of linearly varying thickness resting on two-point parameter foundation (Pasternak foundation) have been studied on the basis of classical plate theory. It is observed that frequency parameter Ω increases with the increase in non-homogeneity parameter μ and aspect ratio *a/b* keeping other plate parameters fixed. Further Ω is found to decrease with the increasing value of density parameter β keeping all other plate parameters fixed for all the three boundary conditions. However, its behaviour with taper parameter α is not monotonous. The frequency parameter Ω is found to increase with the increase in foundation parameters (*K* and *G*) for all the boundary conditions in all the modes considered here. The results will help to design engineers to have desired natural frequency by proper choice of plate parameters.

References

[1] Leissa, A. W., Vibration of Plates: *Washington: office of Technology Utilization. SP, NASA,* 1965.

[2] Leissa, A. W., *Recent research in plane vibrations, Complicating effects. Shock and Vibration Digest,* 9 (1), 21-35, 1977.

[3] Leissa, A. W., Recent research in plane vibrations, 1973-1976, *Complicating effects. Shock and Vibration Digest,* 10 (12), 21-35, 1978.

[4] Leissa, A. W., *Plate vibration research,* 1976-1980, (9),11-22, 1981a.

[5] Leissa, A. W., *Plate vibration research,* 13 (10), 19-36, 1981b.

[6] Leissa, A. W., Recent studies in plate vibrations: part I, Classical theory. *The Shock and Vibration Digest,* 19 (2), 10-24, 1987a

[7] Leissa, A. W., Recent studies in plate vibrations: part II, Complicating effects. *The Shock and Vibration Digest,* 19 (3), 10-24, 1987b.

[8] Rao, G. V., Rao, B. P. and Raju, L. S., Vibrations of inhomogeneous thin plates using a high- precision triangular element. *Journal of Sound and Vibration,* 34(3), 444 – 445, 1974.

[9] Tomar, J. S., Gupta, D. C. and Jain, N. C., Free vibrations of an isotropic non-homogenous infinite plate of linearly varying thickness. *Meccanica, Journal of Italian Association of Theoretical and Applied Mechanics AIMETA,* 18, 30-33, 1983

[10] Gutierrez, R. H. and Laura, P. A. A., Vibrations of rectangular plates with Linearly varying thickness and non-uniform boundary conditions. *Journal of Sound and Vibration,* 178 (4), 563-566, 1994

[11] Lal, R., Gupta, U. S. and Reena, Quintic splines in the study of transverse vibrations of non- uniform orthotropic rectangular plates. *Journal of Sound and Vibration,* 207(1), 1-13, 1997.

[12] Lal, R., Gupta, U.S and Goel, C., Chebyshev polynomials in the study of transverse Vibration of non-uniform rectangular plates. *The Shock and Vibration Digest,* 33 (2), 103-112, 2001.

[13] Gupta, U. S., Lal, R. and Sharma, S., Vibration analysis of non-homogeneous circular plate of non-linear thickness variation by differential quadrature method. *Journal of Sound and Vibration,* 298 (4-5), 892-906, 2006.

[14] Lal, R. and Dhanpati, Transverse vibrations of non-homogeneous orthotropic rectangular plates of variable thickness: A spline technique. *Journal of Sound and Vibration,* 306 (1-2), 203-214, 2007.

[15] Sharma, S., Gupta, U. S and Singhal, P., Vibration analysis of non- homogeneous Orthotropic rectangular plate of variable thickness resting on Winkler foundation, *Journal of Applied Sciences and Engineering,* 15 (3), 291-300, 2012.

[16] Gupta, U. S., Sharma, S. and Singhal, P. (2012). Numerical simulation vibration of non-homogeneous plates of variable thickness. *International Journal of Engineering and Applied Sciences,* 4(4), 26 - 40, 2012.

[17] Selvadurai, A. P. S., Elastic Analysis of Soil-Foundation Interaction. *Elsevier, NY,* 1979.

[18] Kerr, A. D., Elastic and viscoelastic foundation models. *ASME Journal of Applied Mechanics,* 31, 491-498, 1964.

[19] Hetenyi, M., Beams and plates on elastic foundation and related problems. *Applied Mechanics Reviews,* 19, 95-102, 1966.

[20] Yingshi, Z., Vibration of stepped rectangular thin plates on Winkler foundation. *Applied Mathematics and Mechanics,* 20(5), 568-578, 1972.

[21] Raju, K. K. and Rao, G. V., Effect of elastic foundation on the mode shapes in stability and vibration problems of simply supported rectangular plates. *Journal of Sound and Vibration,* 139(1), 170-173, 1990.

[22] Turvey, G. J., Uniformly loaded, simply supported, antisymmetrically laminated, rectangular plate on a Winkler-Pasternak foundation. *International Journal of and Structures,* 13, 437-444, 1977.
[23] Kerr, A. D., Elastic and viscoelastic foundation models. *ASME Journal of Applied Mechanic*, 31, 491-498, 1964.
[24] Katiskadelis, J. T. and Kallivokas, L. F., Clamped plates on Pasternak-type elastic foundation by the boundary element method. *Journal of Applied Mechanics Division, ASME,* 53, 909-917, 1986.
[25] Wang, C. W., Wang, C. and Ang, K. K., Vibration of initially stressed Reddy Plates on a Winkler-Pasternak foundation. *Journal of Sound and Vibration,* 204(2), 203-212, 1997.
[26] Wang, J. G., Wang , X. X . and Haung, M. K., Fundamental solutions and Boundary integral equations for Reissner's plates on two parameter foundations. *International Journal of Solids and Structures,* 29, 1233-1239, 1992.
[27] Xiang, Y., Wang, C. M. and Kitipornchai, S., Exact vibration solution for initially stressed Mindlin plates on Pasternak foundations. *International Journal of Mech. Science,* 36 (4), 311-316, 1994.
[28] Shen, H. S., Postbuckling of orthotropic plates on two-parameter elastic foundation, *Journal of Engineering Mechanics,* 121, 50-56, 1995.
[29] Teo, T. M. and Liew, K. M., Differential cubature method for analysis of shear deformable rectangular plates on Pasternak foundation. *International Journal of Mechanical Sciences,* 44, 1179-1194, 2002.
[30] Omurtag, M. H. and Kadioglu, F., Free vibration analysis of orthotropic plates resting on Pasternak foundation by mixed finite element formulation. *Computers and Structures,* 67 (4), 253-265, 1998.
[31] Malekzadeh, P. and Karami, G., Vibration of non-uniform thick plates on elastic foundation by differential quadrature method. *Engineering Structures,* 26, 1473-1482, 2004.
[32] Leung, A. V. T. and Zhu, B., Transverse vibration of Mindlin plates on two- parameter foundation by analytical trapezoidal p-elements. *Journal of Engineering Mechanics,* 131(11), 1140-1145, 2005.
[33] Lal, R. and Dhanpati., Effect of non-homogeneity on vibration of orthotropic rectangular plates of varying thickness resting on Pasternak Foundation. *ASME Journal of Vibration and Acoustics,* 131, 1-9, 2009.
[34] Biancolini, M. E., Brutti, C. and Reccia, L., Approximate solution for free vibration of thin orthotropic rectangular plates. *Journal of Sound and Vibration,* 228, 321-344, 2005.
[35] Lekhnitskii, S. G., Anisotropic plates. Translated by S.W. Tsai and T. Cheron, *Gorden and Breach,* New York, 1968.
[36] Panc, V., Theories of Elastic Plates. *Noordhoff International Publishing, Leydon, the Netherlands,* 1975.
[37] Shu, C., Differential quadrature and its application in Engineering. *Springer-Verlag, Great-Briatain,* 2000.
[38] Shu, C., Differential quadrature and its application in Engineering. *Springer-Verlag, Great-Briatain,* 2000.
[39] Jain, R. K. and Soni, S. R., Free vibration of rectangular plates of parabolically varying thickness. *Indian Journal of Pure and Applied Mathematics,* 4 (3), 267-277, 1973.