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DYNAMIC RESPONSE OF THICK PLATES ON TWO PARAMETER ELASTIC FOUNDATION UNDER TIME VARIABLE LOADING

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Abstract

In this paper, behavior of foundation plates with transverse shear deformation under time variable loading is presented using modified Vlasov foundation model. Finite element formulation of thick plates on elastic foundation is derived by using an 8-noded finite element based on Mindlin plate theory. Selective reduced integration technique is used to avoid shear locking problem which arises when smaller plate thickness is considered for the evaluation of the stiffness matrices. After comparisons are made with the results given in the literature, the effects of the ratio of plate thickness to shorter span of the plate, aspect ratio, subsoil depth, loaded area, time variable loading type and pulse duration of impulsive load on its responses are analyzed. The results demonstrate that these parameters have significant influence on the dynamic behavior of the plate on elastic foundation.

Keywords: Finite element method; Thick plate; Vlasov model; Time variable loading

1. Introduction

The dynamic analysis of soil-structure interaction problem such as beams or plates on elastic foundation is finding wide application in many engineering field because the solutions of dynamic analysis are guide for engineers in the structural designs. Among the various plate theories available today, the mostly used is the Kirchhoff plate theory where transverse shear deformation effects are neglected. But, the Kirchhoff plate theory is not valid for higher plate thickness. Mindlin plate elements include the effects of transverse shear deformation which became important as the ratio of plate thickness to shorter span increases. These elements based on Mindlin plate theory can be used for the analysis of the thin and the thick plates. However, the stiffness matrix obtained from Mindlin plate elements becomes too stiff and gives zero solutions if the thickness of plate is quite small. This phenomenon is called shear locking problem. Reduced or selective reduced integration techniques are recommended to avoid this problem in many studies. On the other hand the subsoil is usually represented by spring elements in the soil-structure interaction problems. The springs are assumed to be discrete in Winkler model and any interaction between the springs is ignored. However there is not only pressure but also moments or rotations at the point of contact between structure and foundation. For this reason more realistic foundation model is needed for more accurate analysis. Therefore a number of idealized foundation models which represent relationship of soil structure interaction in realistic manner have been developed since the soil exhibits a very complex behavior. Two parameter models such as Pasternak and Vlasov Model are derived as an extension of the Winkler model by assuming the interaction between the spring elements. But, there is not any certain consensus about determination of the soil parameters in two parameter models. Modified Vlasov

Model is used in this study since it leads to more accurate results because a shear interaction between the springs is included in the model and soil parameters are calculated using an iteration technique. The main advantages of Modified Vlasov model is that it considers interaction between the springs and determines the soil parameters depending on the properties of subsoil, loading and surface displacements [1].

In recent years, a lot of studies concerning analysis of plates on elastic foundation are performed. Omurtag et al. [2] developed a mixed finite element formulation based on Gateaux Differential for free vibration analysis of Kirchhoff plates on elastic foundation. Shen et al. [3] performed the free and forced vibration analysis of Reissner-Mindlin plates resting on Pasternak-type elastic foundation. Huang and Thambiratnam [4] studied dynamic response of plates on elastic foundation subjected to moving loads and the investigated effects of velocity, subgrade reaction, moving path and distance between multiple moving loads on responses. Sun [5] derived a closed-form solution of dynamic response of a Kirchhoff plate on a viscoelastic foundation subjected to impulse and harmonic circular loads. Malekzadeh and Karami [6] presented a differential quadrature solution for free vibration analysis of thick plates with continuously varying thickness on two parameter elastic foundation. Celep and Güler [7] studied the static behavior and forced vibration of a rigid circular plate supported by a tensionless Winkler elastic foundation by assuming that the plate is subjected to a uniformly distributed load and a vertical load having an eccentricity. Eröz and Yildiz [8] presented a finite element formulation of forced vibration problem of a prestretched plate resting on rigid foundation. Yu et al. [9] presented the dynamic response of Reissner-Mindlin plate resting on an elastic foundation of the Winkler-type and Pasternak-type using an analytical-numerical method. Wen and Aliabadi [10] used boundary element method for the analysis of Mindlin plates on elastic foundation subjected to dynamic load using Winkler and Pasternak Model. Motaghian et al. [11] studied free vibration problem of thin rectangular plates on Winkler and Pasternak elastic foundation model which is distributed over a particular arbitrary area of the plate. Kutlu et al. [12] presented a method of analysis for investigating the dynamic response behavior of Mindlin plates resting on arbitrarily orthotropic two parameter foundation and partially in contact with a quiescent fluid on its other side. Özdemir [13] developed 17 noded finite element for shear locking free analysis of thick plates resting on elastic foundation using Winkler Model.

In this paper; 8-noded Mindlin plate elements are adopted for the analysis of thick plates resting on Modified Vlasov foundation under time variable loading. For this purpose a computer program is coded in Fortran. Newmark-β method is used for time integration. Selective reduced integration technique is used to obtain stiffness matrix of plates. The effects of the ratio of plate thickness to shorter span, the aspect ratio, the subsoil depth, loaded area, time variable loading type and pulse duration of the impulsive load on its responses are analyzed.

2. Modified Vlasov Model

For a plate on two parameter elastic foundation as shown in Fig.1, if the assumptions with the displacements of the soil $\overline{u}, \overline{v}$ and $\overline{w}$ in *x*, *y* and *z* direction respectively

$$\overline{u}(x, y, z) = 0 \tag{1}$$

$$\overline{v}(x, y, z) = 0 \tag{2}$$

and

$$\overline{w}(x, y, z) = w(x, y)\phi(z) \tag{3}$$

are made, total potential energy of the plate-soil system can be written as

$$\Pi = \frac{1}{2}\int_{\Omega}\left(\frac{\partial^2 w}{\partial x^2},\frac{\partial^2 w}{\partial y^2},2\frac{\partial^2 w}{\partial x\partial y}\right)[D]\left(\frac{\partial^2 w}{\partial x^2},\frac{\partial^2 w}{\partial y^2},2\frac{\partial^2 w}{\partial x\partial y}\right)^T dxdy$$
$$+\frac{1}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left\{kw^2+2t(\nabla w)^2\right\}dxdy-\int_{\Omega} qwdxdy \tag{4}$$

where w, $[D]$, q are displacement of the plate in z direction, the flexural rigidity of the plate and load on plate respectively. k and $2t$ in above expression are the soil parameters and may be defined as

$$k=\int_0^H \frac{E_s(1-\nu_s)}{(1+\nu_s)(1-2\nu_s)}\left(\frac{\partial\phi}{\partial z}\right)^2 dz$$
$$2t=\int_0^H \frac{E_s}{2(1+\nu_s)}\phi^2 dz \tag{5}$$

Where H is the subsoil depth, E_s is modulus of elasticity of the subsoil, and ν_s is the Poisson ratio of the subsoil. $\phi(z)$ is the mode shape function that gives the variation of the deflection in the vertical direction. The boundary conditions of mode shape function $\phi(z)$ are $\phi(0)=1$ and $\phi(H)=0$.

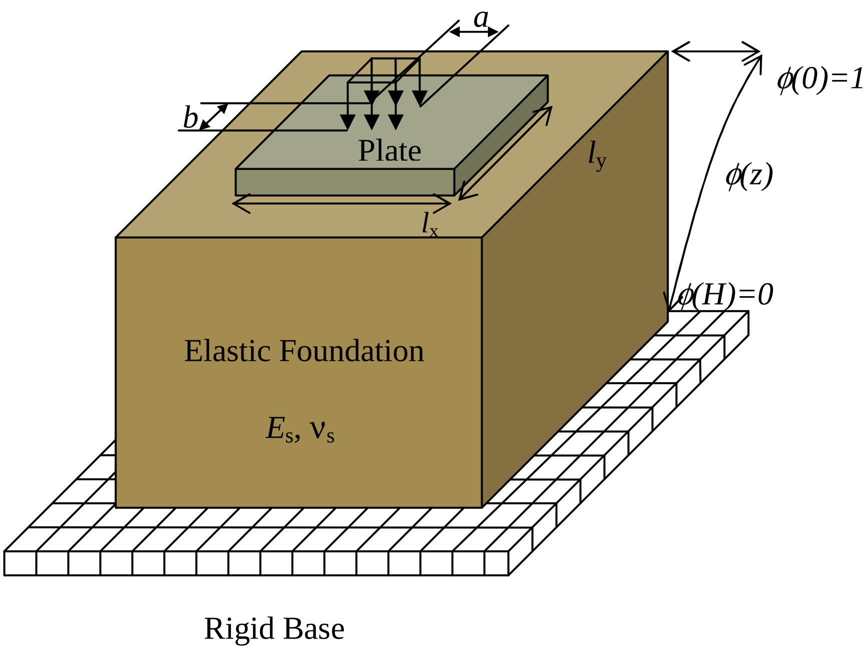


Fig. 1 A plate resting on an elastic foundation

The main field equation of the plate on an elastic foundation can be written as follows

$$D\nabla^4 w-2t\nabla^2 w+kw=q \tag{6}$$

where ∇^2 is the Laplace and ∇^4 is the biharmonic operators. The mode shape function can be expressed as

$$\phi(z) = \frac{\sinh \gamma (1 - \frac{z}{H})}{\sinh \gamma} \tag{7}$$

where γ represents the vertical deformation parameter within the subsoil and is calculated using the equation shown below.

$$\left(\frac{\gamma}{H}\right)^2 = \frac{1-2\nu_s}{2(1-\nu_s)} \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (\nabla w)^2 dxdy}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} w^2 dxdy} \tag{8}$$

As seen from the Eq. (5) and Eq. (8) the modulus of subgrade reaction, *k*, and the second parameter *2t* representing the shear deformation of the soil, are both dependent on the vertical deformation function ϕ and the depth of the soil *H* whereas the value of γ varies with the displacement of the plate and the depth of the subsoil. The parameter γ can be evaluated after determining *w*(x,y) which satisfies differential equation below and around of the plate. It is obvious that the solution technique for γ parameter have to be an iterative procedure. More details for the modified Vlasov model can be found in references [14] and [15].

3. Finite Element Modelling

According to finite element method, the general equation of motion for the plate-soil system is given by

$$[M]\{\ddot{w}\} + [K]\{w\} = \{F\} \tag{9}$$

where [*K*] is the stiffness matrix of the plate-soil system, [*M*] is the mass matrix of the plate-soil system, {*F*} is the applied load vector, w and $\ddot{w}$ are the displacement and acceleration vector of the plate, respectively. In this study the Newmark-β method is used for the time integration of Eq. (9) by using the average acceleration method [16]. Evaluation of the stiffness and mass matrices are given in the following sections for a plate resting on an elastic foundation.

3.1. Evaluation of the Stiffness Matrix

An 8-noded rectangular finite element based on Mindlin theory is used to develop the element stiffness matrices (Fig.2).

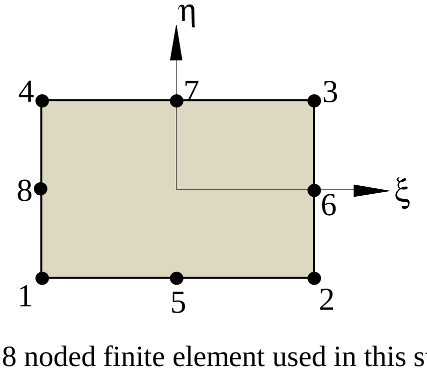


Fig. 2 The 8 noded finite element used in this study

Nodal displacements at each node are

$$w, \varphi_x, \varphi_y \tag{10}$$

where w is the transverse displacement, φ_x, φ_y are the rotations of the normal to the undeformed middle surface. It is assumed that w, φ_x and φ_y varies quadratically over the element so that

$$\begin{aligned} u &= z\varphi_y = z\sum_{i=1}^{n} N_i \varphi_{yi} \\ v &= -z\varphi_x = -z\sum_{i=1}^{n} N_i \varphi_{xi} \\ w &= \sum_{i=1}^{n} N_i w_i \end{aligned} \tag{11}$$

the displacement shape functions are given as

$$[N_i] = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & . & . & . & N_8 & 0 & 0 \end{bmatrix} \tag{12}$$

The shape functions, N_i, are given in [17] and [18].

The stiffness matrices of the plate-soil system can be evaluated as

$$U = \frac{1}{2}\{w_e\}^t \left([k_p] + [k_w] + [k_{2t}] \right) \{w_e\} \tag{13}$$

where $[k_p]$, $[k_w]$ and $[k_{2t}]$ are stiffness matrix of the plate, vertical deflection element stiffness matrix of the foundation and shear deformation element stiffness matrix of the foundation, respectively. $\{w_e\}$ is the nodal displacement vector for an element containing 24 components.

In this study, the selective reduced integration rule on the shear terms is used to obtain the element stiffness matrix of the plate $[k_p]$ to avoid shear locking problem under the thin plate limit. The number of points of integration for shear energy terms of plate stiffness matrix are reduced 2x2 in the selective reduced integration whereas 3x3 of Gauss points are used for both bending and shear energy terms of plate stiffness matrix in the full integration. The element stiffness matrices are given in explicit forms by reference [19] for plate element and by reference [20] for the vertical deflection of foundation and the shear deformation of foundation.

Boundary conditions need to be applied before solving the system of equations. The effect of the infinite soil domain outside the plate is applied as equivalent stiffness parameters on the plate boundary in the modified Vlasov model. Equivalent forces due to surrounding soil domain on the boundary of the plate are computed as a function of the displacement on the boundary [15 and 20].

3.2. Evaluation of the Mass Matrix

According to Hamilton's variational principle, the total kinetic energy of the plate-soil system may be written as

$$\pi_k = \frac{1}{2}\int_{\Omega} \{\dot{w}\}^T [\mu] \{\dot{w}\} d\Omega \tag{14}$$

where [μ] is the mass density matrix and { $\dot{w}$ } represents the partial derivative of the vector of generalized displacement with respect to time variable. The general formula for the consistent mass matrix, [M], can be written by substituting $w = N_1 w_e$ into Eq. (14)

$$[M] = \int_{\Omega} \{N_1\}^T [\mu] \{N_1\} d\Omega \tag{15}$$

The matrix [μ] in Eq. (15) is a square symmetric matrix of the form

$$[\mu] = \begin{bmatrix} \rho_p h + \frac{1}{3}\rho_s H & 0 & 0 \\ 0 & \frac{1}{12}\rho_p h^3 & 0 \\ 0 & 0 & \frac{1}{12}\rho_p h^3 \end{bmatrix} \tag{16}$$

where ρ_p is the plate density, h is the plate thickness and ρ_s is the mass density of the soil. The following expression can be written for each finite piece

$$[N_1] = \begin{bmatrix} N \\ \dfrac{dN}{dy} \\ \dfrac{dN}{dx} \end{bmatrix}. \tag{17}$$

The consistent mass matrix of the plate and the soil can be evaluated after substituting Eq. (17) into Eq. (15). By assembling the element mass matrix obtained, the system mass matrix is evaluated. The matrices are presented in reference [21].

4. Numerical Examples

4.1. Example 1

At first dynamic responses of a plate on elastic foundation subjected to uniformly distributed load and concentrated load are compared with those of reference [16]. Daloglu et al.[16] analyzed the problem by MZC rectangular element based on Kirchhoff plate theory. In this study 8 nodded rectangular elements based on Mindlin plate theory with selective integration techniques are used.

The properties of the plate-soil system are as follows. The modulus of elasticity of the plate is E_p=27000000 kN/m^2, poisson ratio of the plate is ν_p=0.20, the modulus of elasticity of the subsoil is E_s=20000 kN/m^2, poisson ratio of the plate is ν_s=0.25. The mass densities of the plate and subsoil are taken to be ρ_p=2500 kg/m^3 and ρ_s=1700 kg/m^3 respectively. The uniformly distributed load on the plate is 30 kN/m^2 and concentrated load at the center of the plate is 1000 kN. 6 elements are used for 10 m length and 0.01 s time increments for distributed load and a 0.001 s time increment for concentrated load. The shorter span length of the plate is kept constant at 10 m. The maximum displacements for l_y/l_x=1.0 and 2.0, h=0.5 m and H=5 m has been compared first with the results obtained in reference [16] to verify the accuracy of the present formulation in Table 1. As seen from Table 1 the results obtained in this study for both load cases are in a good agreement with the reference results.

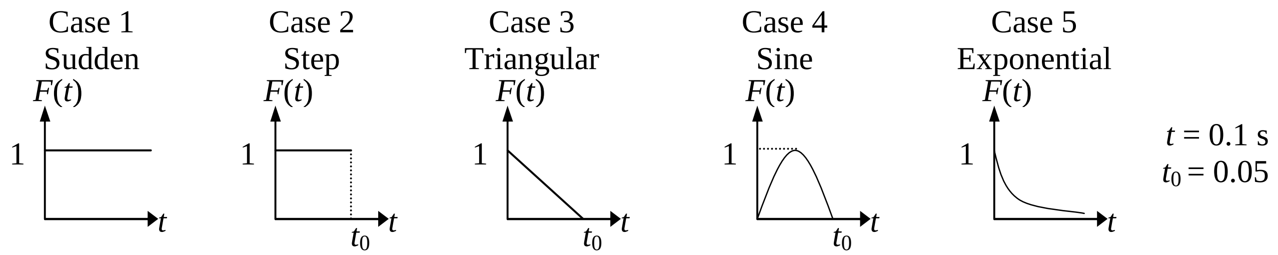
Table 1 Comparison of maximum displacement of plate on elastic foundation

	Distributed Load		Concentrated Load	
	10mx10m plate	10mx20m plate	10mx10m plate	10mx20m plate
	w_{max} (mm)	w_{max} (mm)	w_{max} (mm)	w_{max} (mm)
Ref. [16]	10.170	12.800	6.590	5.800
In this study	10.170	12.840	6.570	5.898

4.2. Example 2

A parametric study is carried out for various values of subsoil depth (H), aspect ratios(l_y/l_x), the ratio of plate thickness to shorter span of the plate (h/l_x), loaded area (a/l_x, b/l_y), time variable loading type and pulse duration of impulsive load. The effects of the parameters mentioned above on maximum displacements and corresponding moments of the plates are presented in graphical form.

The properties of the plate-soil system are as follows. The modulus of elasticity of the plate is E_p=25000000 kN/m^2, poisson ratio of the plate is ν_p=0.20, the modulus of elasticity of the subsoil is E_s=20000 kN/m^2, poisson ratio of the plate is ν_s=0.25. The mass densities of the plate and subsoil are taken to be ρ_p=2500 kg/m^3 and ρ_s=1700 kg/m^3 respectively. Longer span length is considered as 10, 15 and 20 m for l_x/l_y=1.0, 1.5 and 2.0 respectively while the shorter span length of the plate is kept constant at 10 m. The thickness of the plate is considered as 0.5, 1.0, and 2.0 for h/l_x=0.05, 0.10 and 0.20 respectively. The analysis is performed for three subsoil depth, H=3, 5 and 10 m. The time variable load $q=q_0.F(t)$ is applied on the plate, in which maximum amplitude of patch load (q_0) is 30 kN/m^2. F(t) is a unit function of any impulsive load case given Fig.3 in the time domain.



$$F(t)= \quad F(t)=\begin{cases}1, t\le t_0\\ 0, t>t_0\end{cases} \quad F(t)=\begin{cases}1-\dfrac{t}{t_0}, t\le t_0\\ 0, t>t_0\end{cases} \quad F(t)=\begin{cases}Sin\dfrac{\pi t}{t_0}, t\le t_0\\ 0, t>t_0\end{cases} \quad F(t)=e^{-t}$$

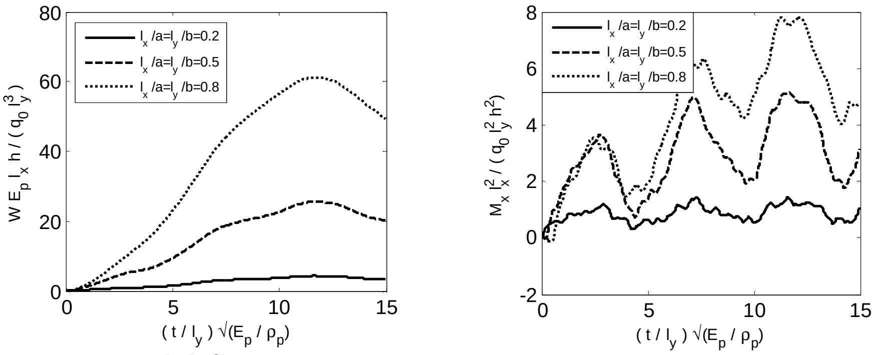
Fig 3 The various types of time variable loads

A convergence study for the mesh size and the time increment is performed first for the sake of accuracy. It is concluded that the results have acceptable error when equally spaced 8 elements for 10 m length if a 0.0001 s time increment are used.

In these examples, dimensionless forms of time, central displacement and bending moment are used and they are defined as follows respectively.

$$\bar{t} = \frac{t}{\ell_y}\sqrt{\frac{E_p}{\rho_p}} \qquad \bar{w} = \frac{w.E_p.\ell_x.h}{q_0.\ell_y^3} \qquad \overline{M_x} = \frac{M_x.\ell_x^2}{q_0.\ell_y^2.h^2}$$

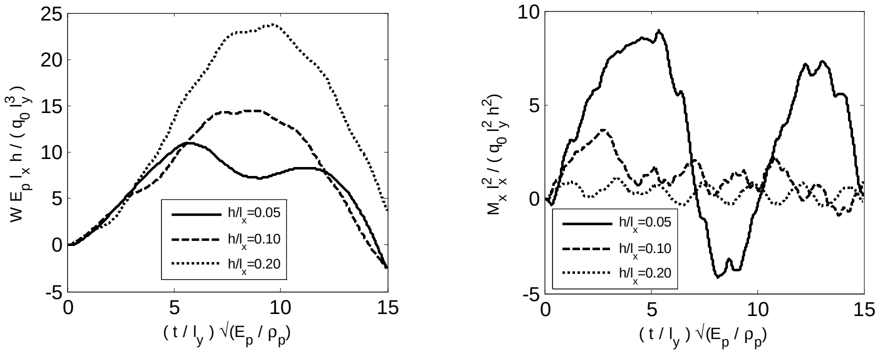
The results are presented in Fig. 4-10. Fig. 4 indicates the effects of loaded area on the maximum displacement and bending moment of the plate subjected to patch load for H=10m, h/l_x=0.10, l_x/l_y=1.0 and sudden load case. The ratio of loaded area dimensions to plate dimensions (a/l_x= b/l_y) are taken as 0.2, 0.5 and 0.8 as the plate dimensions are kept constant. As expected, these results show that the displacement and moment increase as the loaded area increases.



(a) Central deflections (b) Bending moments

Fig. 4 Effect of the loaded area on dynamic behaviors of thick plate

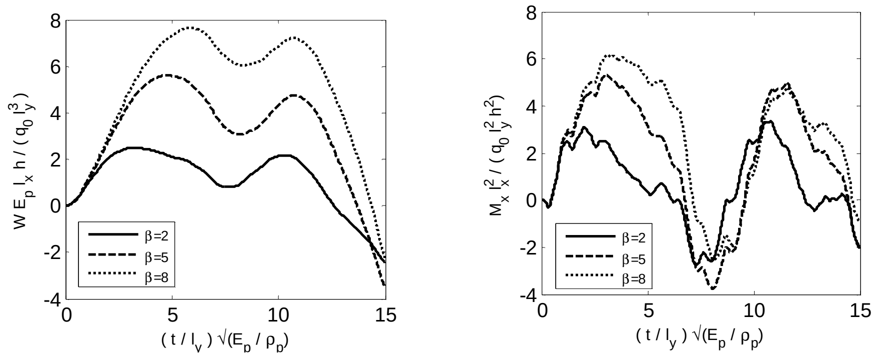
The variation of the central displacement and bending moment of the plate as a function of time for various values of the ratio of plate thickness to shorter span length of the plate (h/l_x) is plotted for H=10m, l_x/l_y=1.0, a/l_x= b/l_y=0.5 and step load case in Fig. 5. The ratio of plate thickness to shorter span length of the plate is taken 0.05, 0.10 and 0.20 while the shorter span length of the plate is kept as 10 m. Here, as seen dimensionless equations, results are affected plate thickness. So, figures needs to be explicated in reverse. The central displacement decreases as the ratio of plate thickness to shorter span length of the plate (h/l_x) increases while bending moment increases with increasing the ratio of plate thickness to shorter span length of the plate (h/l_x). It can be seen that the transverse shear deformation has a significant effect on the dynamic behavior of the plate.



(a) Central deflections (b) Bending moments

Fig. 5 Effect of the plate thickness to length of the plate ratio on dynamic behaviors of thick plate

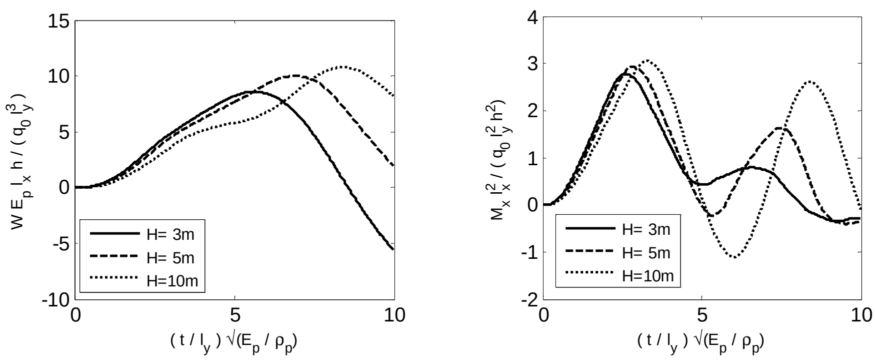
Fig. 6 shows the effects of pulse duration on the central displacement and bending moment of the plate subjected to centrally patch load for H=10m, h/l_x=0.10, l_x/l_y=1.0, a/l_x= b/l_y=0.5 and triangular load case. Here β indicates $(\bar{t}_0/l_y)\sqrt{E_p/\rho_p}$. As expected, results indicate that central displacement and bending moment increase as the pulse duration increases.



(a) Central deflections (b) Bending moments

Fig. 6 Effect of the pulse duration on dynamic behaviors of thick plate

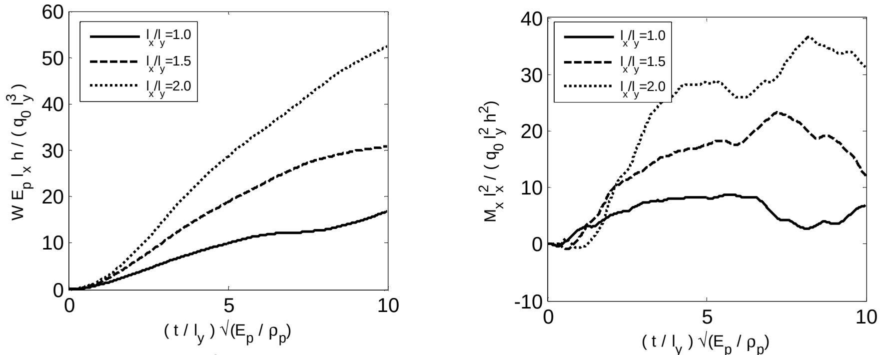
Fig. 7 shows central displacement and bending moment of a square plate subjected to central patch load as a function of time for various subsoil depth. The subsoil depth is 3, 5 and 10 m while other parameters are kept as l_x/l_y=1.0, h/l_x=0.10, a/l_x= b/l_y=0.5 and sine load case. As seen figure, the subsoil depth has a significant effect on the dynamic response of the plate. The central displacement and bending moment increase as the subsoil depth increases. This behavior is understandable in that a plate on elastic foundation with larger subsoil depth becomes more flexible.



(a) Central deflections (b) Bending moments

Fig. 7 Effect of the subsoil depth on dynamic behaviors of thick plate

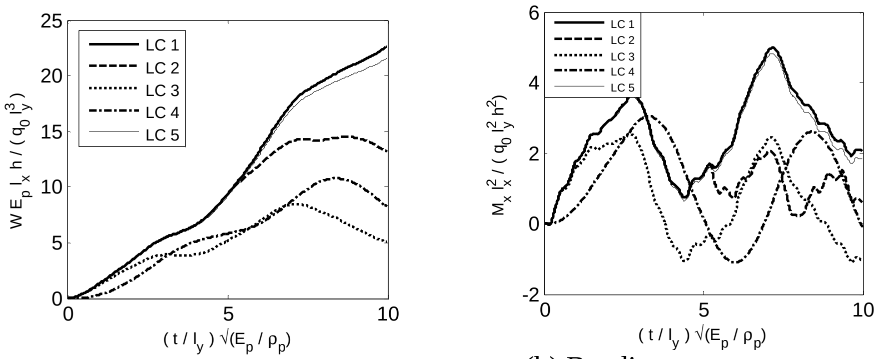
The variation of the central displacement and bending moment of the plate as a function of time for various values of aspect ratio (l_x/l_y) is plotted in Fig. 8 for H=10 m, h/l_x=0.10, a/l_x= b/l_y=0.5 and exponential load case. The aspect ratio is taken 1.0, 1.5 and 2.0 while the shorter span length of the plate is kept as 10 m. The central displacement and bending moment increase as the aspect ratio (l_x/l_y) increases.



(a) Central deflections (b) Bending moments

Fig. 8 Effect of the plate aspect ratio on dynamic behaviors of thick plate

The variation of the central displacement and bending moment of the plate as a function of time for various impulsive loading types such as sudden loads, step loads, triangular loads, sine loads and exponential loads given Fig.3 is plotted in Fig 9 for H=10 m, h/l_x=0.1, a/l_x= b/l_y=0.5 and l_x/l_y=1.0.



(a) Central deflections (b) Bending moments

Fig. 9 Effect of the time variable loading type on dynamic behaviors of thick plate

7. Conclusions

In this study, 8-noded Mindlin plate element are adopted for the dynamic analysis of thick plates on elastic foundations subjected to time variable patch loads and the effects of the thickness/shorter-span ratio (h/l_x), the aspect ratio (l_x/l_y), the subsoil depth (H), the loaded area, time variable loading type and pulse duration of the impulsive load on the central displacement and bending moment of thick plate on elastic foundation subjected to time variable loads are determined using Modified Vlasov Model.

The applicability of the 8-noded element to analysis of thick plates on two parameter elastic foundations subjected to time variable loading using Modified Vlasov model is confirmed in the initial example for the first time. In the latter example, a parametric study has been carried. The following conclusions can be drawn from the results.

i. The results demonstrate that thickness/shorter-span ratio of the plate, aspect ratio; subsoil depth, loaded area, time variable loading type and pulse duration of the impulsive load have significant influence on the characteristics of the dynamic behavior of the plate on elastic foundation.

ii. The displacements and bending moments of the plate increase as the soil depth increases since the elastic foundation become more flexible.

iii. The displacement and bending moments are increased as the pulse duration increases.

iv. It can be noted that the effect of loaded area and aspect ratio plays more crucial role on the dynamic response of the plate compare to other parameters considered.

v. The transverse shear deformation of the plate has a significant effect on the dynamic behavior of the plate.

vi. The dynamic behavior of the plate varies according to the type of loading. Sudden and exponential load types are more effective on the dynamics response.

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