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PREDICTING COMPRESSIVE STRENGTH OF CONCRETE FOR VARYING WORKABILITY USING REGRESSION MODELS

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Abstract

A mathematical analysis, using statistical techniques, for prediction of compressive strength of concrete was performed for the concrete strength data obtained from experimental work conducted under standard conditions in the laboratory. The data on compressive strength was obtained separately for concrete mixes proportioned for medium and high workability. The variables used in the prediction models were the mix proportioning elements, which include water-cement ratio, aggregates to cement ratio, etc. The multiple non-linear regression models developed in this work yielded excellent CODs for prediction of compressive strength at different curing ages (28, 56 and 91 days). The regression model developed for experimental data was compared with those developed by other researchers as well. In general, it was found that both the models developed as a part of this study could predict the compressive strength at 28 and 91 days with more than 95% accuracy. Also, it can be concluded that for better prediction 91 days strength for both medium and high workability mixes, it is desirable to consider the 28 and 56 days strengths in the regression equations.

Keywords: mathematical model, concrete, compressive strength, workability, multiple regression.

1. Introduction

Concrete is a wonder construction material which has changed the way construction is being carried out during the last century. Having the capability to be formed into any shape and size, it has transformed itself, leading to the development of many appealing structures. From a simple material easily formed by just adding coarse aggregates, sand, cement and water in desired proportions, it caught the fascination of researchers many decades ago. By playing around with its basic ingredients, researchers have been able to develop concretes, which not only have high compressive strength, but are durable as well. The results of compressive strengths vary, not only for different concrete mixtures, but for the same mixture as well, which could be attributed to various factors (ACI: 214R-02) [16]. The variations in measured strength could be attributed to various sources, which include, batch-to-batch variations of the proportions and characteristics of the constituent materials; production, delivery, and handling processes; climatic conditions and variations in the sampling, specimen preparation, curing, and testing procedures (within-test). Statistical procedures provide tools of considerable value when evaluating the results of such strength tests. Information derived from such procedures is also valuable in refining the design criteria and specifications. Statistical methods also have the added attraction that once fitted they can be used to perform predictions quickly and are simpler to implement in software. Apart from its speed, statistical modeling has advantages over other techniques that it is rigorous and can be used to define confidence interval for the prediction. In the construction industry, the compressive strength is the primary criterion for selecting concrete for a particular application. Concrete used for construction gains a significant component of its strength during the initial 3 to 4 week period, but continues to do so even for a longer period of time after pouring. Although the characteristic strength of concrete is defined as the compressive strength of a sample that has been cured for 28 days, however, to hasten the construction at a site we must be able to predict the concrete strength based upon the early strength data. Therefore, rapid and reliable prediction for the strength of concrete would be of great significance, as it would provide a chance to make the necessary adjustments to the mix proportions, wherever necessary, specifically for cases where concrete does not reach the required design strength or by avoiding concrete that is unnecessarily strong leading to more economic use of raw material and fewer construction failures (Kheder et al. [4]). Prediction of concrete strength, therefore, has been an active area of research and many a study has been carried out. Attempts have been made by many researchers to obtain a suitable mathematical model, which would be capable of predicting the compressive strength of concrete at various ages with acceptable high accuracy (Zain and Suhad, [14]; Kheder et al. [4]; Zain et al. [15]; Tsivilis and Parissakis, [13]; Zelic et al. [16]; Akkurt et al. [1]; Hwang et al. [3]).

As, for the experimental data obtained in the present study, the data sets on the dependent variable were normally distributed for each of the possible combinations of the level of the X variables, it was deemed suitable to use regression models for prediction of concrete strengths. Bayrak and Akgül [2] predicted the lifetime performance and remaining service life of a bridge system using regression models, so that the best maintenance and repair strategies which kept the system safe can be obtained. Many attempts had earlier been made to obtain a suitable mathematical model that was capable of predicting the strength of concrete at various ages with good accuracy (Popovics, [9]; Namyong et al. [7]; Steven et al. [12]; Mehta and Monteiro, [6]).

In the present study, a regression model was investigated as a performance prediction model for predicting the concrete compressive strength. Moreover, the effects of the changes of the coefficients of regression model of the performance curve were also examined. For this purpose, multiple regression analysis was carried out for predicting the compressive strength of concrete using four variables, namely, water-cementations ratio, fine aggregate-cementitious ratio, coarse aggregate-cementitious ratio and cementitious content. Regression models were developed for concrete with medium and high workability at different curing ages (28, 56 and 91days). For models developed for compressive strength prediction at 56 days and 91 days, the compressive strengths at lower ages were also considered as a parameter.

2. Experimental Dataset

The compressive strength data for the present work was obtained from the experiments conducted by Kumar [5]. For generating a reliable data bank on concrete compressive strength, he had considered five parameters, namely, water-cementitious material ratio, cementitious content, water content, workability, and curing ages in the experimental program. The casting and testing of specimens for generating the data bank were performed in controlled laboratory conditions. “Table 1” shows the variations in the values of parameters as taken by Kumar [5]. A set of 15 cubes for each of mixes so proportioned were cast and tested after 28, 56 and 91 days of curing. This extensive data bank, so generated, for analyzing the compressive strength of concrete was used in the present work.

Table1. Range of various parameters

Water-cementitious ratio	0.42 – 0.55
Cementitious content	350 – 475 @ 25 kg/m^3
Water content	180-230 @ 10 kg/m^3
Workability	Medium and high
Curing ages, days	28, 56, 91

The physical properties of the materials used in the study are shown in "Table 2". Ordinary Portland Cement (OPC) of 43 grade (IS: 8112-1989[20]) having a specific gravity of 3.12 and a compressive strength of 46.50MPa after 28 days of curing was used. The fine aggregates used had a specific gravity of 2.54 and belonged to zone – II of the grading zones as per IS: 383-1970 [18]. Two types of coarse aggregates, one with size 20mm (CA1) and other of 10mm (CA2) size, were used in varying proportions, depending upon the requirements for a particular mix. The 20mm and 10mm coarse aggregates had specific gravity of 2.61 and 2.63, respectively. As the aim of the present work was to study the effect of varying workability on the compressive strength of concrete, different mix proportions were formulated and used.
The details of the mixes using different proportions of water, cement, fine aggregates and coarse aggregates (20mm and 10mm) are shown in "Table 3" and "Table 4". The compressive strength test was performed and evaluated in accordance with IS: 516 [19], after curing the specimens for 28, 56 and 91 days. "Table 5" and "Table 6" show the results of compressive strength at the above ages, for medium and high workability concrete mixes, respectively.

Table 2. Physical Properties of Materials Used

Materials	Properties
Ordinary Portland Cement (OPC)	Grade: 43, as per IS:8112-1989 Specific Gravity: 3.12 7 days compressive strength: 35.50 MPa 28 days compressive strength: 46.50 MPa
Fine aggregates	Zone: III, as per IS: 383-1970 Fineness modulus: 2.09 Specific Gravity: 2.54
Coarse Aggregates – I (20mm size)	Specific Gravity: 2.61
Coarse Aggregates – I (10mm size)	Specific Gravity: 2.63

Table3. Details of proportions for concrete mixes for medium workability

S. No.	Mix designation	W/c ratio	Mix proportions (C: F. AGG: CAI: CAII)	Cement content, Kg/m^3	Workability and Vee-Bee time, seconds
1.	MD-01	0.514	1:1.392:2.181:1.074	350	MED. (7.2)
2.	MD-02	0.543	1:1.497:2.294:1.130	350	MED. (5.2)
3.	MD-03	0.480	1:1.245:2.001:0.986	375	MED. (7.5)
4.	MD-04	0.507	1:1.354:2.134:1.051	375	MED. (6.0)
5.	MD-05	0.450	1:1.100:1.811:0.892	400	MED. (7.8)
6.	MD-06	0.475	1:1.210:1.953:0.962	400	MED. (6.6)
7.	MD-07	0.423	1:0.981:1.650:0.813	425	MED. (8.0)
8.	MD-08	0.447	1:1.087:1.794:0.883	425	MED. (6.9)
9.	MD-09	0.422	1:0.977:1.644:0.810	450	MED. (7.3)
10.	MD-10	0.543	1:1.497:1.712:1.712	350	MED. (5.7)
11.	MD-11	0.507	1:1.354:1.593:1.593	375	MED. (6.4)
12.	MD-12	0.475	1:1.210:1.458:1.458	400	MED. (6.9)
13.	MD-13	0.447	1:1.087:1.339:1.339	425	MED. (7.2)
14.	MD-14	0.422	1:0.977:1.226:1.226	450	MED. (7.5)

3. Modeling of the Data

The most popular regression equation used by researchers for the prediction of compressive strength is the linear regression equation:

$$f_c = a_o + a_1\left(\frac{w}{cm}\right) \tag{1}$$

where, f_c is compressive strength of concrete; $\frac{w}{cm}$ is water to cementitious material ratio; and a_0 and a_1 are regression coefficients. The origin of this equation is Abram's Law, which states that in concrete materials, for a mixture of workable consistency, the strength of concrete is determined by the ratio of water to cementitious material, Popovics and Ujhelyi [8]. According to this law, increasing the $\frac{w}{cm}$ ratio will lead to a decrease in concrete strength. The original formula for Abram is as given by Eq. (2) below:

$$f_c = \frac{A}{B^{w/cm}} \tag{2}$$

where, f_c is again compressive strength of concrete; $\frac{w}{cm}$ is again water to cementitious material ratio; and *A* and *B* are empirical constants. Lyse (Namyong et al. [7]) developed a formula similar to Abram's but related compressive strength to cementitious material/water ratio instead of water/cementitious material ratio.

Table 4. Details of proportions for concrete mixes for high workability

S. No.	Mix designation	W/c ratio	Mix proportions (C: F. AGG: CAI: CAII)	Cement content, Kg/m^3	Workability and Vee-Bee time, seconds
1.	MD-15	0.533	1:1.581:2.042:1.006	375	HIGH (4.5)
2.	MD-16	0.500	1:1.430:1.892:0.932	400	HIGH (5.0)
3.	MD-17	0.525	1:1.543:2.005:0.988	400	HIGH (4.2)
4.	MD-18	0.471	1:1.275:1.725:0.850	425	HIGH (5.2)
5.	MD-19	0.494	1:1.397:1.858:0.915	425	HIGH (4.5)
6.	MD-20	0.444	1:1.140:1.576:0.776	450	HIGH (5.6)
7.	MD-21	0.467	1:1.254:1.702:0.839	450	HIGH (4.9)
8.	MD-23	0.442	1:1.192:1.650:0.813	475	HIGH (5.1)
9.	MD-24	0.533	1:1.581:1.524:1.524	375	HIGH (4.8)
10.	MD-25	0.500	1:1.430:1.412:1.412	400	HIGH (5.0)
11.	MD-26	0.525	1:1.543:1.497:1.497	400	HIGH (4.2)
12.	MD-27	0.471	1:1.275:1.288:1.288	425	HIGH (5.6)
13.	MD-29	0.518	1:1.511:1.473:1.473	425	HIGH (4.0)
14.	MD-31	0.467	1:1.254:1.271:1.271	450	HIGH (5.2)
15.	MD-34	0.442	1:1.192:1.232:1.232	475	HIGH (5.5)
16.	MD-35	0.463	1:1.230:1.255:1.255	475	HIGH (4.8)

According to Lyse (Zain et al. [14]), strength of concrete increases linearly with an increase in the *cm/w* ratio, and a general form of this popular model was:

$$f_c = A + B(\frac{cm}{w}) \tag{3}$$

where, f_c is compressive strength of concrete; $\frac{cm}{w}$ is cementitious material to water ratio; and *A* and *B* are empirical constants. The models as proposed by Abram and Lyse did not account for the quantities of fine aggregates and coarse aggregates for the prediction of concrete strength. So, for various concrete mixes, where their $\frac{w}{cm}$ ratio is constant, the strength will be the same is not true. Thus, it made it imperative to accommodate all the constituent materials into the predicting equation to have more reliable and accurate results for the prediction of concrete strength. For the stated reasons, Abram's Law has been extended by various researchers, to include other variables in the form of multiple linear regression equations and are widely used to predict the compressive strength of various types of concrete.

$$f_c = b_0 + b_1\left(\frac{w}{cm}\right) + b_2\left(\frac{FA}{cm}\right) + b_3\left(\frac{CA}{cm}\right) \tag{4}$$

where, f_c is compressive strength of concrete; $\frac{W}{cm}$ is water to cementitious material; $\frac{FA}{cm}$ is fine aggregate to cementitious material ratio; $\frac{CA}{cm}$ is coarse aggregate to cementitious material ratio; and b_0, b_1, *and* b_2 are the regression coefficients. As per Eq. (4), all variables are related linearly to the compressive strength, but this may not always be true because the variables involved in a concrete mix and affecting the compressive strength are interrelated and the additive action does not always hold true. This highlights the need to look at alternative mathematical models that can reliably predict the compressive strength of concrete with acceptable high accuracy. Thus, a general form of the multiple linear regression as below, is considered:

$$Y = a_0 + a_1x_1 + a_2x_2 + \ldots + a_mx_m \tag{5a}$$

The Eq. (5a) could be transformed back to a form that predicts the dependent variable (*Y*) by taking antilogarithm to yield an equation of type (5b):

$$Y = a_0 x_1^{a_1} x_2^{a_2} x_3^{a_3} \ldots x_m^{a_m} \tag{5b}$$

This equation (Eq. (5b)) is known as the multivariable power equation. In engineering applications, as variables are often dependent on several independent variables, this functional dependency is best characterized by the equation, of the type Eq. (5b), and is said to give results that are more realistic too. The equation had been successfully used to predict the compressive strength of Ordinary Portland Cement Concrete (Kheder et al. [4]) and also for compressive strength prediction of high performance concrete (Zain et al. [14]).

Table 5. Compressive strength data for concrete mixes with medium workability

W/c ratio	28 days curing		56 days curing		91 days curing	
	Mean, MPa	*S.D., MPa*	*Mean, Mpa*	*S.D., MPa*	*Mean, MPa*	*S.D., MPa*
0.514	39.52	0.97	43.31	1.51	46.13	0.99
0.543	31.66	1.4	37.18	1.77	43.92	1.92
0.480	42.73	1.4	48.23	1.07	52.23	1.53
0.507	40.69	1.1	44.46	1.83	46.42	2.03
0.450	47.99	1.7	52.95	1.1	55.51	1.64
0.475	44.89	1.24	51.2	1.36	53.85	1.1
0.423	51.25	1.64	57.55	1.57	59.5	1.86
0.447	49.05	1.38	54.14	1.03	57.35	1.51
0.422	53.69	1.37	57.77	1.35	59.89	1.55
0.543	36.64	0.99	43.46	2.03	46.55	1.81
0.507	41.57	1.37	46.81	1.63	50.04	1.8
0.475	46.22	1.03	52.58	1.61	53.07	1.83
0.447	50.35	1.36	56.02	1.57	58.32	1.62
0.422	54.11	1.42	58.52	1.33	62.28	1.53

In this study, the multivariable power equation was used for prediction of compressive strength of concrete, for varying workability, specifically developed under controlled laboratory conditions. For developing the model, in this study, the mean value of compressive strength of concrete at 28, 56 and 91 days of curing were predicted based upon the input in the form of the ratios as specified earlier.

Table 6. Compressive strength data for concrete mixes with high workability

W/c ratio	28 days curing		56 days curing		91 days curing	
	Mean, MPa	*S.D., MPa*	*Mean, MPa*	*S.D., MPa*	*Mean, MPa*	*S.D., MPa*
0.533	36.84	1.97	40.92	2.21	44.52	1.82
0.500	43.13	1.75	50.22	1.79	51.97	1.95
0.525	38.58	2.04	45.51	2.2	47.49	1.76
0.471	47.16	1.18	51.25	1.2	54.27	1.29
0.494	45.05	1.8	50.72	1.35	52.85	1.42
0.444	49.63	1.5	54.48	1.45	58.04	1.57
0.467	47.42	1.55	51.34	1.81	55.3	1.02
0.442	50.05	1.34	55.72	1.43	58.31	1.32
0.533	37.81	1.19	43.5	1.91	47.55	1.2
0.500	44.11	1.62	50.98	1.62	52.56	1.51
0.525	40.9	1.36	46.56	1.83	51.07	1.19
0.471	47.51	1.31	52.92	1.36	54.47	1.23
0.518	42.54	1.93	49.05	1.83	51.19	1.21
0.467	48.74	1.37	53.42	1.63	55.03	1.36
0.442	53.06	1.32	56.67	1.53	62.57	1.43
0.463	49.18	1.17	54.04	1.37	57.1	1.14

For predicting compressive strength at higher ages of 56 and 91 days, the values of compressive strengths at lower ages were also used as an additional input variable. The experimental data generated and as provided in “Table 3” to “Table 6” (Kumar [5]) is used for regression analysis. The final form of the regression equations for different cases as per Model-1 is given below.

$$f_{c28} = A_0 \left(\frac{w}{cm}\right)^{A_1} \left(\frac{FA}{cm}\right)^{A_2} \left(\frac{CA1}{cm}\right)^{A_3} \left(\frac{CA2}{cm}\right)^{A_4} \tag{6}$$

$$f_{c56} = A_0 \left(\frac{w}{cm}\right)^{A_1} \left(\frac{FA}{cm}\right)^{A_2} \left(\frac{CA1}{cm}\right)^{A_3} \left(\frac{CA2}{cm}\right)^{A_4} \left(\frac{f_{c28}}{cm}\right)^{A_5} \tag{7}$$

$$f_{c91} = A_0 \left(\frac{w}{cm}\right)^{A_1} \left(\frac{FA}{cm}\right)^{A_2} \left(\frac{CA1}{cm}\right)^{A_3} \left(\frac{CA2}{cm}\right)^{A_4} \left(\frac{f_{c28}}{cm}\right)^{A_5} \left(\frac{f_{c56}}{cm}\right)^{A_6} \tag{8}$$

$$f_{c91,28} = A_0 \left(\frac{w}{cm}\right)^{A_1} \left(\frac{FA}{cm}\right)^{A_2} \left(\frac{CA1}{cm}\right)^{A_3} \left(\frac{CA2}{cm}\right)^{A_4} \left(\frac{f_{c28}}{cm}\right)^{A_5} \tag{9}$$

$$f_{c91,56} = A_0 \left(\frac{w}{cm}\right)^{A_1} \left(\frac{FA}{cm}\right)^{A_2} \left(\frac{CA1}{cm}\right)^{A_3} \left(\frac{CA2}{cm}\right)^{A_4} \left(\frac{f_{c56}}{cm}\right)^{A_6} \tag{10}$$

where, f_{c28} is the compressive strength of concrete after 28 days of curing, f_{c56} is the compressive strength of concrete after 56 days of curing, and f_{c91} is the compressive strength of concrete after 91 days of curing. $f_{c91,28}$ is the compressive strength of concrete after 91 days of curing and $\frac{f_{c28}}{cm}$ is engaged as one of the independent variable. $f_{c91,56}$ is the compressive strength of concrete after 91 days of curing and $\frac{f_{c56}}{cm}$ is engaged as one of the independent variable. In Eq. (6) to Eq. (10), in predicting the strength for higher ages, the strength of concrete at lower ages has also been considered in the model developed.

$$f_{c28} = exp[\, B_0 + B_1 \left(\frac{w}{cm}\right) + B_2(cm) + B_3 \left(\frac{cm}{FA + CA1}\right) + B_4 \left(\frac{cm}{FA + CA2}\right)] \tag{11}$$

$$f_{c56} = exp[\, B_0 + B_1 \left(\frac{w}{cm}\right) + B_2(cm) + B_3 \left(\frac{cm}{(FA + CA1)}\right) + B_4 \left(\frac{cm}{FA + CA2}\right) + B_5 \left(\frac{cm}{f_{c28}}\right)] \tag{12}$$

$$f_{c91} = \exp\left[\, B_0 + B_1 \left(\frac{w}{cm}\right) + B_2(cm) + B_3 \left(\frac{cm}{(FA + CA1)}\right) + B_4 \left(\frac{cm}{FA + CA2}\right) + B_5 \left(\frac{cm}{f_{c28}}\right) + B_6 \left(\frac{cm}{f_{c56}}\right)\right] \tag{13}$$

$$\mathrm{f}_{c91,28} = \exp[\, \mathrm{B}_0 + \mathrm{B}_1 \left(\frac{\mathrm{w}}{\mathrm{cm}}\right) + \mathrm{B}_2(\mathrm{cm}) + \mathrm{B}_3 \left(\frac{\mathrm{cm}}{(\mathrm{FA} + \mathrm{CA1})}\right) + \mathrm{B}_4 \left(\frac{\mathrm{cm}}{\mathrm{FA} + \mathrm{CA2}}\right) + \mathrm{B}_5 \left(\frac{\mathrm{cm}}{\mathrm{f}_{c28}}\right)] \tag{14}$$

$$\mathrm{f}_{c91,56} = \exp\left[\, \mathrm{B}_0 + \mathrm{B}_1 \left(\frac{\mathrm{w}}{\mathrm{cm}}\right) + \mathrm{B}_2(\mathrm{cm}) + \mathrm{B}_3 \left(\frac{\mathrm{cm}}{(\mathrm{FA} + \mathrm{CA1})}\right) + \mathrm{B}_4 \left(\frac{\mathrm{cm}}{\mathrm{FA} + \mathrm{CA2}}\right) + \mathrm{B}_6 \left(\frac{\mathrm{cm}}{\mathrm{f}_{c56}}\right)\right] \tag{15}$$

In the second model (Model-2), a separate set of regression equations of the form as given in Eq. (11) to Eq. (15) were used for the development of the model. In this model, the ratio of cement is considered separately with both types of coarse aggregates.

In order to compare the regression equations developed as a part of the study the model as suggested by Namyong et al. [7] Eq. (16) to Eq. (20), was also used, wherein, the ratio of cement to total aggregates (fine and both coarse aggregates) is also considered.

$$f_{c28} = \exp[\, B_0 + B_1 \left(\frac{w}{cm}\right) + B_2(cm) + B_3 \left(\frac{cm}{FA + CA1 + CA2}\right)] \tag{16}$$

$$f_{c56} = exp[\, B_0 + B_1 \left(\frac{w}{cm}\right) + B_2(cm) + B_3 \left(\frac{cm}{(FA + CA1 + CA2)}\right) + B_4 \left(\frac{cm}{f_{c28}}\right)] \tag{17}$$

$$f_{c91} = exp\left[\, B_0 + B_1 \left(\frac{w}{cm}\right) + B_2(cm) + B_3 \left(\frac{cm}{(FA + CA1 + CA2)}\right) + B_4 \left(\frac{cm}{f_{c28}}\right) + B_5 \left(\frac{cm}{f_{c56}}\right)\right] \tag{18}$$

$$f_{c91,28} = exp\left[\, B_0 + B_1 \left(\frac{w}{cm}\right) + B_2(cm) + B_3 \left(\frac{cm}{(FA + CA1 + CA2)}\right) + B_4 \left(\frac{cm}{f_{c28}}\right)\right] \tag{19}$$

$$f_{c91,56} = exp[\, B_0 + B_1 \left(\frac{w}{cm}\right) + B_2(cm) + B_3 \left(\frac{cm}{(FA + CA1 + CA2)}\right) + B_5 \left(\frac{cm}{f_{c56}}\right)] \tag{20}$$

The three models, as developed above, were used for analysis of experimental data generated by Kumar [5] and the regression coefficients so obtained along with other related statistical parameters have been tabulated separately. The regression coefficients so obtained as per the first model Eq. (6) to Eq. (10) for medium and high workability mixes are provided in “Table 7” and “Table 8”.

Table7. Regression coefficients of multiple regression models predicting the compressive strength of concrete with medium workability as per Model–1

Medium Workability Concrete Mixes					
Coefficient	f_{c28}	f_{c56}	f_{c91}	$f_{c91,28}$	$f_{c91,56}$
A_0	0.0222	89.3373	9.4989	32.3254	105.1563
A_1	-9.0712	-0.7887	-2.1096	-0.6680	0.3007
A_2	4.0255	-0.2764	0.3807	-0.3803	-0.8739
A_3	0.1424	0.0111	-0.0107	-0.0374	-0.0390
A_4	0.2142	0.1105	0.0768	0.0719	0.0551
A_5		0.5218	-0.5556	-0.0340	
A_6			0.5524		0.1399
COD	0.9905	0.9746	0.9838	0.9766	0.9781
RMSE	0.0236	0.0208	0.0138	0.0166	0.0161

The regression coefficients so obtained as per the second model Eq. (11) to Eq. (15) for medium and high workability mixes are provided in “Table 9” and “Table 10”, respectively. The regression coefficients so obtained as per the model suggested by Namyong et al. [7] (Model 3), Eq. (16) to Eq. (20) for medium and high workability mixes are provided in “Table 11” and “Table 12”, respectively

Table8. Regression coefficients of multiple regression models predicting the compressive strength of concrete with high workability as per Model–1

High Workability Concrete Mixes					
Coefficient	f_{c28}	f_{c56}	f_{c91}	$f_{c91,28}$	$f_{c91,56}$
A_0	17.3445	31.1750	11.5347	10.8133	11.9447
A_1	-1.4096	-1.5687	-2.2648	-2.3694	-2.2904
A_2	-0.0302	0.3297	0.7271	0.8466	0.7662
A_3	-0.1262	-0.0752	-0.1189	-0.1221	-0.1186
A_4	0.0099	0.0073	-0.0028	-0.0053	-0.0036
A_5		0.3201	-0.0982	0.1417	
A_6			-0.2312		0.1584
COD	0.9688	0.8892	0.9334	0.9310	0.9330
RMSE	0.0227	0.0292	0.0216	0.0219	0.0216

Table9. Regression coefficients of multiple regression models predicting the compressive strength of concrete with medium workability as per Model–2

Medium Workability Concrete Mixes					
Coefficient	f_{c28}	f_{c56}	f_{c91}	$f_{c91,28}$	$f_{c91,56}$
B_0	6.9549	4.5112	4.3499	4.5507	3.8336
B_1	-5.3138	-1.1579	-1.4540	-1.9211	-0.6728
B_2	-0.0004	0.0016	0.0013	0.0009	0.0016
B_3	-0.3221	0.0279	0.1660	0.2186	0.2598
B_4	-0.7736	-0.2334	-0.0662	-0.1884	0.0185
B_5		-0.0672	0.0551	-0.0013	
B_6			-0.0916		-0.0372
COD	0.9650	0.9795	0.9889	0.9743	0.9805
RMSE	0.0277	0.0186	0.0114	0.0174	0.0151

Table10. Regression coefficients of multiple regression models predicting the compressive strength of concrete with high workability as per Model–2

High Workability Concrete Mixes					
Coefficient	f_{c28}	f_{c56}	f_{c91}	$f_{c91,28}$	$f_{c91,56}$
B_0	4.8570	3.3476	3.4957	3.3683	3.9523
B_1	-2.6876	1.3105	0.6762	0.9507	-0.4263
B_2	0.0004	0.0028	0.0023	0.0024	0.0017
B_3	0.3539	-0.1528	0.0989	0.0700	0.2351
B_4	-0.0908	0.0407	0.0527	0.0603	0.0163
B_5		-0.1309	-0.0685	-0.0989	
B_6			-0.0297		-0.0680
COD	0.9632	0.9532	0.9656	0.9625	0.9566
RMSE	0.0202	0.0190	0.0155	0.0162	0.0174

4. Discussion of Results

From the values of the regression coefficients and COD values, as shown in “Table 7” to “Table 12”, for different regression equations as presented in the preceding sections, it was observed that the COD was slightly higher for medium workability mixes as compared to those for high workability mixes at all ages.

From “Table 7”, which shows the regression coefficients for predicting compressive strength of medium workability concrete mixes, and was based upon the presumed Eq. (6) to Eq. (10) (Model-1), it was observed that COD was higher than 0.97 for all ages, indicating the suitability of the derived regression equations. The highest COD of 0.9905 was achieved for 28 days compressive strength, which reduced for f_{c56} to 0.9745, but further increased for prediction of f_{c91} to 0.9837. It could also be seen that for 91 days strength the COD was the highest when the strengths of 28 and 56 days were also considered in the development of the regression model, in addition to other parameters. To summarize, it can be said that, the regression equations as presumed under Model-1 were best suited to predict 28 days strength for medium workability mixes and also the 91 days strength could be better predicted using this model if the compressive strength values at 28 and 56 days ages was also known.

Table11. Regression coefficients of multiple regression models predicting the compressive strength of concrete with medium workability as per Model–3

Medium Workability Concrete Mixes					
Coefficient	f_{c28}	f_{c56}	f_{c91}	$f_{c91,28}$	$f_{c91,56}$
B_0	5.4926	3.7079	3.4846	3.5614	3.4906
B_1	-3.6013	0.0003	0.7057	-0.4983	-0.1543
B_2	0.0004	0.0023	0.0024	0.0018	0.0020
B_3	-0.5411	0.1164	0.2124	0.4842	0.4839
B_4		-0.0830	-0.0703	0.0217	
B_5			-0.0294		-0.0496
COD	0.9421	0.9757	0.9654	0.9641	0.9779
RMSE	0.0356	0.0203	0.0155	0.0205	0.0161

Table12. Regression coefficients of multiple regression models predicting the compressive strength of concrete with high workability as per Model–3

High Workability Concrete Mixes					
Coefficient	f_{c28}	f_{c56}	f_{c91}	$f_{c91,28}$	$f_{c91,56}$
B_0	5.0848	3.4319	3.4846	3.3753	3.9898
B_1	-2.9811	1.0184	0.7057	0.9214	-0.4099
B_2	0.0006	0.0024	0.0023	0.0023	0.0019
B_3	-0.4116	0.1175	0.2123	0.2332	0.0671
B_4		-0.1164	-0.0703	-0.0976	
B_5			-0.0294		-0.0754
COD	0.9492	0.9505	0.9654	0.9622	0.9528
RMSE	0.0238	0.0196	0.0155	0.0162	0.0182

From “Table 8”, that contains the regression coefficients, as per Model-1, for predicting the compressive strength of high workability concrete mixes, it was observed that COD was higher than 0.93 for all ages except for 56 days strength, indicating the suitability of the presumed regression equations mainly for 28 and 91days curing only. Again, as per the trend observed for medium workability mixes, the highest COD of 0.9688 was achieved for 28 days compressive strength, which reduced for f_{c56} to 0.8892, but further increased for prediction of f_{c91} to 0.9334, although these coefficients were less than those achieved for medium workability mixes. Similarly, it could also be seen that for the prediction of 91 days strength the COD was the highest when the strengths of 28 and 56 days were also considered in the development of the regression model.

From “Table 9”, that contains the regression coefficients, as per Model-2, for predicting compressive strength of medium workability concrete mixes and was based upon the presumed Eq. (11) to Eq. (15), it was observed that COD was higher than 0.96 for all ages. The highest COD of 0.9889 was achieved for the prediction of 91 days compressive strength, which reduced to 0.9795 for f_{c56} and 0.9650 for prediction of f_{c28}. It could also be seen that for 91 days strength the COD was the highest when compressive strengths of 28 and 56 days were also considered in the development of the regression model. To summarize, it can be concluded that the regression equations as per Model–2 were best suited to predict 91days compressive strength for medium workability mixes and the same could be better predicted using the model if the strength at 28 and 56 days curing was also used as an input parameter.

From “Table 10”, which encompasses the regression coefficients for predicting the compressive strength of high workability concrete mixes and was based upon the presumed Eq. (11) to Eq. (15), it was observed that COD was higher than 0.95 for all prediction equations, indicating the suitability of the assumed regression equations. On similar lines, as per the trend observed for medium workability mixes, the highest COD of 0.9688 was achieved for 28 days compressive strength, which reduced for f_{c56} to 0.9532, but further increased for prediction of f_{c91} to 0.9656, however, these coefficients were less than the ones achieved for medium workability mixes.

To have a comparative analysis the regression analysis of the experimentally generated data was also carried out using the model developed by Namyong et al. [7]. The results are tabulated in “Table 11” and “Table 12”. From “Table 11”, which shows the regression coefficients for predicting compressive strength of medium workability concrete mixes and was based upon the Eq. (16) to Eq. (20), it was observed that COD was higher than 0.94 for all ages, indicating the suitability of the assumed regression equations. On further analysis, it was observed that the COD achieved for prediction of f_{c28} was the lowest at 0.9421, which increased for f_{c56} to 0.9757 and further increased for prediction of f_{c91} (0.9841). It could also be seen from the analysis that for a prediction of 91 days strength the COD was the highest when the strengths of 28 and 56 days were also considered in the development of the regression model in addition to other parameters laid down in the equations.

Table13. Comparison of results for medium workability mixes

Strength	*Model-1*		*Model-2*		*Namyong et al. [7]*	
	COD	RMSE	COD	RMSE	COD	RMSE
f_{c28}	0.9905	0.0236	0.9650	0.0277	0.9421	0.0356
f_{c56}	0.9746	0.0208	0.9795	0.0186	0.9757	0.0203
f_{c91}	0.9838	0.0138	0.9889	0.0114	0.9654	0.0155
$f_{c91,28}$	0.9766	0.0166	0.9743	0.0174	0.9641	0.0205
$f_{c91,56}$	0.9781	0.0161	0.9805	0.0151	0.9779	0.0161

From "Table 12", which shows the regression coefficients for predicting the compressive strength of high workability concrete mixes and was based upon the assumed Eq. (16) to Eq. (20), it was observed that COD was higher than 0.94 for all ages. Again, as per the trend observed for other models with medium workability mixes, the highest COD of 0.9654 was achieved for the prediction of 91 days compressive strength, which reduced to 0.9505 for f_{c56}, and 0.9492 for f_{c28}, although these coefficients were less than the ones achieved for medium workability mixes. Similarly, it could also be seen that for 91 days strength the COD was the highest when the strengths of 28 and 56 days were also considered in the development of the regression model.

On comparing the regression analysis results achieved using the Model-1 and Model-2 and model as suggested by Namyong et al. [7], as per the details provided in "Table 13" and "Table 14", it could be said that in general both Model-1 and Model-2 as suggested by the author provide better CODs for both medium and high workability mixes.

For medium workability mixes, Model-1 was found to be the best suited for predicting the 28 days compressive strength, whereas, for predicting 56 and 91 days compressive strength, both Model-1 and Model-2 are equally suitable. It was also observed that 91 and 56 days compressive strength can be best predicted if in addition to other parameters, the strength at the preceding ages was also known.

Table14. Comparison of results for high workability mixes

Strength	*Model-1*		*Model-2*		*Namyong et al [7]*	
	COD	RMSE	COD	RMSE	COD	RMSE
f_{c28}	0.9688	0.0227	0.9632	0.0202	0.9492	0.0238
f_{c56}	0.8892	0.0292	0.9532	0.0190	0.9505	0.0196
f_{c91}	0.9334	0.0216	0.9656	0.0155	0.9654	0.0155
$f_{c91,28}$	0.9310	0.0219	0.9625	0.0162	0.9622	0.0162
$f_{c91,56}$	0.9330	0.0216	0.9566	0.0174	0.9528	0.0182

For high workability mixes, Model-1 was again found to be the best suited for predicting the 28 days compressive strength, whereas, for predicting strength at 56 and 91 days, both Model-2 and Namyong et al. [7] model were equally suitable. On similar lines as for medium workability mixes, it was observed that 91 and 56 days compressive strength could be best predicted if in addition to other parameters, the strength at the preceding ages was also known. The variation in the CODs was, however, less for high workability mixes.

5. Conclusions

The work presented in this paper comprises the development of regression models for predicting the compressive strength of concrete at three stage of its curing. The regression models thus developed can reliably predict the compressive strength of various mixes more efficiently. In general, both Model-1 and Model-2 provide better CODs for medium and high workability mixes. For better prediction of the 91 days compressive strength for both medium and high workability mixes, it is recommended to consider the 28 and 56 days strengths in the regression equations. The regression equation as per Model-1 is the best suited scheme for predicting the 28 days strength for medium workability mixes, whereas, the regression equation as per Model-2 is the best suited scheme for predicting 91 days strength for medium workability mixes where the strengths at 28 and 56 days curing are also included in regression equation. The regression equation as per Model-2 is the best suited method for predicting the compressive strength of high workability mixes at all ages as compared to the other regression models.

Notations

Code	Description
f_{c28}	Compressive strength of 28 days
f_{c56}	Compressive strength of 56 days
f_{c91}	Compressive strength of 91days
$f_{c91,28}$	Compressive strength of 91days, also included data set of compressive strength of 28days
$f_{c91,56}$	Compressive strength of 91days, also included data set of compressive strength of 56days
W	Water
CM	Cementitious content or cement content
FA	Fine aggregates
CA1	Coarse Aggregates – I 20mm size Specific Gravity: 2.61
CA2	Coarse Aggregates – I 10mm size Specific Gravity: 2.63
RMSE	Root Mean Square Error
COD	Coefficient of Determination

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