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ANALYTICAL DETERMINATION OF BUCKLING LOAD IN A LAMINATED COMPOSITE PLATE WITH EMBEDDED DELAMINATION

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Abstract

Analytical solution for critical buckling loads in a laminated composite plate with an embedded delamination is obtained using energy based approach. The effect of fiber orientation and delamination size on the local buckling load is calculated. Critical buckling load decreases with increase in the fiber orientation angle and delamination size and the results are in good agreement with the literature results.

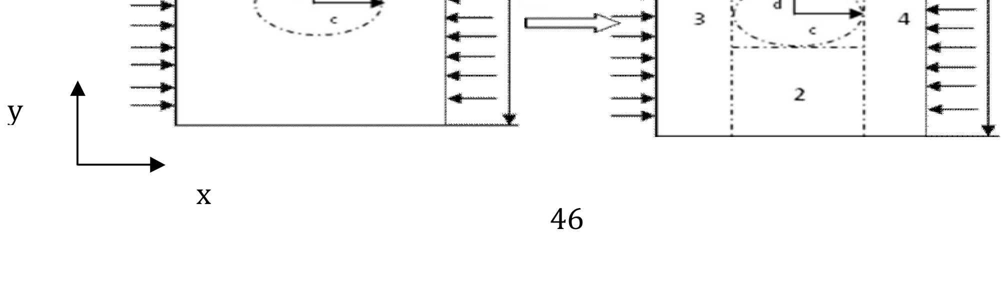
Keywords: Buckling load, circular delamination, analytical solution, composite plate

1. Introduction

Delamination is a common failure mode in layered composite materials which may result from the manufacturing imperfections, lay-up geometries, edge effects and various loadings. The presence of the delamination may cause significant reduction in stiffness and strength of the laminate under compressive loads. Hence, a clear understanding of the compressive failure behavior of the laminate is extremely important. Many investigators reported on the buckling induced delamination propagation of composite laminates in the literature. Whitcomb and Shivakumar [1] studied the delamination growth due to the local buckling of a composite plate with embedded delaminations. Nilsson et al. [2] studied delamination buckling and growth of slender composite panels using both numerical and experimental methods. Riccio et al. [3] investigated the compressive behavior of carbon fiber/epoxy laminated composite panels containing throughthe-width and embedded delaminations. Tafreshi and Oswald [4] developed finite element models to study the global, local and mixed-mode buckling behavior of composite plates with embedded delaminations. Wagner et al. [5] proposed a finite element method to simulate the delamination propagation in plates with circular embedded delamination. A finite element based on the efficient higher-order zig-zag theory with multiple delaminations was developed by Kim and Cho [6]. As a two-dimensional problem, composite laminates with through-the-width delaminations has been investigated extensively. For embedded delaminations, there are also a lot of solutions for delamination analysis of composite plates. However, the most of the previous studies for the case of embedded delaminations were based on Finite element [7,8,9] or other numerical solutions. The effect of geometrical parameters of the delamination on the deflection and the energy release rate for composite plate with multiple through-the width delaminations are calculated using two-dimensional beam theory in [10]. Mixed mode brittle fracture in polycrystalline graphite material is examined using maximum tangential stress (MTS) criterion by Masoud et.al. [11].Most of the previous analytical or numerical studies in the literature on buckling analysis of delaminated laminates extensively used models are essentially one-dimensional have been examined for the case of through-the-width delaminations. A sublaminate with a small out-of-plane thickness ratio may deflect in a local mode, and simultaneously the parent substrate deforms in a global mode. Hence the present study focuses to derive a two-dimensional analytical solutions for laminates with embedded delaminations, considering both the delamination and the parent substrate displacements. Although, finite element (FE) method and other numerical solutions are available for the study of the behavior of delaminated composite panels, they are for a particular material, geometry and laminate scheme and require extensive computing time. This study aims to derive the simple and cost effective analytical solution for the calculation of buckling loads of composite plates containing embedded elliptical and circular delaminations with minimum computational time. The analytical solutions is derived in terms of variables representing geometry, material and laminate properties so that they can be used for various plate and delamination dimensions, material and laminate scheme and can be used for quick parametric assessment of such defects detected during routine material evaluation test. Motivated by the current situation, a two-dimensional analytical model based on classical laminated plate theory is developed to determine the local buckling load of a laminated plate with embedded delamination. The present analytical model determines the buckling load for the case of embedded elliptical and circular delaminations. The limitation of this study includes that it does not consider the contact conditions, the situation of physically inadmissible buckling modes in the delaminated area causing the penetration between the two delaminated layers. In the present work only a single delamination is included and it does not include the study of delamination growth. This model has limited applications to compression–after-impact experiments where multiple delaminations occur through the thickness.

2. Plate with embedded delamination

When an in-plane compressive load is applied in the delaminated composite plates, three different possible modes of instability such as global, mixed, and local buckling modes may be readily identified at the critical load, depending on the delamination length. The buckling load is rapidly decreased when the global buckling mode of the laminated composite plate is transformed into the local buckling mode. The delaminated fiber reinforced composite plate model for the present analytical work is shown in Fig. 1. This model contains an elliptical delamination with radius *c*, *d* and is located at a distance *h* from top surface of the plate. Let *2a* and *2b* are the length and width of the plate. The loaded ends of the plate are clamped. As shown in the figure, the delamination divides the plate into four segments. The region above the delamination plane with thickness *h* is referred to as the ‘‘upper’’ sublaminate, while the part below it having thickness *H* is named the ‘‘lower’’ sublaminate. The sections before and after the delamination, where the plate is intact are referred to as the ‘‘base’’ laminates. Here we consider each of these regions as a separate composite plate. In the present study, an energy-based approach for the determination of the buckling load of the composite plate with embedded delamination is employed and the effects of delamination size and fiber orientation are determined.



3. (Fig.1. Composite plate with elliptical delamination discritized into four segments

In this section we present the equations leading to an eigen value problem for the determination of buckling load of composite plates containing embedded delamination. In particular, the methodology for determining the critical values of constant applied edge loads for the rectangular plate and square plate is described. At the loaded edges, the plate is clamped such that the transverse displacements and the derivatives of the displacements of the plate attain zero values at the edges x=0 and x=2a. The transverse deflection w of the deformed discritized plate segments are assumed to be of the following expressions:

$$W1 = \left(A1 * (\mathrm{Cos}[x] - 1) + A2 * (\mathrm{Cos}[3x] - 1) + A3(\mathrm{Cos}[5x] - 1)\right) * \mathrm{Cos}\left[\frac{\mathrm{Piy}}{2b}\right] \quad \text{-- (1)}$$

$$W2 = (B1 * (\mathrm{Cos}[x] - 1) + B2 * (\mathrm{Cos}[3x] - 1) + B3(\mathrm{Cos}[5x] - 1)) * \mathrm{Cos}\left[\frac{\mathrm{Piy}}{2b}\right] \quad \text{-- (2)}$$

$$W3 = \left(C1 * (\mathrm{Cos}[x] - 1) + C2 * (\mathrm{Cos}[3x] - 1) + C3(\mathrm{Cos}[5x] - 1)\right) * \mathrm{Cos}\left[\frac{\mathrm{Piy}}{2b}\right] \quad \text{---(3)}$$

$$W4 = (D1 * (\mathrm{Cos}[(2a - x)] - 1) + D2 * (\mathrm{Cos}[3 * (2a - x)] - 1) + D3 * (\mathrm{Cos}[5 * (2a - x)] - 1)) * \mathrm{Cos}\left[\frac{\mathrm{Piy}}{2b}\right] \quad \text{--(4)}$$

where Ai,Bi,Ci,Di, i=1,3 are the twelve unknown coefficients to be determined from the boundary conditions and continuity conditions. The continuity conditions between the four integrated plates are given by the following equations:

For the plate segments (1) and (3) at $x = a - c,\ y = b + d$

$$\left.\begin{aligned} W1(x,y) &= W3(x,y) \\ \frac{\partial W1(x,\ y)}{\partial x} &= \frac{\partial W3(x,\ y)}{\partial x} \\ \frac{\partial W1(x,\ y)}{\partial y} &= \frac{\partial W3(x,\ y)}{\partial y} \end{aligned}\right\} \quad \text{----(5)}$$

For the plate segments (2) and (3) at $x = a - c,\ y = b - d$

$$\left.\begin{aligned} W2(x,y) &= W3(x,y) \\ \frac{\partial W2(x,y)}{\partial x} &= \frac{\partial W3(x,y)}{\partial x} \\ \frac{\partial W2(x,\ y)}{\partial y} &= \frac{\partial W3(x,\ y)}{\partial y} \end{aligned}\right\} \quad \text{----(6)}$$

For the plate segments (1) and (4) at $x = a + c, \; y = b + d$

$$\left.\begin{aligned} W1(x,y) &= W4(x,y) \\ \frac{\partial W1(x,\ y)}{\partial x} &= \frac{\partial W4(x,\ y)}{\partial x} \\ \frac{\partial W1(x,\ y)}{\partial y} &= \frac{\partial W4(x,\ y)}{\partial y} \end{aligned}\right\} \quad \text{----(7)}$$

For the plate segments (2) and (4) at $x = a + c \, , y = b - d$

$$\left.\begin{aligned} W2(x,y) &= W4(x,y) \\ \frac{\partial W2(x,\ y)}{\partial x} &= \frac{\partial W4(x,\ y)}{\partial x} \\ \frac{\partial W2(x,\ y)}{\partial y} &= \frac{\partial W4(x,\ y)}{\partial y} \end{aligned}\right\} \quad \text{----(8)}$$

Using the above continuity equations the twelve constants appearing in the displacement functions are determined. All the twelve constants are not independent and the independent constants are determined by applying the condition that the variation of the elastic potential in the buckled state to vanish. Explicit evaluation of this condition leads to the homogeneous equations system that yields the non-trivial solution of the present buckling problem. By using the appropriate stiffness matrix D11, D22, D12, D66 for these discritized undelaminated plate segments(1) to (4) and top and bottom delaminated plate segments, the expressions for strain energy of each of these segments are derived as follows:

The strain energy of the top undelaminated buckled plate region -1 is given by,

$$Ut = \int_{a-c}^{a+c} \int_{b+d}^{2*b} 0.5 * [(D11 * \left\{\frac{\partial^2 W1}{\partial x^2}\right\}^2 + D22 * \left\{\frac{\partial^2 W1}{\partial y^2}\right\}^2] \, dy \, dx + \int_{a-c}^{a+c} \int_{b+d}^{2*b} [(D12 * (\frac{\partial^2 W1}{\partial x^2} * \frac{\partial^2 W1}{\partial y^2}) + 2 * D66 * \left\{\frac{\partial^2 W1}{\partial x \, \partial y}\right\}^2] \, dy \, dx \, ; \quad \text{----(9)}$$

The strain energy of the bottom undelaminated plate region-2 is given by,

$$Ub = \int_{a-c}^{a+c} \int_{0}^{b-d} 0.5 * [D11 * \left\{\frac{\partial^2 W2}{\partial x^2}\right\}^2 + D22 * \left\{\frac{\partial^2 W2}{\partial y^2}\right\}^2] \, dy \, dx + \int_{a-c}^{a+c} \int_{0}^{b-d} [D12 * (\frac{\partial^2 W2}{\partial x^2} * \frac{\partial^2 W2}{\partial y^2}) + 2 * D66 * \left\{\frac{\partial^2 W2}{\partial x \, \partial y}\right\}^2] \, dy \, dx \,; \qquad \text{----(10)}$$

The strain energy of the left undelaminated plate region-3 is given by,

$$Ul = \int_{0}^{a-c} \int_{0}^{2*b} 0.5 * [D11 * \left\{\frac{\partial^2 W3}{\partial x^2}\right\}^2 + D22 * \left\{\frac{\partial^2 W3}{\partial y^2}\right\}^2] \, dy \, dx + \int_{0}^{a-c} \int_{0}^{2*b} [D12 * (\frac{\partial^2 W3}{\partial x^2} * \frac{\partial^2 W3}{\partial y^2}) + 2 * D66 * \left\{\frac{\partial^2 W3}{\partial x \, \partial y}\right\}^2] \, dy \, dx \qquad \text{----(11)}$$

The potential energy of stress resultant in the left undelaminated plate region-3 is given by,

$$Vl = \int_{0}^{a-c} \int_{0}^{2*b} -0.5 * N11 * \{\frac{\partial W3}{\partial x}\}^2 \, dy \, dx \qquad \text{----(12)}$$

The strain energy of the right undelaminated plate region-4 is given by,

$$Ur = \int_{a+c}^{2*a} \int_{0}^{2*b} 0.5 * [D11 * \left\{\frac{\partial^2 W4}{\partial x^2}\right\}^2 + D22 * \left\{\frac{\partial^2 W4}{\partial y^2}\right\}^2] \, dy \, dx + + \int_{a+c}^{2*a} \int_{0}^{2*b} \left[D12 * (\frac{\partial^2 W4}{\partial x^2} * \frac{\partial^2 W4}{\partial y^2}) + 2 * D66 * \left\{\frac{\partial^2 W4}{\partial x \, \partial y}\right\}^2\right] \, dy \, dx \qquad \text{----(13)}$$

The potential energy of the stress resultant in the right undelaminated plate region-4 is given by,

$$Vr = \int_{a+c}^{2*a} \int_{0}^{2*b} -0.5 * N11 * \{\frac{\partial W4}{\partial x}\}^2 \, dy \, dx \,; \qquad \text{----(14)}$$

The transverse deflection W of the delaminated segments is assumed to be as follows:

$$\mathrm{Wep1} = (1 + (1 - \frac{(x-a)^2}{c^2} - \frac{(y-b)^2}{d^2})^2) * (\mathrm{R1} + \mathrm{R2}x^2 + \mathrm{R3}y^2 + \mathrm{R4}xy + \mathrm{R5}xxx + \mathrm{R6}yyy + \mathrm{R7}xxy + \mathrm{R8}xyy) \quad \text{---(15)}$$

The 8 unknown coefficients $\mathrm{R1}, \mathrm{R2}, \mathrm{R3}, \mathrm{R4}, \mathrm{R5}, \mathrm{R6}, \mathrm{R7}$ and $\mathrm{R8}$ appearing in eqn.(15) are determined by the continuity conditions of transverse displacement and slope of the plate and delamination segment at the delamination boundary namely (a-c,0), (a+c,0), (b-d,0) and (b+d,0).

The strain energy of the top delaminated plate region is given by,

$$\mathrm{Ud1} = \int_{a-c}^{a+c} \int_{b-d}^{b+d} 0.5 * (\mathrm{D11e1} * (\mathrm{Wepx}'')^2 + \mathrm{D22e1} * (\mathrm{Wepy}'')^2)\, \mathrm{d}y\, \mathrm{d}x$$
$$+ \int_{a-c}^{a+c} \int_{b-d}^{b+d} (\mathrm{D12e1} * (\mathrm{Wepx}'' * \mathrm{Wepy}'') + 2 * \mathrm{D66e1} * (\mathrm{Wepxy}')^2)\, \mathrm{d}y\, \mathrm{d}x$$

----(16)

The strain energy of the bottom delaminated plate region is given by,

$$\mathrm{Ud2} = \int_{a-c}^{a+c} \int_{b-d}^{b+d} 0.5 * (\mathrm{D11e2} * (\mathrm{Wepx}'')^2 + \mathrm{D22e2} * (\mathrm{Wepy}'')^2)\, \mathrm{d}y\, \mathrm{d}x$$
$$+ \int_{a-c}^{a+c} \int_{b-d}^{b+d} (\mathrm{D12e2} * (\mathrm{Wepx}'' * \mathrm{Wepy}'') + 2 * \mathrm{D66e2} * (\mathrm{Wepxy}')^2)\, \mathrm{d}y\, \mathrm{d}x \quad \text{----(17)}$$

The total elastic potential Π in the buckled state is determined by adding up the contributions of the strain energy of the plate and the energy contribution by the applied load. Since all the constants appearing in equations (1)-(4) are not independent quantities and applying the principle of minimization of potential energy $\frac{\partial \Pi}{\partial A_i} = 0$ with respect to the unknown independent constants, the eigen value solution is obtained. Three examples using the present analytical approach are considered to study the influence of fiber orientation and delamination size on buckling load.

4. Effect of delamination size and the fiber orientation on the buckling load

In the first example elliptically delaminated clamped plate with the material and geometric properties as given by Yeh et.al.,[7] is considered. The plate has 60mm length, 40mm width and 8 layers. Each layer thickness of the carbon-fiber reinforced prepreg is 0.12 mm. The total thickness is 0.96mm. The ratio of the major and minor axes is 1.72. The stacking sequence of the $[+\theta/-\theta]_{2s}$. The longitudinal modulus E11=96.2 GPa, the transverse modulus E22=6.7 GPa, the shear modulus

G12=3.2 GPa, Poisson's ratios ν12= 0.27. The delamination is between second and third layer. Fig.2. shows the effect of delamination size on the normalized buckling load and it is observed that the buckling load decreases as the delamination size increases and the results are in good agreement with those reported in [7] and [9]. In this second example, a clamped plate with four-layer square plate [0//0/0/0] is considered to characterize the effect of an embedded circular delamination on the buckling behaviour of composite plate. The delamination is between first and second layer. The laminate has length L=150mm and thickness H =4x 0.125 mm. The material properties given in [8] are being used for the present work and are given by, E11 =135.4 GPa; E22 =9.6 GPa; G12 =4.8 GPa; ν12 =0.31. Fig.3. represents the variation of buckling load with respect to the ratio of delamination size and the plate size. It is seen that the buckling load decreases with the increase in the delamination size and is compared with the result given in [8]. The results are in good agreement with those given in [8]. From the analytical results (Fig.3), delamination length for which global buckling occurs is less than $a/L = 0.25$; delamination length for mixed buckling is $0.25 < a/L < 0.3$; delamination length for local buckling is larger than 0.3. In these two examples the delamination buckling mode that depends on the delamination size is related to the buckling load. When the global buckling mode occurs, the buckling load does not decrease significantly at the delamination size for which global buckling mode occurs. If the delamination size is larger than the critical delamination size, the buckling load begins to decrease and mixed buckling mode occurs instead of global buckling mode. At the delamination size for which local buckling mode occurs, the buckling load is greatly affected and it drops significantly. In the third example we consider a clamped plate as in the first example for the calculation of critical buckling load for local buckling mode for various fiber orientation angles. The geometry and the material properties are as given by Yeh et.al.,[7] and as used in the first example. Fig.4 represents the graph of normalized buckling load for various fiber orientation angle and it is seen that the buckling load decreases when fiber orientation angle increases from zero degree. It is observed that greater buckling load occurs when the fibers are oriented towards 0 degree due to the flexural stiffness and stability of the sublaminate is higher towards 0 degree orientation. The critical buckling loads are compared with the results given in [7]. The present analytical approach does not use any symmetry conditions in these three examples and hence consistently underestimates the buckling load compared to the references. Higher buckling loads in the reference[7,8,9] results are due to the symmetry model used in the references.

5. Conclusion

Buckling load in a laminated composite plate with an embedded delamination is obtained analytically. Critical buckling load decreases with increase in the fiber orientation angle and delamination size. The delamination buckling mode that depends on the delamination size is related to the buckling load. In the present study, the orthotropic laminate with embedded delamination is considered and three different possible modes of instability are identified at the critical load. Global buckling preceding any other instability mode occurs for a relatively small delamination length. On the other hand, if the length of the delamination is relatively large, then local buckling of the upper sublaminate precedes any other mode of instability because the delamination is sufficiently slender in comparison with the whole laminate. Mixed buckling involving out-of-plane deflections for both the sublaminates and the base laminate, a combination of the two modes, takes place in the intermediate delamination length between global buckling and local buckling. The present analytical approach does not use any symmetry conditions for the calculation of critical buckling loads. The results are in good agreement with those reported in the literature.

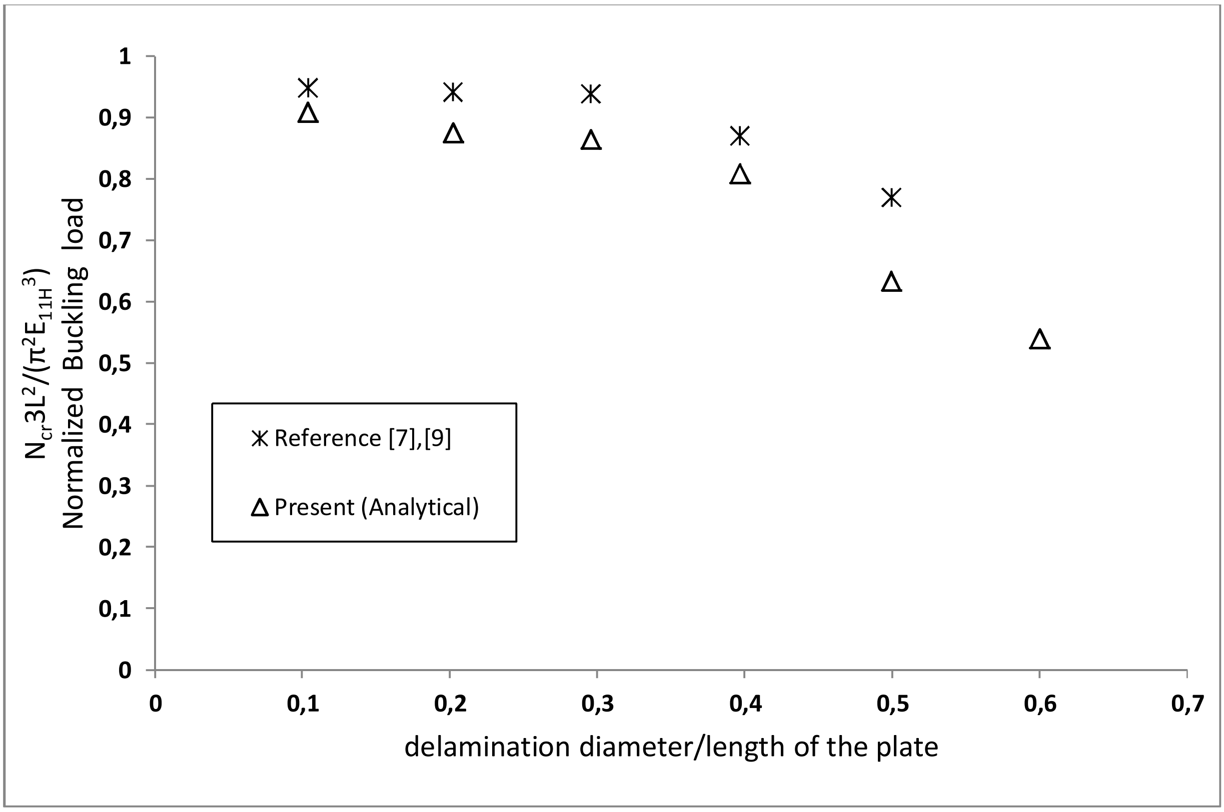


Fig.2. Effect of delamination size on buckling load

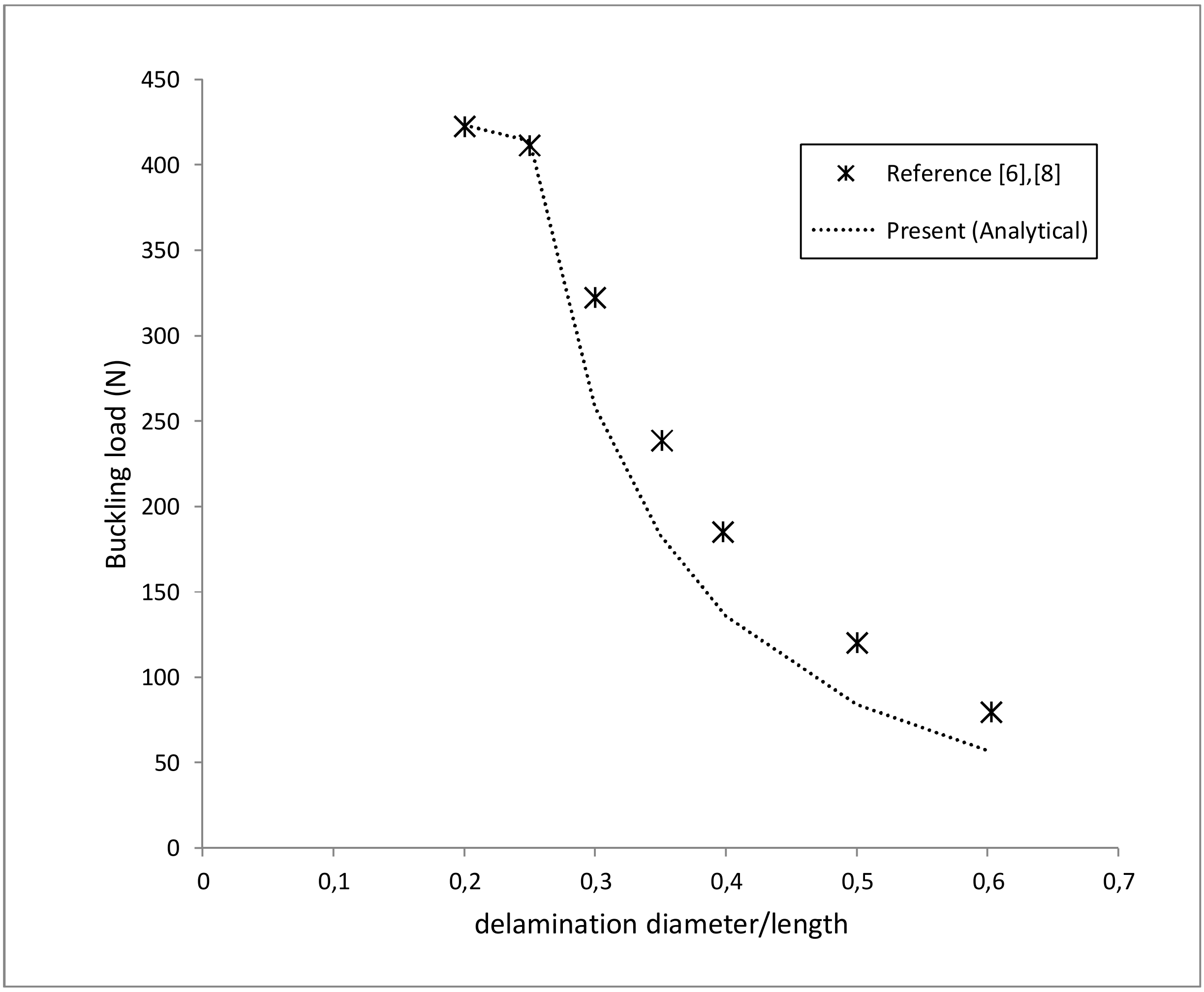


Fig.3. Effect of delamination size on buckling load of a square plate with circular delamination

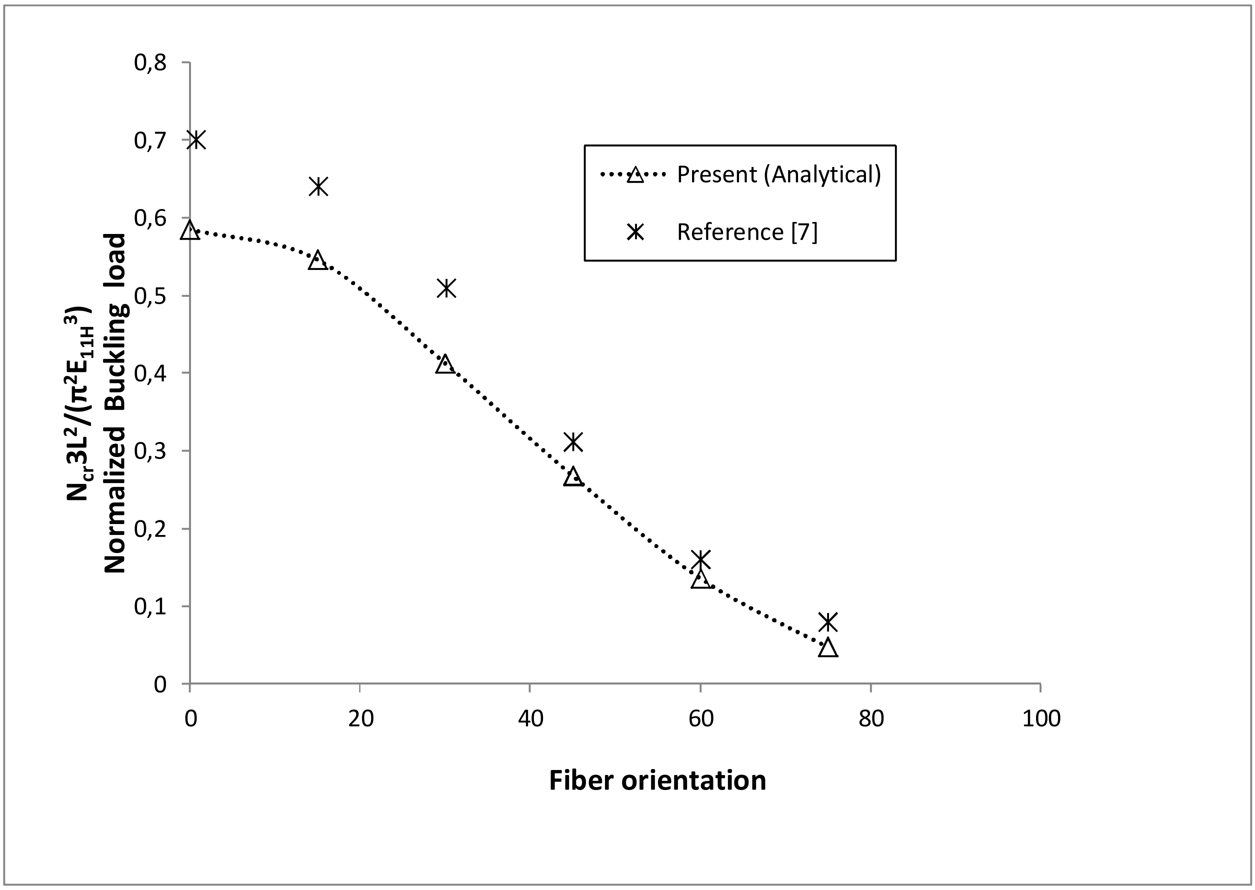


Fig.4. Effect of fiber orientation on buckling load

Acknowledgments

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