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Static Analysis of a Nano Plate by Using Generalized Differential Quadrature Method

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Abstract

In this paper, static analysis of a nano rectangular plate subjected to uniform distributed load is studied based on modified couple stress theory (MSCT) by using Generalized Differential Quadrature (GDQ) Method. The inclusion of an additional material parameter enables the new plate model to capture the size effect. The new non-classical plate model reduces to the classical plate model when the length scale parameter is set to zero. In deriving of the governing equations, the minimum total potential energy principle is used. In the solution of the governing equations, the DQM method is used for simply-supported nano plate within the Kirchhoff-Love plate theory. In the numerical results, the influences the material length scale parameter and the dimension parameters of the plate on the static deflection of the nano plate is presented. Also, the difference between the classical theory (CT) and MSCT is investigated for static responses of nano plate.

Keywords: Nano Plate, Modified couple stress theory, Generalized Differential Quadrature Method, Static Analysis.

1. Introduction

With the great advances in technology in recent years, micro and nano structures have found many applications. In these structures, nano/micro plates are widely used in micro- and nano electromechanical systems (MEMS and NEMS) such as sensors (Zook et al. [1], Pei et al. [2]), actuators (Senturia [3], Rezazadeh et al. [4]). In investigation of micro and nano structures, the classical continuum mechanics which is scale independent theories, are not capable of explanation of the size-dependent behaviors. Nonclassical continuum theories such as higher order gradient theories and the couple stress theory are capable of explanation of the size dependent behaviors which occur in micro-scale structures.

At the present time, the experimental investigations of the micro materials are still a challenge because of difficulties confronted in the micro scale. Therefore, mechanical theories and atomistic simulations have been used for micro structural analysis. The process of the atomistic simulations is very difficult and takes much time. So, continuum theory is the most preferred method for the analysis of the micro and nano structures. Classical continuum mechanics does not contain the size effect, because of its scale-free character. The nonlocal continuum theory initiated by Eringen [5] which has been widely used to mechanical behavior of nano-micro structures.

The size effect plays an important role on the mechanical behavior of microstructures at the micrometer scale that the classic theory has failed to consider when the size reduces from macro to nano (Toupin [6], Mindlin [7], Mindlin [8], Fleck and Hutchinson [9], Yang et al [10], Lam et al. [11]). Therefore, higher-order theories modified couple stress theory (MCST) and modified strain gradient are used in the mechanical model of the nano-micro structures (Yang et al [10], Lam et al. [11]).

The determination of the micro-structural material length scale parameters is very difficult experimentally. So, Yang et al. [10] proposed the modified couple stress theory in which the strain energy has been shown to be a quadratic function of the strain tensor and the symmetric part of the curvature tensor, and only one length scale parameter is included. After this, the MCST and the strain gradient elasticity theories have been widely applied to static, buckling and dynamic analysis of nano/micro plates [12-41]. In these studies, Ansari et al. [37] studied three-dimensional bending and vibration analysis of functionally graded nanoplates, Ghadir et al. [38] investigated thermo-mechanical vibration of orthotropic cantilever nanoplate, Kananipour [39] investigated static analysis of nonlocal nanoplates based Kirchhoff and Mindlin plate theories, Arani and Jafari [40] examined nonlinear vibration analysis of laminated composite Mindlin micro/nano-plates resting on orthotropic Pasternak medium and Pradhan and Kumar [41] investigated vibration analysis of orthotropic graphene sheets using nonlocal elasticity theory differential quadrature method.

In this study, the static bending responses of a simple supported rectangular nano plate subjected to uniform distributed load based on the MCST theory within the Kirchhoff-Love plate theory by using Generalized Differential Quadrature (GDQ) Method. The effect of the material length scale parameter and the dimension parameters of the nano plate on the static responses of the nano plate are investigated in both the CT and MCST.

2. Formulations

Consider a simple supported rectangular nano plate with thickness t in X_3 *direction*, the length of Lx_1 and Lx_2 the in X_1 and X_2 direction, respectively. The nano plate is subjected to uniformly distributed transverse load (q). The modified couple stress theory was proposed by Yang et al. (2002). Based on this theory, the strain energy density for a linear elastic material which is a function of both strain tensor and curvature tensor is introduced for the modified couple stress theory;

$$U = \int_V (\boldsymbol{\sigma}:\boldsymbol{\varepsilon} + \boldsymbol{m}:\boldsymbol{\chi})\, dV \tag{1}$$

where σ is the stress tensor, ε is the strain tensor, m is the deviatoric part of the couple stress tensor, χ is the symmetric curvature tensor, defined by

$$\boldsymbol{\sigma} = \lambda\, tr(\boldsymbol{\varepsilon}) I + 2\mu\boldsymbol{\varepsilon} \tag{2}$$

$$\boldsymbol{\varepsilon} = \frac{1}{2}[\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T] \tag{3}$$

$$\boldsymbol{m} = 2l^2\mu\, \boldsymbol{\chi} \tag{4}$$

$$\boldsymbol{\chi} = \frac{1}{2}[\nabla\boldsymbol{\theta} + (\nabla\boldsymbol{\theta})^T] \tag{5}$$

where λ and μ are Lame's constants, l is a material length scale parameter which is regarded as a material property characterizing the effect of couple stress, $\boldsymbol{u}$ is the displacement vector and $\boldsymbol{\theta}$ is the rotation vector, given by

$$\boldsymbol{\theta} = \frac{1}{2}\mathrm{curl}\, \boldsymbol{u} \tag{6}$$

The parameters λ and μ in the constitutive equation are given by

$$\lambda = \frac{E\ \nu}{(1+\nu)(1-2\nu)}\ ,\ \mu = \frac{E}{2(1+\nu)} \tag{7}$$

where E is the modulus of elasticity and ν is the Poisson ratio.

According to the Kirchhoff-Love plate theory, the axial and the displacement fields are expressed as

$$u(X_1, X_2, X_3) = u_0(X_1, X_2) - X_3 \frac{\partial w(X_1,X_2)}{\partial X_1} \tag{8}$$

$$v(X_1, X_2, X_3) = v_0(X_1, X_2) - X_3 \frac{\partial w(X_1,X_2)}{\partial X_2} \tag{9}$$

$$w(X_1, X_2, X_3) = w_0(X_1, X_2) \tag{10}$$

where u, v, w are X_1, X_2 and X_3 components of the displacements, respectively.

By using equations (3), (8), (9) and (10) and strain- displacement relation can be obtained:

$$\varepsilon_{X_1} = \frac{\partial u}{\partial X_1} = \frac{\partial u_0(X_1,X_2)}{\partial X_1} - X_3 \frac{\partial^2 w(X_1,X_2)}{\partial {X_1}^2} \tag{11a}$$

$$\varepsilon_{X_2} = \frac{\partial v}{\partial X_2} = \frac{\partial v_0(X_1,X_2)}{\partial X_2} - X_3 \frac{\partial^2 w(X_1,X_2)}{\partial {X_2}^2} \tag{11b}$$

$$\varepsilon_{X_1X_2} = \frac{1}{2}\left(\frac{\partial u_0(X_1,X_2)}{\partial X_2} + \frac{\partial v_0(X_1,X_2)}{\partial X_1} - 2X_3 \frac{\partial^2 w(X_1,X_2)}{\partial X_1 \partial X_2}\right) \tag{11c}$$

By using equations (6), (8), (9) and (10),

$$\theta_{X_1} = \frac{\partial w(X_1,X_2)}{\partial X_2} \tag{12a}$$

$$\theta_{X_2} = \frac{\partial w(X_1,X_2)}{\partial X_1} \tag{12b}$$

$$\theta_{X_3} = -\frac{1}{2}\left(\frac{\partial u_0(X_1,X_2)}{\partial X_2} - \frac{\partial v_0(X_1,X_2)}{\partial X_1}\right) \tag{12c}$$

Substituting equation (12) into equation (5), the curvature tensor χ can be obtained as follows

$$\chi_{X_1X_1} = \frac{\partial^2 w(X_1,X_2)}{\partial X_1 \partial X_2} \tag{13a}$$

$$\chi_{X_2X_2} = -\frac{\partial^2 w(X_1,X_2)}{\partial X_1 \partial X_2} \tag{13b}$$

$$\chi_{X_1X_2} = -\frac{1}{2}\left(\frac{\partial^2 w(X_1,X_2)}{\partial {X_1}^2} - \frac{\partial^2 w(X_1,X_2)}{\partial {X_2}^2}\right) \tag{13c}$$

$$\chi_{X_1X_3} = -\frac{1}{4}\left(\frac{\partial^2 u_0(X_1,X_2)}{\partial X_1 \partial X_2} - \frac{\partial^2 v_0(X_1,X_2)}{\partial {X_1}^2}\right) \tag{13d}$$

$$\chi_{X_2X_3} = -\frac{1}{4}\left(\frac{\partial^2 u_0(X_1,X_2)}{\partial {X_2}^2} - \frac{\partial^2 v_0(X_1,X_2)}{\partial X_1 \partial X_2}\right) \tag{13d}$$

The constitutive equations of the nano plate are as follows:

$$\sigma_{\mathrm{ij}} = \frac{E}{(1-\nu^2)}\left[\nu \varepsilon_{kl}\delta_{ij} + (1-\nu)\varepsilon_{ij}\right] \tag{14a}$$

$$m_{\mathrm{ij}} = 2\mu l^2 \chi_{\mathrm{ij}} \tag{14b}$$

According to the minimum total potential energy principle, the first variation of the total potential energy must be zero. That is

In deriving of the governing equations, the Hamilton's principle is used;

$$\delta(U_i + U_e)=0 \tag{15}$$

where *Ui* and *Ue* are the strain energy and the potential energy of the external load, respectively. The first variation of *Ui* and *Ue* are expressed as

$$\delta U_i = \int_V \left(\sigma_{ij}\,\delta\varepsilon_{ij} + m_{ij}\,\delta\chi_{ij}\right)dV = \int_0^{L_{X1}}\int_0^{L_{X2}}\Big[N_1 \frac{\partial \delta u_0(X_1,X_2)}{\partial X_1} - M_1 \frac{\partial^2 \delta w(X_1,X_2)}{\partial {X_1}^2} + 2M_{12}\frac{\partial^2 w(X_1,X_2)}{\partial X_1 \partial X_2} + N_{12}\left(\frac{\partial u_0(X_1,X_2)}{\partial X_2} + \frac{\partial v_0(X_1,X_2)}{\partial X_1}\right) + N_2 \frac{\partial \delta v_0(X_1,X_2)}{\partial X_2} - M_2 \frac{\partial^2 \delta w(X_1,X_2)}{\partial {X_2}^2} + \frac{E_{13}}{2}\left(-\frac{\partial^2 u_0(X_1,X_2)}{\partial X_1 \partial X_2} + \frac{\partial^2 v_0(X_1,X_2)}{\partial {X_1}^2}\right) + E_1 \frac{\partial^2 w(X_1,X_2)}{\partial X_1 \partial X_2} + \frac{E_{23}}{2}\left(-\frac{\partial^2 u_0(X_1,X_2)}{\partial {X_2}^2} + \frac{\partial^2 v_0(X_1,X_2)}{\partial X_1 \partial X_2}\right) + E_{12}\left(-\frac{\partial^2 w(X_1,X_2)}{\partial {X_1}^2} + \frac{\partial^2 w(X_1,X_2)}{\partial {X_2}^2}\right) + E_2\left(-\frac{\partial^2 w(X_1,X_2)}{\partial X_1 \partial X_2}\right)\Big]\,dX_1 dX_2 \tag{16a}$$

$$\delta U_e = -\int_A \left[q(X_1,X_2)\,\delta w(X_1,X_2)\right] dA \tag{16b}$$

where N_1, N_2, N_{12}, M_1, M_2, M_{12}, E_1, E_2, E_{12}, E_{13} and E_{23} are stress resultants, and expressed as follows:

where

$$(N_1, N_2, N_{12}) = \int_{-0.5h}^{0.5h}\left(\sigma_{X_1X_1}, \sigma_{X_2X_2}, \sigma_{X_1X_2}\right)dX_3 \tag{17a}$$

$$(M_1, M_2, M_{12}) = \int_{-0.5h}^{0.5h}\left(\sigma_{X_1X_1}, \sigma_{X_2X_2}, \sigma_{X_1X_2}\right)X_3\,dX_3 \tag{17b}$$

$$(E_1, E_2, E_{12}, E_{13}, E_{23}) = \int_{-0.5h}^{0.5h}\left(m_{X_1X_1}, m_{X_2X_2}, m_{X_1X_2}, m_{X_1X_3}, m_{X_2X_3}\right)dX_3 \tag{17c}$$

Substituting eqs. (16) into eq. (15), and then using integrating by parts, the governing equations of the problem can be obtained as follows;

$$\frac{Et^3}{12(1-\nu^2)}\left(\frac{6l^2(1-\nu)}{t^2}\right)\left(\frac{\partial^4 w(X_1,X_2)}{\partial {X_1}^4} + 2\frac{\partial^4 w(X_1,X_2)}{\partial {X_1}^2 \partial {X_2}^2} + \frac{\partial^4 w(X_1,X_2)}{\partial {X_2}^4}\right) = q(X_1,X_2) \tag{18}$$

The boundary conditions at the simple supported nano plate ends are as follows;

$$w(X_1,0)=w(L_{X1},0)=w(0,X_2)=w(0,L_{X2})=0 \tag{19a}$$

$$E_{12}(X_1,0)-M_2(X_1,0)=0 \tag{19b}$$

$$M_1(L_{X1},X_2)+E_{12}(L_{X1},X_2)=0 \tag{19c}$$

$$E_{12}(X_1,L_{X2})-M_2(X_1,L_{X2})=0 \tag{19d}$$

$$M_1(0,X_2)+E_{12}(0,X_2)=0 \tag{19e}$$

In the solution of the governing equations, the Generalized Differential Quadrature Method is used. In the differential quadrature method, the derivatives of a function are written as linear summation of the values at all points in the domain [42-45];

$$\frac{d^{(p)}w(x_j)}{dx^{(p)}} \approx \sum_{i=1}^{n} B_{ji}^{(p)}\, w(x_i) \tag{20}$$

where n is the number of the points in the domain, p is the order of derivative in the function, $B_{ji}^{(p)}$ is the weighting coefficient with pth derivative of the function with respect to x. The weight coefficients for first-order derivative (p=1) are as follows [42,43];

$$B_{ji}^{(1)}=\begin{cases}\dfrac{\prod_{j=1}^{n}(x_j-x_i)}{(x-x_j)\prod_{j=1}^{n}(x_i-x_j)} & i\neq j\\ -\sum_{j=1,i\neq j}^{n} B_{ji}^{(1)} & i=j\end{cases} \tag{21}$$

For the higher order derivatives, the weight coefficient is expressed as follows:

$$B_{ji}^{(p)}=\sum_{r=1}^{n} B_{jr}^{(1)} B_{ri}^{(p-1)} \qquad (i,j=1,n) \tag{22}$$

For determined the sampling points in the domain, Chebyshev–Gauss–Lobatto grid points is employed[42,43];

$$x_j=\frac{1}{2}\left[1-cos\left(\frac{j-1}{n-1}\pi\right)\right] \qquad (j=1,n_{x1}) \tag{23a}$$

$$x_i=\frac{1}{2}\left[1-cos\left(\frac{i-1}{n-1}\pi\right)\right] \qquad (i=1,n_{x2}) \tag{23b}$$

where n_{x1} and n_{x2} are the number of the grid points in X_1 and X_2 direction, respectively.

Substituting eqs. (20-23) into eq. (18), and then using GDQ discretization, the governing equations of the problem can be obtained as follows;

$$\frac{Et^3}{12(1-\nu^2)}\left(\frac{6l^2(1-\nu)}{t^2}\right)\left(\sum_{k=1}^{n_{x1}} B_{jk}^{(4)}\, w_{kj}+2\sum_{k=1}^{n_{x1}}\sum_{m=1}^{n_{x2}} B_{jk}^{(2)}\, B_{im}^{(2)} w_{km}+\sum_{k=1}^{n_{x2}} B_{ik}^{(4)}\, w_{ki}\right)=q$$

$$(j=1,n_{x1}),\ (i=1,n_{x2}),\ (k=1,\mathrm{p}+1) \tag{24}$$

The dimensionless displacement can be expressed as

$$\bar{w} = w \frac{E\, t^3}{q\, a^4} \tag{25}$$

3. Numerical Results

In this section, various numerical examples are presented and discussed to investigate the static deflections of the nano plate. In order to determine the effects of the material length scale parameter and the dimension parameters on the static bending of the nano plate, result are obtained in conjunctions with the MCST and the CT. The nano plate considered is made of epoxy (*E=1.44 GPa*, v=0.38, l=17.6 μm). Unless otherwise stated, it is assumed that the length of Lx_1=500 μm, the length of Lx_2=500 μm for distributed load q=1 $\mu N/\mu m^2$. In the numerical calculations, the numbers of the grid points are taken as $n_{x1}=n_{x2}=18$.

In figure 1, the effect of the thickness of the nano plate on the maximum the vertical displacements of the nano plate is presented for various values of the distributed load q for CT and MCST.

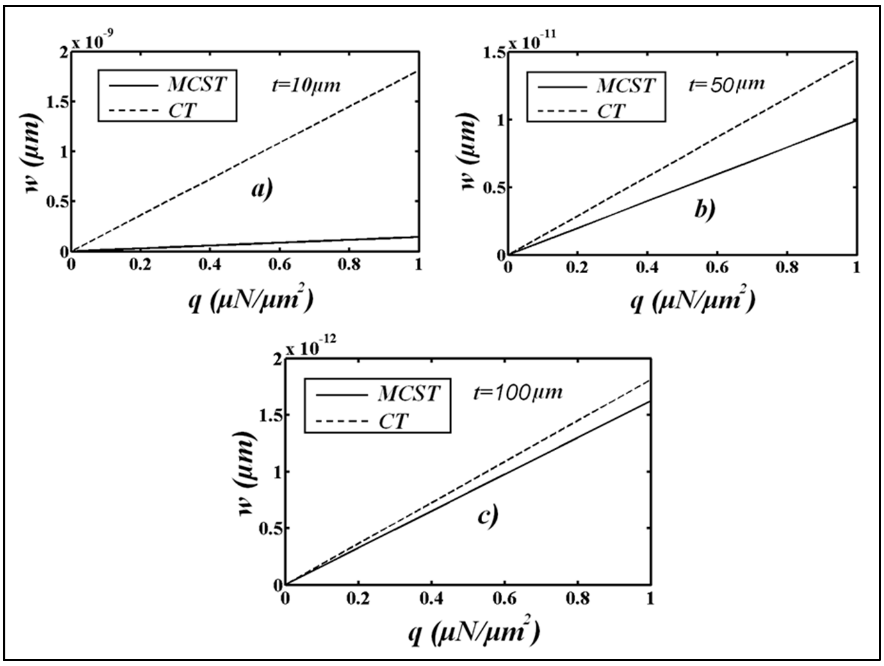

Figure 1: Effect the thickness on the maximum the vertical displacements of the nano plate for CBT and MCST; a) t=10 µm, a) t=50 µm and a) t=100 µm.

It is seen from figure 1, with the increase in the thickness of the nano plate, the difference between the results of the MCST and CT decrease considerably. It shows that an increase in the thickness of the nano plate leads to a decline on effects of size effect and difference between the results of MCST and CBT.

In figure 2, the effect of the dimensionless material length scale parameter (l/t) on the maximum the vertical displacements of the nano plate are presented for CT and MCST. In this figure, for different values of the dimensionless material length scale parameters (l/t), the material length scale parameters (l) is varied when the thickness of the nano plate (t) is keep constant as 10 μm. Therefore, the thickness of the material length scale parameters increases as the dimensionless material length scale parameter (*l/t*) increases.

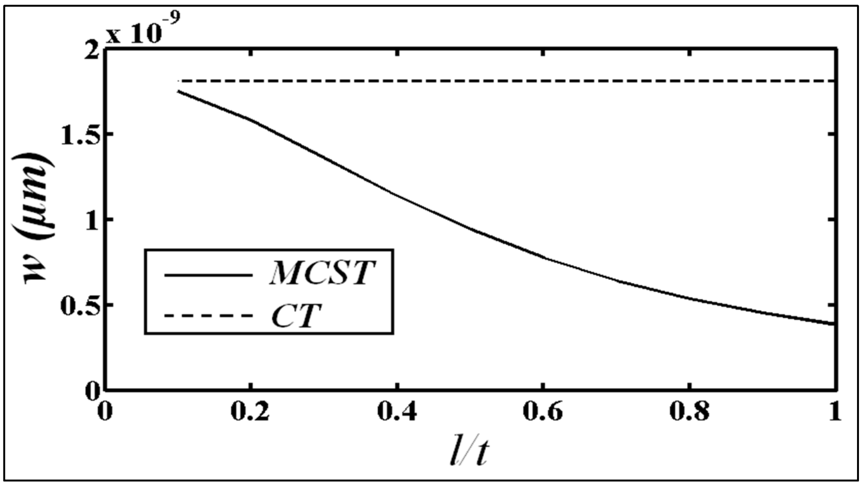

Figure 2: Effect the dimensionless material length scale parameter on the maximum the vertical displacements of the nano plate for CBT and MCST.

As seen from figure 2 that the dimensionless material length scale parameter has no effect on the displacements for the classical theory, which is unable to capture the size effect. However, the displacements of the non classical plate model decreases as the material length scale parameter increases. The static deflections estimated by the CT are always larger than those of the MCST. It is observed from figures that the difference between the two models is significant when the ratio of *l/t* increases. It shows that the material parameter has a very important role on the static responses of the nano plates.

4. Conclusions

In this paper, the static deflections of rectangular nano plate examined based on MCST by using GDQ method. In numerical study, the effect of the material length scale parameter and the dimension parameters of the nano plate on the static responses of the nano plate is presented in both the CT and MCST. Numerical results show that the geometry properties and the dimensionless material length scale parameter have a very important role on the static behavior of the nano plate. MCST displays important size-dependence in higher values of the *l/t* ratios. Also, it is found the numerical results that the displacements of the nano plate by the CT are always larger than those by the MCST.

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