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# Examination of lecturers' content preferences in the teaching of integral: The case of curriculum revision[*]

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**Abstract.** In this study, the content preferences of the lecturers were evaluated in the context of the revision made in the undergraduate mathematics teachers training program in Turkey in 2018. Within this scope, the theorems and examples preferred by the lecturers while teaching the integral subject were evaluated in the context of the shortened time with the revision of the analysis course curriculum. The participants of the study were eight lecturers from different universities. The qualitative data collection procedures were used via document analysis and interviews. The results of the study showed that the participants attached more importance to the pure content in the revised curriculum compared to the previous curriculum, and the time limitations caused a decrease in the applied content in particular. It has been determined that the content in the "Riemann sums" category remained important during the application of both curricula, but after the curriculum revision, the contents in the "Integrability" and "Applications of integral" categories are less placed in the lecture notes. Due to time limitations, some theorems and examples were not included in the teaching of the integral, and this may cause limited understanding for students. The reflections of lecturers' content preferences on student understanding are discussed within the relevant literature.

**Keywords:** Integral, curriculum revision, theorems, examples.

[*] The ethics committee approval for this study was obtained from the Ethics Committee of the Educational Sciences of İstanbul Medeniyet University, dated 03/01/2021 and numbered 2022/01-05.

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## 1. INTRODUCTION

Curriculum revision studies are performed at certain intervals to simplify, deepen or update a specific course or a program in general. In the last three decades, important changes have been made in the analysis course curriculum. One of the most well-known revisions is the calculus reform movement. At this point, it is suggested to present concepts with multiple representations in analysis programs and textbooks, to include productive content that supports conceptual understanding, and to focus on real-life problems that reveal the relationship of mathematics with other disciplines. A similar approach is available in the curriculum revision within Turkish teacher training programs. With the curriculum revision, course credits for mathematics content knowledge decreased and pedagogy knowledge was emphasized more (The Turkish Council of Higher Education [YOK], 2018). In the revised analysis course curriculum, the total credits of the analysis courses (Analysis 1, Analysis 2, and Analysis 3) were reduced from 15 hours to six hours, the applied parts (credits) of the courses have removed, and some contents (e.g., multiple integrals, vectorial analysis etc.) were extracted from the curriculum. The current revision has created a natural study field for researchers. Along with the curriculum revision, it is important to identify the content that the instructors use in their classroom practice because it shows the transitions of aimed and taught knowledge from the instructor's perspectives (Chevallard & Bosch, 2014). While many studies in the mathematics education literature draw attention to the difficulties encountered by calculus students (Hughes-Hallett, 2006; Klymchuk et al., 2010; Tall, 1992), there are few studies on the content preferences of lecturers at the undergraduate level (Bingolbali & Ozmantar, 2009; Sevimli, 2016). However, one of the issues that may cause students’ difficulties is the available teaching content in the classroom. Lecturers are responsible to their students for the content they present in the classroom to be productive and to achieve the aimed learning outcomes.

One of the topics in the analysis course that students have difficulty understanding is integral. Studies present that multiple interpretations and conceptualisations in the integral support understanding more (Ely, 2017; Jones, 2013, 2015a; Sealey, 2014). While some studies focus on interpretation types which are important in integral learning from the student's perspective (Jones, 2015b; Kouropatov & Dreyfus, 2013; Orton, 1983), it is wondered, which interpretation types and which proofs of the integral are prioritised by the instructors in managing the transformation process from scientific knowledge to taught knowledge following the curriculum change. In this study, the reflection of curriculum revision on lecturer content preference was assessed in the context of the integral. The main research problem addressed in the study is to understand how the pure and applied content that lecturers prefer while teaching the integral has changed with the curriculum revision. From this point of view, the research questions are formulated as follows: (*1)* How do the pure and applied content preferred by the lecturers in integral teaching change according to the type of implemeted curriculum? *(2)* How do the theorems preferred by the lecturers vary in terms of definition, proof, and exemplification content, in the teaching of integral? Based on the answers to these questions, the core-content in the teaching of the integral concept was determined according to the lecturers’ teaching practices. Although this study includes local findings in terms of addressing the results of the curriculum reviison in a country; the study results have a wide range of influence in terms of revealing what the core-content should be in teaching the integral concept from the perspective of lecturers. On the other hand, the demand for distance education platforms and the rapid development in the digital transformation of education has made it essential to identify the core content that can be taught in a limited time, for which the study also provides a perspective.

**Analysis Course: Curriculum Revision and Content Preferences**

Analysis courses, forming the foundations of mathematics at the undergraduate level, include advanced mathematical thinking processes such as proof, abstraction, exemplification, and generalisation. The depth of explanations about the theoretical nature of the analysis concepts (such as limit, derivative, integral, series, etc.), and time allocated to these concepts vary according to the programme in which the course takes place. The analysis concepts are introduced at the high school level mathematics curriculum and the theoretical background of these concepts taught in different departments and programmes under different course names (such as Fundamental Mathematics, Advanced Mathematics, Calculus, and Analysis) at the undergraduate level. For instance, the integral concept is taught in two separate courses (theoretical sections in analysis and applications sections in calculus course) in some countries (such as Canada, UK, and the US) while it is lectured under the frame of analysis course (pure and applied parts together) in others (such as Brazil, Germany and Turkey). In some studies, it has been determined that the content supporting conceptual understanding in the traditional classroom environment where calculus subjects are taught is limited and calculation-based approaches are frequently used (Hughes-Hallett, 2006; Oberg, 2000; Orton, 1983; Tall, 1992).

The calculus reform movement started in the late 1980s and early 90s. It is a product of calculus stakeholders who came together for curriculum and teaching content change. The reasons for the reform movement are that students do not see the power of mathematics to unite disparate fields (lack of using the calculus subjects in an unfamiliar situation) and their mathematical knowledge is limited to manipulative techniques (Hughes-Hallett, 2006). As a result of the analysis reform movement, many universities in the USA have revised their teaching content and learning outcomes (Goerdt, 2007). Calculus curriculum begins to include real-life problems, and the teaching contents were re-arranged to allow multiple representations (Hughes-Hallett et al., 2008). As a result of the reform movement, the textbooks developed according to "the rule of four" (use of graphical, numerical, analytical, and verbal interpretations) approaches are used in different countries, so this curriculum and content change have global reflections. Several changes have also been introduced to the Turkish Education System in terms of both mathematics curriculums and teacher training programmes over the past decade. Especially in the mathematics teacher training programme, a radical change was adopted in 2018 with the main philosophy of equipping teachers with higher pedagogical content knowledge during the programme by reducing the rate of pure mathematical content knowledge and increasing the rate of pedagogical content knowledge (YOK, 2018). The reason for this change is to equip mathematics teachers with more qualified pedagogical content knowledge. The most explicit reflection of this change is the reduction of the ratio of the number of analysis course series in the whole programme by 60% compared to the previous programme. In addition, while the learning outcomes for the applied contents were prepared in one-third of the analysis course curriculum, the applied part was not included after the revision.

To monitor the effectiveness of curriculum revisions at undergraduate level, it is necessary to understand how well the instructors could reflect the curriculum into their teaching practice. Instructors' previous experiences and epistemological beliefs may affect the transition process from scholarly knowledge in curriculum or textbooks to the knowledge to be taught (Winslow, 2007). Excluding K-12 levels, although the instructors at the undergraduate level are based on a syllabus, they are more flexible in the allocation of the time to the content and the choice of the textbook used. This situation also provides an opportunity for researchers to determine which learning outcomes are more productive from the instructors' perspective. It is important for the effectiveness of teaching that the instructors include content that will provide more productive definitions and interpretations (Jones, 2015a). According to Hughes-Hallet (2006), the main impact of reform movements is related to its innovative usage and collaboration with other disciplines by the community of mathematicians. Sevimli (2016) addressed the consistency between the content of the textbooks and the traditional and reform calculus approaches from the lecturers' perspectives. The researcher found out that lecturers did not consider the textbooks prepared according to the reform approaches sufficient in terms of supporting the technology integration and conceptual understanding. On the other hand, as a result of interviews with traditional analysis textbook authors and internationally renowned lecturers who actively teach analysis courses, Sofronas et al., (2011) determined that the main achievement aimed in the traditional classroom is to train students who can master derivative-integral calculations and have high operational skills. Some researchers have evaluated the content preferences of university lecturers in pure and applied mathematics according to the undergraduate programme in which the course is taken. For instance, Bingolbali and Özmantar (2009) determined that calculus lecturers, who teach the same topic at different departments, consciously privilege different aspects of mathematics, set different questions on examinations, and follow different textbooks. With the curriculum revision of analysis courses, it is important to investigate how much the lecturers include the pure (theorems) and applied (examples) contents in teaching integral to understand the aimed and taught knowledge in the analysis course. The motivation of this study is to contribute to the literature to fill this gap and to understand the classroom practices.

## 2. METHOD

To answer the research questions addressed in the present study, a research design that provides an opportunity to examine an up-to-date phenomenon (lecturers' content preferences), within its boundaries (within the analysis course), and from a holistic perspective (through the document review and interview data) is needed. Case study design, which is one of the qualitative research methods, was used in the study to evaluate the existing case in-depth with a holistic approach by using different data sources. The case study design is used for evaluation purposes where the phenomenon is interpreted within its boundaries and its effects on other components are examined (Yin, 2009). Comparative case study design, which is the specific type of case study, enables the evaluation of more than one independent case within boundaries systems; it also allows for analytical generalisation (Berg, 2001). In this research, the comparative case study design was used since the preferences of the lecturers for the pure and applied contents of the integral were addressed within the framework of the curriculum revision.

**Participants**

The study includes the follow-up of three semesters of teaching content from the fall semester of 2018 to the spring semester of 2019. The participants are eight lecturers from different universities who have taught analysis courses in the mathematics teachers' training programme. The participants who lecture the analysis course have a Ph.D. in the mathematics or mathematics education departments and are free to choose teaching content and resources during the academic term. It was aimed to improve representativeness of sampling by consider diversity in the entrance scores of the teacher training undergraduate programmes and the accessibility of the lecture notes of the instructors in the study. In addition, the necessity of getting to the lecturers who gave the same course in the previous and current programme to obtain more consistent findings and increase the reliability of the study was effective in the sampling process. Thus, the analysis course notes of the same eight lecturers from the previous and revised curriculum were assessed. The ethics committee approval for this study was obtained from the Ethics Committee of the Educational Sciences of İstanbul Medeniyet University, dated 03/01/2021 and numbered 2022/01-05.

**Settings**

Mathematics teacher training programmes are located within the faculty of education in Turkey, and both pure mathematics and pedagogical courses are completed at the end of a four-year education. A standardised curriculum prepared by the Higher Education Institution of Turkey is followed in mathematics teacher training programmes all over the state which were revised in 2018 (YOK, 2018). The remarkable change between the previous programme used between 2012-2017 and the current mathematics teacher training programme that has been updated in 2018 is the decrease in content knowledge courses (pure mathematics) and the intensity of pedagogical content knowledge in the current programme compared to the former one. The most radical reflection of the change has been observed within the analysis course series: course credit hours have been shortened by 60% compared to the previous programme, and some analysis contents have been removed from the curriculum. Table 1 summarising the comparison of the two curricula according to the credits hours and contents is presented below.

For Analysis 1 and Analysis 2 courses, four hours of theory and two hours of application parts in the Previous Curriculum of Analysis (hereinafter referred to as PCA) have been replaced by two hours of theory in the Revised Curriculum of Analysis (hereinafter referred to as RCA). When Table 1 is analysed, it is obvious that derivative and integral topics were given together in Analysis 1. With the revision, integral (in single variable functions) was given in Analysis 2, and the time allocated to the integral being reduced by a third. Also, multivariable functions' integral and vector analysis topics were removed from the revised curriculum. This revision in the analysis curriculum, which is taught in the first two years of undergraduate mathematics, created a natural research environment. Thus, the opportunity to study with the same lecturers for a long time in the context of two different curricula has emerged. In this sense, the effective content preferred by the lecturers in teaching integral could be observed, and opinions about what the neglected content is and why it is extracted could be determined in a case study design.

Table 1

*The Content and Credit Hours in Previous and Revised Curriculum of Analysis Courses*

| | Previous Curriculum of Analysis | Revised Curriculum of Analysis |
|---|---|---|
| Analysis 1 | (4 hours theory + 2 hours applications)- Limit, continuity concepts, and their applications, the concept of derivative, differentiation rules, theorems of differentiability and its' applications, Riemann sums, definite integral concept, integrability, techniques of integration, applications of integrations in various fields, and improper integrals. | (2 hours theory)- Sets and number systems, types of functions, exponential functions, and logarithmic functions; limit, continuity concepts, and their applications; the concept of derivative, differentiation rules, applications of derivatives, and graphic drawings. |
| Analysis 2 | (4 hours theory + 2 hours applications)- Multivariable function concept, limit and continuity concept in bivariate functions and their applications, partial differentiation, Lagrange multipliers, double integral concept, volume calculations with double and triple integral, and vectorial analysis. | (2 hours theory)- Trigonometric functions, complex numbers, and their properties; Riemann sums, definite integral concept, integrability, techniques of integration, applications of integral, applications of integrations in various fields, improper integrals, and series and convergence tests. |
| Analysis 3 | (3 hours theory)-Concept of series, convergence and divergence in series, alternating series and power series, uniform convergence in function series, generalised convergence tests, Taylor series, and Fourier series | (2 hours theory)-Multivariable functions; IRn's topology, limit, and continuity in bivariate functions, sequences, and series; directional derivative, partial derivative, geometric interpretation of the partial derivative, higher-order derivatives, and chain rule. |

**Data Collection Tools**

Different data collection tools were needed to determine which teaching content the lecturers preferred only after the curriculum revision and to explain the reasons for these preferences. Therefore, document analysis and interviews were used as two different techniques for gathering the data. In the first stage of the data collection process, documents regarding the content used by the lecturers in the teaching practice were examined. While determining the lecture notes, the student notebooks, which are thought to have comprehensive content in the analysis course, were used. In this process, notes of the students selected from different universities with the highest attendance rate were scanned and a dataset covering 1087 pages for application of the PCA and 426 pages for application of the RCA was achieved.

The researcher developed the framework to be used to evaluate the content in these documents (see Appendix A). In this process, the nature of the analysis course was considered. Since the analysis courses are carried out with calculus contents in some countries (like Turkey, where the present study was conducted), both pure and applied contents are presented within the same course. Because the analysis course consists of theoretical and application content and no time is allocated for the application part of the course with the change in the curriculum, the classroom practices of the lecturers were assessed primarily through the pure-applied content balance. In this sense, the knowledge taught under the titles of definition, axiom, theorem, proposition, lemma, and proof in the teaching of integral were coded as a pure content. The contents presented under the examples, exercises, problems, and homework were evaluated in the applied category. In the analysis course, not only theorems but also examples are very important reference sources for understanding the curriculum knowledge preferred by teachers in the classroom. To examine the content preferences of the lecturers in-depth, the used theorems were evaluated under the variables of definition, proof and examples.

In the semi-structured interview, two of the participants (P4 and P6) were asked how the curriculum revision affected the use of examples and theorems in pure and applied contents. In addition, the question of which integral meanings should be understood by the students was directed to the participants. Thus, it is aimed to reveal the contents that are considered productive from the lecturers' perspectives. The answers to the interview questions were used to support the document analysis findings.

**Data Analysis Process**

Theorems were considered to examine the pure contents used in the teaching process. First of all, theorems and propositions concerning the concept of integral in the calculus and analysis textbooks were compiled (Alcock, 2014; Balcı, 2016; Brannan, 2006; Dernek; 2009; Schröder, 2007; Thomas, Weir & Hass, 2013). These textbooks have been chosen because they have international recognition and are shown as reference books by the participants of the study. While categorising the used theorems, the subtitle of the integral unit in the textbooks was considered. Although the number and type of theorems that can be used in teaching the integral concept vary according to the textbook, the theorems in at least two of the different sources nominated by the participants of this study were coded and included in the evaluation framework (see Appendix A). The contents and codes analysed in the process of grouping theorems under the categories are as follows; i) properties of Riemann partitions (T1 and T4), Riemann conditions (T2), upper and lower integrals (T3) under the Riemann sums category, ii) necessary and/or sufficient conditions for integrability (T5 and T6), common limit criterion (T7), integrability of special defined functions (T8) under the Integrability category, iii) linear properties of definite integral (T9), Inequality rule for integrals (T10), FTC (T11), Uniqueness Theorem for Primitives (T12) under the Properties of integrals category, iv) integration by parts (T13), the substitution rule for integration (T14 and T15), improper integral (T16) under the Techniques of integration category, and v) the arc length (T17), the net change theorem (T18), Mean Value Theorem for integral (T19), and theorem of Pappus (T20) under the Applications of integral category. Afterward, the theorems or propositions in the lecture notes of each participant were coded as well, and the numerical equivalent of the coding frequency was determined within the framework of the categories in Appendix A. The coding process was carried out under the supervision of two analysis professors.

While conducting document analyses, different expressions of theorems were considered; hence, the suitability of the content to the relevant category was also considered. For example, there are different theorems in the lecture notes that include the common limit criterion (T7), which are evaluated under the category of "Integrability". Some of these are as follows; i) Let f: [a, b] → IR be a bounded function, if f is integrable on [a, b], then there is a sequence {Pn} of partitions of [a, b] such that $\lim_{n\to\infty} L(f, Pn) = \int_a^b f$ and $\lim_{n\to\infty} U(f, Pn) = \int_a^b f$ and ii) If there is a sequence {Pn} of partitions of [a, b] such that $\lim_{n\to\infty} U(f, Pn) = \int_a^b f$ and $\lim_{n\to\infty} U(f, Pn) = \int_a^b f$ then f is integrable on [a, b] and the common value of these two limits is $\int_a^b f$. If either or both of these two statements are present in the lecture notes, it is confirmed that the common limit criterion content is described. Thus, the differences in expression and detailing in theorems causing data loss were prevented. While analyzing the pure and applied contents included in the teaching of the integral subject, each lecture note is considered in itself in terms of text lengths. Since pure and applied contents can be together in a page, the scanned lecture notes were compared within the standard scale page. Thus, the ratio of pure and applied contents preferred by the instructors in terms of pages was determined.

Theorems or propositions was evaluated according to the variables of definition, proof, and example. For instance, if three of the four theorems (or propositions) evaluated under the "Riemann sums" category were present in a lecture note, it was considered for such lecture note that "the ratio of the content in the Riemann sums category according to the definition variable is ¾". The codes (from T1 to T20) in all lecture notes were analysed descriptively in terms of each variable (definition, proof, and example) in this way, and all findings were presented with percentages over the categories. While the theorems utilized were evaluated in terms of "example" variable; there were some cases where more than one example was given for theorems in various categories or propositions. Therefore, for consistent coding, frequency analysis was performed depending on whether the example was available. The examples quoted from the lecture notes have been translated into English, adhering to the original text, for the readers to better understand the process. The views supporting the findings in the lecture notes have been presented with a direct quotation. A qualitative analysis program (Nvivo 10) which can present lecture notes from different participants with a standard evaluation platform and provide data in terms of the content rate within a page was used to analyse the data.

**Validity and reliability**

The qualitative research approach was used in this study; therefore, credibility instead of validity and dependability instead of reliability were highlighted (Guba & Lincoln, 1994). Initially, the lecture notes in the two settings (applications of PCA and RCA) were obtained from the same participants. thus, other external variables (such as instructor's role and/or teaching environment differences) were taken under control, and analysis was conducted over consistent documents. While obtaining the lecture notes, multiple sources were searched (including the most comprehensive one among the student notebooks), and it was aimed to reach more valid content. The contents of six-different textbooks were referenced (Alcock, 2014; Balcı, 2016; Brannan, 2006; Dernek; 2009; Schröder, 2007; Thomas, Weir & Hass, 2013), and the evaluation framework was formed by consulting two professors who studied in the mathematical analysis field for increasing the credibility of the study. Besides, 112 pages (approximately 10% of the lecture notes) from randomly selected lecture notes were re-coded by two external coders (research assistants), and their compatibility with the researcher's codes was determined by measuring the agreement between them (calculation by dividing the number of matched codes to the total number of codes and multiplying with 100). The agreement rate was found as 84% and 87% for the coding process of pure and applied contents, respectively. The fact that intercoders made consistent encodings for the same contents (inter-rater reliability) proved that the results of the study are dependable (Miles & Huberman, 1994).

## 3. FINDINGS

### Pure and Applied Content Preferences in Teaching Integral

While presenting the research findings, general findings regarding the percentage of pure and applied contents that the participants included in the teaching of the integral subject were first given. Then, the reflections of pure and applied contents on theorem and example usage were analysed. The ratios of pure and applied content used by the participants while teaching the integral subject before and after the curriculum revision were presented in Figure 1.

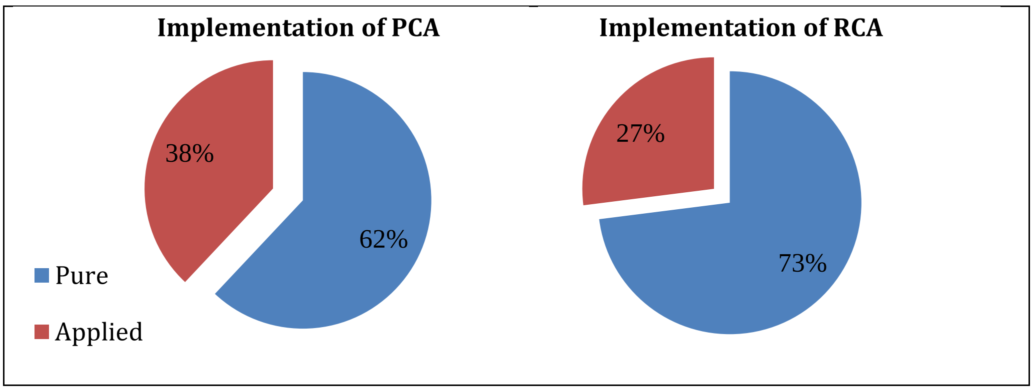


Figure 1. The Distribution of Pure and Applied Content Rates in Lecture Notes

The findings showed that the rate of pure content used in the lecture notes increased (62% in PCA and 73% in RCA) while the rate of applied content decreased (38% in PCA and 27% in RCA) with the curriculum revision. It was determined that the participants proportionally extended the pure content (up to about 11%) and restricted the applied content in the revised curriculum (see Figure 1). Although the teaching content remained the same within the application of RCA, the one-third percentage constraint of course time led to the favoring of pure content in the transitions of aimed knowledge to taught knowledge. As a result of the tasks, examples, or problems that require the application of integration rules necessitating a long time in the class, it was observed that the lecture notes from the implementation of RCA usually contained a prototype example for each integration technique and often the definition of integrating rules and formulas, or theoretical content on how they are obtained (such as functions with non-elementary indefinite integrals).

*P4:* Of course, the best of all is to give the students the theoretical and practical nature of this course in a period in which they can conceptualize it adequately. However, if there is a reduction in terms of time while the topics remain the same, the aim of the course and the target audience should be considered. Analysis course is the basis of advanced mathematical thinking at the undergraduate level and we give our lecture with the aim of training mathematics teachers. So, instead of examples that summarize a few cases, it would be more reasonable to proceed with the theorems that generalize the cases.

In the interviews, when asked why the reduction of the course led to the increased theoretical content in the classroom practice, P4 and P6 stated that they reduced the number of examples in order to finish teaching the content in time, therefore students were assigned problems in homework or worksheets, aiming to eliminate this incompetence in out-of-class learning environments. Also, by making reductions especially in the area or volume calculation topics, which require drawing of the graphs, it was targeted to save time. The following excerpt includes the arguments an instructor considers when balancing pure and applied content. P4, who was cited and included theoretical content in a percentage of 66% in the implementation of PCA, increased the rate of theoretical content to 79% in the implementation of RCA. When the change in the aimed and taught content is analyzed in the context of pure-applied balance, it is seen that P4 tends to prefer pure content, in which symbolical and formalized meanings are prioritized, with the goal of training mathematics teachers. To analyse the change in the pure and applied content through curriculum revision, the findings related to the theorem and example usage are evaluated in more detail.

**The Use of Definition, Proof and Example in Teaching Integral**

The percentage of the theorems used by the participants in the previous and revised curriculum was classified under five categories based on the variables of definition, proof, and example. These findings were presented in Table 2. The most frequently defined theorems were included under the "Properties of integrals" category in the lecture notes where PCA is applied. Definitions of the contents evaluated under this category, such as linear properties of definite integral or definitions of the FTC were included in the application of PCA by 86%. Another theorem category that was frequently defined in the application of PCA was Riemann sums. Examining how often the theorems defined are proved, it was found out that the proofs of the content from "Properties of integrals" and "Integrability" categories were more frequently included in the application of PCA. In the application of PCA, the theorem category that was defined but proven less was "Applications of integral", and approximately one quarter (78% -52% = 26%) of the theorems in this category were not proven by the lecturers.

The participants frequently defined the theorems under the "Properties of integrals (79%)" and the "Riemann sums (74%)" category in the implementation of RCA. It was also determined that the theorems in the "Integrability (53%)" and "Applications of integral (58%)" categories were used less for the definition variable in the lecture notes. When the theorems used by the participants in their lecture notes are evaluated within the proof variable, it is seen that the rates of proved theorems in the "Properties of integrals (70%)" category are higher than other theorems categories. Nevertheless, it was also determined that there was a decrease in the proving of theorems in the categories of "Applications of integral (30%)" and "Techniques of integration" (44%). While the category with the lowest difference between definition and proof variables was "Riemann sums"; the most significant difference was observed in the category of "Applications of integral" in the lecture notes of the RCA.

Table 2

*The Distribution of Use of Definition, Proof and Example in Teaching Integral*

| | Application of PCA | | | Application of RCA | | |
|---|---|---|---|---|---|---|
| Category | Definition | Proof | Example | Definition | Proof | Example |
| Riemann sums | 83% | 70% | 69% | 74% | 68% | 51% |
| Integrability | 78% | 74% | 85% | 53% | 47% | 32% |
| Properties of integrals | 86% | 77% | 82% | 79% | 70% | 71% |
| Techniques of integration | 73% | 63% | 91% | 65% | 44% | 84% |
| Applications of integral | 78% | 52% | 88% | 58% | 30% | 38% |

For the definition of the theorems, the categories that maintain their dominance in both applications were "Properties of integrals" and "Riemann sums". With the curriculum revision, the rate of preference for both definition and proof variables in the "Integrability" and "Integral applications" categories decreased by approximately 20%. The theorems in the "Integrability" category, which was frequently defined by a percentage of 78% in the lecture notes of PCA were represented with a rate of 53% by decreasing a quarter in the lecture notes of RCA. In terms of the proof variable, similar to the definition variable, the content of the course notes included for the RCA decreased by 27% in the category of "Integrability" and 22% in the category of "Applications of integral". When the theorems in the categories were examined particularly, the three theorems used by the participants in the lecture notes of both applications at a high rate (over 75%) were as follows: T2 coded theorem related to conditions of Riemann sums in the "Riemann sums" category, T11 coded theorem related to the FTC in the "Properties of integrals" category and T13 coded theorem related to the partial integration rule in the "Techniques of integration" category. The most reduced (over 50%) theorems in the lecture notes compared to the previous application were T8 coded theorem related to "continuity and integrability" in the "Integrability" category and T17 coded theorem "the arch length formula" in the "Applications of integral" category.

When the contents from the implementation of PCA were analyzed in terms of the "examples" variable, it was observed that the examples of theorems in the "Techniques of integration (91%) " and "Applications of integral (88%) " categories were included more generally. It has also been found that the categories in which the most and the least use of examples were included during the RCA’s implementations were "Techniques of integration (84%)" and "Integrability (32%)", respectively. In the exemplification process of the RCA's implementation, the decreases were determined in all categories, but the notable decrease was in the "Riemann sums" category. Compared to the lecture notes in PCA, the category with the notable decrease in terms of use of examples in the lecture notes in RCA is "Applications of integral", while the category that maintains its importance in terms of use of examples in both curricula is "Techniques of integration". An excerpt from the interviews with P6 is shared below to clarify the reasons for the content change between the two implementations. P6, who often included "Techniques of integration" examples in both implementations but reduced the "Applications of integral" contents in the implementation of RCA stated that the content that analyses the relationship between derivative and integral are more important to mathematics majors for their professional development.

*P6:* It is much more important to teach the Riemann integrability requirements theoretically, and reversible transitions between derivative and integral in practice in the limited course time, because, mathematics teacher candidates will encounter such examples about indefinite integral more frequently in their future professional teaching experiences. Other physics or engineering applications can be taught in other courses.

## 4. RESULTS, DISCUSSIONS AND SUGGESTIONS

After the curriculum revision, the pure and applied contents preferred by the lecturers in integral teaching were evaluated in the present study. Although this study covers local findings in terms of addressing the results of the curriculum revision in a country (Turkey); the results of the study have a wide impact, as it focuses on the instructors' perspective of what the core-content should be in teaching the concept of integral. With the curriculum revision, it was determined that lecturers reduced the applied content, and the dominance of the pure content increased in the lecture notes of the RCA. By the nature of the analysis course, it is expected that the formal and axiomatic language of the integral concept is used more by mathematicians to train mathematics teachers. Still, it is noteworthy that the lecturers tended to present more theoretical content in the teaching practices when the content remained the same, but the time was shortened. While no study in the mathematic education literature investigates the pure-applied content balance within the instructors' perspectives, studies that show the importance attached by the lecturers to the proof processes and the formal language of mathematics also project the teaching practice in analysis courses. For instance, based on the opinions of academicians, Güler (2016) found that proof processes have a key role in the professional lives of pre-service mathematics teachers because it improves problem-solving and advanced mathematical thinking. Despite the fact that the calculus contents are also included in the analysis course in the curriculum where the research was carried out, the lecturers' decision to decrease the calculus contents may be related to their previous learning experiences (training with theorem-proof dominance).

Based on the interview findings and related literature, it is observed that participants focus on the pure content by considering the nature of the course (the combination of algebra and geometry) and the professional development of the students (mathematics teachers' content knowledge) in their content preferences. According to Bingolbali and Özmantar (2009) the lecturers may change their content preferences in regard to the professional life needs of the client students. In this study, the lecturers, who evaluated that definitions and proofs were more important in the analysis course given to pre-service mathematics teachers, may have preferred the pure content more following the revision. However, the decrease in the content of the practices of the integral may lead future teachers not sufficiently to understand the integral concept's interdisciplinary power. Although the theoretical knowledge of the integral concept is not taught at the high school level, it is possible to encounter the accumulated change meanings of the definite integral from geometry (such as the calculation of the approximate value of pies) to physics (such as a transition from position- to velocity- to acceleration) in the curriculum. For instance, Kouropatov and Dreyfus (2013) highlighted that the integral conceptualisation based on the idea of accumulation at the high school level is vital for the proceptual understanding and suggested that curriculums should give more place to these applications. For this reason, the strong content knowledge and pedagogical expertise that mathematics teachers have about integral are allowed to construct meaningful knowledge together with acquiring technical abilities.

When the pure contents in the lecture notes were examined within the theorem usage, it was determined that the "Riemann sums" and "Properties of integrals" categories maintained their place and importance in both implications. Considering the historical development of the concept of integral, it can be said that the current theoretical definitions, which were found sophisticated by mathematicians and emerged by Cauchy, Riemann, and Schwartz in the modern calculus period, were the definitions essentially highlighted by the participants (Bressoud, 2011). This finding also overlaps with definite integral being the introduction in the content order in many analysis and calculus textbooks, and the definition of Riemann, which is the limit of infinite sums being the first unit in these sources (Brannan, 2006; Schröder, 2007; Thomas, Weir & Hass, 2013). If the course time is not enough, the instructor, who presents the knowledge to be taught following the syllabus order, may not be able to lecture the contents at the end of the syllabus in the class. For this reason, the order of teaching of the contents in the textbooks may affect the content selection. However, the lecturers who are the participants of this study have the freedom and flexibility in terms of the content formation and order. At this point, it is thought that the lecturers' beliefs about the productiveness of teaching content may affect the forming of taught knowledge more than the teaching order of the contents in reference books.

Another issue that needs to be discussed regarding the content preferences is that the proofs and examples in the categories of "Integrability" and "Application of integral" are less given with the application of the revised curriculum. The reduction of pure and applied contents regarding the concept of integrability and applications of integral may cause various difficulties. A student who has been learning the integral concept in a course via the FTC dominance that experiences all integrations can be manipulated by using anti-derivatives, may think that the "accumulated changes" meaning is not necessary and is an approximate calculation. In fact, the cardinality of the set of Riemann integrable functions is bigger than that set of the differentiable functions (Dernek, 2009). In other words, every integrable function does not have to be differentiable. Compared to the limited number of functions we can calculate its anti-derivatives currently; more functions need to be calculated with more numerical integration (especially real-world applications.) Many real-world applications involve functions that do not have anti-derivatives which can be expressed in terms of elementary functions. For instance, Sevimli (2018) found that mathematics majors have misconceptions such as not every integrable function is continuous which is associated with the limitations of the lectured examples and counterexamples. In this study, a significant decrease was found regarding the proof of the "Integrability" category in the lecture notes of the RCA, which may cause students to have difficulty understanding the bilateral relations between the concepts of continuity, differentiability, and integrability. The fact that the lecturers decreased the "Application of integral" content after curriculum revision also limited the exemplification of real-world problems in the lecture notes. Using examples revealing a single or dominant conceptualisation (anti-derivatives in this study) when teaching integral may restrict students' theoretical understanding of the integral.

The results of this study pointed out that the reduction in time and content after the curriculum revision has the following three reflections on the lecturers ' content preferences: i) lecturers attached more importance to the pure contents compared to the applied ones during the teaching of the integral, ii) the definitions and proofs of the theorems related to Riemann sums remained important, but there was a significant decrease in proving of the theorems related to integrability and applications of integral contents, iii) While the contents about integration techniques remained important in the exemplification process, the number of examples related to applications of integral decreased in the lecture notes.

The results of present study include recommendations for curriculum developers and researchers. The fact that the time allocated to the topic is insufficient after the curriculum revision in the analysis course causes a decrease in proof of the theorems. To preserve the theoretical knowledge of the students, it is necessary to allocate sufficient time not only to integral but also to other analysis topics. While both the pure and the applied part of the integral should be included in the same course in the curriculum of the study, the reduction of the applied content of the course due to the lack of time might limit the interdisciplinary usage of integral by the students. To prevent this limitation, it is recommended that an applied mathematics course should be included in the teacher training programme, discussing the applications of the analysis course in other disciplines. In this research, the tendencies of the lecturers in content preferences and the contents that should be included in the teaching of the integral were described. Furthermore, it is wondered what the reflection of this content difference on student knowledge and understanding will be. In this context, the effects of preferred content on the students’ conceptualization process and the misconceptions expressed in the integral literature regarding the concept can be investigated in future researches.

## References

Alcock, L. (2014). *How to think about Analysis.* UK, Oxford University Press.

Balcı, M. (2016). *Matematik Analiz 1.* Ankara, Palme Yayıncılık.

Berg, B. L. (2001). *Qualitative Research Methods for the Social Sciences.* Boston, Allyn and Bacon.

Bingolbali, E., & Ozmantar, M. F. (2009). Factors shaping mathematics lecturers’ service teaching in different departments. *International Journal of Mathematical Education in Science and Technology, 40*(5), 597-617.

Brannan, A. D. (2006). *A first course in Mathematical Analysis*. UK, Cambridge University Press.

Bressoud, D. M. (2011). Historical reflections on teaching the Fundamental Theorem of Integral Calculus. *The American Mathematical Monthly, 118*(2), 99-115.

Chevallard, Y., & Bosch M. (2014). Didactic transposition in mathematics education. In: Lerman S. (Eds.), *Encyclopedia of Mathematics Education*. Dordrecht, Springer.

Dernek, A. (2009). *Analiz I* (2nd edition). Ankara, Nobel Yayıncılık.

Ely, R. (2017). Definite integral registers using infinitesimals. *Journal of Mathematical Behavior, 48*, 152-167.

Goerdt, L. S. (2007). *The effect of emphasizing multiple representations on calculus students' understanding of the derivative concept* (Unpublished master's thesis). EdD, The Universty of Minnesota, USA.

Guba, E. G., & Lincoln, Y. S. (1994). Competing paradigms in qualitative research. In N. K. Denzin, & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 105-117). London: Sage Publications.

Güler, G. (2016). The difficulties experienced in teaching proof to prospective mathematics teachers: Academician views. *Higher Education Studiess, 6*, 145–158.

Hughes Hallett, D. (2006). What have we learned from calculus reform? The road to conceptual understanding. *MAA Notes, 69*, 43-47. Retrieved from https://www.math.arizona.edu/~dhh/NOVA/calculus-conceptual-understanding.pdf

Hughes-Hallett, D., Gleason, A. M., McCallum, W. G. et al. (2008). *Calculus: Single variable* (5th edition). New York, Wiley.

Jones, S. (2013). Understanding the integral: Students' symbolic forms. *Journal of Mathematical Behavior, 32*, 122–141.

Jones, S. R. (2015a). Areas, anti-derivatives, and adding up pieces: Definite integrals in pure mathematics and applied science contexts. *Journal of Mathematical Behavior, 38*, 9–28.

Jones, S. R. (2015b). The prevalence of area-under-a-curve and antiderivative conceptions over Riemann sum-based conceptions in students' explanations of definite integrals. *International Journal of Mathematical Education in Science and Technology, 46*(5), 721-736.

Klymchuk, S., Zverkova, T., Gruenwald, N., & Sauerbier, G. (2010). University students' difficulties in solving application problems in calculus: Student perspectives. *Mathematics Education Research Journal, 22*, 81–91.

Kouropatov, A., & Dreyfus, T. (2013). Constructing the integral concept on the basis of the idea of accumulation: suggestion for a high school curriculum. *International Journal of Mathematical Education in Science and Technology, 44*(5), 641-651.

Miles, M. B., & Huberman, A. M., (1994). *Qualitative data analysis: An expanded sourcebook*. Thousand Oaks: Sage Publications.

Oberg, R. (2000). *An investigation of under graudate calculus students understanding of the definite integral* (Unpublished master's thesis). EdD., The Univesrty of Montana, USA.

Orton, A. (1983). Student's understanding of integration. *Educational Studies in Mathematics, 14*(1), 1-18.

Sealey, V. (2014). A framework for characterizing student understanding of Riemann sums and definite integrals. *Journal of Mathematical Behavior, 33*(1), 230–245.

Schröder, B. (2007). *Mathematical analysis-A concise introduction.* New Jersey, Wiley-Interscience.

Sevimli, E. (2016). Evaluating views of lecturers on the consistency of teaching content with teaching approach: traditional versus reform calculus. *International Journal of Mathematical Education in Science and Technology, 47*(6), 877-896.

Sevimli, E. (2018). Understanding students' hierarchical thinking: a view from continuity, differentiability and integrability. *Teaching Mathematics and its Applications: An International Journal of the IMA*, *37*(1), 1-16.

Sofronas, K.S., De Franco, T. C., Vinsonhaler, C., Gorgievski, N., Schroeder, L. & Hamelin, C. (2011). What does it mean for a student to understand the first-year calculus? Perspectives of 24 experts. *The Journal of Mathematical Behavior*, *30*, 131-148.

Tall, D. (1992). Students' difficulties in calculus. In K-D. Graf, N. Malara, N. Zehavi, & J. Ziegenbalg (Eds.), *Proceedings of Working Group 3 at ICME–7, Québec 1992* (pp. 13–28). Berlin, Freie Universität Berlin.

Thomas, G. B., Weir, M. D., & Hass, J. (2013). *Thomas' calculus* (13th edition). Boston, MA, Pearson.

Winslow, C. (2007). Didactics of mathematics: An epistemological approach to mathematics education. *The Curriculum Journal, 18*(4), 523-536.

Yin, R. K. (2009). *Case study research: Design and methods* (4th edition). Thousand Oaks, Sage Publications.

YOK [Council of Higher Education in Turkey], (2018). İlköğretim matematik öğretmenliği lisans programı [Mathematics teacher training program]. Retrieved from https://www.yok.gov.tr/Documents/Kurumsal/egitim_ogretim_dairesi/ Yeni-Ogretmen-Yetistirme-Lisans-Programlari/Ilkogretim_Matematik_Lisans_Programi.pdf.

**Appendix A.** Evaluation Framework for Used Theorems

| Category | The code for theorems in the textbooks about integral |
|---|---|
| ***Riemann Sums*** | $T_1$: Let *f*: [a, b] → *IR be* a bounded function. Let *P* and *P′* be partitions of [a, b], where *P′* is a refinement of *P* that contains just one additional partition point. Then *L(f, P)≤L(f, P')* and *U(f, P')≤U(f, P).*<br>$T_2$: Let *f*: [a, b] → *IR be* a bounded function. Then f is Riemann integrable on [a,b] iff for all *ε*>0, there is a partition *P* of [a, b] for which *L(f, P) - U(f, P)* < *ε*.<br>$T_3$: Let *f*: [a, b] → *IR be* a bounded function. Then: the lower integral $\int_a^b f$ and the upper integral $\int_a^b f$ both exist.<br>$T_4$: Let *f*: [a, b] → *IR* be bounded and let $P = \{a = x_0, . . . , x_n= b\}$ be a partition of [a, b]. Then for all associated evaluation sets $T = \{t_1, . . . , t_n\}$ the inequalities *L(f,P)≤R(f,P,T)≤U(f,P)* hold. |
| ***Integrability*** | $T_5$: Let *f*: [a, b] → *IR* be a bounded function. Then *f* is Riemann integrable.<br>$T_6$: Let *f*: [a, b] → *IR* be a bounded function. If *f* is monotonic on [a, b], then it is integrable on [a, b].<br>$T_7$: Let *f*: [a, b] → *IR be* a bounded function. If f is integrable on [a, b], then there is a sequence *{Pn}* of partitions of [a, b] such that $\lim_{n\to\infty} L(f, Pn) = \int_a^b f$<br>$T_8$: Let *f*: [a, b] → *IR* be continuous. Then *f is* Riemann integrable. |
| ***Properties of integrals*** | $T_9$: Let $c_1f$ and $c_2g$ be integrable on [a, b] (for any two constants $c_l$ and $c_2$). Then so are: Sum Rule $\int_a^b (c_1\mathrm{f} + c_2\mathrm{g})(x)dx = c_1\int_a^b f(x)dx + c_2\int_a^b g(x)dx$<br>$T_{10}$: Let *f*, *g*: [a, b] → *IR* be integrable. If *f (x) ≤ g(x)* for all *x ϵ* [a,b], then $\int_a^b f(x)\,dx \le \int_a^b g(x)dx$.<br>$T_{11}$: Let *f*: [a, b] → *IR* be a continuous functions. If $F(x) = \int_a^x f(t)dt$ Then $F'(x) = f(x)$ and $\int_a^b f(x)dx = F(b) - F(a)$.<br>$T_{12}$: Let $F_1$ and $F_2$ be primitives of a function *f* on an interval *I*. Then there exists some constant c such that $F_2$(x)= $F_1$(x)+ for all *cϵI* |
| ***Techniques of integration*** | $T_{13}$: If *f* and *g* are differentiable on an interval [a, b], and $f'$ and $g'$ are continuous on [a, b], then $\int_a^b f(x).g'(x)dx = f(x).g(x)\Big\{_a^b - \int_a^b f'(x).g(x)dx$<br>$T_{14}$: If *u=g(x)* is a differentiable function whose range is an interval *I* and is continuous on *I* , then $\int f((g(x))g'(x)dx = \int f(u)du$<br>$T_{15}$: If *f* is continuous on [a, b], *g* differentiable on[c, d], $g'$ continuous on [c, d], and *g*([c, d]) ⊆ [a, b], then $\int_{g(c)}^{g(d)} f(x)dx = \int_c^d f\big(g(t)\big)(g'(t)dt$<br>$T_{16}$: $\int_1^\infty \frac{1}{x^p}dx$ is convergent if *p>1* and divergent if *p≤1.* |
| ***Applications of integral*** | $T_{17}$: If is $f'$ continuous on [a, b] , then the length of the curve y=f(x), $a \le x \le b$, is $L = \int_a^b \sqrt{1 + [f'(x)]^2}$<br>$T_{18}$: The integral of a rate of change is the net change: $\int_a^b F'(x)dx = F(b) - F(a)$.<br>$T_{19}$: Let *f: [a, b]* → *IR* be continuous. Then there is a *c ϵ(a, b)* so that $\int_a^b f(x)dx = f(c)(b-a)$ , it is also equal to the average value of *f* on *[a, b]* ( $f_{ave} = \frac{1}{b-a}\int_a^b f(x)dx$).<br>$T_{20}$: Let *R* be a plane region that lies entirely on one side of a line in the plane. If *R* is rotated about, then the volume of the resulting solid is the product of the area *A* of *R* and the distance travelled by the centroid of. |

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