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Initial Coefficients Estimate and Fekete-Szegö Problems for Two New Subclasses of Bi-Univalent Functions

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Abstract

In the present investigation, making use of definition of the generalized Bivariate Fibonacci-Like polynomials that include polynomials such as Horadam, Chebyshev polynomials two new subclasses of bi-univalent functions are introduced. Then, some bounds are determined for the initial Taylor-Maclaurin coefficients of the functions belonging to these new subclasses. Further, the well-known Fekete-Szegö problem is discussed for the defined subclasses. Lastly, several remarks are indicated for the some special values of variables.

Keywords: *Bi-univalent function; bivariate Fibonacci polynomials; coefficient estimates; Fekete-Szegö functional; Horadam polynomials.*
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1. Introduction

The usages some special polynomials such as Chebyshev, Faber, Horadam, Lucas and Fibonacci in geometric function theory has made considerable effect on the many mathematicians. As a result of this effect numerious subclasses of univalent and bi-univalent functions which are holomorphic in the unit disk were introduced in the literature. By using the well-known subordination notion and some basic rules in geometric function theory of complex analysis, certain famous problems like initial coefficients estimate, Fekete-Szegö and Hankel determinant problems were considered for these new subclasses of analytic functions. Motivated by the relations between special polynomials and analytic function classes, we introduce two new subfamilies of analytic and bi-univalent functions in this paper. Later on, we obtain some upper bounds for the first two coefficients of functions belonging to these new subclasses. Also, we deal with the Fekete-Szegö problems for the defined function classes. Furthermore, we present several corollaries and remarks at the end of the main results.
It is worth to mention here that special polynomials reduce to the some well-known special number sequences for certain values of the parameters. Thus, we find opportunity to study few function subclasses connected with the special polynomials and number sequences together. That is, our main results are generalized certain earlier results given in some previous paper which will be mentioned in the sequel.
This paper is organized as follow: Section 1 is divided into three subsections. Some basic definitions of geometric function theory is remembered in the first subsection, while we present knowledge about the generalized bivariate Fibonacci-like polynomials in the second subsection. In third subsection, two new function subfamilies of analytic bi-univalent functions is introduced by making use of generalized bivariate Fibonacci-like polynomials. In Section 2 and Section 3, we determine some upper bounds for the second and third coefficients of the functions belonging to the subclasses introduced. In Section 4, the well-known Fekete-Szegö problem is discussed for these new subclasses. At the end of the last three section, we also present some remarks and corollaries for the initial coefficient estimations and Fekete-Szegö inequalities.

1.1. Some basic concepts in Geometric Function Theory

Let $\mathscr{A}$ denote the class of all holomorphic functions of the form

$$f(z)=z+a_2z^2+\cdots=z+\sum_{n=2}^{\infty}a_nz^n, \tag{1.1}$$

in the open unit disk $\mathscr{U}=\{z\in\mathbb{C}:|z|<1\}$ normalized by the conditions $f(0)=f'(0)-1=0$. Let $\mathscr{S}$ show the subfamily of $\mathscr{A}$ consisting of functions univalent in $\mathscr{A}$. Due to the well-known Koebe one quarter theorem (see [12]), it can be easily said that if $f\in\mathscr{S}$, then there

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exists the inverse function f^{-1} satisfying

$$f^{-1}(f(z)) = z, (z \in \mathscr{U}) \text{ and } f\left(f^{-1}(w)\right) = w, \left(|w| < r_0(f), r_0(f) \geq \frac{1}{4}\right),$$

where

$$f^{-1}(w) = w - a_2w^2 + \left(2a_2^2 - a_3\right)w^3 - \left(5a_2^3 - 5a_2a_3 + a_4\right)w^4 + \cdots =: g(w). \tag{1.2}$$

It is well-known that if both functions f and f^{-1} are univalent in $\mathscr{U}$, then the function $f \in \mathscr{A}$ is called bi-univalent function in $\mathscr{U}$. In general, bi-univalent functions' class is shown by Σ. Very recently Srivastava *et al.* gave comprehensive informations about bi-univalent functions in their pioneering work [22]. Interested readers can find some interesting examples and a short history about the class Σ in this paper.
In the literature several important coefficient estimates of the analytic bi-univalent functions can be found. For example, Lewin proved the first estimation as $|a_2| < 1.51$, while Brannan and Clunie presented a bound for the second coefficient as $|a_2| \leq \sqrt{2}$ in [17] and [7], respectively. Further, Netenyahu found that $\max|a_2| = \frac{4}{3}$ and Tan showed that $|a_2| \leq 1.485$ for $f \in \Sigma$ in [19] and [27], respectively. In addition, Brannan and Taha studied on some subclasses of bi-univalent functions and presented some coefficient estimates in [8]. In Geometric Function Theory, the problem of the finding a coefficient estimate on $|a_n|$ for $n \in \mathbb{N}, n \geq 3$, is still an open problem. Nowadays, many mathematicians focused on this problem to solve it. For this purpose, so many new subclasses of analytic bi-univalent functions is defined and also investigated coefficient bounds for these subclasses. For instance, motivated by the some earlier works (like [7, 8, 17] and [22] the authors defined some new subclasses of bi-univalent functions and presented certain non-sharp estimates on the initial coefficients of these subclasses in [1, 2, 3, 4, 5, 6, 9, 10, 11, 13, 14, 15, 23, 24, 25, 26, 28] and references therein. In the above papers, the authors utilize generally the principle known as subordination. Now, we would like to remind this principle as follow:
If the functions f and $F \in \mathscr{A}$, then f is said to be subordinate to F if there exists a Schwarz function w such that

$$w(0) = 0, |w(z)| < 1 \text{ and } f(z) = F(w(z)) \qquad (z \in \mathscr{U}).$$

This subordination is shown by

$$f \prec F \quad \text{or} \quad f(z) \prec F(z) \quad (z \in \mathscr{U}).$$

If F is univalent function in $\mathscr{U}$, then this subordination is equivalent to

$$f(0) = F(0), \quad f(\mathscr{U}) \subset F(\mathscr{U}).$$

There are comprehensive informations about the subordination notion in Monographs written by Miller and Mocanu(see [18]). Also, a function $f \in S$ is said to be Bazilevic function, if it satisfies (see [21]):

$$R\left\{\frac{z^{1-\lambda}f'(z)}{(f(z))^{1-\lambda}}\right\} > 0, z \in \mathscr{U}, \lambda \geq 0.$$

This class of functions was denoted by B_λ. It is observed that for $\lambda = 0$, we have the class of starlike functions.

1.2. Generalized bivariate Fibonacci-like polynomial

Chebyshev, Faber, Horadam, Lucas, Fibonacci polynomials and their generalizations have great importance in applied sciences such as physics, engineering and so on. The Fibonacci polynomial is one of the very important special polynomials. Its several generalizations is defined in the literature since this polynomial has common usage in the applied sciences (see [16]). The Fibonacci numbers are defined as the following recurrence relation:

$$F_n = F_{n-1} + F_{n-2}, F_0 = 0, F_1 = 1$$

for $n \geq 2$. In [20] and its references, the readers may find a short history and comprehensive informations about the generalized bivariate Fibonacci polynomial. Also, the authors gave a new generalization of the Fibonacci polynomial which is called generalized bivariate Fibonacci-like polynomial in [20].

Let p, q be positive integers and x, y be real numbers. For, $n \geq 2$, the generalized bivariate Fibonacci-like polynomials are defined by the recurrence relation:

$$H_n(x,y) = pxH_{n-1}(x,y) + qyH_{n-2}(x,y), \tag{1.3}$$

where $H_0(x,y) = a$, $H_1(x,y) = b$ and px, $qy \neq 0$, $p^2x^2 + 4qy \neq 0$. The generating functions of generalized bivariate Fibonacci-like polynomials is (see[20])

$$H^{(x,y)}(z) = \sum_{n=0}^{\infty} H_n(x,y)z^n = \frac{a + (b - apx)z}{1 - pxz - qyz^2}. \tag{1.4}$$

For the different choosing p, q, a, b and y, we obtain different polynomial sequences by using recursive relation. These polynomial sequences are given in the Table 1 below:

(p,q)	(a,b)	(x,y)	$H_n(x,y)$
$(1,1)$	$(0,1)$	(x,y)	Bivariate Fibonacci, $F_n(x,y)$
$(1,1)$	$(0,1)$	$(x,1)$	Fibonacci, $F_n(x)$
$(2,1)$	$(0,1)$	$(x,1)$	Pell, $P_n(x)$
$(1,1)$	$(2,x)$	(x,y)	Bivariate Lucas, $L_n(x,y)$
$(2,1)$	$(1,2t)$	$(t,-1)$	Chebyshev of the second kind, $U_n(x)$
(p,q)	(a,bx)	$(x,1)$	Horadam, $H_{n+1}(x)$

Table 1: Special cases of the generalized bivariate Fibonacci-like polynomials

1.3. Two new subclasses of Bi-univalent functions

In this part, two new subclasses of bi-univalent functions are introduced as follows:

Definition 1.1. *Let $0 \leq \gamma \leq 1$. A function $f \in \Sigma$ of the form (1.1) is said to be in the class $\mathscr{H}_{n,\Sigma,\gamma}^{(p;q;x;y)}(h(z))$ if the following subordinations hold true:*

$$\frac{zf'(z)+\gamma z^2 f''(z)}{(1-\gamma)f(z)+\gamma zf'(z)} \prec h(z) = H^{(x,y)}(z)+1-a \tag{1.5}$$

and

$$\frac{wg'(w)+\gamma w^2 g''(w)}{(1-\gamma)g(w)+\gamma wg'(w)} \prec h(w) = H^{(x,y)}(w)+1-a, \tag{1.6}$$

where $p^2x^2+4qy>0$, $z,w \in \mathscr{U}$ and the function g is of the form (1.2).

Definition 1.2. *Let $\lambda \geq 0$ and $0<\delta \leq 1$. A function $f \in \Sigma$ of the form (1.1) is said to be in the class $\mathscr{H}_{n,\Sigma,\lambda,\delta}^{(p;q;x;y)}(h(z))$ if the following subordinations hold true:*

$$\frac{1}{2}\left[\frac{z^{1-\lambda}f'(z)}{(f(z))^{1-\lambda}}+\left(\frac{z^{1-\lambda}f'(z)}{(f(z))^{1-\lambda}}\right)^{\frac{1}{\delta}}\right] \prec h(z) = H^{(x,y)}(z)+1-a \tag{1.7}$$

and

$$\frac{1}{2}\left[\frac{w^{1-\lambda}g'(w)}{(g(w))^{1-\lambda}}+\left(\frac{w^{1-\lambda}g'(w)}{(g(w))^{1-\lambda}}\right)^{\frac{1}{\delta}}\right] \prec h(w) = H^{(x,y)}(w)+1-a, \tag{1.8}$$

where $p^2x^2+4qy>0$, $z,w \in \mathscr{U}$ and the function g is of the form (1.2).

Remark 1.3. *It is important to emphasize here that it can be obtained more subclasses of analytic bi-univalent functions (like bi-Bazilevič, bi-Starlike, bi-Convex and so on) by giving certain special values to the parameters p,q,γ,λ and δ in the Definition 1.1 and Definition 1.2.*

If we take $\gamma = 0$ in Definiton 1.1, we obtain the following definition:

Definition 1.4. *A function $f \in \Sigma$ of the form (1.1) is said to be in the class $\mathscr{H}_{n,\Sigma,0}^{(p;q;x;y)}(h(z))$ if the following subordinations hold true:*

$$\frac{zf'(z)}{f(z)} \prec h(z) = H^{(x,y)}(z)+1-a \tag{1.9}$$

and

$$\frac{wg'(w)}{g(w)} \prec h(w) = H^{(x,y)}(w)+1-a, \tag{1.10}$$

where $p^2x^2+4qy>0$, $z,w \in \mathscr{U}$ and the function g is of the form (1.2).

If we take $\gamma = 1$ in Definiton 1.1, we obtain the following definition:

Definition 1.5. *A function $f \in \Sigma$ of the form (1.1) is said to be in the class $\mathscr{H}_{n,\Sigma,1}^{(p;q;x;y)}(h(z))$ if the following subordinations hold true:*

$$1+\frac{zf''(z)}{f'(z)} \prec h(z) = H^{(x,y)}(z)+1-a \tag{1.11}$$

and

$$1+\frac{zg''(w)}{g'(w)} \prec h(w) = H^{(x,y)}(w)+1-a, \tag{1.12}$$

where $p^2x^2+4qy>0$, $z,w \in \mathscr{U}$ and the function g is of the form (1.2).

If we take $\delta = 1$ in Definiton 1.2, we obtain the following definition:

Definition 1.6. *A function $f \in \Sigma$ of the form (1.1) is said to be in the class $\mathscr{H}_{n,\Sigma,\lambda,1}^{(p;q;x;y)}(h(z))$ if the following subordinations hold true:*

$$\frac{z^{1-\lambda}f'(z)}{(f(z))^{1-\lambda}} \prec h(z) = H^{(x,y)}(z)+1-a \tag{1.13}$$

and

$$\frac{w^{1-\lambda}g'(w)}{(g(w))^{1-\lambda}} \prec h(w) = H^{(x,y)}(w)+1-a, \tag{1.14}$$

where $p^2x^2+4qy>0$, $z,w \in \mathscr{U}$ and the function g is of the form (1.2).

If we take $\lambda = 0$, $\delta = 1$ in Definiton 1.2, we obtain the Definition 1.4.
If we take $\lambda = 1$, $\delta = 1$ in Definiton 1.2, we obtain the following definition:

Definition 1.7. *A function $f \in \Sigma$ of the form (1.1) is said to be in the class $\mathscr{H}_{n,\Sigma,1,1}^{(p;q;x;y)}(h(z))$ if the following subordinations hold true:*

$$f'(z) \prec h(z) = H^{(x,y)}(z) + 1 - a \tag{1.15}$$

and

$$f'(w) \prec h(w) = H^{(x,y)}(w) + 1 - a, \tag{1.16}$$

where $p^2x^2 + 4qy > 0$, $z, w \in \mathscr{U}$ and the function g is of the form (1.2).

2. Coefficient estimates for the function class $\mathscr{H}_{n,\Sigma,\gamma}^{(p;q;x;y)}(h(z))$

In this section, the upper bound estimates for the coefficients a_2 and a_3 of functions in the class $\mathscr{H}_{n,\Sigma,\gamma}^{(p;q;x;y)}(h(z))$ are presented.

Theorem 2.1. *Let $f(z) = z + \sum_{n\geq 2} a_n z^n$ be in the class $\mathscr{H}_{n,\Sigma,\gamma}^{(p;q;x;y)}(h(z))$. Then*

$$|a_2| \leq \frac{|b|\sqrt{|b|}}{\sqrt{|b^2(1+2\gamma-\gamma^2) - (pbx + aqy)(1+\gamma)^2|}} \tag{2.1}$$

and

$$|a_3| \leq \frac{|b|}{2+4\gamma} + \frac{b^2}{(1+\gamma)^2}. \tag{2.2}$$

Proof. Let $f(z) = z + \sum_{n\geq 2} a_n z^n$ be in the class $\mathscr{H}_{n,\Sigma,\gamma}^{(p;q;x;y)}(h(z))$. Then, there exist analytic functions $u, v : \mathscr{U} \to \mathscr{U}$ given by

$$u(z) = \sum_{k=1}^{\infty} u_k z^k \text{ and } v(w) = \sum_{k=1}^{\infty} v_k w^k \tag{2.3}$$

such that

$$|u_k| \leq 1 \text{ and } |v_k| \leq 1, \tag{2.4}$$

where $z, w \in \mathscr{U}$. Then, from the Definition 1.1, we can write that

$$\frac{zf'(z) + \gamma z^2 f''(z)}{(1-\gamma)f(z) + \gamma z f'(z)} = h(u(z)) \tag{2.5}$$

and

$$\frac{wg'(w) + \gamma w^2 g''(w)}{(1-\gamma)g(w) + \gamma w g'(w)} = h(v(w)). \tag{2.6}$$

On the other hand, we know that

$$\frac{zf'(z) + \gamma z^2 f''(z)}{(1-\gamma)f(z) + \gamma z f'(z)} = 1 + (1+\gamma)a_2 z + [2(1+2\gamma)a_3 - (1+\gamma)^2 a_2^2]z^2 + \cdots$$

and

$$\frac{wg'(w) + \gamma w^2 g''(w)}{(1-\gamma)g(w) + \gamma w g'(w)} = 1 - (1+\gamma)a_2 w - [2(1+2\gamma)a_3 + (\gamma^2 - 6\gamma - 3)a_2^2]w^2 + \cdots.$$

So, we may write that

$$h(u(z)) = 1 + H_1(x,y)u(z) + H_2(x,y)(u(z))^2 + H_3(x,y)(u(z))^3 + \cdots \tag{2.7}$$

and

$$h(v(w)) = 1 + H_1(x,y)v(w) + H_2(x,y)(v(w))^2 + H_3(x,y)(v(w))^3 + \cdots. \tag{2.8}$$

By comparing the coefficients of the equations (2.5) and (2.7), we can write

$$(1+\gamma)a_2 = H_1(x,y)u_1, \tag{2.9}$$

$$2(1+2\gamma)a_3 - (1+\gamma)^2 a_2^2 = H_1(x,y)u_2 + H_2(x,y)u_1^2 \tag{2.10}$$

and also, using the equations (2.6) and (2.8), we get

$$-(1+\gamma)a_2 = H_1(x,y)v_1, \tag{2.11}$$

$$-2(1+2\gamma)a_3-(\gamma^2-6\gamma-3)a_2^2=H_1(x,y)v_2+H_2(x,y)v_1^2. \quad (2.12)$$

From the equations (2.9) and (2.11), we can write

$$u_1=-v_1 \quad (2.13)$$

and

$$a_2^2=\frac{H_1^2(x,y)(u_1^2+v_1^2)}{2(1+\gamma)^2}. \quad (2.14)$$

By adding the equation (2.10) to the equation (2.12), we deduce

$$2(1+2\gamma-\gamma^2)a_2^2=H_1(x,y)(u_2+v_2)+H_2(x,y)(u_1^2+v_1^2). \quad (2.15)$$

By putting (2.14) in (2.15), we have

$$a_2^2=\frac{H_1^3(x,y)(u_2+v_2)}{2H_1^2(x,y)(1+2\gamma-\gamma^2)-2H_2(x,y)(1+\gamma)^2}. \quad (2.16)$$

By considering the inequalities in (2.4), we obtain

$$|a_2|\leq\frac{|b|\sqrt{|b|}}{\sqrt{|b^2(1+2\gamma-\gamma^2)-(pbx+aqy)(1+\gamma)^2|}}. \quad (2.17)$$

By subtracting (2.10) and (2.12), we have

$$4(1+2\gamma)[a_3-a_2^2]=H_1(x,y)(u_2-v_2). \quad (2.18)$$

Then, from the inequalities in (2.4) and equation (2.14), we deduce

$$|a_3|\leq\frac{|H_1(x,y)|}{2+4\gamma}+\frac{b^2}{(1+\gamma)^2}. \quad (2.19)$$

□

Remark 2.2. *By giving different values to the paramaters in Theorem 2.1, we obtain some bounds on the coefficients $|a_2|$ and $|a_3|$ of bi-starlike and bi-convex functions defined by the generalized bivariate Fibonacci-like polynomials, respectively.*

i. *For $\gamma=0$, let $f(z)=z+\sum_{n\geq2}a_nz^n$ be in the class $\mathscr{H}_{n,\Sigma,0}^{(p;q;x;y)}(h(z))$. Then*

$$|a_2|\leq\frac{|b|\sqrt{|b|}}{\sqrt{|b^2-pbx-aqy|}}$$

and

$$|a_3|\leq\frac{|b|}{2}+b^2.$$

ii. *For $\gamma=1$, let $f(z)=z+\sum_{n\geq2}a_nz^n$ be in the class $\mathscr{H}_{n,\Sigma,1}^{(p;q;x;y)}(h(z))$. Then*

$$|a_2|\leq\frac{|b|\sqrt{|b|}}{\sqrt{|2b^2-4pbx-4aqy|}}$$

and

$$|a_3|\leq\frac{|b|}{6}+\frac{b^2}{4}.$$

Remark 2.3. *By giving $b=bx, y=1$ in class $\mathscr{H}_{n,\Sigma,\gamma}^{(p;q;x;y)}(h(z))$ of Theorem 2.1, we obtain some bounds on the coefficients $|a_2|$ and $|a_3|$ of bi-starlike and bi-convex functions defined by the Horadam polynomials, respectively.*

i. *For $\gamma=0$, let $f(z)=z+\sum_{n\geq2}a_nz^n$ be in the class $\mathscr{H}_{n,\Sigma,0}^{(p;q;x;1)}(h(z))$. Then*

$$|a_2|\leq\frac{|bx|\sqrt{|bx|}}{\sqrt{|b^2x^2-pbx^2-aq|}}$$

and

$$|a_3|\leq\frac{|bx|}{2}+b^2x^2.$$

ii. *For* $\gamma = 1$, *let* $f(z) = z + \sum_{n\geq 2} a_n z^n$ *be in the class* $\mathscr{H}_{n,\Sigma,1}^{(p;q;x;1)}(h(z))$. *Then*

$$|a_2| \leq \frac{|bx|\sqrt{|bx|}}{\sqrt{|2b^2x^2 - 4pbx^2 - 4aq|}}$$

and

$$|a_3| \leq \frac{|bx|}{6} + \frac{b^2x^2}{4}.$$

Remark 2.4. *By giving* $p = 2, q = 1, a = 1, b = 2t, x = t, y = -1$ *in class* $\mathscr{H}_{n,\Sigma,\gamma}^{(p;q;x;y)}(h(z))$ *of Theorem 2.1, we obtain some bounds on the coefficients* $|a_2|$ *and* $|a_3|$ *of bi-starlike and bi-convex functions defined by the Chebyshev polynomials which these results can be found as Corollary 2 in [1], while Corollary 2.1 in [5].*

i. *For* $\gamma = 0$, *let* $f(z) = z + \sum_{n\geq 2} a_n z^n$ *be in the class* $\mathscr{H}_{n,\Sigma,0}^{(2;1;t;-1)}(h(z))$. *Then*

$$|a_2| \leq |2t|\sqrt{|2t|}$$

and

$$|a_3| \leq |t| + 4t^2.$$

ii. *For* $\gamma = 1$, *let* $f(z) = z + \sum_{n\geq 2} a_n z^n$ *be in the class* $\mathscr{H}_{n,\Sigma,1}^{(2;1;t;-1)}(h(z))$. *Then*

$$|a_2| \leq \frac{|t|\sqrt{|2t|}}{\sqrt{|1 - 2t^2|}}$$

and

$$|a_3| \leq \frac{|t|}{3} + t^2.$$

Remark 2.5. *By giving* $p = q = 1, a = 0, b = 1, y = 1$ *in class* $\mathscr{H}_{n,\Sigma,\gamma}^{(p;q;x;y)}(h(z))$ *of Theorem 2.1, we obtain some bounds on the coefficients* $|a_2|$ *and* $|a_3|$ *of bi-starlike and bi-convex functions defined by the Fibonacci polynomials, respectively.*

i. *For* $\gamma = 0$, *let* $f(z) = z + \sum_{n\geq 2} a_n z^n$ *be in the class* $\mathscr{H}_{n,\Sigma,0}^{(1;1;x;1)}(h(z))$. *Then*

$$|a_2| \leq \frac{1}{\sqrt{|1 - x|}}$$

and

$$|a_3| \leq \frac{3}{2}.$$

ii. *For* $\gamma = 1$, *let* $f(z) = z + \sum_{n\geq 2} a_n z^n$ *be in the class* $\mathscr{H}_{n,\Sigma,1}^{(1;1;x;1)}(h(z))$. *Then*

$$|a_2| \leq \frac{1}{\sqrt{|2 - 4x|}}$$

and

$$|a_3| \leq \frac{5}{12}.$$

3. Coefficient estimates for the function class $\mathscr{H}_{n,\Sigma,\lambda,\delta}^{(p;q;x;y)}(h(z))$

In this section, the upper bound estimates for the coefficients a_2 and a_3 of functions in the class $\mathscr{H}_{n,\Sigma,\lambda,\delta}^{(p;q;x;y)}(h(z))$ are presented.

Theorem 3.1. *Let* $f(z) = z + \sum_{n\geq 2} a_n z^n$ *be in the class* $\mathscr{H}_{n,\Sigma,\lambda,\delta}^{(p;q;x;y)}(h(z))$. *Then*

$$|a_2| \leq \frac{2|b|\sqrt{|b|}\delta}{\sqrt{|b^2(\lambda+1)[\delta(\lambda+2)(\delta+1)+(\lambda+1)(1-\delta)] - (pbx+aqy)(\lambda+1)^2(\delta+1)^2|}} \tag{3.1}$$

and

$$|a_3| \leq \frac{2|b|\delta}{(\lambda+2)(\delta+1)} + \frac{4b^2\delta^2}{(\lambda+1)^2(\delta+1)^2}. \tag{3.2}$$

Proof. Let suppose that $f(z)=z+\sum_{n\geq 2}a_n z^n \in \mathscr{H}^{(p;q;x;y)}_{n,\Sigma,\lambda,\delta}(h(z))$. Then, there are two analytic functions $u,v:\mathscr{U}\to\mathscr{U}$ given by

$$u(z)=\sum_{k=1}^{\infty}u_k z^k \text{ and } v(w)=\sum_{k=1}^{\infty}v_k w^k \tag{3.3}$$

such that

$$|u_k|\leq 1 \text{ and } |v_k|\leq 1, \tag{3.4}$$

where $z,w\in\mathscr{U}$. From the Definition 1.2, we have

$$\frac{1}{2}\left[\frac{z^{1-\lambda}f'(z)}{(f(z))^{1-\lambda}}+\left(\frac{z^{1-\lambda}f'(z)}{(f(z))^{1-\lambda}}\right)^{\frac{1}{\delta}}\right]=h(u(z)) \tag{3.5}$$

and

$$\frac{1}{2}\left[\frac{w^{1-\lambda}g'(w)}{(g(w))^{1-\lambda}}+\left(\frac{w^{1-\lambda}g'(w)}{(g(w))^{1-\lambda}}\right)^{\frac{1}{\delta}}\right]=h(v(w)). \tag{3.6}$$

Also, we have

$$\begin{aligned}\frac{1}{2}\left[\frac{z^{1-\lambda}f'(z)}{(f(z))^{1-\lambda}}+\left(\frac{z^{1-\lambda}f'(z)}{(f(z))^{1-\lambda}}\right)^{\frac{1}{\delta}}\right]&=1+\frac{(\lambda+1)(\delta+1)}{2\delta}a_2 z\\&+\left[\frac{(\lambda+2)(\delta+1)}{4\delta}(2a_3+(\lambda-1)a_2^2)+\frac{(\lambda+1)^2(1-\delta)}{4\delta^2}a_2^2\right]z^2+\cdots\end{aligned}$$

and

$$\begin{aligned}\frac{1}{2}\left[\frac{w^{1-\lambda}g'(w)}{(g(w))^{1-\lambda}}+\left(\frac{w^{1-\lambda}g'(w)}{(g(w))^{1-\lambda}}\right)^{\frac{1}{\delta}}\right]&=1-\frac{(\lambda+1)(\delta+1)}{2\delta}a_2 w\\&+\left[\frac{(\lambda+2)(\delta+1)}{4\delta}((\lambda+3)a_2^2-2a_3)+\frac{(\lambda+1)^2(1-\delta)}{4\delta^2}a_2^2\right]w^2+\cdots.\end{aligned}$$

Then, we get

$$h(u(z))=1+H_1(x,y)u_1 z+(H_1(x,y)u_2+H_2(x,y)u_1^2)z^2+\cdots \tag{3.7}$$

and

$$h(v(w))=1+H_1(x,y)v_1 w+(H_1(x,y)v_2+H_2(x,y)v_1^2)w^2+\cdots. \tag{3.8}$$

By using the equations (3.5) and (3.7), we deduce

$$\frac{(\lambda+1)(\delta+1)}{2\delta}a_2=H_1(x,y)u_1, \tag{3.9}$$

$$\frac{(\lambda+2)(\delta+1)}{4\delta}(2a_3+(\lambda-1)a_2^2)+\frac{(\lambda+1)^2(1-\delta)}{4\delta^2}a_2^2=H_1(x,y)u_2+H_2(x,y)u_1^2 \tag{3.10}$$

and from the equations (3.6) and (3.8), we obtain

$$-\frac{(\lambda+1)(\delta+1)}{2\delta}a_2=H_1(x,y)v_1 \tag{3.11}$$

and

$$\frac{(\lambda+2)(\delta+1)}{4\delta}((\lambda+3)a_2^2-2a_3)+\frac{(\lambda+1)^2(1-\delta)}{4\delta^2}a_2^2=H_1(x,y)v_2+H_2(x,y)v_1^2. \tag{3.12}$$

Comparing the equations (3.9) and (3.11) imply that

$$u_1=-v_1 \tag{3.13}$$

and

$$a_2^2=\frac{2H_1^2(x,y)\delta^2(u_1^2+v_1^2)}{(\lambda+1)^2(\delta+1)^2}. \tag{3.14}$$

By summing the equations (3.10) and (3.12), we get

$$\left[\frac{(\lambda+2)(\delta+1)(\lambda+1)}{2\delta}+\frac{(\lambda+1)^2(1-\delta)}{2\delta^2}\right]a_2^2=H_1(x,y)(u_2+v_2)+H_2(x,y)(u_1^2+v_1^2). \tag{3.15}$$

By putting (3.14) in (3.15), we have

$$a_2^2 = \frac{2H_1^3(x,y)\delta^2(u_2+v_2)}{H_1^2(x,y)(\lambda+1)\left[\delta(\lambda+2)(\delta+1)+(\lambda+1)(1-\delta)\right]-H_2(x,y)(\lambda+1)^2(\delta+1)^2}. \tag{3.16}$$

By considering the inequalities in (3.4), we get

$$|a_2| \leq \frac{2|b|\sqrt{|b|}\delta}{\sqrt{|b^2(\lambda+1)\left[\delta(\lambda+2)(\delta+1)+(\lambda+1)(1-\delta)\right]-(pbx+aqy)(\lambda+1)^2(\delta+1)^2|}}. \tag{3.17}$$

By subtracting (3.10) and (3.12), we obtain

$$\frac{(\lambda+2)(\delta+1)}{\delta}[a_3-a_2^2] = H_1(x,y)(u_2-v_2). \tag{3.18}$$

Then, from the inequalities in (3.4) and the equation (3.14), we have

$$|a_3| \leq \frac{2|H_1(x,y)|\delta}{(\lambda+2)(\delta+1)} + \frac{4b^2\delta^2}{(\lambda+1)^2(\delta+1)^2}. \tag{3.19}$$

□

Remark 3.2. *By giving some special values to the paramaters in Theorem 3.1, we obtain some bounds on the coefficients $|a_2|$ and $|a_3|$ of bi-univalent function classes (like Bi-Bazilevič and Bi-starlike) related with the generalized bivariate Fibonacci-like polynomials.*

i. *For $\delta = 1$, let $f(z) = z + \sum_{n\geq 2} a_n z^n$ be in the class $\mathscr{H}_{n,\Sigma,\lambda,1}^{(p;q;x;y)}(h(z))$. Then*

$$|a_2| \leq \frac{2|b|\sqrt{|b|}}{\sqrt{|2b^2(\lambda+1)(\lambda+2)-4(pbx+aqy)(\lambda+1)^2|}}$$

and

$$|a_3| \leq \frac{|b|}{\lambda+2} + \frac{b^2}{(\lambda+1)^2}.$$

ii. *For $\lambda = 0$, $\delta = 1$, the obtained bounds coincide with the condition **i.** in Remark 2.2.*

iii. *For $\lambda = \delta = 1$, let $f(z) = z + \sum_{n\geq 2} a_n z^n$ be in the class $\mathscr{H}_{n,\Sigma,1,1}^{(p;q;x;y)}(h(z))$. Then*

$$|a_2| \leq \frac{|b|\sqrt{|b|}}{\sqrt{|3b^2-4pbx-4aqy|}}$$

and

$$|a_3| \leq \frac{|b|}{3} + \frac{b^2}{4}.$$

Remark 3.3. *By giving $b = bx, y = 1$ in class $\mathscr{H}_{n,\Sigma,\lambda,\delta}^{(p;q;x;y)}(h(z))$ of Theorem 3.1, we obtain some bounds on the coefficients $|a_2|$ and $|a_3|$ of bi-univalent function classes (like Bi-Bazilevič and Bi-starlike) related with the Horadam polynomials which results are also found in [2] and [24].*

i. *For $\delta = 1$, let $f(z) = z + \sum_{n\geq 2} a_n z^n$ be in the class $\mathscr{H}_{n,\Sigma,\lambda,1}^{(p;q;x;1)}(h(z))$. Then*

$$|a_2| \leq \frac{2|bx|\sqrt{|bx|}}{\sqrt{|2b^2x^2(\lambda+1)(\lambda+2)-4(pbx^2+aq)(\lambda+1)^2|}}$$

and

$$|a_3| \leq \frac{|bx|}{\lambda+2} + \frac{b^2x^2}{(\lambda+1)^2}.$$

ii. *For $\lambda = 0$, $\delta = 1$, the obtained bounds coincide with the condition **i.** in Remark 2.3.*

iii. *For $\lambda = \delta = 1$, let $f(z) = z + \sum_{n\geq 2} a_n z^n$ be in the class $\mathscr{H}_{n,\Sigma,1,1}^{(p;q;x;1)}(h(z))$. Then*

$$|a_2| \leq \frac{|bx|\sqrt{|bx|}}{\sqrt{|3b^2x^2-4pbx^2-4aqy|}}$$

and

$$|a_3| \leq \frac{|bx|}{3} + \frac{b^2x^2}{4}.$$

Remark 3.4. *By giving $p = 2, q = 1, a = 1, b = 2t, x = t, y = -1$ in the class $\mathscr{H}_{n,\Sigma,\lambda,\delta}^{(p;q;x;y)}(h(z))$ of Theorem 3.1, we obtain some bounds on the coefficients $|a_2|$ and $|a_3|$ of bi-univalent function classes (like Bi-Bazilevič and Bi-starlike) related with the Chebyshev polynomials which these results may be found as Corollay 3 in [1], while Corollary 2 in [9].*

i. *For* $\delta = 1$, *let* $f(z) = z + \sum_{n\geq 2} a_n z^n$ *be in the class* $\mathscr{H}_{n,\Sigma,\lambda,1}^{(2;1;t;-1)}(h(z))$. *Then*

$$|a_2| \leq \frac{4|t|\sqrt{|2t|}}{\sqrt{|4t^2(\lambda+1)(\lambda+2) - 4(4t^2-1)(\lambda+1)^2|}}$$

and

$$|a_3| \leq \frac{|2t|}{\lambda+2} + \frac{4t^2}{(\lambda+1)^2}.$$

ii. *For* $\lambda = 0$, $\delta = 1$, *the obtained bounds coincide with the condition* ***i.*** *in Remark 2.4.*

iii. *For* $\lambda = \delta = 1$, *let* $f(z) = z + \sum_{n\geq 2} a_n z^n$ *be in the class* $\mathscr{H}_{n,\Sigma,1,1}^{(2;1;t;-1)}(h(z))$. *Then*

$$|a_2| \leq \frac{|t|\sqrt{|2t|}}{\sqrt{|1-t^2|}}$$

and

$$|a_3| \leq \frac{|2t|}{3} + t^2.$$

Remark 3.5. *By giving* $p = 1, q = 1, a = 0, b = 1, y = 1$ *in class* $\mathscr{H}_{n,\Sigma,\lambda,\delta}^{(p;q;x;y)}(h(z))$ *of Theorem 3.1, we obtain some bounds on the coefficients* $|a_2|$ *and* $|a_3|$ *of bi-univalent function classes (like Bi-Bazilevič and Bi-starlike) related with the Fibonacci polynomials.*

i. *For* $\delta = 1$, *let* $f(z) = z + \sum_{n\geq 2} a_n z^n$ *be in the class* $\mathscr{H}_{n,\Sigma,\lambda,1}^{(1;1;x;1)}(h(z))$. *Then*

$$|a_2| \leq \frac{2}{\sqrt{|2(\lambda+1)(\lambda+2) - 4x(\lambda+1)^2|}}$$

and

$$|a_3| \leq \frac{1}{\lambda+2} + \frac{1}{(\lambda+1)^2}.$$

ii. *For* $\lambda = 0$, $\delta = 1$, *the obtained bounds coincide with the condition* ***i.*** *in Remark 2.5.*

iii. *For* $\lambda = \delta = 1$, *let* $f(z) = z + \sum_{n\geq 2} a_n z^n$ *be in the class* $\mathscr{H}_{n,\Sigma,1,1}^{(1;1;x;1)}(h(z))$. *Then*

$$|a_2| \leq \frac{1}{\sqrt{|3-4x|}}$$

and

$$|a_3| \leq \frac{11}{12}.$$

4. Fekete-Szegö inequality for the class $\mathscr{H}_{n,\Sigma,\gamma}^{(p;q;x;y)}(h(z))$ and $\mathscr{H}_{n,\Sigma,\lambda,\delta}^{(p;q;x;y)}(h(z))$

In this section, the upper bound estimates for the Fekete-Szegö functional $|a_3 - \mu a_2^2|$ of functions in the classes $\mathscr{H}_{n,\Sigma,\gamma}^{(p;q;x;y)}(h(z))$ and $\mathscr{H}_{n,\Sigma,\lambda,\delta}^{(p;q;x;y)}(h(z))$ are presented.

Theorem 4.1. *Let suppose that* $f(z) = z + \sum_{n\geq 2} a_n z^n \in \mathscr{H}_{n,\Sigma,\gamma}^{(p;q;x;y)}(h(z))$. *Then*

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{|b|}{2(1+2\gamma)}, & |\mu-1| \leq \frac{|b^2(1+2\gamma-\gamma^2)-(pbx+aqy)(1+\gamma)^2|}{2b^2(1+2\gamma)} \\ \frac{|1-\mu||b^3|}{|b^2(1+2\gamma-\gamma^2)-(pbx+aqy)(1+\gamma)^2|}, & |\mu-1| \geq \frac{|b^2(1+2\gamma-\gamma^2)-(pbx+aqy)(1+\gamma)^2|}{2b^2(1+2\gamma)} \end{cases}$$

Proof. By using the equations (2.16) and (2.18), we have

$$\begin{aligned} a_3 - \mu a_2^2 =& (1-\mu)\frac{(u_2+v_2)b^3}{2b^2(1+2\gamma-\gamma^2) - 2(pbx+aqy)(1+\gamma)^2} + \frac{b(u_2-v_2)}{4(1+2\gamma)} \\ =& \left[\frac{(1-\mu)b^3}{2b^2(1+2\gamma-\gamma^2) - 2(pbx+aqy)(1+\gamma)^2} + \frac{b}{4(1+2\gamma)}\right] u_2 \\ &+ \left[\frac{(1-\mu)b^3}{2b^2(1+2\gamma-\gamma^2) - 2(pbx+aqy)(1+\gamma)^2} - \frac{b}{4(1+2\gamma)}\right] v_2. \end{aligned}$$

Hence, we get

$$a_3 - \mu a_2^2 = \left[h(\mu) + \frac{b}{4(1+2\gamma)}\right] u_2 + \left[h(\mu) - \frac{b}{4(1+2\gamma)}\right] v_2, \tag{4.1}$$

where

$$h(\mu)=\frac{(1-\mu)b^3}{2b^2(1+2\gamma-\gamma^2)-2(pbx+aqy)(1+\gamma)^2}. \tag{4.2}$$

Then, by taking modulus of the equation (4.1), we conclude that

$$|a_3-\mu a_2^2|\leq\begin{cases}\frac{|b|}{2(1+2\gamma)}, & 0\leq|h(\mu)|\leq\frac{|b|}{4(1+2\gamma)}\\ 2|h(\mu)|, & |h(\mu)|\geq\frac{|b|}{4(1+2\gamma)}.\end{cases} \tag{4.3}$$

□

Remark 4.2. *Let $f(z)=z+\sum_{n\geq2}a_nz^n$ be in the class $\mathscr{H}_{n,\Sigma,\gamma}^{(p;q;x;y)}(h(z))$. By taking some special values in the parameters in Theorem 4.1 we obtain the followings:*

i. *For $\mu=1$,*

$$|a_3-a_2^2|\leq\frac{|b|}{2(1+2\gamma)}.$$

ii. *For $\gamma=0$,*

$$|a_3-\mu a_2^2|\leq\begin{cases}\frac{|b|}{2}, & |\mu-1|\leq\frac{|b^2-pbx-aqy|}{2b^2}\\ \frac{|1-\mu||b^3|}{|b^2-pbx-aqy|}, & |\mu-1|\geq\frac{|b^2-pbx-aqy|}{2b^2}.\end{cases}$$

iii. *For $\gamma=1$,*

$$|a_3-\mu a_2^2|\leq\begin{cases}\frac{|b|}{6}, & |\mu-1|\leq\frac{|b^2-2pbx-2aqy|}{3b^2}\\ \frac{|1-\mu||b^3|}{|2b^2-4pbx-4aqy|}, & |\mu-1|\geq\frac{|b^2-2pbx-2aqy|}{3b^2}.\end{cases}$$

Theorem 4.3. *Let $f(z)=z+\sum_{n\geq2}a_nz^n$ be in the class $\mathscr{H}_{n,\Sigma,\lambda,\delta}^{(p;q;x;y)}(h(z))$. Then*

$$|a_3-\mu a_2^2|\leq\begin{cases}\frac{2|b|\delta}{(\lambda+2)(\delta+1)}, & |\mu-1|\leq A\\ \frac{4|1-\mu||b^3|\delta^2}{|b^2(\lambda+1)[\delta(\lambda+2)(\delta+1)+(\lambda+1)(1-\delta)]-(pbx+aqy)(\lambda+1)^2(\delta+1)^2|}, & |\mu-1|\geq A\end{cases}$$

and

$$A=\frac{|b^2(\lambda+1)[\delta(\lambda+2)(\delta+1)+(\lambda+1)(1-\delta)]-(pbx+aqy)(\lambda+1)^2(\delta+1)^2|}{2b^2\delta(\lambda+2)(\delta+1)}.$$

Proof. Since the proof is similar to the proof of Theorem 4.1 we omitted the details. □

Remark 4.4. *Let $f(z)=z+\sum_{n\geq2}a_nz^n$ be in the class $\mathscr{H}_{n,\Sigma,\gamma}^{(p;q;x;y)}(h(z))$. By taking some special values in the parameters in Theorem 4.3 we obtain the followings:*

i. *For $\mu=1$, we obtain*

$$|a_3-a_2^2|\leq\frac{2|b|\delta}{(\lambda+2)(\delta+1)}.$$

ii. *For $\delta=1$, we have*

$$|a_3-\mu a_2^2|\leq\begin{cases}\frac{|b|}{\lambda+2}, & |\mu-1|\leq\frac{|2b^2(\lambda+1)(\lambda+2)-4(pbx+aqy)(\lambda+1)^2|}{4b^2(\lambda+2)}\\ \frac{4|1-\mu||b^3|}{|2b^2(\lambda+1)(\lambda+2)-4(pbx+aqy)(\lambda+1)^2|}, & |\mu-1|\geq\frac{|2b^2(\lambda+1)(\lambda+2)-4(pbx+aqy)(\lambda+1)^2|}{4b^2(\lambda+2)}.\end{cases}$$

iii. *For $\lambda=0$, $\delta=1$, the obtained results coincide with the condition **ii.** in Remark 4.2.*

iii. *For $\lambda=\delta=1$, we get*

$$|a_3-\mu a_2^2|\leq\begin{cases}\frac{|b|}{3}, & |\mu-1|\leq\frac{|3b^2-4pbx-4aqy|}{3b^2}\\ \frac{|1-\mu||b^3|}{|3b^2-4pbx-4aqy|}, & |\mu-1|\geq\frac{|3b^2-4pbx-4aqy|}{3b^2}.\end{cases}$$

Remark 4.5. *As the above remarks, the solving of Fekete-Szegö problem can be given for Horadam, Chebyshev and Fibonacci polynomials by taking p,q,a and b as special values. For example,*

i. *if we take $\gamma=0,p=2,q=1,a=1,b=2t,x=t,y=-1$ in Theorem 4.1, we obtain the result related to Fekete-Szegö problem of Corollary 2 in [1].*

ii. *if we take $\lambda=\delta=1,p=2,q=1,a=1,b=2t,x=t,y=-1$ in Theorem 4.3, we obtain the result related to Fekete-Szegö problem of Corollary 3 in [1].*

Conclusion

In this paper, by making use of the generalized bivariate Fibonacci-like polynomials two new subclasses of analytic bi-univalent functions are introduced. Firstly, initial coefficients estimates are discussed and then the well-known Fekete-Szegö problem for these subclasses are solved. Note that, if we take p, q, x, y, a and b be as certain values, our evaluations cover most studies in the literature.

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