International Journal of Engineering & Applied Sciences (IJEAS)
Vol.14, Issue 1 (Special Issue: Advance in Nano Scaled Mechanics) (2022) 1-14
http://dx.doi.org/10.24107/ijeas.1064690
Int J Eng Appl Sci 14(1) (2022) 1-14

A Finite Element Solution for Bending Analysis of a Nanoframe using Modified Couple Stress Theory

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Received date: 28.01.2022
Accepted date: 31.05.2022

Abstract

In this work, a finite element formulation for a size dependent frame system is presented. Size dependency is discussed via the modified couple stress theory. The nodal displacement and rotation analyses of a frame system with total of three elements, including two columns and one beam element connecting these two columns, are considered. The classical stiffness and size dependent stiffness matrices of frame system are derived. Then, solution procedure for this problem is explained. Lastly, a numerical application is realized and effect of material length scale parameter on nodal displacements and rotations is discussed. To present the numerical application, it is assumed that the elements of the nanoframe are composed of silicon carbide nanotubes.

Keywords: Modified couple stress theory, Frame system, Static, Finite element method

1. Introduction

Nanotechnology covers the manufacture and understanding of the nanoscale materials /elements / systems. Nanotechnology controls materials and devices at atomic and molecular levels, enabling them to be arranged or reconstructed. The properties of enormously small materials, which are much better than expected, have attracted great interest. Thus, studies on the discovery of new nanomaterials and the understanding of the properties of discovered nanomaterials have begun to increase. Especially, carbon nanotubes [1] have attracted much attention and many studies have been carried out on the usage areas and synthesis methods of such nanomaterials. These studies have accelerated and the synthesis of various nanomaterials and nanostructures has begun. Recently, it has been seen that studies [2-4] on more complicated nanostructures (nanoframes) have been presented. In these studies, various properties such as the architecture, oxygen reaction reduction activity, the exciton decay time, and catalytic performance of various nanoframe structures were discussed.

Studies have revealed that nanoscale materials are affected by some parameters and manipulations that do not affect conventional materials. In nanomaterials, parameters such as the changed length, the number of atoms that constitute them etc., can vary the behavior of these material. This is called the size effect. The size effect is important for nanomaterials. Because in order to design correctly applications of nano-electromechanical systems/micro-electromechanical systems (NEMS/MEMS), it is important to know the behavior of the materials that constitute them. Continuum mechanics theories based on the size effect have an important place among the practical methods that contribute to the revealing of these behaviors. Some of these size dependent theories are: nonlocal elasticity theory, modified couple stress theory, modified strain gradient theory, doublet mechanics theory, nonlocal strain gradient theory, surface elasticity theory.

Structures such as rods, beams, plates, and frames have recently been modeled at nano and micro scales and their various analyzes like static, vibration and buckling have been carried out based on the mentioned size dependent theories. Navier's method [5-11], Fourier sine solution [12-17], finite element method [18-29], separation of variable procedure [30, 31], generalized differential quadrature method [32-36], Ritz method have been frequently used by scholars to present the mechanical responses of various small scale structures such as nanobeam, nanoframe, nanotruss, nanorod, nanoplate, cracked microbeam with functionally graded material, cracked nanobeam, functionally graded nanobeam, porous nanotube etc. The above-mentioned solution methods and others have been also utilized for macro-dimensional porous plate [37-39], porous beam [40, 41], beam [42], reinforced plate [43, 44], functionally graded and laminated beam [45-50], shell [51], functionally graded and laminated plate [52], functionally graded shell [53], reinforced beam [54], frame [55] structures.

In this study, the analyzes are carried out using the finite element method. The finite element method has attracted attention with its practical solution and applicability to a wide variety of problems. The finite element method, which is used for the analysis of macro-scale elements and structures, has recently been used frequently for the analysis of nano- and micro-scale elements and structures after the size dependent elasticity theories. As can be understood from the above-mentioned papers, structures such as beams, plates, rods, and frames have been modeled at nano and micro scales and their theoretical analyzes have been carried out. When we look at the literature, it is understood that there are very few studies presenting the size effect analyzes of frame systems. To the best of the authors knowledge, this work for the first time, discusses the effect of material length scale parameter on the nodal displacement and rotation of a nano-sized frame in the context of modified couple stress theory. In this study, a size-dependent finite element formulation based on the modified couple stress theory that calculates the nodal displacement and rotation values of a nanoscale frame system is presented.

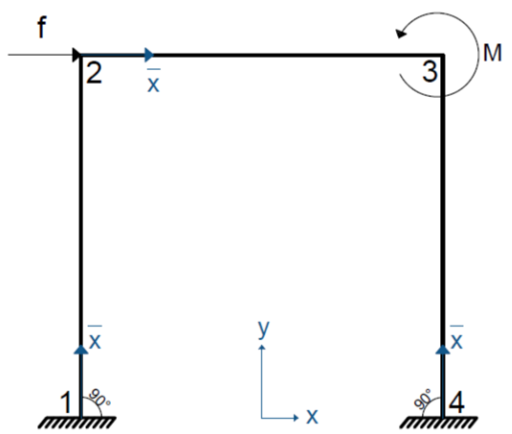


Fig.1. A nano-sized frame system

2. Modified Couple Stress Theory

Modified couple stress theory (MCST) is one of the higher-order elasticity theories used to perform size-based analyzes of nano- and micro-scale structures. For this theory proposed by Yang et al. [56], the strain energy U is expressed as follows:

$$U=\frac{1}{2}\int_V\left(\sigma_{ij}\varepsilon_{ij}+m_{ij}\chi_{ij}\right)dV \tag{1}$$

In the strain energy equation, σ_{ij} , ε_{ij} m_{ij} and χ_{ij} represent the the classical stress tensor, strain tensor, the symmetric couple stress tensor and symmetric rotation gradient tensor, respectively. Also, V denotes the volume occupied by body. After some mathematical procedures, the strain energy can be written as follows.

$$U=\frac{1}{2}\int_0^L\left(EA\left(\frac{\partial u}{\partial x}\right)^2+\left(EI+GAl^2\right)\left(\frac{\partial^2 w}{\partial x^2}\right)^2\right)dx \tag{2}$$

In which, L, I, A denote the length, moment of inertiea and cross-sectional areas, respectively. While E represents the Young's modulus, G denotes the shear modulus. u and w represent axial and transverse displacement fields, respectively. Lastly, l specifies the material length scale parameter. Material length scale parameter is a small size parameter. Thanks to this parameter, the analysis can be realised based on the size effect. Neglecting this parameter in the equations reduces the problem to classical theory and the problem becomes independent of the small size effect.

3. Finite Element Formulation Based on MCST for a Nanoframe System

The finite element solution, which allows us to find nodal displacements and rotations, is expressed as follows [57]:

$$\{F\}=[K]\{\bar{d}\} \tag{3}$$

In the above equation, {F}, [K] and { $\bar{d}$ } specify the global nodal force vector, stiffness matrix and global nodal displacement vector, respectively. Interpolation functions are used to derived the stiffness matrix of the elements that constitute the frame system. These interpolation functions for the axial and transverse displacements are as follows [20, 22, 57]:

$$\psi^u=\begin{bmatrix}\psi_1^u\\ \psi_2^u\end{bmatrix} \tag{4}$$

In which,

$$\psi_1^u = 1 - \frac{x}{L_e}$$
$$\psi_2^u = \frac{x}{L_e} \tag{5}$$

Ψ^u is defined as interpolation functions for axial displacement. In addition to the axial displacement, interpolation functions for transverse displacement should be defined. The shape functions of transverse displacement indicated by Ψ^w are given as follows [22, 57]

$$\psi^w = \begin{bmatrix} \psi_1^w \\ \psi_2^w \\ \psi_3^w \\ \psi_4^w \end{bmatrix} \tag{6}$$

Here,

$$\psi_1^w = 1 - \frac{3x^2}{L_e^2} + \frac{2x^3}{L_e^3},$$
$$\psi_2^w = x - \frac{2x^2}{L_e} + \frac{x^3}{L_e^2}$$
$$\psi_3^w = \frac{3x^2}{L_e^2} - \frac{2x^3}{L_e^3}, \tag{7}$$
$$\psi_4^w = -\frac{x^2}{L_e} + \frac{x^3}{L_e^2}$$

In the above equations, L_e represent the length of a finite element. Stiffness matrices are written via the obtained strain energy expression and interpolation functions given previous equations:

$$K_u = \int_0^{L_e} EA \left(\frac{\partial \left[\psi^u \right]}{\partial x} \right)^T \frac{\partial \left[\psi^u \right]}{\partial x} dx = EA \begin{bmatrix} \frac{1}{L_e} & \frac{-1}{L_e} \\ \frac{-1}{L_e} & \frac{1}{L_e} \end{bmatrix} \tag{8}$$

$$K_w = \int_0^{L_e} EI \left(\frac{\partial^2 \psi^w}{\partial x^2} \right)^T \frac{\partial^2 \left[\psi^w \right]}{\partial x^2} dx + \int_0^{L_e} GAl^2 \left(\frac{\partial^2 \psi^w}{\partial x^2} \right)^T \frac{\partial^2 \left[\psi^w \right]}{\partial x^2} dx$$
$$= \frac{EI}{L_e^3} \begin{bmatrix} 12 & 6L_e & -12 & 6L_e \\ 6L_e & 4L_e^2 & -6L_e & 2L_e^2 \\ -12 & -6L_e & 12 & -6L_e \\ 6L_e & 2L_e^2 & -6L_e & 4L_e^2 \end{bmatrix} + \frac{GAl^2}{L_e^3} \begin{bmatrix} 12 & 6L_e & -12 & 6L_e \\ 6L_e & 4L_e^2 & -6L_e & 2L_e^2 \\ -12 & -6L_e & 12 & -6L_e \\ 6L_e & 2L_e^2 & -6L_e & 4L_e^2 \end{bmatrix} \tag{9}$$

By assembling the K_u and K_w matrices given above, the stiffness matrix of a nanoframe element is obtained as follows:

$$\bar{K}=\begin{bmatrix} \frac{EA}{L_e} & 0 & 0 & -\frac{EA}{L_e} & 0 & 0 \\ 0 & 12\frac{EI}{L_e^3}+12\frac{GAl^2}{L_e^3} & 6\frac{EI}{L_e^2}+6\frac{GAl^2}{L_e^2} & 0 & -12\frac{EI}{L_e^3}-12\frac{GAl^2}{L_e^3} & 6\frac{EI}{L_e^2}+6\frac{GAl^2}{L_e^2} \\ 0 & 6\frac{EI}{L_e^2}+6\frac{GAl^2}{L_e^2} & 4\frac{EI}{L_e}+4\frac{GAl^2}{L_e} & 0 & -6\frac{EI}{L_e^2}-6\frac{GAl^2}{L_e^2} & 2\frac{EI}{L_e}+2\frac{GAl^2}{L_e} \\ -\frac{EA}{L_e} & 0 & 0 & \frac{EA}{L_e} & 0 & 0 \\ 0 & -12\frac{EI}{L_e^3}-12\frac{GAl^2}{L_e^3} & -6\frac{EI}{L_e^2}-6\frac{GAl^2}{L_e^2} & 0 & 12\frac{EI}{L_e^3}+12\frac{GAl^2}{L_e^3} & -6\frac{EI}{L_e^2}-6\frac{GAl^2}{L_e^2} \\ 0 & 6\frac{EI}{L_e^2}+6\frac{GAl^2}{L_e^2} & 2\frac{EI}{L_e}+2\frac{GAl^2}{L_e} & 0 & -6\frac{EI}{L_e^2}-6\frac{GAl^2}{L_e^2} & 4\frac{EI}{L_e}+4\frac{GAl^2}{L_e} \end{bmatrix} \tag{10}$$

The above matrix includes both bending and axial effects. Buckling, vibration and bending analysis of nanobeams via matrices that include these both effects were presented by Akbaş [20] using modified couple stress theory. This matrix, which can be used for the solution of a straight nanobeam, is used for the elements of the nanoframe in this study. As it is known, frame systems are structures formed by connecting more than one element to each other at different angles. In order to realize the solutions of these structures, we should first write them on the same axis set, taking into account the orientations of all the elements. For this, transformation matrix (*T*) is used [22, 57]:

$$T=\begin{bmatrix} C & S & 0 & 0 & 0 & 0 \\ -S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & S & 0 \\ 0 & 0 & 0 & -S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

C and S define cosine and sine, respectively. C and S are the angle between the local axis of the element and the global axis. To obtain the global stiffness matrix for a nanoframe element, we need to utilize the following equation [22, 55, 57]:

$$K_{mcst}=T^T\bar{K}T \tag{12}$$

The stiffness matrix of a single element of the frame system obtained the following form when the transformation matrix and equation (12) are used:

$$K_{mcst}=\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} \end{bmatrix} \tag{13}$$

The elements of K_{mcst} matrix are given as follows:

$$K_{11} = K_{44} = \frac{EA}{L_e}C^2 + \left(12\frac{EI}{L_e^3} + 12\frac{GAl^2}{L_e^3}\right)S^2 \tag{14}$$

$$K_{12} = K_{21} = \frac{EA}{L_e}CS - \left(12\frac{EI}{L_e^3} + 12\frac{GAl^2}{L_e^3}\right)CS \tag{15}$$

$$K_{13} = K_{31} = -\left(6\frac{EI}{L_e^2} + 6\frac{GAl^2}{L_e^2}\right)S \tag{16}$$

$$K_{14} = K_{41} = -\frac{EA}{L_e}C^2 - \left(12\frac{EI}{L_e^3} + 12\frac{GAl^2}{L_e^3}\right)S^2 \tag{17}$$

$$K_{15} = K_{51} = -\frac{EA}{L_e}CS + \left(12\frac{EI}{L_e^3} + 12\frac{GAl^2}{L_e^3}\right)CS \tag{18}$$

$$K_{16} = K_{61} = -\left(6\frac{EI}{L_e^2} + 6\frac{GAl^2}{L_e^2}\right)S \tag{19}$$

$$K_{22} = K_{55} = \left(12\frac{EI}{L_e^3} + 12\frac{GAl^2}{L_e^3}\right)C^2 + \frac{EA}{L_e}S^2 \tag{20}$$

$$K_{23} = K_{32} = K_{26} = K_{62} = \left(6\frac{EI}{L_e^2} + 6\frac{GAl^2}{L_e^2}\right)C \tag{21}$$

$$K_{24} = K_{42} = \left(12\frac{EI}{L_e^3} + 12\frac{GAl^2}{L_e^3}\right)CS - \frac{EA}{L_e}CS \tag{22}$$

$$K_{25} = K_{52} = -\left(12\frac{EI}{L_e^3} + 12\frac{GAl^2}{L_e^3}\right)C^2 - \frac{EA}{L_e}S^2 \tag{23}$$

$$K_{33} = K_{66} = 4\frac{EI}{L_e} + 4\frac{GAl^2}{L_e} \tag{24}$$

$$K_{34} = K_{43} = K_{46} = K_{64} = \left(6\frac{EI}{L_e^2} + 6\frac{GAl^2}{L_e^2}\right)S \tag{25}$$

$$K_{35} = K_{53} = K_{56} = K_{65} = -\left(6\frac{EI}{L_e^2} + 6\frac{GAl^2}{L_e^2}\right)C \tag{26}$$

$$K_{36} = K_{63} = \left(2\frac{EI}{L_e} + 2\frac{GAl^2}{L_e}\right) \tag{27}$$

$$K_{45} = K_{54} = -\left(12\frac{EI}{L_e^3} + 12\frac{GAl^2}{L_e^3}\right)CS + \frac{EA}{L_e}CS \tag{28}$$

As can be understood, the matrix given in equation (13) is the stiffness matrix of a nanoframe element based on the modified couple stress theory. In this size-dependent stiffness matrix, besides the material length scale parameter l, there are C and S expressions. If we neglect the material length scale parameter in the stiffness matrix based on the modified couple stress theory, the stiffness matrix based on the classical elasticity theory is obtained. The stiffness matrix of a classical frame element is as follows [57]

$$K_{cl} = \frac{E}{L_e}\begin{bmatrix} AC^2 + \frac{12I}{L_e^2}S^2 & \left(A - \frac{12I}{L_e^2}\right)CS & -\frac{6I}{L_e}S & -\left(AC^2 + \frac{12I}{L_e^2}S^2\right) & -\left(A - \frac{12I}{L_e^2}\right)CS & -\frac{6I}{L_e}S \\ \left(A - \frac{12I}{L_e^2}\right)CS & AS^2 + \frac{12I}{L_e^2}C^2 & \frac{6I}{L_e}C & -\left(A - \frac{12I}{L_e^2}\right)CS & -\left(AS^2 + \frac{12I}{L_e^2}C^2\right) & \frac{6I}{L_e}C \\ -\frac{6I}{L_e}S & \frac{6I}{L_e}C & 4I & \frac{6I}{L_e}S & -\frac{6I}{L_e}C & 2I \\ -\left(AC^2 + \frac{12I}{L_e^2}S^2\right) & -\left(A - \frac{12I}{L_e^2}\right)CS & \frac{6I}{L_e}S & AC^2 + \frac{12I}{L_e^2}S^2 & \left(A - \frac{12I}{L_e^2}\right)CS & \frac{6I}{L_e}S \\ -\left(A - \frac{12I}{L_e^2}\right)CS & -\left(AS^2 + \frac{12I}{L_e^2}C^2\right) & -\frac{6I}{L_e}C & \left(A - \frac{12I}{L_e^2}\right)CS & AS^2 + \frac{12I}{L_e^2}C^2 & -\frac{6I}{L_e}C \\ -\frac{6I}{L_e}S & \frac{6I}{L_e}C & 2I & \frac{6I}{L_e}S & -\frac{6I}{L_e}C & 4I \end{bmatrix} \tag{29}$$

4. Application Procedure

A finite element solution of nodal displacement and rotation analysis based on modified couple stress theory of a nanoscale frame structure is shown. As can be understood from the previous formulations, the size effect takes place in the stiffness matrix. In the solution based on the modified couple stress theory, only the stiffness matrix is affected by the material length scale parameter, which gives the size effect. To perform the solution of a nanoframe system like in Figure 1, first the stiffness matrix of each element is obtained. Then these stiffness matrices are assembled. A global nodal force vector and a global nodal displacement vector are written.

$$\left[K_{mcst}^{global}\right]_{12x12}\begin{Bmatrix} d_{1x} \\ d_{1y} \\ \theta_1 \\ d_{2x} \\ d_{2y} \\ \theta_2 \\ d_{3x} \\ d_{3y} \\ \theta_3 \\ d_{4x} \\ d_{4y} \\ \theta_4 \end{Bmatrix} = \begin{Bmatrix} f_{1x} \\ f_{1y} \\ M_1 \\ f_{2x} \\ f_{2y} \\ M_2 \\ f_{3x} \\ f_{3y} \\ M_3 \\ f_{4x} \\ f_{4y} \\ M_4 \end{Bmatrix} \tag{30}$$

In the above equation, d and θ indicate the nodal displacements and rotations, respectively. In the global nodal force vector, those expressed by f and M are the applied nodal force and moment, respectively. The numbers (*1,2,3,4*) given as subscript indicate the node number. As a final process, the boundary conditions are applied to the required nodes and the results are obtained.

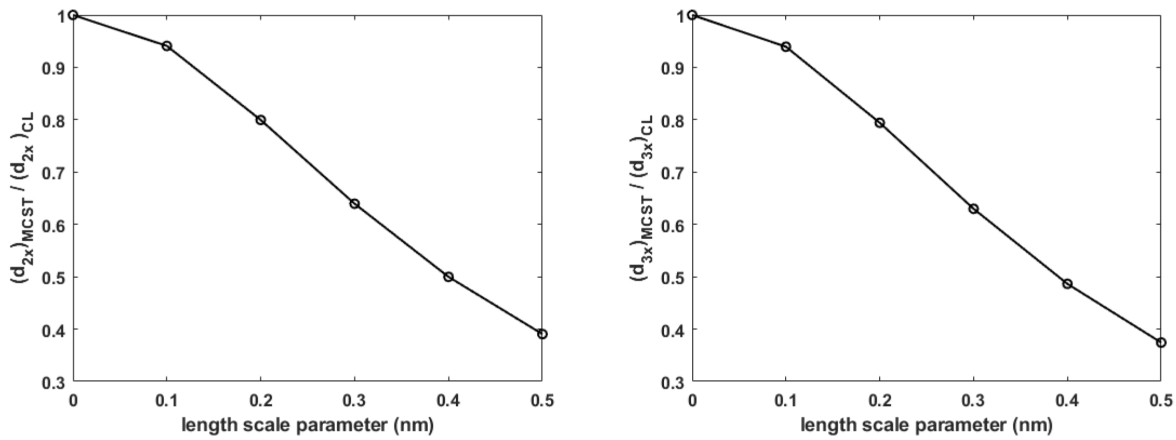
5. Numerical Result and Discussion

In this section of the study, a numerical application is realized. For this purpose, it is assumed that the elements of the nanoframe given in figure 1 are composed of silicon carbide nanotubes. The Young's modulus and Poisson's ratio of silicon carbide nanotube are as follows: $E=0.45$ *TPa*, $v=0.27$ [58, 59]. The shear modulus of silicon carbide nanotube is calculated by the following relation:

$$G = \frac{E}{2(1+\nu)} \tag{31}$$

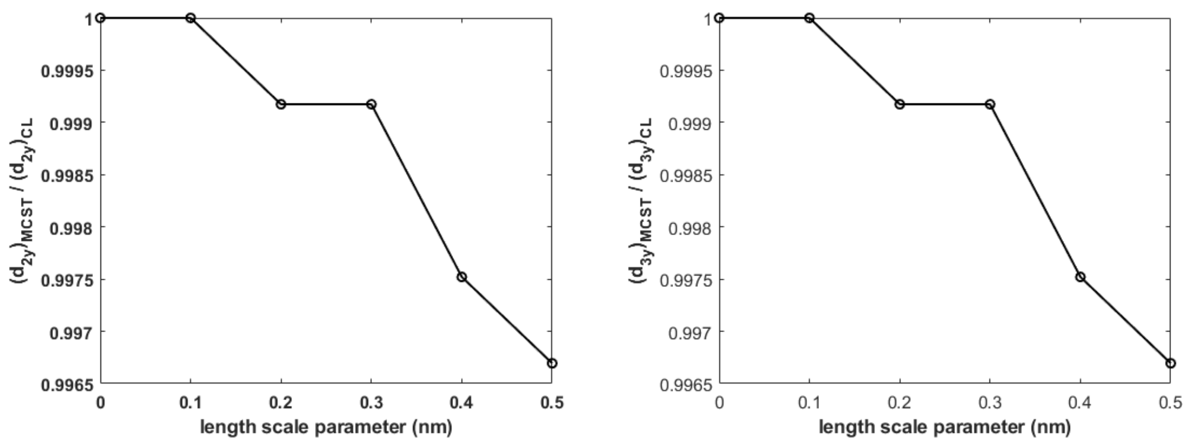
The geometric properties of the beam and columns that build up the nanoframe are equal to each other. The length of the nanoframe elements is $L=10$ *nm*, while their diameter is $d=1$ *nm*. Lastly, the nodal force and nodal moment are: $f_{2x}=1$ *nN*, $M_3=10$ *nNnm*.

Numerical results are given in normalized form to show the effect of material length scale parameter. Normalized displacements and rotations are expressed as the ratio of the results obtained with the modified couple stress theory to the results obtained with the classical theory of elasticity (*CL*). The material length scale parameter values are changed from 0 nm to 0.5 nm in 0.1 increments. It should be reminded once again that if material length scale parameter set to zero, the solution gives the results of classical elasticity theory. Figure 2 shows the x-direction normalized displacements of node 2 and node 3, while figure 3 shows those in the y-direction. As can be seen figure 2, as the material length scale parameter value increases, there is a decrease in the normalized displacement values. This means that higher displacements occur when the size effect is not taken into account. In other words, lower displacements occur for the x-direction when the size effect is considered. In Figure 3, the effect of the material length scale parameter on displacement in the y-direction is shown. When the normalized displacement values are examined, it is understood that the effect of the material length scale parameter on the displacements in the y-direction is much less. Figure 4 is plotted to examine the effect of the small scale parameter on nodal rotations. It is understood from this figure that the modified couple stress theory reduces the rotation values. As the material length scale parameter increases, the rotation values decrease.



(a) (b)

Fig.2. The x-direction normalized displacements a) node 2 b) node 3



(a) (b)

Fig.3. The y-direction normalized displacements a) node 2 b) node 3

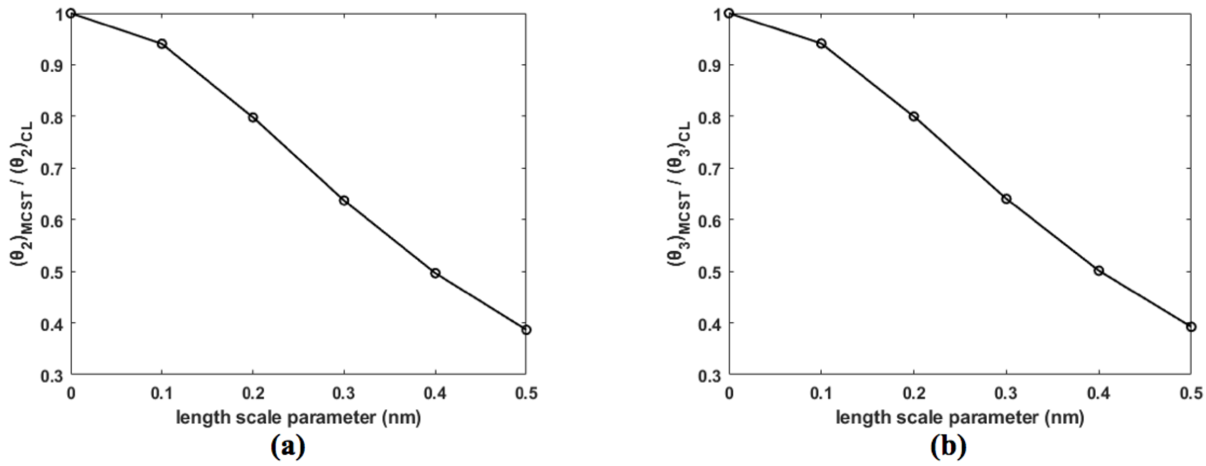


Fig.4. The normalized rotations a) node 2 b) node 3

6. Conclusions

In this work, a size-dependent finite element formulation of nanoframe is presented in the context of the modified couple stress theory. It is important to carry out theoretical analyzes that can reflect correct and actual behaviors of nanoscale structures. Theoretical analyzes have several advantages in terms of both time and cost. When literature is reviewed, it can be seen that the majority of the analyses are on one-dimensional nanoscale elements. This work presents a finite element solution that gives nodal displacements and rotations of a two-dimensional nanoscale frame. Modified couple stress theory, which is one of the elasticity theories that can theoretically present small-scale effects, is the subject of this study. The importance of the material length scale parameter is demonstrated in many scientific articles. When the results of this study are examined, it is understood that the size effect based on the material length scale parameter is important for bending of nanoframe system. Knowing how the material length scale parameter is included in the calculations for the solution of a frame system is important for engineering and device applications of such structures via obtaining numerical result.

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