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A NEW ALGORITHM BASED ON THE DECIC (TENTH DEGREE) B-SPLINE FUNCTIONS FOR NUMERICAL SOLUTION OF THE EQUAL WIDTH EQUATION

Melis Zorsahin Gorgulu

Department of Mathematics and Computer Sciences, Eskişehir Osmangazi University, Eskişehir, Turkey mzorsahin@ogu.edu.tr

Abstract

In this study, a new algorithm is introduced for the numerical solution of equal width (EW) equation. This algorithm is created by using the collocation finite element method based on decic B-spline functions for the space discretization of the EW equation and the Crank-Nicolson method for the time discretization of his equation. The obtained results are compared with the previous ones to see the efficiency and accuracy of the proposed method.

Keywords: Collocation method, Crank-Nicolson method, decic (tenth degree) B-spline function, equal width equation

1. Introduction

The frequently occurred nonlinear phenomena in the mathematical models of some nonlinear evolution equations are the propagation of the solitary waves in nonlinear dispersive media. This dispersive waves can be described using the following EW equation with the positive parameters ε and μ [1]:

$$u_t + \varepsilon u u_x - \mu u_{xxt} = 0. \tag{1}$$

The EW equation appears as an alternative form of the regularized long wave and Korteweg-de Vriese equations in the literature. The solutions of the EW equation have been studied for many years by researchers. Mostly the Galerkin [2-8] and collocation [9-14] finite element methods based on various B-spline functions have been used. Also some other methods are proposed to solve the EW equation numerically as the finite differences method, method of lines based on Runge-Kutta integration, least square method, Galerkin method, Petrov-Galerkin method, RBF-PS scheme, meshless kernel-based method, multi quadratic quasi-interpolation method, Haar Wavelet method, numerical method using polynomial scaling functions [15-25].

In this study, the decic (tenth degree) B-spline function is used as a trial function for obtaining the numerical solution of Eq. (1) by the collocation method. Although the using of the high degree B-spline functions are not as famous as the low degree B-spline functions in the literature, this study is a new experiment to get the numerical solutions of the nonlinear partial differential equations. The main aim in this paper is to see that when the collocation method based on decic B-spline functions is applied for the numerical solution of the EW equation, what is the effect of using the high degree B-spline functions in the space discretization of the equation.

The organization of paper is as follows. In section 2, the time discretization of the Eq. (1) is obtained by using the Crank-Nicolson method and then collocation method based on the decic B-spline functions is described for getting the fully discretized form of the EW equation. In section 3, two test problems are considered to see the efficiency of the proposed method. Lastly, the results obtained by the proposed method for the numerical experiments are compared with each other and with other methods in the literature.

2. Decic B-spline collocation methods

Let consider a solution domain $[\alpha_1, \alpha_2]$ by the equal space step h and the following conditions for the Eq.(1):

$$\begin{aligned}
&u(\alpha_1, t) = a, u(\alpha_2, t) = b,\\
&u_x(\alpha_1, t) = 0, u_x(\alpha_2, t) = 0, t \in (0, T],\\
&u(x, 0) = f(x), x \in [\alpha_1, \alpha_2].
\end{aligned} \tag{2}$$

The analytical solutions of the EW equation are described as follows

$$u(x_r, t_s) = u_r^s, r = 0,1, \dots, N; s = 0,1,2, \dots$$

where $x_r = \alpha_1 + rh,\ t_s = s\Delta t$ and the the numerical value of u_r^s at the grid points are shown by the notation U_r^s.
Applying the Crank-Nicolson method to the Eq. (1) to find the time discretized form of the EW equation, the following equation is obtained

$$u^{s+1} + \varepsilon \frac{\Delta t}{2}(u^{s+1} u_x^{s+1}) - \mu u_{xx}^{s+1} = u^s - \varepsilon \frac{\Delta t}{2}(u^s u_x^s) - \mu u_{xx}^s. \tag{3}$$

The decic B-spline $\varphi_r(x)$ the details of which are produced in [26] is defined as

$$\varphi_r(x) = \frac{1}{h^{10}} \begin{cases} \sigma_1 & , x_{r-5} \le x < x_{r-4} \\ \sigma_2 & , x_{r-4} \le x < x_{r-3} \\ \sigma_3 & , x_{r-3} \le x < x_{r-2} \\ \sigma_4 & , x_{r-2} \le x < x_{r-1} \\ \sigma_5 & , x_{r-1} \le x < x_{r} \\ \sigma_6 & , x_{r} \le x < x_{r+1} \\ \sigma_7 & , x_{r+1} \le x < x_{r+2} \\ \sigma_8 & , x_{r+2} \le x < x_{r+3} \\ \sigma_9 & , x_{r+3} \le x < x_{r+4} \\ \sigma_{10} & , x_{r+4} \le x < x_{r+5} \\ \sigma_{11} & , x_{r+5} \le x < x_{r+6} \\ 0 & , \text{otherwise} \end{cases} \tag{4}$$

where

$$\begin{aligned}
\sigma_1 &= g_{m-5}^{10}(x), \\
\sigma_2 &= h^{10} + 10h^9 g_{m-4}(x) + 45h^8 g_{m-4}^2(x) + 120h^7 g_{m-4}^3(x) + 210h^6 g_{m-4}^4(x) \\
&\quad +252h^5 g_{m-4}^5(x) + 210h^4 g_{m-4}^6(x) + 120h^3 g_{m-4}^7(x) + 45h^2 g_{m-4}^8(x) \\
&\quad +10h g_{m-4}^9(x) - 10 g_{m-4}^{10}(x), \\
\sigma_3 &= 1013h^{10} + 5010h^9 g_{m-3}(x) + 11025h^8 g_{m-3}^2(x) + 14040h^7 g_{m-3}^3(x) \\
&\quad +11130h^6 g_{m-3}^4(x) + 5292h^5 g_{m-3}^5(x) + 1050h^4 g_{m-3}^6(x) \\
&\quad -360h^3 g_{m-3}^7(x) - 315h^2 g_{m-3}^8(x) - 90h g_{m-3}^9(x) + 45 g_{m-3}^{10}(x), \\
\sigma_4 &= 47840h^{10} + 141060h^9 g_{m-2}(x) + 171000h^8 g_{m-2}^2(x) \\
&\quad +100080h^7 g_{m-2}^3(x) + 16800h^6 g_{m-2}^4(x) - 13608h^5 g_{m-2}^5(x) \\
&\quad -8400h^4 g_{m-2}^6(x) - 720h^3 g_{m-2}^7(x) + 900h^2 g_{m-2}^8(x) \\
\sigma_5 &= 455192h^{10} + 736260h^9 g_{m-1}(x) + 327600h^8 g_{m-1}^2(x) \\
&\quad -95760h^7 g_{m-1}^3(x) - 8400h^4 g_{m-2}^6(x) - 720h^3 g_{m-2}^7(x) \\
&\quad +900h^2 g_{m-2}^8(x) - 8400h^4 g_{m-2}^6(x) - 720h^3 g_{m-2}^7(x) + 900h^2 g_{m-2}^8(x) \\
&\quad +360h g_{m-2}^9(x) - 120 g_{m-2}^{10}(x), \\
\sigma_6 &= 1310354h^{10} + 679560h^9 g_m(x) - 509670h^8 g_m^2(x) - 312480h^7 g_m^3(x) \\
&\quad +91140h^6 g_m^4(x) + 69552h^5 g_m^5(x) - 9660h^4 g_m^6(x) - 10080h^3 g_m^7(x) \\
&\quad +630h^2 g_m^8(x) + 1260h g_m^9(x) - 252 g_m^{10}(x), \\
\sigma_7 &= 1310354h^{10} - 679560h^9 g_{m+1}(x) - 509670h^8 g_{m+1}^2(x) \\
&\quad +312480h^7 g_{m+1}^3(x) + 91140h^6 g_{m+1}^4(x) - 69552h^5 g_{m+1}^5(x) \\
&\quad -9660h^4 g_{m+1}^6(x) + 10080h^3 g_{m+1}^7(x) + 630h^2 g_{m+1}^8(x) \\
&\quad -1260h g_{m+1}^9(x) + 210 g_{m+1}^{10}(x), \\
\sigma_8 &= 455192h^{10} - 736260h^9 g_{m+2}(x) + 327600h^8 g_{m+2}^2(x) \\
&\quad +95760h^7 g_{m+2}^3(x) - 119280h^6 g_{m+2}^4(x) + 13608h^5 g_{m+2}^5(x) \\
&\quad +16800h^4 g_{m+2}^6(x) - 5040h^3 g_{m+2}^7(x) - 1260h^2 g_{m+2}^8(x) \\
&\quad +840h g_{m+2}^9(x) - 120 g_{m+2}^{10}(x), \\
\sigma_9 &= 47840h^{10} - 141060h^9 g_{m+3}(x) + 171000h^8 g_{m+3}^2(x) \\
&\quad -100080h^7 g_{m+3}^3(x) + 16800h^6 g_{m+3}^4(x) + 13608h^5 g_{m+3}^5(x) \\
&\quad -8400h^4 g_{m+3}^6(x) + 720h^3 g_{m+3}^7(x) + 900h^2 g_{m+3}^8(x) \\
&\quad -360h g_{m+3}^9(x) + 45 g_{m+3}^{10}(x), \\
\sigma_{10} &= 1013h^{10} - 5010h^9 g_{m+4}(x) + 11025h^8 g_{m+4}^2(x) - 14040h^7 g_{m+4}^3(x) \\
&\quad +11130h^6 g_{m+4}^4(x) - 5292h^5 g_{m+4}^5(x) + 1050h^4 g_{m+4}^6(x)
\end{aligned}$$

$$+360h^3 g_{m+4}^7(x) - 315h^2 g_{m+4}^8(x) + 90h g_{m+4}^9(x) - 10 g_{m+4}^{10}(x),$$
$$\sigma_{11} = [h - g_{m+5}(x)]^{10}.$$

To set up the space discretization of the Eq. (3), the approximate solution U is obtained in terms of the decic B-splines φ as

$$U(x_r) = U_r = \sum_{i=r-5}^{r+5} \varphi_i(x)\delta_i, \tag{5}$$

where δ_i are time dependent unknowns which will be calculated. So, the approximate solution, its first and second derivatives at the knots can be written by the help of the Eq. (4) as

$$\begin{aligned}
U_r &= (\delta_{r-5} + 1013\delta_{r-4} + 47840\delta_{r-3} + 455192\delta_{r-2} + 1310354\delta_{r-1} \\
&\quad +1310354\delta_r + 455192\delta_{r+1} + 47840\delta_{r+2} + 1013\delta_{r+3} + \delta_{r+4}), \\
U_r' &= \frac{10}{h}(-\delta_{r-5} - 501\delta_{r-4} - 14106\delta_{r-3} - 73626\delta_{r-2} - 67956\delta_{r-1} \\
&\quad +67956\delta_r + 73626\delta_{r+1} + 14106\delta_{r+2} + 501\delta_{r+3} + \delta_{r+4}), \\
U_r'' &= \frac{90}{h^2}(\delta_{r-5} + 245\delta_{r-4} + 3800\delta_{r-3} + 7280\delta_{r-2} - 11326\delta_{r-1} \\
&\quad -11326\delta_r + 7280\delta_{r+1} + 3800\delta_{r+2} + 245\delta_{r+3} + \delta_{r+4}),
\end{aligned}$$

Using the above equations in Eq. (3), a fully discretized form of the Eq. (1) is obtained as

$$\begin{aligned}
&(1+\gamma_1^{s+1})\delta_{r-5}^{s+1} + (1013+\gamma_2^{s+1})\delta_{r-4}^{s+1} + (47840+\gamma_3^{s+1})\delta_{r-3}^{s+1} + (455192+\gamma_4^{s+1})\delta_{r-2}^{s+1} \\
&\quad +(1310354+\gamma_5^{s+1})\delta_{r-1}^{s+1} + (1310354+\gamma_6^{s+1})\delta_r^{s+1} + (455192+\gamma_7^{s+1})\delta_{r+1}^{s+1} \\
&\quad +(47840+\gamma_8^{s+1})\delta_{r+2}^{s+1} + (1013+\gamma_9^{s+1})\delta_{r+3}^{s+1} + (1+\gamma_{10}^{s+1})\delta_{r+4}^{s+1} \\
&= (1-\gamma_1^s)\delta_{r-5}^s + (1013-\gamma_2^s)\delta_{r-4}^s + (47840-\gamma_3^s)\delta_{r-3}^s + (455192-\gamma_4^s)\delta_{r-2}^s \\
&\quad +(1310354-\gamma_5^s)\delta_{r-1}^s + (1310354-\gamma_6^s)\delta_r^s + (455192-\gamma_7^s)\delta_{r+1}^s \\
&\quad +(47840-\gamma_8^s)\delta_{r+2}^s + (1013-\gamma_9^s)\delta_{r+3}^s + (1-\gamma_{10}^s)\delta_{r+4}^s
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
&\beta_1 = \frac{5\varepsilon\Delta t}{h}\eta^j, \beta_2^j = \frac{-90\mu}{h^2}, \gamma_1^j = -\beta_1^j + \beta_2, \gamma_2^j = -501\beta_1^j + 245\beta_2, \\
&\gamma_3^j = -14106\beta_1^j + 3800\beta_2, \gamma_4^j = -73626\beta_1^j + 7280\beta_2, \gamma_5^j = -67956\beta_1^j - 11326\beta_2, \\
&\gamma_6^j = 67956\beta_1^j - 11326\beta_2, \gamma_7^j = 73626\beta_1^j + 7280\beta_2, \gamma_8^j = 14106\beta_1^j + 3800\beta_2, \\
&\gamma_9^j = 501\beta_1^j + 245\beta_2, \gamma_{10}^j = \beta_1^j + \beta_2, \\
&\eta^j = \delta_{m-5}^j + 1013\delta_{m-4}^j + 47840\delta_{m-3}^j + 455192\delta_{m-2}^j + 1310354\delta_{m-1}^j\big) \\
&\qquad +1310354\delta_m^j + 455192\delta_{m+1}^j + 47840\delta_{m+2}^j + 1013\delta_{m+3}^j + \delta_{m+4}^j, j = s+1, s\big).
\end{aligned}$$

Thus, we have a system of $N+1$ equation and $N+10$ unknowns. The conditions $u(\alpha_1,t) = a$, $u(\alpha_2,t) = b$ are help us to eliminate the parameters $\delta_{-5}, \delta_{-4}, \delta_{-3}, \delta_{-2}, \delta_{-1}, \delta_{N+2}, \delta_{N+3}, \delta_{N+4}$ and δ_{N+5} so that we have a solvable matrix system of $N+1$ equation and $N+1$ unknowns which is easily solved with Matlab packet program. To start the iteration of system, δ^0 has been determined by using the conditions in Eq. (2), so then we can obtain iteratively the δ^s at time $t^s = s\Delta t$. Also an inner iteration is used due to the system (6) is an implicit system according to the term δ.

4. Test problems

To demonstrate the efficiency of the proposed algorithm, two test problems, namely the motion of the single solitary wave and the interaction of two solitary waves, are studied. The conservation laws satisfied by the EW equation:

$$C_1 = \int_{-\infty}^{+\infty} u dx \, , C_2 = \int_{-\infty}^{+\infty} (u^2 + \mu (u_x)^2) dx \, , C_3 = \int_{-\infty}^{+\infty} (u^3) dx \qquad (7)$$

are calculated by use of the trapezoidal rule in the numerical experiments. The space convergence order of the method is determined by the following formula:

$$\text{order} = \frac{log\left|\frac{(L_\infty)_{h_r}}{(L_\infty)_{h_{r+1}}}\right|}{log\left|\frac{h_r}{h_{r+1}}\right|}. \qquad (8)$$

The accuracy of the method is calculated by the error norm:

$$L_\infty = \max_r |u_r - U_r|. \qquad (9)$$

4.1 Test problem 1 (Motion of single solitary wave)

The exact solution of EW equation is

$$u(x,t) = 3c sech^2(k[x - \bar{x}_0 - vt]) \qquad (10)$$

where $k = \sqrt{\frac{1}{4\mu}}$. The initial condition is worked out by taking $t = 0$ in Eq. (10). We have used the values of the boundary conditions as zero.The values of the parameters are chosen as $c = 0.1$, $\bar{x}_0 = 10$, $\varepsilon = 1$ and $\mu = 1$. The motion of the obtained single solitary wave with these parameters has been come about the interval $-15 \leq x \leq 45$ in time period $0 \leq t \leq 80$. The solution profiles are depicted in Figure 1 with $h = 0.03$ and $\Delta t = 0.05$. In this figure, it can be said that the peak of the solitary wave remained almost the same throughout the study period.

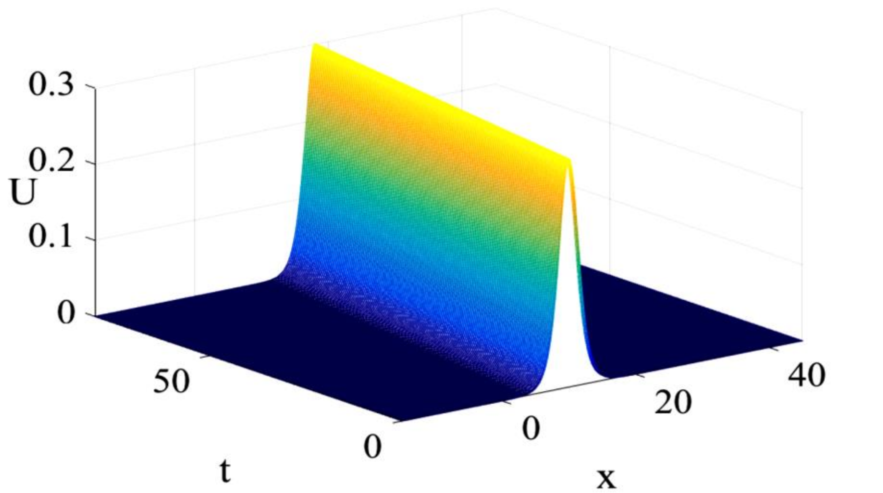


Figure 1. The numerical solutions at the various times.

The distribution of absolute errors at $t = 80$ with $h = 0.03$ and $\Delta t = 0.05$ is given in Figure 2.

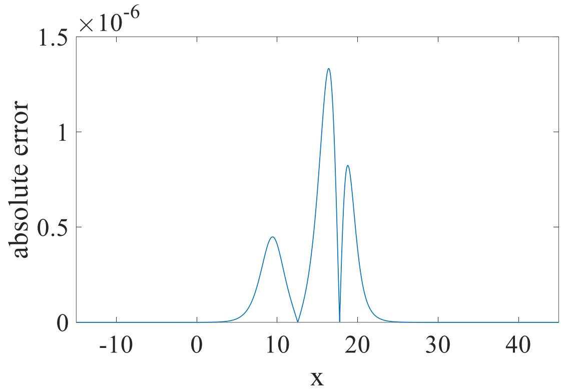


Figure 2. Absolute error with $\boldsymbol{h = \Delta t = 0.05}$ for the proposed methods

To make a comparison with some early studies, the absolute error norms and the calculated conservation invariants C_1, C_2, C_3 of the proposed method and the previous studies are listed at the time $t = 80$ and the space domain $[0,30]$ in Table 1. From this table, it can be seen that the decic B-spline collocation method is one of the most accurate method. The analytical values of the conservation constants for a solitary wave with amplitude $3c$ and width depending on k can evaluated as in [2] by

$$C_1 = \frac{6c}{k}, \quad C_2 = \frac{12c^2}{k} + \frac{48kc^2\mu}{5}, \quad C_3 = \frac{144c^3}{5k}.$$

The numerical values of invariants obtained by the decic B-spline method are $C_1 = 1.19999$, $C_2 = 0.28800$ and $C_3 = 0.05760$ at the time $t = 80$ while their analytical values obtained from Eq. (11) are $C_1 = 1.2$, $C_2 = 0.288$ and $C_3 = 0.0576$ for $c = 0.1$.

Table 1. Invariants and error norms with $\boldsymbol{c = 0.1, \Delta t = 0.05, h = 0.03, 0 \le x \le 30}$

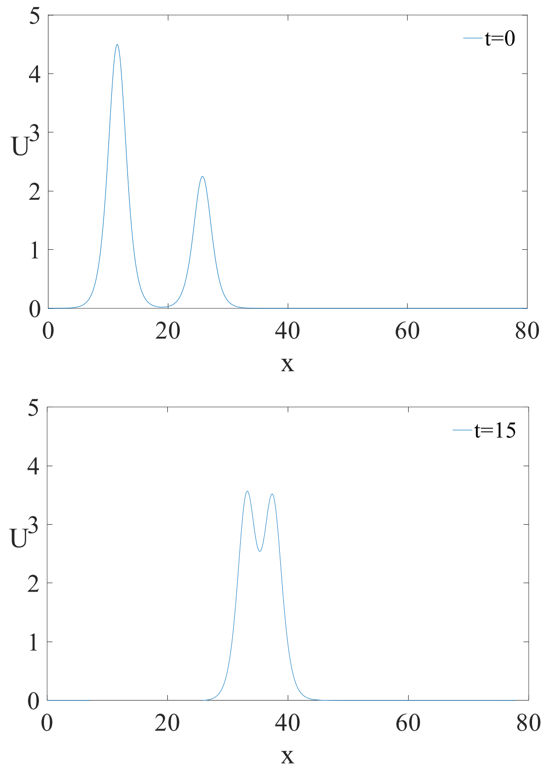
Method	$\boldsymbol{L_\infty}$**x**$\boldsymbol{10^4}$	$\boldsymbol{C_1}$	$\boldsymbol{C_2}$	$\boldsymbol{C_3}$
Decic B-spline collocation method	0.0737	1.19999	0.28800	0.05760
Exponential cubic B-spline collocation (p=1) [11]	0.54	1.1999	0.2880	0.05760
Exponential cubic B-spline collocation (p=0.0000340714) [11]	0.073	1.2000	0.2880	0.05760
Quartic B-spline Galerkin method [6]	0.0737	1.20000	0.28800	0.05760
Differential quadrature method [6]	0.07373	1.19999	0.28800	0.05760
Meshless method with radial-basis functions [6]	0.20296	1.20003	0.28801	0.05761
Petrov-Galerkin method [2]	26.46	1.1910	0.2855	0.05582
Cubic spline collocation method [13]	0.53	1.20005	0.28800	0.05760
Cubic B-spline collocation method [10]	0.53	1.19998	0.28798	0.05759
Lumped Galerkin method [8]	0.21	1.19995	0.28798	0.05759
Galerkin method [18]	164.25	1.23387	0.29915	0.06097
Cubic B-spline collocation method [14]	0.336	1.19999	0.28800	0.05756
Exact		1.2	0.288	0.0576

4.2 Test problem 2 (Interaction of two solitary wave)

Secondly, by using the following initial condition

$$u(x,0) = 3\phi_1 sech^2\big(k_1(x - \overline{x}_1 - \phi_1)\big) + 3\phi_2 sech^2\big(k_2(x - \overline{x}_2 - \phi_2)\big) \qquad (11)$$

the problem in which two single solitary waves is interacted, is considered. The boundary conditions for this problem are chosen as $u(0,t) = u(80,t) = 0$. In consistent with the previous studies, the other parameters are taken as $\mu = 1$, $\varepsilon = 1$, $k_1 = 0.5$, $k_2 = 0.5$, $\overline{x}_1 = 10$, $\overline{x}_2 = 25$, $\phi_1 = 1.5$ and $\phi_2 = 0.75$ in the calculations. The analytical values of the invariants can be found from Eq. (11) as $C_1 = 27$, $C_2 = 8$ and $C_3 = 218.7$. The simulation of the interaction of two single solitary waves can observed in Figure 3 with the time step $\Delta t = 0.1$ and the space step $h = 0.1$ over the space domain $[0,80]$. From the figure, it can be seen that the rise of the two single solitary waves, then the waves interact and finally the waves occur their original shapes. The Table 2 is given to see the effect of choosing the smaller step sizes for the calculations of the invariants. From the table it is seen clearly that the obtained conservation constants by choosing smaller values for the time and space steps gives nearly the same values with the exact ones.



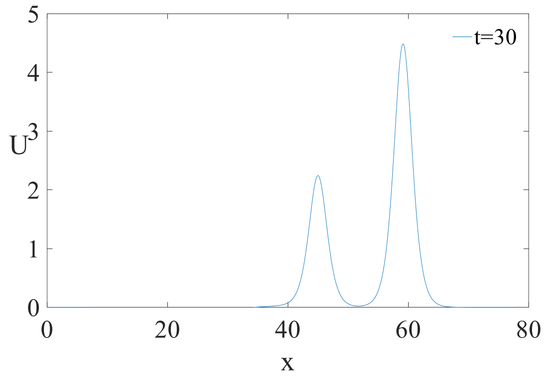


Figure 3. The simulation of the interacted wave solutions.

Table 2. Invariants for the various values of the space and time steps

$h = \Delta t = 0.1$			
t	C_1	C_2	C_3
1	26.993302378591750	80.991136106784495	218.660792457023400
5	26.985543145640346	80.954025171222781	218.494098806768280
10	26.976201331621599	80.914847637768275	218.311510240538810
15	26.970649371206846	80.941311146473225	218.365614712728900
20	26.965596614631067	80.866647798397310	218.101358339337030
25	26.956455573585203	80.814638952319768	217.879664665904900
30	26.946745247721164	80.767859034837628	217.671712922420880
$h = \Delta t = 0.05$			
t	C_1	C_2	C_3
1	26.990461050320302	80.999371980781348	218.697544042151320
5	26.989499238661029	80.994707811248304	218.676538542650550
10	26.988249282271042	80.990557601268890	218.656042086919340
15	26.987485867694875	81.000608829039905	218.684248795519240
20	26.986763460419446	80.984584309319814	218.629651649602890
25	26.985527466074423	80.977048015978198	218.598548488278770
30	26.984220259670970	80.971057218965811	218.571956196134580
$h = \Delta t = 0.01$			
t	C_1	C_2	C_3
1	26.953994253516079	81.002573270696047	218.702757879260470
5	26.952646451957293	81.002552372020247	218.702591702971490
10	26.950797888136012	81.002573139408995	218.702589848684800
15	26.948880619314313	81.003091871507038	218.704173167737140
20	26.946887864441052	81.002563709534769	218.702396599911480
25	26.944811921568093	81.002468848617525	218.701957922997790
30	26.942652695479943	81.002451995919174	218.701735540488670

5. Conclusion

The collocation finite element method based on the decic B-spline functions as trial functions for space discretization and Crank-Nicolson method for time discretization have been proposed to get numerical solution to the EW equation. By investigating the motion of the single solitary wave and the interaction of the two solitary waves problems to see the effectiveness and accurate of the proposed method, it is seen that the proposed method has the reliable results.

References

[1] Morrison, P.J., Meiss, J.D., Carey, J.R., "Scattering of RLW solitary waves", Physica 11D (1984) : 324-336.

[2] Gardner, L.R.T., Gardner, G.A., "Solitary waves of the equal width wave equation", Journal of Computational Physics 101 (1992) : 218-223.

[3] Irk, D., "B-Spline Galerkin solutions for the equal width equation", Physics of Wave Phenomena 20(2) (2012) : 122-130.

[4] Dağ, İ., Saka, B., Irk, D., "Galerkin method for the numerical solution of the RLW equation using quintic B-splines", Journal of Computational and Applied Mathematics 190 (2006) : 532-547.

[5] Saka, B., "A finite element method for equal width equation", Applied Mathematics and Computation 175 (2006) : 730-747.

[6] Saka, B., Dag, I., Dereli, Y., Korkmaz, A., "Three different methods for numerical solution of the EW equation", Engineering analysis with boundary elements 32 (2008) : 556-566.

[7] Dag, I., Irk, D., Boz, A., "Simulation of EW wave generation via quadratic B-spline finite element method", International Journal of Mathematics and Statistics 1 (A07) (2007) : 46-59.

[8] Esen, A., "A numerical solution of the equal width wave equation by a lumped Galerkin method", Applied Mathematics and Computation 168 (2005) : 270-282.

[9] Raslan, K.R., "Collocation method using quartic B-spline for the equal width (EW) equation", Applied Mathematics and Computation 168(2) (2005) : 795-805.

[10] Dağ, İ., Saka, B., "A cubic B-spline collocation method for the EW equation", Mathematical and Computational Applications 90 (2004) : 381-392.

[11] Dag, I., Ersoy, O., "The exponential cubic B-spline algorithm for equal width equation", Advanced Studies in Contemporary Mathematics 25(4) (2015) : 525-535.

[12] Yağmurlu, N.M., Karakaş, A.S., "Numerical solutions of the equal width equation by trigonometric cubic B-spline collocation method based on Rubin-Graves type linearization", Numerical Methods for Partial Differential Equations 36(5) (2020) : 1170-1183.

[13] Irk, D., Saka, B., Dağ, İ., "Cubic spline collocation method for the equal width equation", Hadronic Journal Supplement 18 (2003) : 201-214.

[14] Saka, B., Irk, D., Dağ, İ., "A numerical study of the equal width equation", Hadronic Journal Supplement 18 (2003) : 99-116.

[15] Banaja, M.A., Bakodah, H.O., "Runge-Kutta integration of the equal width wave equation

using the method of lines", Mathematical Problems in Engineering 2015 (2015) : Article ID 274579.

[16] Inan, B., Bahadır, A.R., "A numerical solution of the equal width wave equation using a fully implicit finite difference method", Turkish Journal of Mathematics and Computer Science 2(1) (2014) : 1-14.

[17] Zaki, S.I., "A least-squares finite element scheme for the EW equation", Communications in Numerical Methods in Engineering 189 (2000) : 587-594.

[18] Doğan, A., "Application of Galerkin's method to equal width wave equation", Applied Mathematics and Computation 160 (2005) : 65-76.

[19] Roshan, T., "A Petrov-Galerkin method for equal width equation", Applied Mathematics and Computation 218(6) (2011) : 2730-2739.

[20] Gardner, L.R.T., Gardner, G.A., Ayoub, F.A., Amein, N.K., "Simulations of the EW undular bore", Communications in Numerical Methods in Engineering 13 (1997) : 583-592.

[21] Uddin, M., "RBF-PS scheme for solving the equal width equation", Applied Mathematics and Computation 222 (2013) : 619-631.

[22] Dereli, Y., Schaback, R., "The meshless kernel-based method of lines for solving the equal width equation", Applied Mathematics and Computation 219(10) (2013) : 5224-5232.

[23] Dhawan, S., Ak, T., Apaydin, G., "Algorithms for numerical solution of the equal width wave equation using multi-quadric quasi-interpolation method", International Journal of Modern Physics C 30(11) (2019) : 1950087.

[24] Ghafoor, A., Haq, S., "An efficient numerical scheme for the study of equal width equation", Results in Physics 9 (2018) : 1411-1416.

[25] Oruç, Ö., Esen, A., Bulut, F., "Highly accurate numerical scheme based on polynomial scaling functions for equal width equation", Wave Motion 105 (2021) : 102760.

[26] Koyulmuş, B., "On high degree B-spline functions", Master thesis, Eskişehir Osmangazi University, Eskişehir, Turkey, 2021 (in Turkish).