



On the Co-occurrence of Different Magnitude Earthquakes: Southwestern China Case

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Received:29/04/2016 Revised:02/06/2016 Accepted: 06/06/2016

ABSTRACT

In this study, we build a Markov chain model for the earthquakes in Southwestern China by following the maximum entropy principle while the states of Markov chain are the co-occurrence of earthquakes with different magnitudes. In this case, an approximation is to focus on just the occurrences of the most serious magnitude earthquakes and neglect the others. Our approximation to this situation is to take all magnitude earthquakes into account if they occur at least once in any given period. In order to reveal the feature of this Markov chain in respect of the first passage time distributions, we run a long-term simulation with the occurrences of all the 3 categories of earthquakes. Finally, we give both the fitted distributions and multinomial approximations for the distribution of the first passage time for some states.

Key words: *Discrete-time Markov chain, Entropy, First passage time, Multinomial distribution approximation, Earthquakes in southwestern China*

1. INTRODUCTION

Consisting of Tibet Autonomous Region, Sichuan Province, Chongqing Municipality, Yunnan Province and Guizhou Province, southwestern China is the area that most suffered from earthquakes in China through 1970 to 2015. While the Sichuan-Yunnan region in southwest China lays on the boundary between Tibetan Plateau and South China platform, this region becomes an earthquake prone area in Southwestern China [1]. Moreover, Tibet Autonomous Region takes a part of the Alpine-Himalayan seismic belt which makes Southwestern China more vulnerable to earthquakes.

Nowadays it is widely accepted that it is impossible to predict the exact time, place and magnitude of a coming

earthquake. However, we can still use statistical tools in cooperation with the researchers in the field of seismology and geology to estimate seismic activity in a certain period. In recent years there are lots of studies focusing on to estimate the probability of earthquakes' occurrence in certain periods. Using Markov chain model for earthquake analysis becomes popular in earthquake forecasting studies recently. In 20th century many researchers treated earthquake sequencing as a Markov process to explore the aftershocks [2] [3] [4]. In the beginning of 21st century Markov chain modelling started to be used as a tool in several studies in regional earthquake forecasting [5] [6] [7]. Also there are some attempts to view the global seismic activities, Vasudevan and Cavers constructed a directed graph to show a Markov chain of global earthquake sequence [8] [9].

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On the other hand, in order to build a suitable Markov chain model, a reasonable time interval, which we see it as a single period, is needed to be found. By applying the maximum entropy principle Ünal and Çelebioğlu obtained a suitable time interval calculated for constructing a Markov chain with a finite state space [10]. In this study we are going to follow the maximum entropy principle as well.

Another interesting and meaningful topic that we focus on is the first passage time between two different states, or for the same state that is time of recurrence. Harris has investigated the first passage time and recurrence distributions in several special cases like random walks and large n -state finite Markov chains [11]. Harrison and Knottenbelt also explored the characteristics of passage time distributions in large n -state Markov chains, and applied their implementation in substantial Markov chains with over 1 million states and semi-Markov chains [12]. Gül and Çelebioğlu have obtained distribution of the first passage time relating to lumped states for an irreducible Markov chain with finite states [13]. And in this study we find the approximate distributions of first passage times and recurrence times of an irreducible Markov chain with finite states.

In the introduction section our research problem has been put up. Our data source and methodology are given in Section 2. In Section 3 our modelling procedure of Markov chain application on earthquakes occurring in southwestern China is shown and meanwhile the obtained results also take a part in Section 3. Conclusions and further discussions are given in Section 4.

2. DATA AND METHODOLOGY

2.1. Data Source

In this study our data source is obtained from China Earthquakes Networks Center (CENC). According to the data source during Jan 1, 1970 and Dec 31, 2015, there were a total of 11007 earthquake occurrences in

southwestern China. Considering about the magnitude of those earthquakes occurred in SE China, we can figure out that there are 8674 out of 11007 times that the magnitudes are no more than 4, and 2024 times that the magnitudes of earthquakes $4 < M \leq 5$, while $M > 5$ earthquakes which could make a great damage to human beings occurred 309 times during that period. Inspired by the idea of Ünal and Çelebioğlu, we classified those earthquakes into 3 different types by their magnitudes as we mentioned before [10]. The application of this classification is given in Section 3, and it is an important step of building our model.

2.2. Methodology

2.2.1. Markov Chain

In probability theory, a stochastic process is a collection of random variables considering of the time. The time will either be a subset of natural numbers or a subset of $[0, \infty)$, which will be nonnegative real numbers [14]. Named after the Russian mathematician Andrey A. Markov, Markov process is a stochastic process that satisfies the Markov property, which can be presented as follows: Let (Ω, \mathcal{F}, P) be a probability space with a filtration $(\mathcal{F}_t, t \in T)$, for some (totally ordered) index set T ; and let (S, \mathcal{S}) be a measure space. Then an S -valued stochastic process $X = (X_t, t \in T)$ adapted to the filtration is said to possess the Markov property with respect to the $\{\mathcal{F}_t\}$, if for each $A \in \mathcal{S}$ and each $s, t \in T$ with $s < t$,
$$P(X_t \in A | \mathcal{F}_s) = P(X_t \in A | X_s).$$
And for discrete-time case, S is a discrete set with the discrete sigma algebra, T is the set of natural numbers and then we have:

$$P(X_n = j | X_{n-1} = i, X_{n-2} = x_{n-2}, \dots, X_0 = x_0) = P(X_n = j | X_{n-1} = i).$$

From this we can easily see that the probability of being at the state j at n th step only depends on the state at $(n-1)$ st step, so here we can describe Markov chain with the word "memoryless" [15].

Definition 2.2.1.1 $p_{ij}^{(n)} = \Pr(X_n = j | X_0 = i), i, j \in S, n \in \mathbb{N}$ is n -step transition probability from state i to state j , and the process begins at $t=0$. When $n=1$, it becomes single-step probability from state i to state j . And when the transition probabilities are independent of the time parameter n , which means $\forall i, j \in S$,
$$\Pr(X_1 = j | X_0 = i) = \Pr(X_2 = j | X_1 = i) = \dots = \Pr(X_n = j | X_{n-1} = i), n \in \mathbb{N}.$$

Any Markov chain which satisfies this property is called time-homogeneous, or stationary Markov chain.

The transition matrix P is the $N \times N$ matrix whose elements are p_{ij} which is the one-step transition probability from state i to state j , and it is a stochastic matrix, i.e., $\forall i, j \in S, 0 \leq p_{ij} \leq 1$, and $\forall i \in S, \sum_{j \in S} p_{ij} = 1$.

Theorem 2.2.1.1 For a Markov chain X , $\forall m \in \mathbb{N}^+$, we have $P\{X_1 = x_1, \dots, X_{m-1} = x_{m-1}, X_m = x_m \mid X_0 = x_0\} = P_{x_0x_1} P_{x_1x_2} \dots P_{x_{m-1}x_m}$.

From Theorem 2.2.1.1 we can obtain the following corollary.

Corollary 2.2.1.1 For a Markov chain, we denote $\pi_0(i)$ as the probability of the initial state i , $\forall i \in S$. Hence we have $P\{X_0 = x_0, X_1 = x_1, \dots, X_{m-1} = x_{m-1}, X_m = x_m\} = \pi_0(x_0) P_{x_0x_1} P_{x_1x_2} \dots P_{x_{m-1}x_m}$.

Definition 2.2.1.2 For any two states i and j , we denote $f_{ij}^{(n)}$, $n \in \mathbb{N}^+$ to be the first passage time probability that the chain spends n steps for passing to state j from state i for the first time.

From this definition we can easily get that

$$f_{ij}^{(n)} = \begin{cases} P_{ij} & n = 1 \\ \sum_{b \in S - \{j\}} P_{ib} f_{bj}^{(n-1)} & n = 2, 3, 4, \dots \end{cases}$$

Definition 2.2.1.3 $f_{ij} = \sum_{n=1}^{\infty} f_{ij}^{(n)}$ is called the ever reaching probability, which is the probability of reaching state j from state i in finite steps.

Definition 2.2.1.4 A state j is said to be accessible from a state i (written $i \rightarrow j$), if $\exists n \in \mathbb{N}$, $p_{ij}^{(n)} > 0$ [16].

Definition 2.2.1.5 A state i is said to communicate with state j (written $i \leftrightarrow j$), if both $i \rightarrow j$ and $j \rightarrow i$ [16].

Definition 2.2.1.6 A set of states C is a communicating class, if $\forall i, j \in C, i \leftrightarrow j$ [16].

Definition 2.2.1.7 A Markov chain is said to be irreducible, if its state space is a single communicating class [16].

It is obvious that if it is possible to reach to any state from any state, that Markov chain is an irreducible Markov chain.

Definition 2.2.1.8 A state i is said to be transient, if $f_{ii} < 1, i \in S$.

Definition 2.2.1.9 A state i is said to be recurrent, if it is not a transient state. Recurrent states which have an infinite mean recurrence time are null recurrent. Recurrent states which have a finite mean recurrence time are positive recurrent.

Definition 2.2.1.10 A state i has period k if any return to state i must occur in multiples of k time steps, i.e., $k = \gcd\{n > 0 : \Pr(X_n = i \mid X_0 = i) > 0\}$. If $k = 1$, then the state is said to be aperiodic, which means the returns to state i can occur at irregular times. When $k \geq 2, k \in \mathbb{N}^+$, the state i is said to be periodic

with period k . A Markov chain is said to be aperiodic if every state is aperiodic, i.e., $k = 1$.

Theorem 2.2.1.2 If a Markov chain is irreducible aperiodic with finite states, and P is the transition matrix of the Markov chain, then the system of equations

$$\begin{cases} \pi' P = \pi' \\ \sum_{i \in S} \pi_i = 1 \end{cases}$$

has a unique positive solution. This solution is called the limit distribution of Markov chain.

Then we have $\lim_{n \rightarrow \infty} p_{ij}^{(n)} = \pi_j$ and

$$\lim_{n \rightarrow \infty} p_{ii}^{(n)} = \frac{1}{\sum_{n=0}^{\infty} n f_{ii}^{(n)}} = \frac{1}{\mu_i} = \pi_i$$

, where μ_i is the mean return time of state i .

2.2.2. Entropy

Entropy is a concept first used in Physics to describe the system complexity. In information theory, entropy is the average information contained in each message. In 1948 Shannon brought a study with the idea that entropy plays an important role in measuring the information choice and uncertainty, and the source with the maximum entropy subject to the statistical conditions is preferred to be retained [17]. In 1956, Jaynes's study showed that the maximum-entropy estimate is the least biased estimate possible on the given information [18].

For the discrete random variables entropy is defined as follows:

Definition 2.2.2.1 Let X be a random variable having the values $\{x_1, x_2, \dots, x_n\}$ and $p_i, i = 1, 2, \dots, n$ represents the probability of $X = x_i$ respectively. The entropy of X is defined as

$$H(X) = -c \sum_{i=1}^n p_i \log p_i$$

, where c is an arbitrary positive constant, and is taken as $c = 1$ when the logarithm base is 2. In addition, $\log 0$ is regarded as 0 when we calculate.

Moreover entropy has an application in Markov chains [19].

Definition 2.2.2.2 Suppose Markov chain X is irreducible and aperiodic and recurrent, and has a stationary distribution. Let S be the state space of the Markov chain X with the transition matrix $P = [p_{ij}]_{N \times N}$, and π , its stationary distribution. Then the entropy of Markov chain is given by:

$$H(X) = - \sum_{i,j \in S} \pi_i p_{ij} \log_2 p_{ij} .$$

3. APPLICATION TO EARTHQUAKE DATA

3.1. Aim and Content of the Application

In this study, it is aimed to estimate the seismic risk in southwestern China. In order to accomplish this task we make use of the application of Markov chain model and entropy theory.

3.2. Application Procedure

As mentioned before, there were a total of 11007 earthquake occurrences in southwestern China in the years 1970-2015. We classified those earthquakes by

their magnitudes M into 3 classes: class I, $M \leq 4$; class II, $4 < M \leq 5$; and class III, $M > 5$.

For the purpose of building a Markov chain, it should be decided the state space of the Markov chain. Here we use the number 1 to show that there is at least once the certain class earthquake occurrence in the given period, otherwise we record the earthquake situation as 0 in that period. For example, if the class III earthquake didn't occur in a certain period, then we record the class III earthquake occurrence with the number 0. Since we categorized earthquakes into 3 classes, we have a series of binary numbers, which is from 000 to 111, to express all the states in the given period. For example, state 101 means the class III and class I earthquake occurred at least once, and class II did not occurred even once in that period. Therefore state space of the Markov Chain is $S = \{0,1,2,...7\}$, as in Table 1.

Table 1. List of all states.

State	Class III	Class II	Class I
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

And now our work becomes to find a suitable time interval Δt for obtaining realization steps of Markov chain. Finding a suitable Δt follows these 3 principles: 1) Δt should not be so small that there are too many transitions from state 0 to state 0; 2) Δt should not be too large that there are over transitions from state 7 to state 7; 3) Δt should be with a big entropy of the Markov chain. At the same time we also need to keep in mind that as Δt increases, the number of

sampled transition gets diminishing, which makes the result less robust [6].

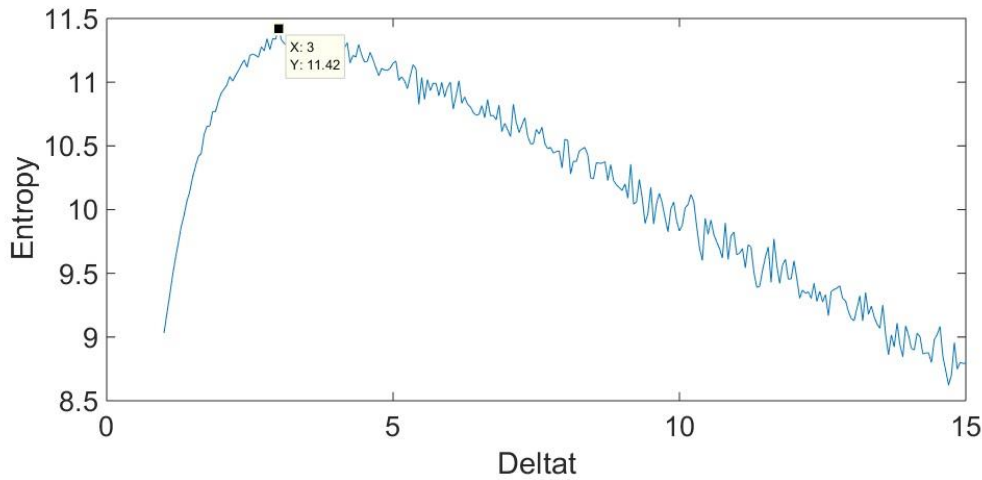


Figure 1. Plot of entropy versus Δt .

By those 3 principles, we finally found a $\Delta t = 3$ days that fits to our Markov chain model, and according to this time interval there are a total of 5600 periods through 1970-2015. With this Δt we obtained the matrix of transition frequencies N , and the estimated transition matrix P of Markov chain, respectively, as follows:

$$N = \begin{bmatrix} 1039 & 466 & 172 & 157 & 45 & 14 & 21 & 20 \\ 492 & 1168 & 106 & 378 & 5 & 20 & 11 & 25 \\ 181 & 104 & 52 & 46 & 11 & 2 & 6 & 3 \\ 148 & 409 & 43 & 179 & 1 & 7 & 3 & 22 \\ 38 & 8 & 8 & 3 & 5 & 0 & 6 & 0 \\ 11 & 18 & 5 & 11 & 0 & 0 & 0 & 1 \\ 18 & 7 & 14 & 6 & 0 & 1 & 4 & 1 \\ 7 & 25 & 5 & 32 & 1 & 2 & 0 & 6 \end{bmatrix};$$

$$P = \begin{bmatrix} 0.5372 & 0.2410 & 0.0889 & 0.0812 & 0.0233 & 0.0072 & 0.0109 & 0.0103 \\ 0.2231 & 0.5297 & 0.0481 & 0.1714 & 0.0023 & 0.0091 & 0.0050 & 0.0113 \\ 0.4469 & 0.2568 & 0.1284 & 0.1136 & 0.0272 & 0.0049 & 0.0148 & 0.0074 \\ 0.1823 & 0.5037 & 0.0530 & 0.2204 & 0.0012 & 0.0086 & 0.0037 & 0.0271 \\ 0.5588 & 0.1176 & 0.1176 & 0.0441 & 0.0735 & 0 & 0.0882 & 0 \\ 0.2391 & 0.3913 & 0.1087 & 0.2391 & 0 & 0 & 0 & 0.0217 \\ 0.3529 & 0.1373 & 0.2745 & 0.1176 & 0 & 0.0196 & 0.0784 & 0.0196 \\ 0.0897 & 0.3205 & 0.0641 & 0.4103 & 0.0128 & 0.0256 & 0 & 0.0769 \end{bmatrix}.$$

3.3. Markov Chain Analysis

For the elements of transition probability matrix we can give some comments which reveal the co-occurrence of earthquakes. For example, for the state 5 we have $p_{55} = 0$, which means that an occurrence of state 5 is almost not followed by itself in the next period, but the transition probability $p_{51} = 0.3913$ points out that state 1 is the most probable among others after state 5. In respect of cases, the foregoing comment is identical to say: cases III and I together is not followed by itself, but case I is the most probable case, which is consistent with the fact that the seismicity is reduced after high magnitude earthquakes.

After obtaining the transition matrix, it is necessary to verify whether this process is a Markov chain. Besides this, we still need to move on the first passage time distribution problem.

3.3.1. Chi-Square Test

For the purpose of testing whether our observed data can be accepted to be a Markov chain, here we use a chi-square test with the hypothesis: H_0 : Our observed data can be accepted as a Markov chain with transition matrix P; H_1 : Our observed data cannot be accepted as a Markov chain with transition matrix P.

Here we have the expected transition frequency matrix:

$$\begin{bmatrix} 1022 & 486 & 172 & 146 & 39 & 13 & 15 & 10 \\ 490 & 1166 & 114 & 407 & 6 & 14 & 13 & 22 \\ 177 & 104 & 55 & 56 & 11 & 0 & 7 & 6 \\ 147 & 413 & 39 & 179 & 2 & 8 & 6 & 29 \\ 30 & 8 & 11 & 2 & 2 & 0 & 9 & 0 \\ 8 & 19 & 5 & 2 & 0 & 0 & 0 & 1 \\ 22 & 4 & 17 & 6 & 0 & 0 & 4 & 1 \\ 6 & 33 & 3 & 25 & 2 & 0 & 0 & 5 \end{bmatrix}$$

The transition frequency matrix of our observed data has been shown in Section 3.2. And now we have got a chi-square test result $\chi_{cal}^2 = 58.508 < \chi_{43,0.05}^2 = 59.304$, and that means H_0 is accepted.

3.3.2. Distribution of First Passage Time and Recurrence Time

We obtained the limiting distribution of our Markov chain as follows:

$$[0.3454 \ 0.3938 \ 0.0723 \ 0.1450 \ 0.0122 \ 0.0082 \ 0.0091 \ 0.0139]$$

For states 4, 5, 6 and 7, their probabilities in limiting distribution are less than 0.015 which can be regarded as rare states. And the numbers of recurrences for states 4, 5, 6 and 7 during 5,600 periods are 66, 45, 49 and 76, respectively. So in order to figure out the features of this Markov chain, we run a 1,000,000-period simulation. By the result of simulation, the number of recurrences for states 4, 5, 6 and 7 during 1,000,000 periods are 11968, 8246, 9091 and 14017 times, respectively.

Now we focus on state 4, which there only has occurrences of class III earthquakes. We have 2 ways to find a fit distribution of recurrence time for state 4:1)

Using software like EasyFit to fit a distribution to our data; 2) Figuring out a distribution with its original definition and test it through hypotheses. For fitting distribution by using software, the result is shown as follows: The recurrence time distribution of state 4 for simulated data is said to be a Dagum distribution with parameters

$p = 0.22627, \alpha = 2.8959, \beta = 162.4$. This result is accepted by Kolmogorov-Smirnov statistic value which is 0.0356 while the p-value is 0.2205. The expectation of this Dagum distribution is 86.4, which means for approximately every 259.2 days a recurrence for state 4 is expected in southwestern China.

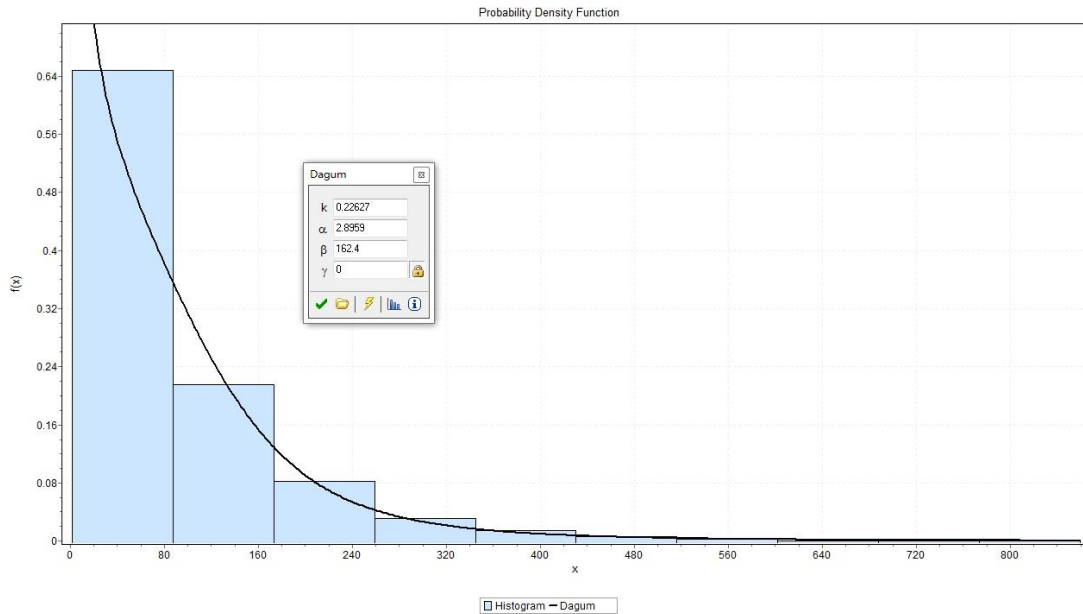


Figure 2. The fitting curve of recurrence time for state 4.

From Table 2 in which expected frequencies of recurrence time of state 4 for simulated data is presented, we should notice that the expected frequency is no less than 5 until recurrence time is over 286 periods. Furthermore, expected frequency is a decreasing series as recurrence time increases. Thus it is reasonable for us to combine those recurrence times whose expected frequencies are less than 5 into a single situation. By doing this we are supposing to give an approximation to the distribution of recurrence time for state 4. We come up with a multinomial distribution with parameters as

$$\frac{f_{ij}^{(1)}}{f_{ij}}, \frac{f_{ij}^{(2)}}{f_{ij}}, \dots, \frac{f_{ij}^{(n-1)}}{f_{ij}}, 1 - \frac{\sum_{k=1}^{n-1} f_{ij}^{(k)}}{f_{ij}}, \text{ and } n=286$$

for state 4 case. Here we use a chi-square test to verify the validity of this approximation to the distribution of

recurrence time for state 4, and the hypotheses are: H_0 : Approximation to the distribution of recurrence time of state 4 can be accepted as a multinomial distribution

$$M\left(\frac{f_{44}^{(1)}}{f_{44}}, \frac{f_{44}^{(2)}}{f_{44}}, \dots, \frac{f_{44}^{(285)}}{f_{44}}, 1 - \frac{\sum_{k=1}^{285} f_{44}^{(k)}}{f_{44}}\right); H_1:$$

Approximation to the distribution of recurrence time of state 4 cannot be accepted as a multinomial distribution

$$M\left(\frac{f_{44}^{(1)}}{f_{44}}, \frac{f_{44}^{(2)}}{f_{44}}, \dots, \frac{f_{44}^{(285)}}{f_{44}}, 1 - \frac{\sum_{k=1}^{285} f_{44}^{(k)}}{f_{44}}\right). \text{ Since}$$

the chi-square statistic values are

$$\chi_{cal}^2 = 310.2958 < \chi_{284,0.05}^2 = 324.3051, H_0 \text{ is accepted.}$$

Table 2. Expected frequency of recurrence time for state 4.

Recurrence Time	Expected Frequency	Recurrence Time	Expected Frequency
1	880	280	5.30
2	197.70	281	5.24
3	158.23	282	5.18
4	134.41	283	5.12
5	124.41	284	5.07
6	119.82	285	5.01
7	117.27	286	4.95
8	115.50	287	4.90
9	114.03	288	4.84

10	112.68	289	4.79
...

Although the sample space of recurrence time for state 4 in the long term is the set of positive integers, we still give a fitted approximation for state 4. From this approximation we can see that the probability of single step recurrence for state 4 is $f_{44}^{(1)} = 7.35\%$, while the probability of recurrence time being more than one period is 92.65%. Moreover according to the limiting distribution, the expectation of the recurrence time for state 4 is 82.3 periods, which means for approximately every 246.9 days a recurrence for state 4 is expected in southwestern China, meanwhile for the fitted

multinomial distribution the expectation is 79.1 periods, which is about 237.3 days.

Then we continue to focus on the first passage time from state 4 to state 7, which is the occurrence of class III earthquake only and the co-occurrence of all three category earthquakes. From 1,000,000-period simulation, we can see that there are a total of 5908 first passage times from state 4 to state 7. In Table 3 it shows expected frequency of first passage time from state 4 to state 7 in 5908-time first passage case.

Table 3. Expected frequency of first passage time from state 4 to state 7.

First Passage Time	Expected Frequency	First Passage Time	Expected Frequency
1	0	207	5.30
2	64.45	208	5.23
3	70.70	209	5.16
4	72.70	210	5.10
5	73.01	211	5.03
6	72.58	212	4.97
7	71.84	213	4.90
8	70.99	214	4.84
9	70.10	215	4.78
10	69.21	216	4.71
...

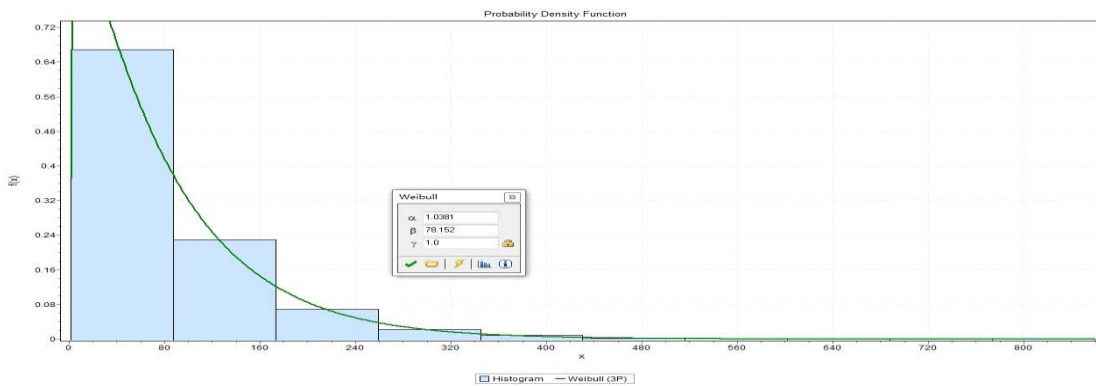


Figure 3. The fitting curve of first passage time from state 4 to state 7

As shown in Figure 3, we found a 3-parameter Weibull distribution fitted for the distribution of first passage time from state 4 to state 7. This Weibull distribution

with parameters $\alpha = 0.46037, \beta = 1.0827, \gamma = 1$ is accepted by

Kolmogorov-Smirnov statistic which is 0.01294 while the p-value is 0.99841. The expectation of this 3-parameter Weibull distribution is 78.0 periods, in other words, for nearly every 234 days a first passage from state 4 to state 7 is expected.

Meanwhile we also give a multinomial approximation to the distribution of first passage time from state 4 to state 7. This time we should notice that probability of single-step transition from state 4 to state 7 is 0, thus our multinomial approximation is a distribution with

$$\text{parameters } \frac{f_{47}^{(2)}}{f_{47}}, \frac{f_{47}^{(3)}}{f_{47}}, \dots, \frac{f_{47}^{(211)}}{f_{47}}, 1 - \frac{\sum_{k=2}^{211} f_{47}^{(k)}}{f_{47}},$$

for $x=2,3,\dots,212$. This time we also use a chi-square test to verify the validity of this approximation to the distribution of first passage time from state 4 to state 7, and the hypotheses are: H_0 : The approximation to the distribution of first passage time from state 4 to state 7 can be accepted as a multinomial distribution

$$M\left(\frac{f_{47}^{(2)}}{f_{47}}, \frac{f_{47}^{(3)}}{f_{47}}, \dots, \frac{f_{47}^{(211)}}{f_{47}}, 1 - \frac{\sum_{k=2}^{211} f_{47}^{(k)}}{f_{47}}\right); H_1: \text{The}$$

approximation to the distribution of first passage time from state 4 to state 7 cannot be accepted as a multinomial distribution

$$M\left(\frac{f_{47}^{(2)}}{f_{47}}, \frac{f_{47}^{(3)}}{f_{47}}, \dots, \frac{f_{47}^{(211)}}{f_{47}}, 1 - \frac{\sum_{k=2}^{211} f_{47}^{(k)}}{f_{47}}\right). \text{ The}$$

chi-square statistic values are

$$\chi_{cal}^2 = 209.4914 < \chi_{209,0.05}^2 = 243.7272,$$

which means H_0 is accepted.

From this multinomial distribution, it can be seen that the 5th period has the most possible probability of first passage time from state 4 to state 7, and in this case the probability increases before the 5th period and decreases after the 5th period. Furthermore, 74.8 periods is the expectation of the multinomial approximation for this case, which means for about every 224.4 days a first passage from state 4 to state 7 is expected.

4. CONCLUSION AND DISCUSSIONS

In this study we have built a Markov chain model based on the earthquake data of southwestern China. It is verified that the observed data follows a Markov chain. Moreover in order to figure out features of this Markov chain we have run a 1,000,000-period simulation. And simulated data are used to find a reasonable distribution of recurrence time and first passage time. By using EasyFit software we find a fitted distribution of recurrence time for state 4, which is a Dagum distribution with parameters

$p = 0.22627, \alpha = 2.8959, \beta = 162.4$ and we give a multinomial approximation

$$M\left(\frac{f_{44}^{(1)}}{f_{44}}, \frac{f_{44}^{(2)}}{f_{44}}, \dots, \frac{f_{44}^{(285)}}{f_{44}}, 1 - \frac{\sum_{k=1}^{285} f_{44}^{(k)}}{f_{44}}\right) \text{ to the}$$

distribution of recurrence time for state 4. Furthermore, we also explored the distribution of first passage time from state 4 to state 7. A 3-parameter Weibull distribution with parameters

$$\alpha = 0.46037, \beta = 1.0827, \gamma = 1 \text{ is fitted by}$$

EasyFit software, and a multinomial distribution

$$M\left(\frac{f_{47}^{(2)}}{f_{47}}, \frac{f_{47}^{(3)}}{f_{47}}, \dots, \frac{f_{47}^{(211)}}{f_{47}}, 1 - \frac{\sum_{k=2}^{211} f_{47}^{(k)}}{f_{47}}\right) \text{ is given}$$

as a multinomial approximation to the distribution of first passage time from state 4 to state 7. The difference found between the results fitted by software and given by approximation is needed to be discussed in a future study. Moreover some interesting facts are needed to be discussed such as the single-step transition probability from state 4 to state 7 is 0 and the 5th period holds the largest probability of first passage time from state 4 to state 7, from the seismicity perspective.

In this study we have investigated the distributions of the recurrence time of a rare state 4, the first passage time from a rare state 4 to a rare state 7. The recurrence time of non-rare states, first passage time from a non-rare state to a rare state and the first passage time from a rare state to a non-rare state in this southwestern China case will be discussed in our further study.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

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