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Banhatti-Sombor Index over a Graph of a Special Class of Semigroup

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Abstract

Kulli introduced the first Banhatti-Sombor index in [20] which is outlined as

$$BSO_1(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{(d_u)^2+(d_v)^2}}.$$

Our research will be calculated on an algebraic formation, utilising the chief principals of Banhatti-Sombor index of monogenic semigroup graphs which was first studied by [14].

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1. Introduction

The authors inspiration behind monogenic semigroup graphs emanated from zero divisor graphs. Our initial focus will be zero divisor graphs before venturing into monogenic semigroup graphs (see [9, 10, 11, 12]). The researchers [16, 17] has provided research on zero divisor graphs which are commutative and non commutative. The adjacement rule of vertices has been utilized in this instance. The research derived from [16, 17] a new graph with relation to monogenic semigroup graph which is defined in [14]. Necessary and sufficient condition for two different vertices $x^i, x^j \in S_M$ to be connected is $i+j>n$ $(1\leq i,j\leq n)$. Our indepth research on monogenic semigroup graph continues in [2, 3, 4, 5, 6].

Topological indices insurmountable value in a wide range of different disciplines of science is immeasurable. Structural property and chemical structure are connected to topological descriptor values. Many graph indices defined as molecular graphs originated from molecules modified by atoms with vertices and bonds between them with edge. Molecular structures need graphs to obtain data in relation to the essence of molecules.

These indices contain values associated with structural property and molecular structure. Several graph indices have been specified as molecular graphs which emanated from molecules changed by atoms with vertices and bonds between them with edge. As a whole chemical molecules utilise graphs to acquire data pertaining to the nature of molecules. Topological indices are frequently used in chemical graph theory to analyze chemical structure (see [27]). The Sombor index which is a vertex degree based molecular structure descriptor lower and upper bounds was initiated by Gutman [18]. Recently, many studies have been done on the chemical applications of this index, see [7, 8, 13, 21, 26]. For lower and upper bounds over the Sombor index of a graph see [15, 19, 22, 25] and elsewhere.

Our research on this study depicts G set of graph, $V(G)$ exhibits vertex set of G and $E(G)$ exhibits edge set of G. Gutman in [18] has defined the Sombor index as given below:

$$SO(G) = \sum_{uv\in E(G)} \sqrt{(d_u)^2+(d_v)^2}.$$

Subsequently, Kulli defined Banhatti-Sombor indices in [20] and calculated certain formulas on some nanostructures. The author defined the first Banhatti-Sombor index as

$$BSO_1(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{(d_u)^2+(d_v)^2}}.$$

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In [23], [24] the author determined an equation of Sombor index and an equation of Nirmala index of a graph of a monogenic semigroup respectively. In this paper we estimate the Banhatti-Sombor index of a graph of a monogenic semigroup.
By the way, for a positive integer n, we have

$$\left\lfloor \frac{n}{2} \right\rfloor = \begin{cases} \frac{n}{2} & if \quad \text{n} \quad is \quad even \\ \frac{n-1}{2} & if \quad \text{n} \quad is \quad odd \end{cases} \tag{1.1}$$

In this study the exact equation of Banhatti-Sombor index of a graph of a monogenic semigroup is determined.

2. An Algorithm

The researchers in [2] presented an algorithm for the neighborhood of vertices to facilitate their computation. We will also use this algorithm to calculate the first Banhatti Sombor index of the graph of a monogenic semigroup. The following notations such as $x^n, x^{n-1}, x^{n-2}, \dots, x^1$ given below represent the elements of the vertex set.

I_n: x^n is adjacent to x^{i_1} $(1 \le i_1 \le n-1)$.

I_{n-1}: x^{n-1} is adjacent to x^{i_2} $(2 \le i_2 \le n-2)$.

I_{n-2}: x^{n-2} is adjacent to x^{i_3} $(3 \le i_3 \le n-3)$.

Continuing the algorithm in this way, we obtain the following conclusion.

If n is even:

$I_{\frac{n}{2}+2}$: $x^{\frac{n}{2}+2}$ is adjacent not just to $x^{\frac{n}{2}-1}$, $x^{\frac{n}{2}}$ and $x^{\frac{n}{2}+1}$ also adjacent to x^n, x^{n-1}, x^{n-2}, $\dots$, $x^{\frac{n}{2}+3}$.

$I_{\frac{n}{2}+1}$: $x^{\frac{n}{2}+1}$ is adjacent not just to $x^{\frac{n}{2}}$ also adjacent to x^n, x^{n-1}, x^{n-2}, $\dots$, $x^{\frac{n}{2}+2}$.

If n is odd:

$I_{\frac{n+1}{2}}$: $x^{\frac{n+1}{2}+2}$ is adjacent not just to $x^{\frac{n+1}{2}-2}$, $x^{\frac{n+1}{2}-1}$, $x^{\frac{n+1}{2}}$ and $x^{\frac{n+1}{2}+1}$ also adjacent to x^n, x^{n-1}, x^{n-2}, $\dots$, $x^{\frac{n+1}{2}+3}$.

$I_{\frac{n+1}{2}+1}$: $x^{\frac{n+1}{2}+1}$ is adjacent not just to $x^{\frac{n+1}{2}-1}$ and $x^{\frac{n+1}{2}}$ also adjacent to x^{n-1}, x^{n-2}, $\dots$, $x^{\frac{n+1}{2}+2}$.

It is possible to reach some of the studies on degree series from [1, 14] sources and see also the references cited in these works. The proof of the following lemma given in [14], as given in the above algorithm ([2]).

Lemma 2.1. *Let $d_1, d_2, \dots, d_n$ be the degrees of vertices $x^1, x^2, \dots, x^n$ in a monogenic semigroup graph $(\Gamma(S_M))$, respectively. Then we have*

$$d_1 = 1, \quad d_2 = 2, \quad \dots \quad , d_{\lfloor \frac{n}{2} \rfloor} = \left\lfloor \frac{n}{2} \right\rfloor, \quad d_{\lfloor \frac{n}{2} \rfloor + 1} = \left\lfloor \frac{n}{2} \right\rfloor, \quad d_{\lfloor \frac{n}{2} \rfloor + 2} = \left\lfloor \frac{n}{2} \right\rfloor + 1, \quad \dots \quad , d_n = n-1. \tag{2.1}$$

Remark 2.2. *If we consider the repeated terms in the above lemma which are listed below*

$$d_{\lfloor \frac{n}{2} \rfloor} = \left\lfloor \frac{n}{2} \right\rfloor = d_{\lfloor \frac{n}{2} \rfloor + 1}.$$

we see the degree of d_n is denoted by $n-1$, granted that the vertices number is n.

3. Banhatti-Sombor Index over a Monogenic Semigroup Graph

In this part an explicit equation of the first Banhatti-Sombor index over a graph of a monogenic semigroup is calculated.

Theorem 3.1. *Let S_M be a monogenic semigroup. The first Banhatti-Sombor index of $\Gamma(S_M)$ is*

$$BSO_1(\Gamma(S_M)) = \begin{cases} \sum_{k=1}^{\frac{n}{2}-1} \sum_{i=k}^{n-k-1} \frac{1}{\sqrt{(n-k)^2+i^2}} + \sum_{k=1}^{\frac{n}{2}} \frac{1}{\sqrt{(n-k)^2+\left(\frac{n}{2}\right)^2}} & if \quad n \quad is \quad even \\ \sum_{k=1}^{\frac{n-1}{2}} \sum_{i=k}^{n-k-1} \frac{1}{\sqrt{(n-k)^2+i^2}} + \sum_{k=1}^{\frac{n-1}{2}} \frac{1}{\sqrt{(n-k)^2+\left(\frac{n}{2}\right)^2}} & if \quad n \quad is \quad odd \end{cases}$$

Proof. The idea is to methodically prepare $BSO_1(\Gamma(S_M))$ in terms of the sum of degrees. In our calculations we will utilise the algorithm given because it uses a structured method to sum up the degrees of vertices adding equations. We also need the equations (1.1), (2.1) and Remark 2.2.

If n is odd:

$$\begin{aligned}
[BSO_1](\Gamma(S_M) = & \frac{1}{\sqrt{d_n^2+d_1^2}} + \frac{1}{\sqrt{d_n^2+d_2^2}} + \frac{1}{\sqrt{d_n^2+d_3^2}} + \ldots + \frac{1}{\sqrt{d_n^2+d_{n-2}^2}} + \frac{1}{\sqrt{d_n^2+d_{n-1}^2}} + \\
& + \frac{1}{\sqrt{d_{n-1}^2+d_2^2}} + \frac{1}{\sqrt{d_{n-1}^2+d_3^2}} + \ldots + \frac{1}{\sqrt{d_{n-1}^2+d_{n-2}^2}} + \\
& + \ldots + \\
& + \frac{1}{\sqrt{d_{\frac{n+1}{2}+2}^2+d_{\frac{n+1}{2}-2}^2}} + \frac{1}{\sqrt{d_{\frac{n+1}{2}+2}^2+d_{\frac{n+1}{2}-1}^2}} + \frac{1}{\sqrt{d_{\frac{n+1}{2}+2}^2+d_{\frac{n+1}{2}}^2}} + \frac{1}{\sqrt{d_{\frac{n+1}{2}+2}^2+d_{\frac{n+1}{2}+1}^2}} \\
& + \frac{1}{\sqrt{d_{\frac{n+1}{2}+1}^2+d_{\frac{n+1}{2}-1}^2}} + \frac{1}{\sqrt{d_{\frac{n+1}{2}+1}^2+d_{\frac{n+1}{2}}^2}}
\end{aligned}$$

Consequently, the first Banhatti-Sombor index of $\Gamma(S_M)$ is obtained as the following sum

$$[BSO_1](\Gamma(S_M) = \sum_{ij\in E(G)} \frac{1}{\sqrt{d_i^2+d_j^2}} = [BSO_1]_n + [BSO_1]_{n-1} + \ldots + [BSO_1]_{\frac{n+1}{2}+2} + [BSO_1]_{\frac{n+1}{2}+1}$$

We will calculate each Banhatti-Sombor index separately in the sum given above. Besides, we consider the equation $\lfloor \frac{n}{2} \rfloor = \frac{n-1}{2}$ given in (1.1) where n is odd.

$$\begin{aligned}
[BSO_1]_n = & \frac{1}{\sqrt{(n-1)^2+1^2}} + \frac{1}{\sqrt{(n-1)^2+2^2}} + \frac{1}{\sqrt{(n-1)^2+3^2}} + \ldots + \frac{1}{\sqrt{(n-1)^2+\lfloor \frac{n}{2} \rfloor^2}} + \ldots + \\
& + \frac{1}{\sqrt{(n-1)^2+(n-2)^2}} + \frac{1}{\sqrt{(n-1)^2+\lfloor \frac{n}{2} \rfloor^2}} \\
= & \sum_{i=1}^{n-2} \frac{1}{\sqrt{(n-i)^2+i^2}} + \frac{1}{\sqrt{(n-1)^2+(\frac{n-1}{2})^2}}
\end{aligned}$$

If the same procedure is applied in $[BSO_1]_n$ applied to $[BSO_1]_{n-1}, \ldots, [BSO_1]_{\frac{n+1}{2}+2}, [BSO_1]_{\frac{n+1}{2}+1}$ we have

$$[BSO_1]_{n-1} = \sum_{i=2}^{n-3} \frac{1}{\sqrt{(n-i)^2+i^2}} + \frac{1}{\sqrt{(n-2)^2+(\frac{n-1}{2})^2}},$$

$$[BSO_1]_{\frac{n+1}{2}+2} = \frac{1}{\sqrt{(\frac{n+3}{2})^2+(\frac{n-3}{2})^2}} + \frac{1}{\sqrt{(\frac{n+3}{2})^2+(\frac{n-1}{2})^2}} + \frac{1}{\sqrt{(\frac{n+3}{2})^2+(\frac{n-1}{2})^2}} + \frac{1}{\sqrt{(\frac{n+3}{2})^2+(\frac{n+1}{2})^2}}$$

hence

$$[BSO_1]_{\frac{n+1}{2}+1} = \frac{1}{\sqrt{(\frac{n+1}{2})^2+(\frac{n-1}{2})^2}} + \frac{1}{\sqrt{(\frac{n+1}{2})^2+(\frac{n-1}{2})^2}}.$$

Hence

$$[BSO_1]_n + [BSO_1]_{n-1} + \ldots + [BSO_1]_{\frac{n+1}{2}+2} + [BSO_1]_{\frac{n+1}{2}+1} = \sum_{k=1}^{\frac{n-1}{2}} \sum_{i=k}^{n-k-1} \frac{1}{\sqrt{(n-k)^2+i^2}} + \sum_{k=1}^{\frac{n-1}{2}} \frac{1}{\sqrt{(n-k)^2+\frac{n}{2}^2}}.$$

□

Similarly, if n is even, the following sum can be found.

$$[BSO_1]_n + [BSO_1]_{n-1} + \ldots + [BSO_1]_{\frac{n}{2}+2} + [BSO_1]_{\frac{n}{2}+1} = \sum_{k=1}^{\frac{n}{2}-1} \sum_{i=k}^{n-k-1} \frac{1}{\sqrt{(n-k)^2+i^2}} + \sum_{k=1}^{\frac{n}{2}} \frac{1}{\sqrt{(n-k)^2+\frac{n}{2}^2}}$$

The example given below is showing the calculation of the first Banhatti-Sombor index of $\Gamma(S_{M_6})$ to support the main theorem.

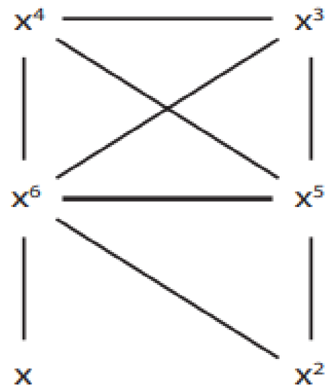

Figure 3.1: S_{M_6} monogenic semigroup graph

Example 3.2. *Let S_{M_6} be a monogenic semigroup that given below.*

$$S_{M_6} = \left\{x, x^2, x^3, x^4, x^5, x^6\right\} \cup \{0\}$$

Now we will calculate the first Banhatti-Sombor index of $\Gamma(S_{M_6})$ graph by using the technique given in Theorem 3.1.

$$\begin{aligned} BSO_1(\Gamma(S_M)) &= \sum_{k=1}^{2}\sum_{i=k}^{5-k}\frac{1}{\sqrt{(6-k)^2+i^2}} + \sum_{k=1}^{3}\frac{1}{\sqrt{(6-k)^2+(3)^2}} \\ &= \frac{1}{\sqrt{5^2+1^2}} + \frac{1}{\sqrt{5^2+2^2}} + \frac{1}{\sqrt{5^2+3^2}} + \frac{1}{\sqrt{5^2+4^2}} + \frac{1}{\sqrt{4^2+2^2}} + \frac{1}{\sqrt{4^2+3^2}} \\ &+ \frac{1}{\sqrt{5^2+3^2}} + \frac{1}{\sqrt{4^2+3^2}} + \frac{1}{\sqrt{3^2+3^2}} \end{aligned}$$

As will be understood, the Banhatti-Sombor index of a graph of a monogenic semigroup is found easily by considering the exact formula obtained in the main theorem.

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