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RESEARCH ARTICLE / ARAŞTIRMA MAKALESİ

The Swept Volume of a Circular Paraboloid End Mill Moving Along a Helical Path

Helisel Bir Yörüngede Hareket Eden Dairesel Paraboloit Parmak Frezenin Süpürme Hacmi

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Abstract
To obtain complex part geometries at one pass using machining processes, it is important to employ the tools with non-conventional geometries. A circular paraboloid is a solid of revolution, which can be obtained by rotating a parabola. The swept volume of an end mill can be defined as the unification of all sets of points on the tool for every instant as it moves, and its derivation is an obligation to determine the machined part geometry prior to an actual machining process. After derivation of the swept volume of the tool, machined part geometry is obtained by subtracting the swept volume of the tool from the volume of the initial workpiece. However, derivation of the swept volume of the tool is not a straightforward task. In this work, an analytical model was introduced to derive a complete set of points on the machined part by means of well-defined and constrained tool geometry and tool path. In the model, a plane that passes through the screw axis was used to observe the instant cross-section of the tool as it moves along the helical path. By overlapping the instant cross-sections of the tool in the plane, the final cross-section was derived. Since all cross-sections that pass through the screw axis are identical, the method gives an entire set of points on the machined surface. To validate the model, a computer-aided design program was utilized.
Keywords: Milling, Circular paraboloid end mill, Helical path, Swept volume.

Öz
Talaşlı imalat yöntemlerini kullanarak tek pasoda karmaşık parça geometrileri elde etmek için, geleneksel olmayan geometrilere sahip takımların kullanılması önemlidir. Dairesel paraboloit, bir parabolün döndürülmesiyle elde edilebilen katı bir cisimdir. Bir parmak frezenin süpürme hacmi, takım hareket ettikçe üzerindeki tüm noktaların her an için birleştirilmesi olarak tanımlanabilir ve bunun elde edilmesi gerçek bir talaşlı imalat işleminden önce işlenmiş parça geometrisini belirlemek için bir zorunluluktur. Takımın süpürme hacminin türetilmesinden sonra, işlenmiş parça geometrisi takımın süpürme hacminin ilk iş parçasının hacminden çıkarılmasıyla elde edilir. Ancak takım süpürme hacminin elde edilmesi basit değildir. Bu çalışmada, iyi tanımlanmış ve kısıtlanmış takım geometrisi ve takım yolu kullanılarak işlenmiş parça üzerindeki tüm noktaları elde etmek için analitik bir model sunulmuştur. Modelde takımın helisel yörünge boyunca hareket ederken anlık kesitini gözlemlemek için vida ekseninden geçen bir düzlem kullanılmıştır. Bu düzlemde takımın anlık kesitleri üst üste getirilerek, son kesit elde edilmiştir. Vida ekseninden geçen tüm kesitler özdeş olduğundan, yöntem işlenmiş parça yüzeyi üzerindeki noktaların kümesini vermektedir. Modeli doğrulamak için bilgisayar destekli tasarım programı kullanılmıştır.
Anahtar Kelimeler: Frezeleme, Dairesel paraboloit parmak freze, Helisel yörünge, Süpürme hacmi.

I. INTRODUCTION

There are several milling processes in which the tool path is a helix. One of them is the helical milling process. The helical milling is usually utilized as an alternative hole-making process that provides some advantages as compared to conventional drilling such as less tool wear, low cutting forces, better chip evacuation, and improved hole quality [1]. Another milling process that uses the helical path is thread milling which brings versatility to the thread-making as compared to thread tapping. Some of the advantages of thread milling are as follows. The threads with different diameters can be produced by the same tool as long as the tool diameter is less than the thread diameter, and thread milling provides better chip evacuation as compared to thread tapping [2]. In addition, spindle speed – feed synchronization is not required in the thread milling, thus higher cutting speeds can be employed [3]. Apart from hole-making and thread-making, a milling process with a helical path can be used to create internal and external helical grooves when the cutting tools that can machine undercuts are employed. A circular paraboloid end mill is one that can achieve this requirement. In a milling process with a helical tool path, the cross-section of the machined part is not merely determined by the tool geometry. An overcut caused by the helical path also affects the cross-section. This issue was pointed out for the thread milling process in which the cutting tool can machine undercuts [4]. A similar overcut occurs when the helical grooves are milled by using a helical path.

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Swept volume generation is a common problem for many different areas. It is required for the path planning of the robots [5], collision detection [6], verification of NC machining [7], and solid modeling [8]. In the literature, several approaches have been introduced to derive the swept volumes such as Jacobian rank deficiency method [9], Sweep-envelope differential equation method [10], and Envelope theory [11]. However, the methods based on the solution of complex differential equations are not appropriate in application due to the fact that they require numerical calculations resulting in high computation cost and time [12]. Approximate solution techniques based on time and tool discretization were also proposed for the simulation of machining processes [13, 14]. However, an approximation error exists for such methods. An analytical model for the swept volume of a solid is very difficult to obtain when considering an arbitrary path for the solid. In this work, it was possible due to well-defined and constrained tool geometry and tool path.

The proposed method in this paper gives the equation of the exact cross-sectional profile of the machined surface without numerical calculation when a circular paraboloid end mill is used along a helical path. Since the path is a helix and the tool is axis-symmetrical, every cross-section of the machined part is identical except for a shift in the cross-section along the screw axis. Therefore, the equation of the cross-sectional profile in a plane is once found, this can be used to derive the entire set of points on the machined part surface. The model utilizes a fixed observer plane ($x-y$) that passes through the screw axis. As the tool rotates about the screw axis along the helical path, it intersects the observer plane. At every value of the tool rotation angle, a different cross-section of the tool occurs in the observer plane. The cross-sectional profile of the machined part can be obtained by overlapping the instant cross-sections of the tool.

In the following sections, first, the analytical model was introduced. Then, simulation work was presented. In the results and discussion section, analytical results were compared to those of simulation, and the cross-section of the swept volume was evaluated. Finally, the outcomes of the work were given in the section of conclusions.

II. ANALYTICAL MODEL

Figure 1 illustrates the helical milling with a circular paraboloid end mill for both internal and external helical grooves. The tool contains upper and lower circular paraboloids that are symmetrical about the horizontal axis. For the model, only the upper portion was considered due to symmetry. The general equation of a circular paraboloid is given in Equation (1).

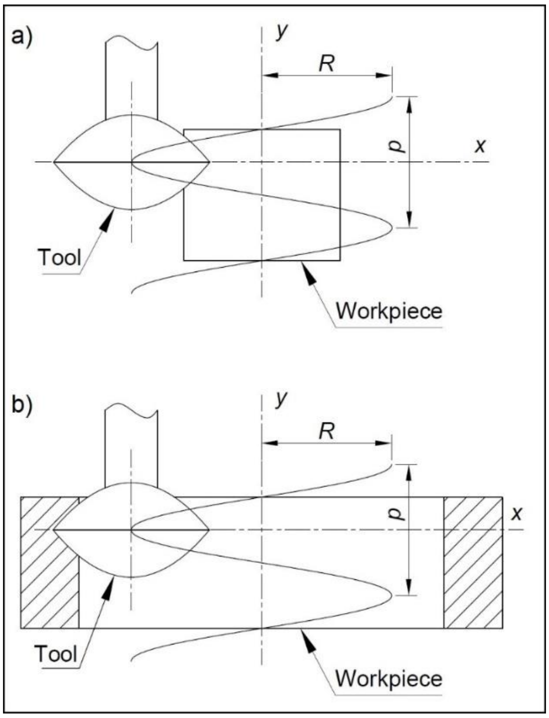

Figure 1. The helical milling with a circular paraboloid end mill. a) External groove. b) Internal groove.

$$(y-y_0)=a(x-x_0)^2+a(z-z_0)^2+b \tag{1}$$

where a and b are the paraboloid constants and x_0, y_0, and z_0 are center coordinates of the tool. As the tool moves along the helical path, the center coordinates of the circular paraboloid change with respect to the tool rotation angle, φ (Figure 2). These coordinates are expressed as the functions of φ in Equations (2-4).

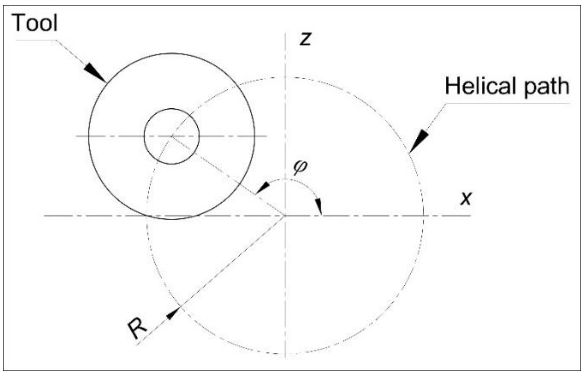

Figure 2. Illustration of the tool rotation angle.

$$x_0=R\cos\varphi \tag{2}$$

$$y_0=\frac{p}{2}-\frac{p\varphi}{2\pi} \tag{3}$$

$$z_0=R\sin\varphi \tag{4}$$

where p is the pitch of the helix and R is the radius of the helix. The first term on the right side of Equation (3) is the initial tool position. Substituting the right sides of the Equations (2-4) into x_0, y_0, and z_0 in Equation (1), Equation (5) is derived.

$$\left(y-\frac{p}{2}+\frac{p\varphi}{2\pi}\right)=a(x-R\cos\varphi)^2+a(z-R\sin\varphi)^2+b \quad (5)$$

Equation (5) is the equation of the circular paraboloid with the variable center for the helical path. Since the equation of the cross-section of the circular paraboloid is required in $x-y$ plane, z in Equation (5) is set to zero. Finally, Equation (6) is derived by arranging Equation (5).

$$y=ax^2-2axR\cos\varphi+aR^2+b+\frac{p}{2}-\frac{p\varphi}{2\pi} \quad (6)$$

Since the upper portion of the tool is investigated, the constant a must be negative for the concave downward curve. Equation (6) must be constrained along y axis by Equation (7) that passes through the center of the tool. Otherwise, it produces an infinitive curve along the negative direction of y axis.

$$y=\frac{p}{2}-\frac{p\varphi}{2\pi} \quad (7)$$

As φ varies in Equation (6), a different curve occurs in $x-y$ plane belonging to the instant cross-section of the tool. To determine the curve that surrounds the curves, Fermat's theorem was applied to Equation (6). According to this, the critical value of φ that maximizes y value can be found by taking the partial derivative of Equation (6) with respect to φ and then equating to zero in Equation (8). Equation (9) is derived by solving Equation (8) for the second quadrant of $x-y$ plane. Substituting the right side of Equation (9) into φ in Equation (6), the equation of the cross-section of the machined part can be found in Equation (10) for the second quadrant of $x-y$ plane.

$$\frac{\partial y}{\partial\varphi}=0 \quad (8)$$

$$\varphi=\pi-\sin^{-1}\left(\frac{p}{4\pi aRx}\right) \quad (9)$$

$$y=ax^2+2axR\sqrt{1-\left(\frac{p}{4\pi aRx}\right)^2}+aR^2+b+\frac{p}{2\pi}\sin^{-1}\left(\frac{p}{4\pi aRx}\right) \quad (10)$$

However, Equation (10) was derived without constraining Equation (6). Therefore, the path of the intersection point of Equation (6) and Equation (7) was used in the second quadrant of $x-y$ plane in order to obtain the remaining portion of the cross-section. φ in Equation (7) was derived as a function of y in Equation (11). Then, the right side of Equation (11) was substituted into φ in Equation (6). Finally, Equation (12) was derived in the second quadrant of $x-y$ plane, which gives the path of the intersection point.

$$\varphi=\pi-\frac{2\pi}{p}y \quad (11)$$

$$y=\frac{p}{2\pi}\cos^{-1}\left(-\frac{ax^2+aR^2+b}{2axR}\right) \quad (12)$$

The entire cross-section of the machined part is mostly formed by Equation (10), and Equation (12) gives the remaining portion. The intersection point of these curves can be found by equating Equation (10) and Equation (12).

III. SIMULATION WORK

Helical milling with a circular paraboloid end mill resembles creating swept volume by using a solid of revolution along a helix in terms of created geometry. Therefore, a computer-aided design program was used to compare the analytical results. Many computer-aided design programs do not have a feature to create the swept volume of a solid. On the other hand, some of them provide the swept volume for limited solids. Solidworks 2016 is one of them that gives the approximate remaining volume after subtracting the swept volume of a solid by means of swept cut command, but the limitation is that the solid must be created by the revolution of lines and arcs rather than a parabola. Since a circular paraboloid is investigated, the command can be used after approximating the parabola. The parabola was created by line segments with constant intervals along x-axis. The interval was chosen as 0.2 mm for the simulation work. According to this value, the maximum deviation from the parabola was 0.0005 mm along y-axis. In Table 1, selected values of the parameters are shown for both analytical model and simulation work.

Table 1. Selected values of the parameters.

Parameter	Value
R (mm)	20
a (1/mm)	-0.05
b (mm)	7.2
p (mm)	20

In Figure 3, steps are shown for the simulation work. First, the circular paraboloid was created by rotating the approximated parabola. This was followed by the creation of the helical path. Since the command only works for subtracting the swept volume, an initial workpiece volume was created. Finally, the remaining volume was obtained by using the swept cut command. Calculation time was 558 seconds with an Intel i5 8265 1.6 GHz processor - 8 GB ram computer.

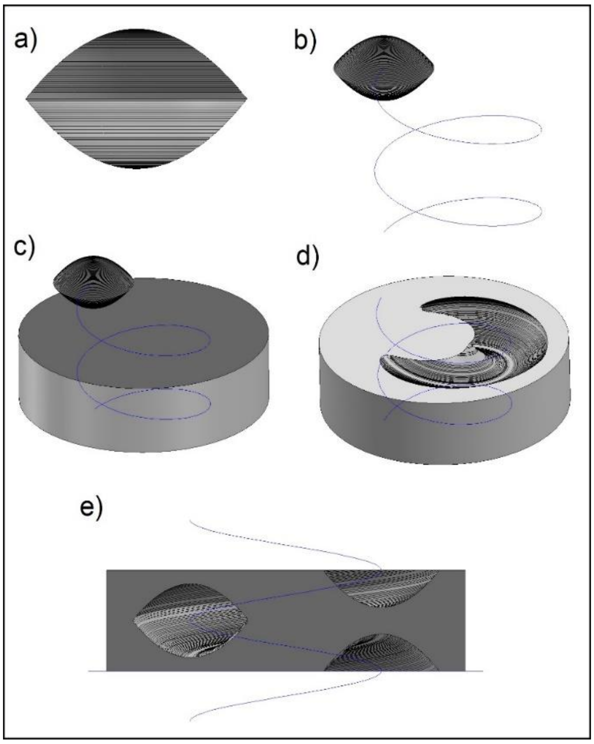

Figure 3. Simulation steps. a) Circular paraboloid was created by rotating the approximated parabola. b) Helical path was created. c) Initial workpiece volume was created. d) After swept cut command. e) Cross-sectional view.

IV. RESULTS AND DISCUSSION

The cross-sections which were obtained by both analytical model and computer-aided design program are shown in Figure 4. Some regions of the overlapped curves are scaled 20 times in the balloons. When the cross-sections are compared, it is observed that a deviation occurs in the cross-section of the swept volume derived by the computer-aided design program. This deviation cannot be attributed to the approximation error for the parabola because the deviation is greater than the approximation error.

The cross-section of the swept volume created by the computer-aided design program consists of the joined splines. Therefore, the form error between the curves created by both methods cannot be compared directly. For the comparison, coordinates of the points which were located with 0.1 mm interval along the horizontal axis on the curve were collected. The comparison was done for the upper portion of the cross-section since it is symmetrical about the horizontal axis. After collecting the points, the sum of squares of the errors along the vertical axis with respect to the curve derived by the analytical model was calculated, and the coefficient of determination (R^2) was obtained as 0.999 which indicates the goodness of the fit. Apart from the fit of the curve, the maximum deviations from the analytical curve were 0.008 mm for the upper bound and 0.052 mm for the lower bound. Another comparison between the curves is the intersection points of the curve along x-axis, which should be -8 and -32 for the investigated case. However, these values were found as -8.040 and -31.999 in the simulation. The highest relative percentage form error is 0.5% for the end points. Based on these findings, it reveals that the deviation is relatively high in the end portions of the curve with respect to the remaining portions. Generation of swept volume in a CAD program is based on meshing. If the mesh size is decreased, a better approximation can be obtained. However, this increases the calculation time. On the other hand, the introduced model directly gives the cross-section of the swept volume.

Analytical model can be used to investigate the overcuts caused by the helical path. In Figure 5, the cross-section of the circular paraboloid end mill is compared to that of the swept volume. In this figure, overcuts produced by the tool can be observed in grey. Based on this illustration, it can be revealed that although the cross-section of the tool is symmetrical about the vertical axis, the cross-section of the swept volume is not symmetrical about the vertical axis. Besides, the overcut is higher for the external groove. It should be noted that the diameter of the internal groove is higher than that of external groove as seen in Figure 1.

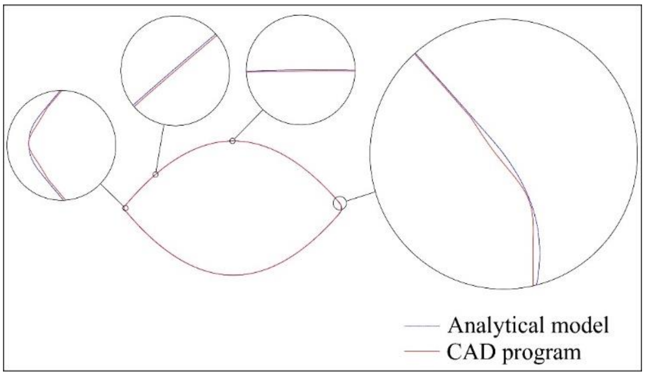

Figure 4. Overlapped image of the cross-sectional profiles derived by analytical model and computer-aided design program.

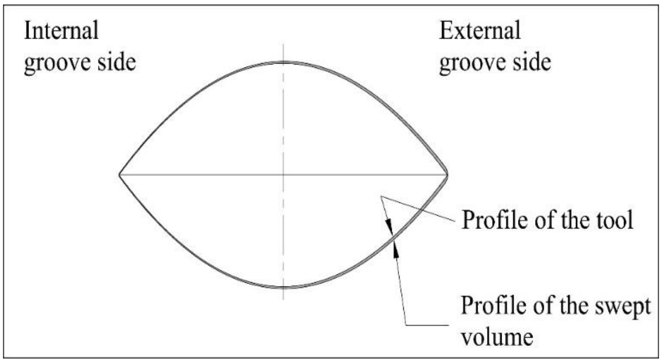

Figure 5. Overlapped image of the cross-section of the circular paraboloid end mill and the cross-section of the swept volume.

Although the helical path of the cutting tool was considered in this work, Equation 10 can be used to obtain the cross-section of the swept volume for the cutting tool moving along a circular path by substituting 0 into p. The approach can also be applicable to the case in which the tool path is linear. In such a case, the linear tool path should be fitted by a helix to obtain the cross-section of the swept volume approximately.

V. CONCLUSIONS

In this work, swept volume of the circular paraboloid end mill for a helical path was derived analytically. A case study was also presented for the selected values of the parameters of the circular paraboloid end mill and tool path. A computer-aided design program was used to evaluate the analytical model. It was observed that the calculation time was very high, 558 seconds for the generated swept volume when a computer-aided design program was used. The reason for the high calculation time was that the parabolic tool profile was approximated by using lines. On the other hand, the exact profile of the swept volume could be derived immediately by using the introduced model. It was found that a deviation occurs in the cross-section of the swept volume when the computer-aided design program is used. The analytical model also enables the investigation of the overcuts.

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