Konuralp Journal of Mathematics, 11 (2) (2023) 131-140

Konuralp Journal of Mathematics

Research Paper

Journal Homepage: www.dergipark.gov.tr/konuralpjournalmath

e-ISSN: 2147-625X

Fractional SIQRV Model for COVID-19 and Numerical Solutions

Zafer Öztürk[1*], Halis Bilgil[2] and Sezer Sorgun[3]

[1]*Institute of Science, Nevşehir Hacı Bektaş Veli University, Nevşehir, Türkiye*
[2]*Faculty of Engineering, Architecture and Design, Kayseri University, Kayseri, Türkiye*
[3]*Department of Mathematics, Nevşehir Hacı Bektaş Veli University, Nevşehir, Türkiye*
[*]*Corresponding author*

Abstract

There is a pandemic situation caused by the COVID-19 epidemic almost all over the world. Despite the measures taken to prevent this epidemic and the vaccine studies developed, the course of the epidemic changes period as the virus mutates and changes its structure. The common opinion of the experts is that the most important weapon in the fight against the epidemic is the vaccine.In this paper, a new fractional SIQRV model was obtained by adding a class of vaccinated individuals to the SIQR (Susceptible-Infected-Quarantine-Recovered) epidemic disease model. Fractional derivatives are used in the sense of Caputo. In the newly created fractional SIQRV model, the total population is divided into five parts. Mathematical analyses were performed on susceptible individuals (S), infective individuals (I), quarantined individuals (Q), recovered individuals (R) and vaccinated individuals (V). Numerical results were got with the help of Generalized Euler Method and graphs were drawn.

Keywords: *Fractional SIQRV Model, Pandemic, Generalized Euler Method, Quarantine, Vaccination, Caputo Fractional Derivative*
2010 Mathematics Subject Classification: *60G25, 34B60, 68U01*

1. Introduction

Epidemiology is a branch of science that provides the detection of health-related situations and events in certain societies. The purpose of epidemiology is to investigate the causes of health problems and health-related conditions and to develop appropriate preventive methods [1]. A pandemic is a general name given to an epidemic that spreads over an extensive area and affects over one country or continent in the world. A pandemic is declared by the World Health Organization (WHO). The COVID-19 pandemic unfolded in December 2019 in the city of Wuhan, Hubei province, China. It has continued to spread since then, causing a pandemic that is still ongoing. The COVID-19 pandemic has been declared an international pandemic worldwide because of its spread and high mortality rate. Mathematical and numerical sciences make valuable contributions to the identification, modelling, forecasting of behaviour, control and treatment of epidemic diseases that humanity has been struggling with for centuries. Recently, it has been observed that mathematical models are increasingly being used in epidemics. Mathematical modelling of infectious diseases is an excellent tool used to predict and control the occurrence, spread of diseases and the future course of the epidemic. One of the most important areas of interest in mathematical epidemiology is the control of the epidemic and its subsequent complete destruction (see, [2]). In the last century, differential equations based on pandemic models have played an important role in the control and complete eradication of epidemics. Mathematical modelling is used to study on the spread and control of the disease, vaccination and taking the measures. Mathematical modelling of epidemics provides an understanding of the mechanisms that affect the spread of a disease, as well as suggests disease and control strategies. For this reason, mathematical models and computer simulations have become important tools for studies related to the study and control of the spread of diseases. Technology, architecture, engineering, health-economic policies, emergency planning, risk assessment, Control, Program Evaluation and optimization in many areas such as mathematical modeling are used in [3]. Modeling of epidemics happens with the division of the population into certain classes and many modeling processes begin in this way. This classification divides individuals into distinct classes, such as susceptible, infected, quarantined, exposed, maternally immunized, recovered. Epidemic models have been changed and improved. It included studies on epidemic diseases in the sources in the literature [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. Epidemic diseases have been categorized in the literature using abbreviations such as SI, SIS, SIR, SIRS, SEIS, SEIR, MSIR, MSEIR, SIQR, SEIQR, SVIR and different. It has created mathematical models for each of them. Kermack and McKendrick have developed the first known classical model of epidemic dynamics in the late 1920s [1]. It uses many effective strategies, vaccines and drug treatment methods to destroy or control the epidemic. A vaccine is a

Email addresses: zaferozturk@aksaray.edu.tr (Z. Öztürk), halis@aksaray.edu.tr (H. Bilgil), srgnrzs@gmail.com (S. Sorgun)

kind of medicine that develops a body immune system before catching the disease. Vaccine and drug therapy are the most important field of interest in mathematical biology and mathematical epidemiology. With the help of the vaccine, it is supported to protect the individual against epidemics that may occur because of environmental factors and to gain immunity. The most fundamental aim of vaccination studies is community immunity. All members of the society are also protected, since the probability of an epidemic is reduced if a critical proportion of the population is vaccinated against infectious diseases. Therefore, it is necessary to achieve high vaccination rates in order to control vaccine-preventable diseases at the community level. In this paper, we have created a new fractional SIQRV mathematical model by adding the V compartment (vaccine) to the SIQR model. This model has revealed the importance of reducing the number of infected by vaccination and controlling the epidemic at an early stage [11, 12, 13, 14]. Mathematical models to understand the effect of vaccination, the efficiency of vaccination, the vaccination campaign on COVID-19 have been developed and analyzed by the authors [20-33]. Mathematical models are the foremost technique to investigate the transmission dynamics of COVID-19 to control and develop a new strategy or policy to prevent the spread of the disease. In this paper, a new fractional model is introduced by adding a V (vaccinated individuals) compartment model to the SIQR model. The main purpose of this study is to examine the dynamics of the spread of COVID-19, to predict the course of the disease in the future. The COVID-19 model with the Caputo operator is solved numerically and the graphical impact of various parameters is depicted for different values of fractional order. This paper comprises four sections. In the 1st section, the importance of mathematical modeling and literature information have given. In the second section, the formation of the fractional SIQRV model and the Generalized Euler Method are given. In the third section, numerical results of the fractional SIQRV model and their graphs. The main results are given in the fourth and last section.

2. Fractional Derivation and Fractional SIQRV Model

The most commonly used definitions of the fractional derivative are Riemann-Liouville, Caputo, Atangana-Baleanu and the Conformable derivative. In this study, because the classical initial conditions are easily applicable and provide ease of calculation, the Caputo derivative operator was preferred and modeling was created. The definition of the Caputo fractional derivative is given below.

Definition 2.1. *([16]) Let $f(t)$ be a function that can be continuously differentiable n times. The value of the function $f(t)$ for the value of α that satisfies the condition $n-1<\alpha<n$. The Caputo fractional derivative of $\alpha-th$ order $f(t)$ is defined by $D_t^{\alpha} f(t)=\frac{1}{\Gamma(n-\alpha)}\int_a^t (t-x)^{(n-\alpha-1)} f^n(x)dx$ where $\Gamma(.)$ is the Gamma function.*
The main advantage of Caputo's approach is that the initial conditions for fractional differential equations with Caputo derivatives take on the same form as for integer-order differential equations, i.e. contain the limit values of integer-order derivatives of unknown functions at the lower terminal $t=\alpha$.
Fractional differential operators with non-singular kernel have captured the minds of several researchers because of its applicability to almost all fields of science, engineering and technology with different scales. Riemann-Liouville and Caputo fractional differential operators could not model the complex realworld problems as they have a singular kernel in their integral, whereas the fractional differential operators with non-singular kernel have removed this issue, as they are able to incorporate the effect of memory and long-range dependence into the mathematical formulation.
Fabrizio and Caputo operators have more widespread advantages than the integer order operator since their nonlocal kernels are defined. The memory and history of any physical process are preserved by the fractional operators. The convolution of the kernels is a topic of interest to many fractional operators [20-31].

Definition 2.2. *([16]) The Riemann-Liouville (RL) fractional-order integral of a function $A(t)\in C_n$ $(n\geq -1)$ is given by*

$$J^{\gamma}A(t)=\frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{(\gamma-1)}A(s)ds, J^0A(t)=A(t). \tag{2.1}$$

Definition 2.3. *([16]) The series expansion of two-parametrized form of Mittag-Leffler function for $a,b>0$ is given by*

$$E_{a,b}(t)=\sum_{t=0}^{\infty}\frac{t^i}{\Gamma(ai+b)}. \tag{2.2}$$

2.1. Fractional SIQRV Model with Vaccination

The fractional SIQRV model basically divides a community into five primary groups. The first are susceptible individuals, the second are infected individuals, the third are individuals who are in quarantine, the fourth individuals who have recovered and the fifth are individuals who have been vaccinated.In the fractional systems, dimensionally consistent is a very important tool, in which the units of measurement from the left- and righthand sides of the equations are coherent. This consistent can be provided by modifying the parameters involved in the right-hand side of the equations, e.g. raising them to power α. The expression of the SIQRV model as a system of fractional differential equations is given the following.

$$\begin{aligned}
\frac{d^{\alpha}S}{dt^{\alpha}} &= b^{\alpha}N-b^{\alpha}S-\frac{\beta^{\alpha}SI}{N}-\sigma^{\alpha}S+\theta^{\alpha}V+\nu^{\alpha}R\\
\frac{d^{\alpha}I}{dt^{\alpha}} &= \frac{\beta^{\alpha}SI}{N}-b^{\alpha}I-\gamma^{\alpha}I-k^{\alpha}I+\frac{\beta^{\alpha}IV}{N}\\
\frac{d^{\alpha}Q}{dt^{\alpha}} &= k^{\alpha}I-b^{\alpha}Q-\gamma^{\alpha}Q\\
\frac{d^{\alpha}R}{dt^{\alpha}} &= \gamma^{\alpha}I+\gamma^{\alpha}Q-b^{\alpha}R-\nu^{\alpha}R\\
\frac{d^{\alpha}V}{dt^{\alpha}} &= \sigma^{\alpha}S-\theta^{\alpha}V-b^{\alpha}V-\frac{\beta^{\alpha}IV}{N}.
\end{aligned} \tag{2.3}$$

Here $\frac{d^\alpha}{dt^\alpha}$ is a derivative of Caputo fraction according to t time.
Initial values are defined as $S(0)=S_0$,$I(0)=I_0$, $Q(0)=Q_0$, $R(0)=R_0$, $V(0)=V_0$ for $\alpha\in(0,1]$. It is easy to see that $S(t)+I(t)+Q(t)+R(t)+V(t)=N(t)$ from the equation (2.1).
In addition, it is clear that

$$\frac{d^\alpha N}{dt^\alpha}=\frac{d^\alpha S}{dt^\alpha}+\frac{d^\alpha I}{dt^\alpha}+\frac{d^\alpha Q}{dt^\alpha}+\frac{d^\alpha R}{dt^\alpha}+\frac{d^\alpha V}{dt^\alpha}. \tag{2.4}$$

The new form of the fractional SIQRV model is written as follows. In time-variable events, fractional models show more realistic and accurate results than models of integer because they have memory. Therefore, the established model was created as fractional order [17]. In this (2.3) system, $\alpha=1$ is reduced from a fractional to a differential equation from a precise level.

Table 1: Variables used in the systems and their meanings

Variables used in the systems	Meaning
$S(t)$	Susceptible individuals in the population
$I(t)$	Infected individuals in the population
$Q(t)$	Quarantined individuals in the population
$R(t)$	Recovered individuals in the population
$V(t)$	Vaccinated individuals in the population
$N(t)$	Total population

Table 2: Parameters and their meanings

Parameters	Meaning
β	The rate of transition of susceptible individuals to the infected compartment (Daily)
σ	Vaccination rate (Daily)
γ	The rate of recovery (Daily)
b	Birth and death rate (Daily)
θ	The rate of decline of vaccine protection (Daily)
ν	The rate of loss of immunity of those who have recovered (Daily)
k	The rate of those who moved from the infected group to the quarantine group (Daily)

There is no external migration intake or external migration in the population. In addition, it has been accepted that every individual in the population has the same probability of transmission. Age, gender, social status and race do not affect the likelihood of being infected. There is no hereditary immunity. Natural birth and death rates were considered equal in the model. All births are considered to have entered the susceptible class [6, 7]. The N population was dimensionalized and created as follows with the help of new variables.

$$s=\frac{S}{N}, i=\frac{I}{N}, q=\frac{Q}{N}, r=\frac{R}{N}, v=\frac{V}{N}$$

where $s+i+q+r+v=1$. The new form of the fractional SIQRV model is written as follows.

$$\begin{aligned}
D^\alpha s(t)&=b^\alpha-b^\alpha s(t)-\beta^\alpha s(t)i(t)-\sigma^\alpha s(t)+\theta^\alpha v(t)+\nu^\alpha r(t)\\
D^\alpha i(t)&=\beta^\alpha s(t)i(t)-b^\alpha i(t)-\gamma^\alpha i(t)-k^\alpha i(t)+\beta^\alpha i(t)v(t)\\
D^\alpha q(t)&=k^\alpha i(t)-b^\alpha q(t)-\gamma^\alpha q(t)\\
D^\alpha r(t)&=\gamma^\alpha i(t)+\gamma^\alpha q(t)-b^\alpha r(t)-\nu^\alpha r(t)\\
D^\alpha v(t)&=\sigma^\alpha s(t)-\nu^\alpha r(t)-b^\alpha v(t)-\beta^\alpha i(t)v(t).
\end{aligned} \tag{2.5}$$

2.2. Generalized Euler Method

In this subsection, we use the Generalized Euler method to solve the initial value problem with the Caputo fractional derivative. Many of the mathematical models comprise non-linear systems, and finding solutions to these systems can be quite complicated. In most cases, analytical solutions cannot be found and it should consider a numerical approach for this. So, one of these approaches is the Generalized Euler method [18].

Let $D^\alpha y(t)=f(t), y(0)=y_0, 0<\alpha<1, t>0$ consider the initial value problem. Let $[0,b]$ the interval over which we want to find the solution of the problem. For convenience subdivide the interval $[0,b]$ into n sub-intervals $[t_j,t_{j+1}]$, where $h=\frac{b}{n}$ is the step size, $j=0,1,..,n-1$. Suppose that $y(t), D^\alpha y(t)$ and $D^{2\alpha}y(t)$ are continuous in range $[0,b]$ and using the generalized Taylor's formula, the following equality is obtained [18].

$$y(t_1)=y(t_0)+\frac{h^\alpha}{\Gamma(\alpha+1)}f(t_0,y(t_0)). \tag{2.6}$$

This process will be repeated to create an array. Let $t_j = t_{j+1} + h$ such that

$$y(t_{j+1}) = y(t_j) + \frac{h^\alpha}{\Gamma(\alpha+1)} f(t_j, y(t_j)) \tag{2.7}$$

$j = 0, 1, .., n-1$ the generalized formula in the form is obtained. For every $k = 0, 1, .., n-1$ with step size h we get

$$\begin{aligned}
S(k+1) &= S(k) + \frac{h^\alpha}{\Gamma(\alpha+1)} \left(bN - bS(k) - \frac{\beta S(k) I(k)}{N} - \sigma S(k) + \theta V(k) + \nu R(k) \right) \\
I(k+1) &= I(k) + \frac{h^\alpha}{\Gamma(\alpha+1)} \left(\frac{\beta S(k) I(k)}{N} - bI(k) - \gamma I(k) - kI(k) + \left(\frac{\beta I(k) V(k)}{N} \right) \right) \\
Q(k+1) &= Q(k) + \frac{h^\alpha}{\Gamma(\alpha+1)} (kI(k) - bQ(k) - \gamma Q(k)) \\
R(k+1) &= R(k) + \frac{h^\alpha}{\Gamma(\alpha+1)} (\gamma I(k) + \gamma Q(k) - bR(k) - \nu R(k)) \\
V(k+1) &= V(k) + \frac{h^\alpha}{\Gamma(\alpha+1)} \left(\sigma S(k) - \theta V(k) - bV(k) - \frac{\beta I(k) V(k)}{N} \right).
\end{aligned} \tag{2.8}$$

An extra dose of BioNTech may be required to strengthen the immune system. It has been observed that the Pfizer vaccine protection rate dropped below 50 percent within 6 months. It has been established that the protection rate of the BioNTech-Pfizer 2 overdose has decreased from 88 percent to 47 percent. While it provided 93 percent protection against the Delta variant in the first month, this rate decreased to 53 percent after 4 months [19].

Theorem 2.4. *$\forall\, t \geq 0$, $S(0) = S_0 \geq 0$, $I(0) = I_0 \geq 0$, $Q(0) = Q_0 \geq 0$, $R(0) = R_0 \geq 0$, $V(0) = V_0 \geq 0$ the solutions of the system in (2.3) with initial conditions $(S(t), I(t), Q(t), R(t), V(t)) \in R^5_+$ are not negative [20-31].*

3. Numerical Simulation of the Fractional SIQRV Model

In this section, numerical simulation and graphs of the fractional SIQRV model are shown. Now let us get a numerical simulation of the fractional SIQRV model using the Generalized Euler method. We consider the parameters $N = 2000, S = 1300, I = 250, Q = 100, R = 200, V = 150, \beta = 0.6, \gamma = 0.25, \nu = 0.02, k = 0.001, b = 0.02, \theta = 0.005, \sigma = 0.55$ and let's take size of step $h = 0.1$. Hence we get the following results and tables. Using the Euler method, we obtain the following tables for given α.

According to these tables, the graphs of the change of the all compartments model with respect to time are the following (see Fig (1)-(5)):

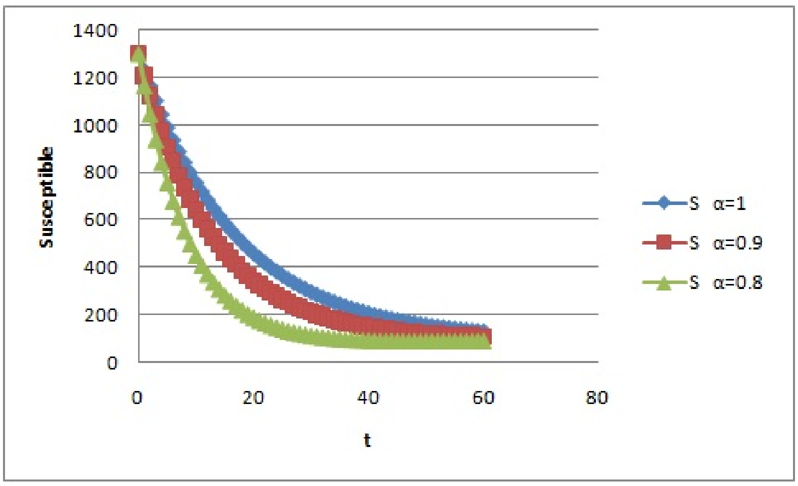

Figure 3.1: Fig 1. The graph of change of the S compartment model.

t	$S(t)$	$I(t)$	$Q(t)$	$R(t)$	$V(t)$
0	1300,00	250,00	100,00	200,00	150,00
1	1230,05	243,58	97,55	207,95	221,08
2	1164,16	237,33	95,15	215,64	288,13
3	1102,1	231,25	92,82	223,07	351,37
4	1043,65	225,32	90,55	230,27	411,03
5	988,59	219,54	88,33	237,22	467,31
6	936,73	213,91	86,16	243,95	520,41
7	887,89	208,42	84,05	250,44	570,52
8	841,88	203,08	81,99	256,72	617,80
9	798,54	197,87	79,98	262,78	662,44
10	757,72	192,80	78,02	268,64	704,57
11	719,28	187,86	76,10	274,29	744,35
12	683,06	183,04	74,24	279,74	781,91
13	648,95	178,35	72,41	285,01	817,38
14	616,83	173,77	70,64	290,08	850,88
15	586,56	169,32	68,90	294,98	882,53
16	558,06	164,98	67,21	299,69	912,44
17	531,22	160,75	65,56	304,24	940,70
18	505,93	156,63	63,95	308,62	967,41
19	482,11	152,62	62,38	312,84	992,67
20	459,68	148,71	60,85	316,89	1016,55
21	438,55	144,90	59,36	320,80	1039,14
22	418,65	141,18	57,90	324,55	1060,52
23	399,91	137,57	56,48	328,16	1080,74
24	382,26	134,04	55,09	331,63	1099,89
25	365,63	130,61	53,73	334,96	1118,01
26	349,97	127,26	52,41	338,15	1135,18
27	335,22	124,00	51,13	341,21	1151,45
28	321,33	120,83	49,87	344,15	1166,86
29	308,25	117,73	48,64	346,97	1181,48
30	295,92	114,72	47,45	349,66	1195,34

Table 3: The values of S, I, Q, R and V at the moment t for $\alpha = 1$.

t	$S(t)$	$I(t)$	$Q(t)$	$R(t)$	$V(t)$
0	1300,00	250,00	100,00	200,00	150,00
1	1208,43	241,6	96,49	210,4	243,05
2	1123,83	233,49	93,11	220,36	329,17
3	1045,67	225,65	89,85	229,90	408,90
4	973,44	218,08	86,71	239,02	482,73
5	906,71	210,76	83,67	247,74	551,09
6	845,06	203,68	80,74	256,08	614,41
7	788,09	196,85	77,91	264,05	673,08
8	735,45	190,24	75,19	271,66	727,44
9	686,82	183,85	72,55	278,92	777,83
10	641,89	177,69	70,01	285,85	824,54
11	600,37	171,72	67,56	292,46	867,86
12	562,01	165,96	65,20	298,76	908,05
13	526,57	160,39	62,91	304,76	945,34
14	493,82	155,01	60,71	310,47	979,96
15	463,56	149,81	58,59	315,91	1012,11
16	435,61	144,79	56,54	321,07	1041,97
17	409,79	139,93	54,56	325,98	1069,72
18	385,92	135,24	52,65	330,64	1095,53
19	363,88	130,70	50,80	335,06	1119,53
20	343,51	126,32	49,02	339,24	1141,88
21	324,70	122,08	47,31	343,20	1162,68
22	307,32	117,99	45,65	346,95	1182,07
23	291,26	114,04	44,05	350,49	1200,14
24	276,42	110,21	42,51	353,83	1217,00
25	262,71	106,52	41,02	356,97	1232,75
26	250,05	102,95	39,59	359,93	1247,45
27	238,36	99,5	38,20	362,71	1261,21
28	227,55	96,16	36,86	365,32	1274,08
29	217,57	92,94	35,57	367,76	1286,13
30	208,35	89,83	34,33	370,04	1297,43

Table 4: The values of S, I, Q, R and V at the moment t for $\alpha = 0.9$.

t	$S(t)$	$I(t)$	$Q(t)$	$R(t)$	$V(t)$
0	1300,00	250,00	100,00	200,00	150,00
1	1164,93	239,42	95,44	213,52	269,11
2	1044,41	228,14	91,10	227,06	373,46
3	936,94	218,26	86,96	240,62	464,59
4	841,12	210,45	83,01	254,21	543,88
5	755,75	202,57	79,24	267,82	612,57
6	679,70	194,38	75,65	281,48	671,80
7	612,00	186,82	72,23	295,18	722,56
8	551,76	172,53	68,96	307,92	765,78
9	498,17	164,70	65,84	316,71	802,25
10	450,53	156,89	62,87	326,54	832,73
11	408,19	149,98	60,03	336,42	857,86
12	370,59	142,13	57,33	346,35	878,25
13	337,20	136,47	54,75	356,33	894,42
14	307,58	129,04	52,29	364,35	906,87
15	281,31	122,86	49,95	373,40	916,02
16	258,02	116,74	47,71	382,49	922,25
17	237,38	110,73	45,58	390,61	925,93
18	219,11	104,67	43,55	398,76	927,36
19	202,94	99,78	41,61	406,92	926,82
20	188,64	94,53	39,76	412,10	924,57
21	175,99	89,71	37,99	418,28	920,83
22	164,83	84,93	36,31	424,46	915,81
23	154,97	79,49	34,71	430,63	909,69
24	146,27	74,89	33,18	436,77	902,63
25	138,61	69,58	31,72	442,89	894,78
26	131,86	65,82	30,33	448,98	886,27
27	125,92	61,79	29,00	454,98	877,22
28	120,70	57,96	27,73	460,01	867,72
29	116,12	53,69	26,53	465,91	857,88
30	112,10	49,92	25,37	470,76	847,78

Table 5: The values of S, I, Q, R and V at the moment t for $\alpha = 0.8$.

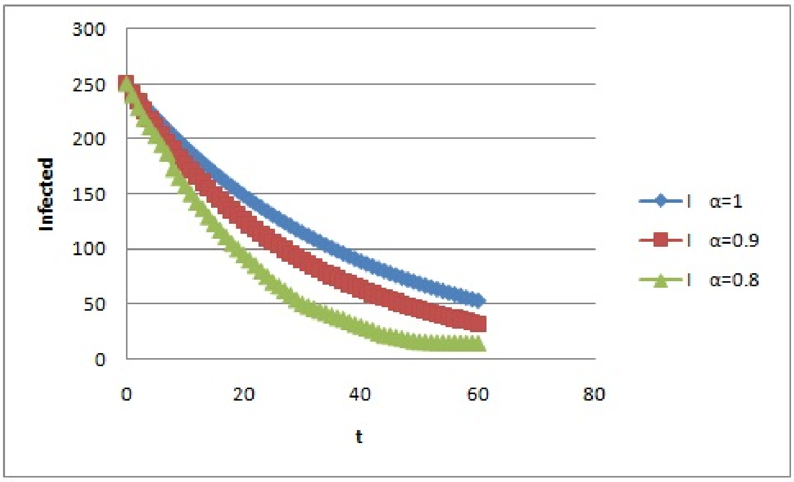

Figure 3.2: Fig 2. The graph of change of the I compartment model.

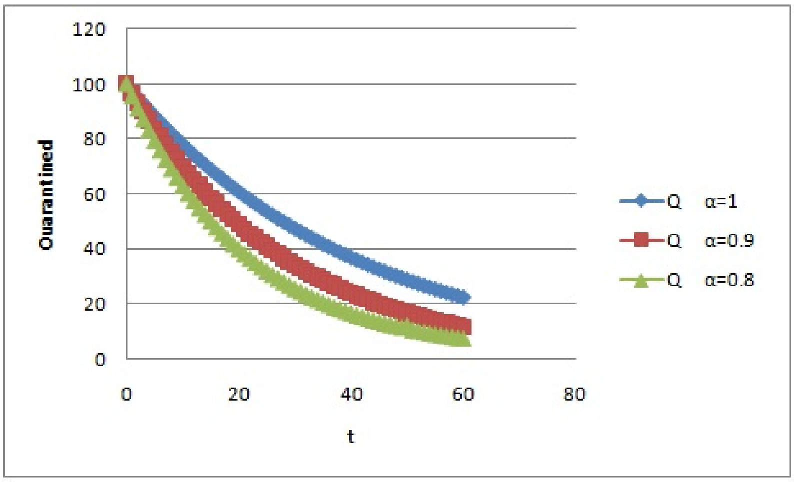

Figure 3.3: Fig 3. The graph of change of the Q compartment model.

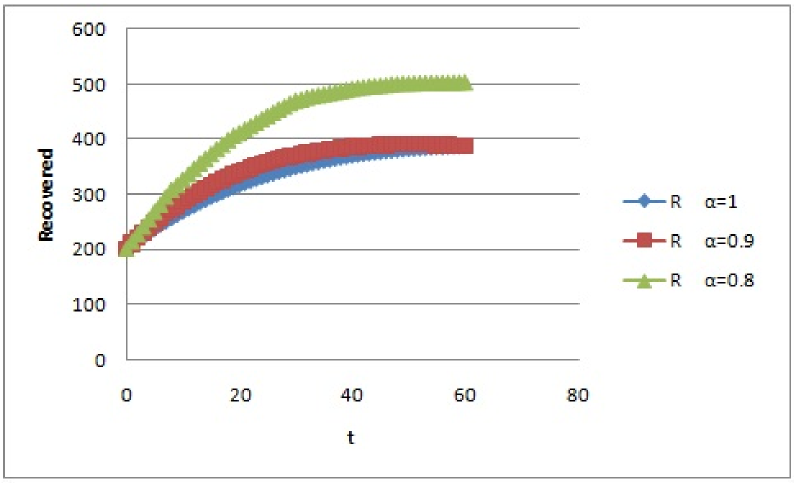

Figure 3.4: Fig 4. The graph of change of the R compartment model.

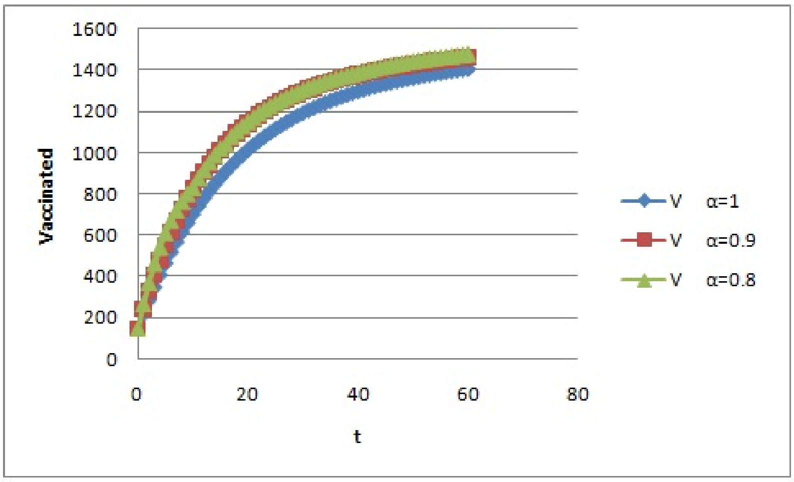

Figure 3.5: Fig 5. The graph of change of the V compartment model.

By the above figures, we observe the following highlights:

* It is observed that susceptible individuals decrease over time (Fig-1).
* It is observed that infected individuals decrease over time (Fig-2).
* It is observed that the individuals in quarantine decrease over time (Fig-3).
* It is observed that recovered individuals increase over time (Fig-4).
* It is observed that vaccinated individuals increase over time (Fig-5).

4. Conclusions

Epidemiological models which incorporate the impact of vaccination can be useful in helping determine effective ways of controlling the spread of disease. The rate of protection against infections, delta and other variants was high shortly after the full vaccination, but after a certain time the rate of protection decreased. The vaccination rate and the vaccine protection loss rate are included in the modeling and the way the epidemic changes with vaccination is examined with the help of numerical simulations.The main advantage of this fact is that fractional order models provide the hereditary and memory effects that naturally arise in physical materials and dynamical systems. The memory trace shows us all of the past activities and a long-term history of our model. For $\alpha = 1$, the memory trace is 0 for any time t. The dynamics of the memory trace is highly time-dependent. When the fractional order α decreases from unity, the memory trace increases nonlinearly from 0. This tells us how different the fractional order model is from the integer order model. From the graphs obtained, susceptible individuals are decreasing over time. Infected individuals decrease over time. Individuals in quarantine decrease over time. Recovered individuals increase over time. Vaccinated individuals increase over time. In addition, an important step will be taken in the fight against the epidemic with an increase in the vaccination rate.The numerical results obtained show that there is a decrease in infected and quarantined individuals with an increase in the vaccination rate. It shows that there is an increase in recovered individuals.

Article Information

Acknowledgements: The authors would like to express their sincere thanks to the editor and the anonymous reviewers for their helpful comments and suggestions.

Author's contributions: All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

Conflict of Interest Disclosure: No potential conflict of interest was declared by the author.

Supporting/Supporting Organizations: No grants were received from any public, private or non-profit organizations for this research.

Ethical Approval and Participant Consent: It is declared that during the preparation process of this study, scientific and ethical principles were followed and all the studies benefited from are stated in the bibliography.

Plagiarism Statement: This article was scanned by the plagiarism program. No plagiarism detected.

Availability of data and materials: Not applicable.

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