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Evaluation of Statistical Power in Random Effect Meta Analyses for Correlation Effect Size

Burçin ÖNER*[1]

Abstract

In meta-analysis, numerical index is used as an estimate of effect size to describe the results of each study and thereafter these estimates of across studies are combined to obtain summary of results.

It should be known that calculations of the power of statistical tests are important in planning research studies and for interpreting situations in which a result has not proven to be statistically significant. Although statistical power is often considered in the design of primary research studies, it is rarely considered in meta-analysis. Despite the importance of statistical power, few studies have been examined the performance of simulated power in meta-analysis. (In this study, calculations of statistical power for statistical tests that are used for unequal sample size on random effects model in meta-analysis using correlation coefficient as effect size were conducted.)

The power of the test for the overall effect size was calculated by using both analytical method and simulation method. Thus, it was investigated whether there was any difference between the simulation power and analytical power in random effects meta-analysis by using correlation coefficient as an effect size.

Keywords: Meta-analysis, simulation power, correlation, analytical power

1. INTRODUCTION

Meta-analysis is a statistical method helpful for qualitatively and quantitatively combining the results of the studies conducted in the same subject in different place, time and centers, as well as reaching a general result in that issue [1].

Today, more research is needed for the number of the studies conducted on the same subject in different areas and for the related subject in every study. The research results with quantitatively

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large sample size give reliability to the researchers in reaching the judgment that the study is definitely correct. In this meaning, there could be no need for conducting additional studies regarding the subject. Besides; conduction of wide-scope studies will be very challenging for the researcher due to the fact that they are expensive, time-consuming and have complex applications. However; the usage of an alternative method like meta-analysis is important in terms of overcoming these hardships for the researcher.

Meta-analysis is a method frequently applied in the fields such as health and social sciences. It combines the summary results of various studies previously conducted on the interested research subject by joining them together under the convenient conditions. In this way, it could be asserted that it is an efficient way in reaching a general result in the related subject.

Meta-analysis is a very popular method in today's scientific world. When an inference is desired to be made with this analysis method, the metrics called "effect size" are used to define the result of each study to be used in the meta-analysis. These scales could be the numeric values representing the means, ratios or relations. Thanks to the combination of these used effect sizes, general effect size will also be able to be calculated. In this way; the hypothesis tests belonging to the query that "whether general effect is existent or not" being one of the main objectives of meta-analysis will also be able to be conducted.

Although the inference procedures belonging to the significance tests of the effect sizes are existent for long years in meta-analysis, relatively fewer studies have been conducted in the issue of the calculation of the power of the statistical tests. Power calculations related to the statistical tests are always an important part of the sound statistical planning [2]. The requirement for the attainment of statistical power estimations is an issue showing an increase in the meta-analysis studies in recent years.

First, Field conducted studies on different effect sizes for the models in meta-analysis under various conditions, and showed under which conditions effect sizes yield stronger results [3]. Hedges and Pigott presented procedures for analytical power calculations for fixed and random effect models [2]. Analytical power is the power calculated with the help of the sampling distribution of the statistics used as the effect size. Cohn and Becker emphasized that one of the frequently encountered problems in meta-analysis is the low statistical power and they sought ways to increase the power [4]. Hedges and Pigott also dealt with statistical power calculations of fixed and mixed effects moderator tests in their meta-analysis [5]. Cafri and Kromrey developed a software for these calculations, since regular applications of statistical power calculations in the meta-analysis in the literature require technical expertise and take a long time [6]. Valentine, Pigott and Rothstein tried to determine the required sample size to achieve high statistical power in meta-analysis [7]. Liu investigated the differences between analytical power and simulated power for standardized mean difference effect size in fixed and random effects meta-analysis models, and the effect of unbalanced design and unequal sample sizes on statistical power [8]. Simulated power is the calculated power with the help of software programs under conditions such as number of studies, sample sizes, effect size, type 1 error and number of simulations. Liu and Pan argued that there is not enough evidence for the accuracy of the formulas in the literature for statistical power in meta-analysis and they used simulation studies as an alternative way to calculate the power of the test [9].

It is seen in the studies conducted regarding the statistical power in meta-analysis that the power calculations are generally conducted with analytical way and sometimes with Monte Carlo simulation studies [2, 10]. The information and related formula regarding how the analytic process will be for the statistical power calculations in meta-analysis have been given by Hedges and Pigott [2] the information regarding how the statistical software items (SAS, R etc.) will be carried out for these calculations have been given by Liu [8].

The sampling distribution of the statistics belonging to the effect size used in the related

meta-analysis is benefited while calculating the power in analytic way. Depending on the effect size preferred in meta-analysis; the sampling distribution of the statistics being the estimation of this effect size is affected from: (i) the study patterns applied in the combination of the studies (such as one-sample, independent or dependent two samples and the independent samples with two values), (ii) assumptions regarding the distribution of the group or groups according to the interested variable/variables, (iii) degree of the effect size (especially the correlation coefficient and odds ratio) and (iv) the size of the sample sizes in the study.

It is also an inevitable result that the analytic power of the test related to the statistical test used in the meta-analysis will also be affected due to these conditions efficient on the sampling distribution of the statistics. Especially in the meta-analyses in which relation scales (correlation coefficient, odds ratio etc.) are used as the effect size, the result may occur that the calculated analytic power does not reflect the real power sometimes due to the relation degree and sometimes due to the small sample [2, 8]. Because the sampling distribution of the statistics used as the effect size in the large samples converges to a known theoretical probability distribution, the usage of this probability distribution becomes possible in the calculation of the analytic power. However; the analytic power that will be calculated with the same probability distribution also in small samples will not show the real power. This is because the formulas for the sampling distribution of the relevant effect size statistics are not valid for small sample sizes. At this situation; it will be more convenient to prefer the way of simulation in the calculation of the power of the test on condition that the conditions necessary for the power calculation will remain the same. Because; when the conditions are the same, the power value attained with the simulation way comes out higher than the value attained with analytic power way.

In the literature, considering this feature of the correlation coefficient effect size, it has been determined that there is no comparison of analytical and simulation-based studies of statistical power in the meta-analysis. For this reason, in this study, the power of test calculations belonging to the statistical tests used for the situation of an unequal sample size in random effects model have been handled in a meta-analysis in which the correlation coefficient has been taken as the effect size. The power of test calculations belonging to the general effect size from them have been attained with the use of analytic methods and simulation method. In this way; the issue that whether there is a difference between the statistical power simulation and analytic power in a meta-analysis in which the correlation coefficient has been used as the effect size metrics has been examined for both models.

Required encodings were/has been written in R program for the analytic power and simulation power calculations. Analytic power and simulation power methods have been used in the power calculation. Within the scope of the study, the research question “Is there any difference between the analytic power and simulation power calculated for the random effects model in meta-analyses in which correlation coefficient has been used in effect size?” has been addressed.

2. MATERIAL AND METHOD

For a meta-analysis study to be conducted, the aim of the relevant study, study design, and data format are to be guide in the selection of the effect size. The effect size to be used in the studies assessing the relation among the variables without conducting causal direction inferences could be the relation scales like correlation coefficient. In this study, correlation coefficient has been used as the effect size. For this reason; correlation coefficient statistics and its sampling distribution have been focused in this part.

If the correlation coefficient is used as the effect size in a meta-analysis, the study pattern is the one-group studies. In the studies in which correlation coefficient expressing the direction and degree of the linear relation between two continuous variables is used, correlation parameter is shown with θ for the population and the sample correlation coefficient statistics which is the estimation of this parameter is shown with

r. This statistic takes a value within the range of [-1, +1] and is defined as follows:

$$r = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \overline{x})^2 \sum_{i=1}^{n}(y_i - \overline{y})^2}} \quad (1)$$

The sampling distribution of this statistics does not show a normal distribution. Especially on condition that $\theta_0 \epsilon [-1, +1]$; the sampling distribution of r statistics shows a skew distribution towards right ($\theta_0 > 0$) or left ($\theta_0 < 0$) while $H_0: \theta = \theta_0$. Also; the expected value of r statistics is equal to the parameter (namely θ), but its variance is a function of itself and the sample size [11].

For the sample correlation coefficient r statistics; the expected value and variance is given with the following equation respectively:

$$E(r) = \theta \text{ and } V_r = \frac{(1 - r^2)^2}{n - 2} \quad (2)$$

For this reason; in the meta-analysis studies in which the correlation coefficient is used as the effect size, r statistics is not directly used for the statistical inferences such as especially the hypothesis test and the power of test. At this situation, processes are conducted by applying Fisher Z transformation on the correlation coefficient. Fisher Z transformation is the following equation:

$$Z = \frac{1}{2} \times \ln\left(\frac{1 + r}{1 - r}\right) \quad (3)$$

and the sampling distribution of this Z statistics shows a normal distribution [12, 13] whose parameters are respectively;

$$E(Z) = \frac{1}{2} ln\left(\frac{1 + \theta}{1 - \theta}\right) \text{ and } V(Z) = \frac{1}{n - 3} \quad (4)$$

Consequently; correlation values could be attained again by applying inverse transformation on Eq. (3).

Let's assume that there are k independent studies on a particular topic has been handled. Let θ_i be the population effect size parameter belonging to i^{th} study. Let effect size estimations be attained from these studies and let us show the estimator of θ_i parameter with T_i. In this situation; an assumption belonging to T_i is as follows;

$$T_i \sim N(\theta_i, v_i); \quad i = 1,2, \dots, k \quad (5)$$

In the case where the correlation coefficient is used as an estimation of the effect size, vi is a known variance and dependent on the sample size. According to Hedges and Olkin [14], this assumption is always valid for effect sizes such as the correlation coefficient transformed with Fisher's Z transformation. However; correlation coefficient which has not been transformed is not completely valid for the effect size; it is only accepted to be correct for large samples [2]. Thus; for i[th] study, $i = 1,2, \dots, k$; $T_i = Z_i$ from (3) and $v_i = \frac{1}{n_i - 3}$ from (4).

2.1. Analytic Power in Random Effect Model

When the studies are combined in meta-analysis according to the random effect model and the correlation coefficient belonging to the studies are taken as the effect size parameter, the model equation defined under Fisher Z transformation is given with the following equation;

$$Z_i = \xi_i + \varepsilon_i = \mu + \eta_i + \varepsilon_i \ ; \ i = 1,2, \dots , \ k \quad (6)$$

In this model; for $i = 1,2, \dots , k$; the effect size ξ_i belonging to i^{th} study (correlation coefficient transformed for the population) is a random variable.

According to the related model; ε_i is the random error of $Z_i$and η_i is the random error of ξ_i, these are both random errors. For these random errors; it is assumed that these are distributed as; $\varepsilon_i \sim N(0, v_i)$ and $\eta_i \sim N(0, \tau^2)$, $(i = 1, 2, \dots , k)$ and independent. In this situation; as the linear functions of the normals, both Z_i being the sample effect size estimation attained with Fisher Z transformation of the correlation coefficient belonging to i[th] study and ξ_i being the effect size belonging to i[th] study (correlation coefficient transformed for the population) have normal distribution. Such that; for $i = 1, 2, \dots , k$; $\xi_i \sim N(\mu , \tau^2)$ while $Z_i \sim N(\xi_i , v_i)$. Here; v_i is the

conditional sampling variance of Z_i and it is calculated with $v_i = \frac{1}{n_i - 3}$, $i = 1, 2, \dots, k$.

Also; τ^2 known as the component of variance between studies could be estimated with $\hat{\tau}^2 = \frac{Q-(k-1)}{c}, Q > (k-1)$. Here; Q is Cochran Q test statistic and c is a constant that is a function of the weights. Thus; statistical analyses could be conducted in the meta-analysis on the mean effect size parameter (μ) belonging to the random effect model, the variance of the random effect size or the component of variance (τ^2) between studies. In these analyses, both the hypotheses about parameters could be tested and the power calculations for these tests could also be conducted.

When the combination of the studies in meta-analysis is conducted according to the random effect model, the effect size has a unique distribution due to the fact that it is a random variable.

When the population correlation coefficient $(\theta_i, i = 1, 2, \dots, k)$ is taken as random effect size for the studies, the effect size transformed under Fisher Z transformation is $\xi_i = \frac{1}{2}\left\{\ln\left[\frac{1+\theta_i}{1-\theta_i}\right]\right\}, i = 1, 2, \dots, k$ random variable and $\xi_i \sim N(\mu, \tau^2)$.

Here; μ is the mean of the effect size distribution and it is known as the mean effect size parameter. Statistical inferences could be made about the effect size with a hypothesis test related to the mean effect size parameter.

The hypotheses to be tested in this test process are formed as follows:

$$\begin{aligned} &\textbf{a)}\ H_0: \mu = \mu_0 \ ;\ H_1: \mu \neq \mu_0 \\ &\textbf{b)}\ H_0: \mu = \mu_0 \ ;\ H_1: \mu < \mu_0 \\ &\textbf{c)}\ H_0: \mu = \mu_0 \ ;\ H_1: \mu > \mu_0 \end{aligned} \quad (7)$$

Here; μ_0 is a known real number. $\bar{Z}_.^* = \frac{\sum_{i=1}^{k} w_i^* Z_i}{\sum_{i=1}^{k} w_i^*}$ that is an unbiased estimator of the mean effect size parameter and it also is the weighed mean statistic of random effects estimations. The sampling distribution of this statistic is shown with $\bar{Z}_.^*$ and has a normal distribution with mean μ and variance $v_.^*$ for the large samples. Such that, here $v_.^*$ is the unconditional sampling variance of Z_i and is calculated with $v_.^* = v_i + \tau^2$. In this case, the test statistic to test the H_0 hypothesis will be the Z statistics which has standard normal distribution and given by the following;

$$Z = \frac{\bar{Z}_.^* - \mu}{\sqrt{v_.^*}} \sim N(0,1) \quad (8)$$

The value of the test statistics is $Z = \frac{\bar{Z}_.^* - \mu_0}{\sqrt{v_.^*}}$ when H_0 is correct. Regarding the decision to be reached as a result of the test process; H_0 is rejected if $|Z| > C_{\alpha/2}$ for the two-sided test and H_0 is rejected if $|Z| > C_\alpha$ for the one-sided tests.

The calculation of the power of test will be able to be a matter in the conditions in which H_0 hypothesis is rejected. In this situation, the distribution of Z test statistics according to the alternative hypothesis will show a normal distribution whose mean is λ^* and variance is 1. Here; the real value of the mean effect size according to the H_1 is $\mu_G = E(\bar{Z}_.^*)$, and parameter λ^* is calculated with the following equation;

$$\lambda^* = \frac{\mu_G - \mu_0}{\sqrt{v_.^*}} \quad (9)$$

The power function which will occur as a result of the rejection of H_0 hypothesis at a given significance level at α will be

$$1 - \beta = 1 - \emptyset(C_\alpha - \lambda^*) \quad (10)$$

for the one-sided tests; and

$$1 - \beta = 1 - \emptyset\left(C_{\alpha/2} - \lambda^*\right) + \emptyset(-C_{\alpha/2} - \lambda^*) \quad (11)$$

for the two-sided test. Here; $\emptyset(x)$ shows the cumulative standard normal distribution function.

2.2. Simulation Power in Meta-Analysis

In the meta-analyses in which some effect sizes such as correlation coefficient and odds ratio have been used, the sampling distributions of the sample effect size statistics could be calculated approximately and asymptotically for the large

samples. For this reason; the analytic power calculations are made using these approximate distributions under the alternative hypotheses for the statistical power related to the tests in meta-analysis. Thereof; it is thought that considering the analytic power values attained regarding the mentioned statistical tests not as the real power, but as the approximate value will be more correct.

According to Liu [8]; it is not known whether the accuracy of the analytic power formulas taking place in the literature is certain or not. The fact that there are some assumptions in the application of the formulas may cause to some biases in the results. Some of the mentioned assumptions could be given as; (i) accepting the within study variances as equal, (ii) preference of the smallest sample size among groups in which small sample sizes are intensive for the purpose of determining a more consistent within study sample size in the combined studies and (iii) preference of the smallest study number among the study numbers to determine a more consistent study number etc. [2, 8].

However; when the correlation coefficient is used as the effect size, the sampling distributions of the test statistics related to the statistical tests may be formed asymptotically for the large samples and the analytic power calculation could also be made with the help of these formed distributions. Therefore; it is highly likely that the analytic power values attained from especially small samples are the biased when compared to the real values.

In this study; statistical power calculation with simulation is suggested as an alternative method in the meta-analyses in which correlation coefficient is taken as the effect size in terms of being able to eliminate the problems in the analytic power calculations for the statistical power related to the statistical tests in meta-analysis.

The formulas used in the analytic power calculations are dependent on the parameters of the population effect size, statistical significance level, sample size, number of studies to be included in the analysis and the component of variance between studies. For this reason; these parameters should be taken into consideration also in the simulation power calculations.

Sample size may show difference from one to the other study included in the meta-analysis. As in the primary studies, the population effect size and statistical power are correlated within the same direction also in the meta-analysis. In other words; as the population effect size increases, statistical power also increases. Generally, the standardized or transformed effect size values are used to combine the measurement scales between studies in meta-analysis; because these operations make the measurement units of the variables independent from each other.

The number of studies to be included in meta-analysis is also in a positive relation with the statistical power when other parameters are equal. Namely; as the number of studies included in meta-analysis increases, statistical power will also increase.

In addition; a simulation study is a very advantageous method both in terms of checking the correctness of the statistical power in a real meta-analysis study and emphasizing its relation with the analytic power. While calculating the population variance in power formulas, the variance of every study is accepted to be equal to one another. This approach is not a correct method in practice although it is frequently used in the power calculation process. So; it could be said that simulation power will give more correct results when compared to the analytic power. Also; the comparison of these two calculated powers provides the opportunity of being able to check the correctness of the calculation findings. In addition, the differences between analytical power and simulation power provide the opportunity for defining a potential bias existent in the power formulas [8].

As in the analytic power calculation, same conditions will be taken into consideration also in the simulation power calculation. These are the sample size, number of studies, population effect size, type I error ratio and model type. In addition to these conditions, also the number of simulations will be considered in the calculation of the simulation power.

3. RESULTS

In this study; the power of test calculations belonging to the statistical tests used in random effects model have been focused in a meta-analysis in which correlation coefficient is considered as the effect size. For this; the examination of both the simulation power and analytic power of a meta-analysis planned on a simulation data have been conducted and the results attained according to two methods have been compared.

The focused research question has been determined as "Is there any difference between the calculated analytic power and simulation power for the random effect models in meta-analysis?"

Power simulation has been conducted under various conditions by taking into account the factors affecting the statistical power and the simulation status and afterwards, analytic power has been calculated using the existent power formulas and the attained two statistical power results have been compared. R programming language has been benefited for these processes. Simulation conditions have been based on similar studies [8].

The conditions have been determined as follows:

Average sample size: The condition showing variability in different meta-analysis studies changes between 20 and 100 (20, 30, 40, 50, 75, 100) in this study. In a real meta-analysis study, the mean sample size is expected to be rather large. However; when the other parameter values are equal, the sample sizes larger than 100 have been neglected in this study due to the fact that the large sample sizes have a statistical power enhancing effect. Besides; the study concerns with the impact of small sample sizes on statistical power.

In reality, the sample sizes among the primary studies are not equal to one another. Thereof; the truncated binomial distribution has been used to meet the sample sizes and to be able to produce positive integer numbers in the simulation study. The maximum value taking place in the distribution has been changed and the variety of the sample size has been ensured. The sample size of each study has been diversified based on different ratios [8].

Population effect sizes: In this study, correlation coefficient (r) has been discussed as the population effect size. Respectively $r = 0$ (no effect), $r = 0{,}1; 0{,}2$ (little effect), $r = 0{,}5$ (medium effect) and $r = 0{,}8$ (high effect) values have been determined for these selected effect sizes and included in the study.

The principles of Cohen [15] and the values taking place in the study of Field [16] who determined as the low, medium and high degree correlation coefficients have been taken into consideration for the determination of these effect sizes.

The sampling distribution of the correlation coefficient shows a distribution which is both not normal and dependent on the unknown population correlation coefficient. Therefore; correlation coefficient is not directly used in the meta-analysis and power calculations.

In this situation, Fisher Z transformation being a logarithmic transformation is applied on the correlation coefficient. This transformation extends $[-1\,;\,+1]$ interval being the interval of values that could be taken by the correlation coefficient into the interval of $(-\infty\,;\,+\infty)$. Also; the sample distribution of the new statistics defined with Fisher Z transformation will approach to the normal distribution [2, 14, 17].

Number of Studies: The numbers of studies determined for this condition shows varieties between 5 and 75 (5, 10, 20, 30, 45, 60, 75). These numbers have been selected by basing them to the real meta-analysis studies.

Moreover; the fact that there is a positive relation between the number of studies and the statistical power when other parameters are equal gives rise to reaching a satisfying power in the numbers of studies higher than 75. For this reason; the numbers of studies higher than 75 have not been included in the power calculation conditions.

Number of Repetitions: Meta-analysis study has been repeated for 10 000 times for the purpose of attaining a constant simulation result.

Type I error ratio: As the related ratio, the ratio of $0{,}01$ used commonly in the literature has been determined in terms of the reliability of the statistical hypothesis test for this study.

According to all these, total simulation scenario is dependent on 3 different factors. These are 5 population effect sizes (0; $0{,}1$; $0{,}2$; $0{,}5$; $0{,}8$), 7 sample sizes ($20, 30, 40, 50, 60, 75, 100$) and 7 numbers of studies ($5, 10, 20, 30, 45, 60, 75$). Sample size and number of studies have 49 combinations. 10 000 Monte Carlo tests have been conducted for each combination

3.1. Type I Error Ratio Control

What is necessary to be conducted before performing the processes for the research question is type I error control. In the condition where the population effect size is zero in the model, the probability of rejecting the zero hypothesis gives the real type I error ratio. This control is necessary, because type I error ratio may affect the type II error ratio, therefore, the statistical power [8]. Related results are given in Table 1 for $0{,}01$ significance level.

Table 1
Type I error control in random effects model (for $\alpha = 0{,}01$)

	Number of Studies						
ASS*	5	10	20	30	45	60	75
20	0,006	0,007	0,007	0,007	0,008	0,007	0,009
30	0,005	0,007	0,007	0,007	0,009	0,007	0,008
40	0,007	0,008	0,007	0,008	0,008	0,009	0,009
50	0,005	0,007	0,008	0,006	0,007	0,008	0,008
60	0,009	0,008	0,009	0,009	0,007	0,008	0,009
75	0,007	0,008	0,009	0,008	0,009	0,008	0,007
100	0,006	0,006	0,009	0,009	0,008	0,008	0,009

* Average Sample Size

3.2. Statistical Power and Simulation under Random Effects Model

When the situation in which the sample size among studies is not equal to one another is taken into consideration, the requirement that the sample sizes to be produced to calculate the simulation power should be positive integer numbers occurs.

In this situation, truncated binomial distribution is benefited to produce various sample sizes between studies. The sample sizes produced thanks to this distribution have turned to the positive integer numbers with a certain mean and standard deviation.

As the standard deviation of the binomial distribution changes, maximum sample size has also been changed. Maximum sample size is attained by multiplying the mean sample size with a certain number.

In this study, it has been selected as" the mean sample size *3". Different tests have been conducted for a larger maximum sample size, but similar results have been attained. For this reason; only the condition "$maximum\ sample\ size = mean\ sample\ size * 3$" has been adopted in the study.

The simulation power and analytic power values calculated by considering the criteria of the number of studies, sample size and different population effect sizes for the random effects model under the design of the inequality of the sample sizes are given in Table 2.

The results related to the power calculations have been attained by coding them in R program.

Table 2
Statistical power in random effects model while type I error ratio is 0,01 ($Maximum\ sample\ size = average\ sample\ size * 3$)

ASS[1]	Simulation Power							Analytical Power						
	Number of Studies													
	5	10	20	30	45	60	75	5	10	20	30	45	60	75
Population Effect Size=0,1														
20	0.036	0.079	0.194	0.332	0.528	0.703	0.824	0.040	0.085	0.202	0.337	0.535	0.698	0.815
30	0.056	0.140	0.359	0.554	0.787	0.924	0.968	0.062	0.144	0.353	0.556	0.782	0.907	0.964
40	0.081	0.208	0.500	0.734	0.926	0.976	0.994	0.086	0.211	0.497	0.728	0.913	0.976	0.994
50	0.109	0.283	0.635	0.849	0.973	0.997	0.999	0.113	0.280	0.624	0.843	0.968	0.994	0.999
60	0.153	0.349	0.743	0.924	0.991	0.999	0.999	0.141	0.352	0.729	0.915	0.989	0.999	0.999
75	0.191	0.474	0.847	0.972	0.998	1.000	1.000	0.187	0.459	0.842	0.968	0.998	0.999	1.000
100	0.289	0.634	0.949	0.996	1.000	1.000	1.000	0.268	0.616	0.943	0.995	0.999	1.000	1.000
Population Effect Size=0,2														
20	0.186	0.450	0.838	0.967	0.998	0.999	1.000	0.178	0.442	0.826	0.962	0.997	0.999	1.000
30	0.326	0.698	0.972	0.998	1.000	1.000	1.000	0.313	0.684	0.968	0.998	1.000	1.000	1.000
40	0.475	0.848	0.996	1.000	1.000	1.000	1.000	0.447	0.842	0.995	0.999	1.000	1.000	1.000
50	0.593	0.931	0.999	1.000	1.000	1.000	1.000	0.571	0.927	0.999	1.000	1.000	1.000	1.000
60	0.703	0.970	0.999	1.000	1.000	1.000	1.000	0.677	0.969	0.999	1.000	1.000	1.000	1.000
75	0.817	0.992	1.000	1.000	1.000	1.000	1.000	0.798	0.992	1.000	1.000	1.000	1.000	1.000
100	0.921	0.9993	1.000	1.000	1.000	1.000	1.000	0.916	0.999	1.000	1.000	1.000	1.000	1.000
Population Effect Size =0,5														
20	0.969	1.000	1.000	1.000	1.000	1.000	1.000	0.972	1.000	1.000	1.000	1.000	1.000	1.000
30	0.997	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
40	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
50	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
60	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Population Effect Size=0,8														
20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
100	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Furthermore; the graphical demonstration of these attained simulation and analytic power results have been drawn in MATLAB R2018b program and the attained curve is given in Figure 1.

The straight lines in the graphics show the analytic power values and the dashed lines shows the simulation power values and the colors of red, yellow, blue, green, black, purple and light blue respectively express the numbers of studies (5, 10, 20, 30, 45, 60, 75).

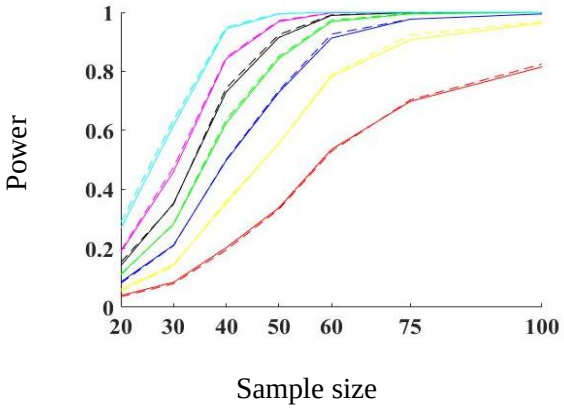

Figure 1 Statistical power in random effects model when type I error ratio is 0,01

Figure 1 shows the power curve in the situation of the inequality of the sample sizes in the random effects model when the type I error ratio is 0,01. It has been observed in the graphics that there are differences between the two power values although the simulation power and analytic power values are very close to each other. The graphics has been drawn only for this effect size due to the fact that the widest differences could be determined for 0,1 population effect size and very close values have been attained for the other designs.

3.3. Differences between the Analytic Power and Simulation Power

In this part, firstly the differences of between analytical power and simulation power under different conditions in random effects model have been examined. As it could be seen from Table 2, simulation power and analytic power have taken values very close to each other under all conditions. So; it cannot be said that there is a systematic difference between them, namely power has underestimation or overestimation different conditions.

It could be theoretically said that statistical power increases as the number of studies, population effect size and average sample size expanded. Also; when the population effect size is 0,8; it has been observed that the power has taken the value of 1 for two random effects model. It is understood here that when the population effect size is very high, other parameters affecting the statistical power become unimportant. However; it cannot be said that the reverse situation is correct. For instance; power could take very low values when the average sample size starts from the value of 100 and number of studies and population effect size take very low values.

The differences in the random effects model have been checked by comparing the power estimations attained from Table 2 for the significance level of 0,01 in the event of the unequal sample size and these comparison results are given in Table 3.

Table 3
Differences between analytic power and simulation power in random effects model ($\alpha = 0{,}01$)

Average Sample Size	Number of Studies						
	5	10	20	30	45	60	75
	Population Effect Size=0,1						
20	-0,004	-0,006	-0,008	-0,005	-0,007	0,005	0,009
30	-0,006	-0,004	0,006	-0,002	0,005	0,017	0,004
40	-0,005	-0,003	0,003	0,006	0,013	0	0
50	-0,004	0,003	0,011	0,006	0,005	0,003	0
60	0,012	-0,003	0,014	0,009	0,002	0	0
75	0,004	0,015	0,005	0,004	0	0,001	0
100	0,021	0,018	0,006	0,001	0,001	0	0

In the case of sample size is not equal, the number of studies, the average sample size and the values of population effect size required to reach a desired statistical power, for the level of Type I

error rate at 0.01, according to the model type are given in Table 4.

Table 4
Average sample size amount necessary for a statistical power at the level of 0,80 and above in random effects model while type I error ratio is 0,01.

	Random Effect Model ($\alpha = 0{,}01$ için)						
Population Effect Size	Number of Studies						
	5	10	20	30	45	60	75
0,1	>100	>100	75	50	>30	30	20
0,2	75	40	20	20	20	20	20
0,5	20	20	20	20	20	20	20
0,8	20	20	20	20	20	20	20

4. CONCLUSION AND DISCUSSION

As seen in Table 1; type 1 error ratio has been kept under control using 0,01 significance level and limited to 1% level for the random effects model. It means that the model has an error ratio at 1% level. In other words; β is one (1), $1-\beta$ is zero (0). Because the population effect size takes the value of zero (0) here, H_0 hypothesis is not wrong and therefore, it requires no rejection. Also; if an examination is conducted according to the average sample size when the numbers of studies are kept stable and according to the number of studies when the average sample size is kept stable, it has been observed that there have been increases and decreases in the values. This could be interpreted as the fact that there is no systematic increase or decrease in the random effects model.

When the situation in which population effect size is 0,1 is considered according to Table 2, the number of studies necessary to attain a power at a level of 80% and above is sufficient to be around 75 according to both simulation and analytic calculations for the average sample size of 20 units is used. When the sample size of 30 units is used, the number of studies necessary to attain a power at a level of 80% and above should approach 60 according to both the simulation and analytic calculations. When the sample size of 40 units is used, the number of studies necessary to attain a power at a level of 80% and above should approach 45 according to both the simulation and analytic calculations. When the sample size of 50 and 60 units is used, the number of studies necessary to attain a power at a level of 80% and above should be around 30 according to both the simulation and analytic calculations. When the sample size of 75 and 100 units is used, the number of studies necessary to attain a power at a level of 80% and above should approach 20 according to both the simulation and analytic calculations.

When the population effect size is 0,2; while the sample size amount necessary to attain a power at a level of 80% and above is sufficient to be 75 in a meta-analysis in which there are 5 studies according to the simulation calculations, the sample size necessary to attain a power at a level of 80% and above should approach to 100 according to the analytic calculations. The sample size necessary to attain a power at a level of 80% and above is around 40 according to both the simulation and analytic calculations in a meta-analysis in which there are 10 studies. The sample size necessary to attain a power at a level of 80% and above is sufficient to be around 20 according to both the simulation and analytic calculations in a meta-analysis in which there are 20, 30, 45, 60 and 75 studies.

When the population effect size is 0,5 and 0,8; sample sizes and numbers of studies with very little size are sufficient for the statistical power to come out very high.

When Figure 1 is examined; while the difference between the simulation power and analytic power is very little in the places in which the sample size is between 60 and 75 for the number of studies is 5, the differences have been observed to have become clearer in other sample size values. It has been seen that the differences have become clearer when the sample size has taken values higher than 60 in the places in which the number of studies is 10, but the power values have come so much closer to each other in the sample size at a level of 100 and higher. The differences have been observed to come closer to each other very much in the event that the sample size has taken a value between 50 and 75 in the places in which the number of studies is 20. No clear difference has been observed between two powers in the places in which the number of studies is 30. While

the differences have become clearer when the sample size has taken a value between 40 and 60 in the places in which the number of studies is 45, it has been monitored that the clear differences have occurred in the sample sizes less than 40 in the meta-analyses to which 60 and 75 studies have been included. When examination has been conducted on the values in which the sample size has increased on the Figure, it has been observed that the two power values have come closer to each other and even reached the integer value of 1.

When Table 3 has been examined; it has been seen that the difference between the simulation power and analytic power is too little at the significance level of 0,01 and it has also been observed that these differences become clearer when the population effect size is 0,1 and 0,2.

For the numbers of studies are respectively 60, 45, 20 and 10 while the population effect size is 0,1 and the average sample size is 30, 40, 50, 75 and 100; also, for the numbers of studies are respectively 5 and 20 and the average sample size is 60; differences exceeding 0,01 have been observed. It has been seen that especially for the number of studies is 10 and average sample size is 100, this difference has approached 0,02 and it has exceeded 0,02 for the number of studies is 5 and the average sample size is 100. It has been determined that there have been differences exceeding 0,01 in the event that the number of studies is 5 while the population effect size is 0,2 in all average sample sizes except for the sample sizes of 20 and 100 units; and for the average sample size has been respectively 30 and 20 in the situations in which the numbers of studies are 10 and 20. It has been observed that the difference has approached 0,03 for the sample sizes are respectively 40, 50 and 60 while especially the number of studies is 5.

It has been seen while the population effect size is 0,5 and 0,8 that the difference between the simulation power and analytic power has remain at a very little level in the random effects model in the unequal sample size. The interpretations belonging to the impact of the unequal sample sizes among studies on the statistical power have been made upon the simulation power although the analytic power and simulation power have given results close to each other. As it could be seen from the power tables, the unequal sample size among studies and the population effect size have had an increasing impact on the statistical power. Because the situation in which population effect size is 0,8 presents 1 integer power, it has been kept out of observation and interpretations have been made upon the other effect sizes.

The results expected to be attained from this study could be sequenced as follows: (a) What the parameters of average sample size, number of studies and population effect size necessary for a meta-analysis study to be conducted to have a statistical power at the desired level at a certain type I error level will be should be decidable, (b) Some differences could be found between the results of analytic power and simulation power attained within the frame of the statistical power calculation in meta-analysis. This difference may stem from the fact that analytic power is based on certain formulas and the differences at or below the level of type I error ratio which has been determined are assumed to be acceptable and (c) Because the differences may show underestimations or overestimations, it cannot be said that there is a systematic increase or decrease in the analytic power.

This study presents the results attained along the simulation conditions that could affect the power belonging to random effects meta-analysis method. The study has provided a wider point of view about the power estimations of the meta-analysis procedures. Two estimations have been supported. These are: (1) Very few differences have been observed to be existent between simulation power and analytic power calculated by taking the determined criteria into consideration. The differences lower than type I error ratio are the neglectable differences. In this situation, it could be said that simulation power and analytic power calculations complete each other. In contrast; in the situation in which type I error ratio is 0,01; it has been observed that the differences are a little higher than this error ratio for some scenarios. Here; it could be inferred that the analytic power estimation will estimate the real power more deficiently as the difference

between the two powers increases. In addition; as the related parameters (mean sample additional volume, population effect size and number of studies) are changed, the differences between analytic power and simulation power have decreased to an acceptable level. (2) As it could be understood from the power tables, no systematic bias has been observed. Both underestimations and overestimations have been detected.

Consequently; in this study it is aimed to determine which method gives better results in the analytical and simulation-based comparison of statistical power in meta-analysis, considering the characteristics of the sampling distribution of the correlation coefficient effect size. Also, it gives researchers an idea about the sample size, the number of studies and the population effect size required for a meta-analysis study in which correlation coefficient will be used as the effect size to have a statistical power at the level of 80% and above.

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The Declaration of Research and Publication Ethics

The authors of the paper declare that they comply with the scientific, ethical and quotation rules of SAUJS in all processes of the paper and that they do not make any falsification on the data collected. In addition, they declare that Sakarya University Journal of Science and its editorial board have no responsibility for any ethical violations that may be encountered, and that this study has not been evaluated in any academic publication environment other than Sakarya University Journal of Science.

REFERENCES

[1] F. Mosteller, G.A. Colditz, "Understanding research synthesis (Meta–analysis)", Annual review of public health, vol. 17, no. 1, pp. 1-23, 1996.

[2] L. V. Hedges, T. D. Pigott, "The power of statistical tests in meta-Analysis", The American Psychological Association, vol. 6, no.3, pp. 203-217, 2001.

[3] A. P. Field, "Meta-analysis of correlation coefficients: A Monte Carlo comparison of fixed- and random-effects methods", Psychological Methods, vol. 6, no. 2, pp. 161-180, 2001.

[4] L. D. Cohn, B. J. Backer, "How meta-analysis increases statistical power", The American Psychological Assosiation, vol. 8, no. 3, pp. 243-253, 2003.

[5] L. V. Hedges, T. D. Pigott, "The power of statistical tests for moderators in meta-analysis", The American Psychological Association, vol. 9, no. 4, pp. 426–445, 2004.

[6] G. Cafri, J. D. Kromrey, M. T. Brannick, "A SAS macro for statistical power calculations in meta-analysis" Behavior Research Methods, vol. 41, no. 1, pp. 35-46, 2009.

[7] J. C. Valentin, T. D. Pigott, H. R. Rothstein, "How many studies do you need? A primer on statistical power for meta-analysis", Journal of Educational and Behavioral Statistics, vol. 35, no. 2, pp. 215-247, 2010.

[8] J. Liu, "Statistical power in meta-analysis", Ph.D. dissertation, Dept. Educational Studies, South Carolina University, Carolina, USA, 2015.

[9] J. Liu, F. Pan, "A SAS Macro to investigate statistical power in meta-analysis", in Paper presented at the Southeast SAS Users Group, Savannah, GA, pp. 109-114., 2015.

[10] A. P. Field., "Meta-analysis of correlation coefficients: A Monte Carlo comparison of fixed- and random-effects methods", Psychological Methods, vol. 6, no. 2, pp. 161-180, 2001.

[11] H. Gamgam, Ö. Ünver, B. Atunkaynak, "Temel istatistik yöntemler", Seçkin Yayıncılık, Ankara, 2011.

[12] S. C. Chow, J. Shao, H. Wang, "Considerations Prior to Sample Size Calculation" in Sample size calculations in clinical research, Second ed., New York, USA, Boca Raton: Taylor & Francis, 2008, 25-49.

[13] F. Schmidt, "What do data really mean? Research findings, meta-analysis, and cumulative knowledge in psychology", American Psychologist, vol. 47, no. 10, pp. 1173-1181, 1992.

[14] L. Hedges, I. Olkin, "Statistical methods for meta-analysis", San Diego, CA, USA, Academic Press, 1985, pp. 224-237.

[15] J. Cohen, "Statistical power analysis for the behavioral sciences", Hillsdale, NJ, USA, Erlbaum, 1988, pp. 47-51.

[16] A.P. Field, "The problems of using fixed-effects models of meta-analysis on real-world data", Psychological Methods, vol. 6, no. 2, pp.161-18, 2003.

[17] M. Borenstein, L. V. Hedges, J. P. T. Higgins, H. R. Rothstein, "Effect Sizes Based on Correlations" in Introduction to meta-analysis, First ed., United Kingdom, John Wiley & Sons, 2009, pp. 41-44.