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CORRIGENDUM TO "ON RINGS WHERE LEFT PRINCIPAL IDEALS ARE LEFT PRINCIPAL ANNIHILATORS"

V. Camillo and W. K. Nicholson

[Int. Electron. J. Algebra, 17(2015), 199-214]

Abstract. We provide here correct versions of both Lemma 5.6 and Theorem 5.7 in the paper [Int. Electron. J. Algebra, 17(2015), 199-214]. Both Lemma 5.6 and Theorem 5.7 are false as stated, a counterexample in both cases being any regular ring that is not semisimple.

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Unless otherwise noted, every ring R is associative with unity. The left and right socles of R are denoted by S_l and S_r, and the left and right singular ideals by Z_l and Z_r. We write the left and right annihilators in R of a set X as $\mathtt{l}(X)$ and $\mathtt{r}(X)$, respectively.

There is an error in the paper [1] and both Lemma 5.6 and Theorem 5.7 are false as stated. These errors can be corrected as follows.

[1, Lemma 5.6] becomes true if we replace "left finitely Kasch" by "left Kasch". Hence it reads as

Lemma 5.6 Every left nonsingular, left Kasch ring R is semisimple.

Proof. Assume that $Z_l = 0$. If L is any left ideal of R we show that L is a direct summand of R. By Zorn's Lemma choose a left ideal M such that $L \oplus M$ is an essential left ideal in R; we show that $\mathtt{r}(L \oplus M) \subset Z_l$. If $a \in \mathtt{r}(L \oplus M)$, then $L \oplus M \subset \mathtt{l}(a)$. It follows that $\mathtt{l}(a)$ is essential left ideal in R; that is $a \in Z_l$. Hence $a = 0$ and so $\mathtt{r}(L \oplus M) = 0$. If $L \oplus M$ is a proper left ideal in R, $\mathtt{r}(L \oplus M) = 0$ is in contradiction with [2, Corollary 8.28]. Thus $L \oplus M = R$, as required. □

In 1968, Yohe [3, Theorem II] proved that a semiprime ring in which every one-sided ideal is principal is semisimple. The following Theorem 5.7 extends this if we add the hypothesis that the ring has the ascending chain condition on left principal annihilators $\mathtt{l}(a)$ for $a \in R$. Hence it reads as

Theorem 5.7 Let R be a semiprime, left pseudo-morphic ring. If R has the ascending chain condition on left principal annihilators $\mathtt{l}(a)$ for $a \in R$, then R is semisimple.

Proof. By Lemma 6.5 in [1], R becomes left Noetherian in this case and so it is left Kasch, being left finitely Kasch by Theorem 5.4 in [1]. Hence by Lemma 5.6, it suffices to show that $Z_l = 0$. Suppose that $0 \neq a \in Z_l$. Then $\mathtt{l}(a)$ is an essential left ideal in R, so $Ra \cap \mathtt{l}(a) \neq 0$. But $[Ra \cap \mathtt{l}(a)]^2 \subset (Ra)\mathtt{l}(a) = 0$, a contradiction because R is semiprime. □

References

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V. Camillo
Department of Mathematics
University of Iowa
Iowa City Iowa, 52242
e-mail: camillo@math.uiowa.edu

W. K. Nicholson
Department of Mathematics
University of Calgary
Calgary T2N 1N4, Canada
e-mail: wknichol@ucalgary.ca