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Fast Computation of Parameters of the Random Variable that is Logarithm of Sum of Two Independent Log-normally Distributed Random Variables

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Abstract

In this paper, two fast methods are proposed for computation of mean and variance of a random variable which is logarithm of two log-normally distributed random variables. It is shown that mean and variance can be computed using only one dimensional numerical integration method. The speed of the proposed algorithms is compared with the baseline algorithm. Simulation results showed that the first proposed method decreases the execution time by an average of 43.98 %. Simulation results also showed that the second proposed method is faster than the first proposed method for the variances greater than 0.325.

Keywords: Sum of log-normally distributed random variables, Parallel model combination, Numerical integration, Robustness.

İki Bağımsız Log-Normal Dağıtılmış Rastgele Değişkenin Toplamının Logaritması Olan Rastgele Değişkenin Parametrelerinin Hızlı Hesaplanması

Öz

Bu çalışmada, iki log-normal dağılımlı rasgele değişkenin logaritması olan rasgele değişkenin ortalama ve varyansını hesaplamak için iki hızlı metot sunulmuştur. Ortalama ve varyansın sadece bir boyutlu nümerik integral metodu ile hesaplanabileceği gösterilmiştir. Önerilen algoritmanın hızı temel algoritmanın hızı ile karşılaştırılmıştır. Benzetim sonuçları önerilen ilk yöntemin çalışma zamanını ortalama %43,98 azalttığını göstermiştir. Benzetim sonuçları ayrıca önerilen ikinci metodun 0,325'ten büyük varyanslar için birinci yöntemden daha hızlı olduğunu göstermiştir..

Anahtar Kelimeler: Log-normal dağılımlı rasgele değişkenlerin toplamı, Paralel model kombinasyonu, Nümerik integral, Gürbüzlük

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1. INTRODUCTION

The parameters of a random variable that represents the log of sum of two log-normally distributed random variables, are required to be estimated for some signal processing applications. These parameters can be used for estimating the distribution of sum of log normally distributed random variables [1,2], and for the Parallel Model Combination (PMC) [3-6] which is our main case for developing the methods proposed in this paper.

The sum of log-normal random variables has applications in many fields such as telecommunication [1,7,8], financial modelling [9], physics [10], and so forth. Many techniques have been developed for estimating distribution of sum of log-normally distributed random variables [1,2, 7,8]. Schwartz-Yeh [1] method and the method proposed in [2] need to use parameters of log of sum of log-normally distributed random variables. Therefore, methods proposed in this paper for estimating the parameters of the log of sum of two log-normally distributed random variables can be used for estimating the distribution of sum of log-normally distributed random variables [1,2].

The PMC is a technique for estimating the noisy speech models using the noise and clean speech models. Noise severely degrades the performance of speech recognition systems [11]. The PMC is one of the most effective techniques used for speech recognition under noisy conditions. In PMC, the noisy speech model parameters are estimated using the clean speech models and noise model. Estimating the noisy speech model parameters is almost the same as estimating the parameters of a random variable which is obtained by taking the logarithm of the sum of two log-normally distributed random variables. Therefore, the method proposed in this paper can be used as a part of numerical integration based PMC.

There are three different PMC techniques which are log-normal approximation [3], data-driven approach [4,5] and numerical integration [6]. The numerical integration technique estimates the noisy speech model parameters with the highest accuracy among the other PMC methods but demands the highest computation time. In this paper, we propose two new fast methods which can be used in PMC, for estimating the parameters (mean and variance) of logarithm of random variable which is obtained by adding two log-normally distributed random variables. Numerical integration-based PMC method is explained in [6], however, the accuracy of the estimated parameters and computational complexity of the numerical integration method are not discussed in this paper. In this paper, we discuss the accuracy and computational complexity of the proposed numerical integration methods.

2. ADDING TWO LOG-NORMALLY DISTRIBUTED RANDOM VARIABLES

Let $\boldsymbol{S_i}$and $\boldsymbol{N_i}$ be two independent Gaussian random variables with means $\boldsymbol{\mu_{s_i}}$, $\boldsymbol{\mu_{n_i}}$and variances $\boldsymbol{\sigma_{s_i}}$, $\boldsymbol{\sigma_{n_i}}$, respectively. We define a new random variable $\boldsymbol{O_i}$ such that

$$O_i = \log\left(e^{S_i} + e^{N_i}\right) = S_i + log\left(1 + e^{X_i}\right) \tag{1}$$

where $X_i=N_i-S_i$. X_i is also a Gaussian random variable with mean $\mu_{X_i}=\mu_{n_i}-\mu_{s_i}$ and variance $\sigma^2_{X_i}=\sigma^2_{n_i}+\sigma^2_{s_i}$ since S_i and N_i are Gaussian random variables. We want to compute the mean and variance of the random variable O_i. There is no closed form of solution for mean and variance. Two dimensional numerical integration can be used to compute mean and variance. However, dimension of integration can be reduced to one as follows. Let us drop the index i for the sake of simplicity. The mean is

$$\mu_o=\mu_s+E\left[\log(1+e^X)\right] \tag{2}$$

The variance is;

$$\begin{aligned}\sigma_o^2 &= E\left[\left(S+\log(1+e^X)\right)^2\right]-\mu_o^2\\ &=\sigma_s^2+E\left[2(S-\mu_s)\log(1+e^X)+\left(\log(1+e^X)\right)^2\right]\\ &\quad-\left(E\left[\log(1+e^X)\right]\right)^2 \qquad (3)\\ &=\sigma_s^2+E\left[2\rho^2(\mu_x-X)\log(1+e^X)+\left(\log(1+e^X)\right)^2\right]\\ &\quad-\left(E\left[\log(1+e^X)\right]\right)^2\end{aligned}$$

where $\rho=\frac{E\left[(X-\mu_x)(S-\mu_s)\right]}{\sigma_x\sigma_s}=\frac{-\sigma_s}{\sigma_x}$

3. COMPUTING THE MEAN AND VARIANCE USING GAUSS-HERMITE QUADRATURE

If the function f(x) is well approximated by a polynomial of order 2N-1, then Gauss-Hermite quadrature is a good estimate of the integral $\int_{-\infty}^{+\infty} f(x)e^{-x^2}$.

$$\int_{-\infty}^{+\infty} f(x)e^{-x^2} \approx \sum_{i=1}^{N} w_i f(x_i) \qquad (4)$$

In this case where x_i and w_i are Gauss-Hermite abscissa and weights, respectively [12] and N is the number of abscissa and weights used. It is known that if x_i is an abscissa then $-\boldsymbol{x_i}$ is also an abscissa [12]. This property of abscissa reduces the number of exponents by almost a factor of two since $e^{x_i}=1/e^{-x_i}$. The accuracies of μ_o and σ_o^2 which are computed using Equation 4 depend on how well the function $f(x)$ is approximated by a polynomial of order $2N$-*1*. In order to compute μ_o and σ_o^2 using Equation 4, We need to compute the following expectations:

$$E\left[log(1+e^X)\right]=\int_{-\infty}^{+\infty}\frac{log\left(1+e^{\mu_x+\sqrt{2}\sigma_x x}\right)}{\sqrt{\pi}}e^{-x^2}dx \qquad (5)$$

$$E\left[\left(\log(1+e^X)\right)^2\right]=\int_{-\infty}^{+\infty}\frac{\left(\log\left(1+e^{\mu_x+\sqrt{2}\sigma_x x}\right)\right)^2}{\sqrt{\pi}}e^{-x^2}dx \qquad (6)$$

and

$$E\left[(X-\mu_x)\log(1+e^X)\right]=\int_{-\infty}^{+\infty}\sqrt{\frac{2}{\pi}}\sigma_x x\frac{\log\left(1+e^{\mu_x+\sqrt{2}\sigma_x x}\right)}{\sqrt{\pi}}e^{-x^2}dx \qquad (7)$$

Computations of exponents and logarithms demand most of the computation time in computing μ_o and σ_o^2 using Gauss-Hermite quadrature. Therefore, we consider comparing only the number of exponents and logarithms. In order to compute μ_o and σ_o^2 using Equations 2-7 with N abscissa, computations of N logarithms, and $(\lfloor N/2\rfloor+1)$ exponents are required, where $\lfloor x\rfloor$ is the floor of x. In this paper, the algorithm which uses Equations 2-7 to compute μ_o and σ_o^2 is referred as the baseline method.

3. FAST COMPUTATION OF MEAN AND VARIANCE

In this paper, two methods for fast computation of the mean (μ_o) and variance (σ_o^2) are proposed. The first method is based on approximating the function $\log(1+e^x)$ for computing mean and variance using Gauss-Hermite quadrature. The latter method is based on approximating the functions $\log(1+e^x)$, $(\log(1+e^x))^2$, and the complementary error function erfc(x) for computing μ_o and σ_o^2.

We need to decide on the error criterion for approximating these functions. In this paper, maximum relative error is minimized to find approximate expressions for these functions. If f(x) is the function and $\hat{f}(x)$ is the approximation of f(x), then the relative error is defined as

$$\left|\frac{f(x)-\hat{f}(x)}{f(x)}\right| \qquad (8)$$

$\log(1+e^x)$ can be approximated as

$$log(1+e^x) \approx \sum_{i=1}^{K} a_i e^{ix} = \sum_{i=1}^{K} a_i \left(e^x\right)^i \quad for\; x \leq 0 \qquad (9)$$

where a_i's are chosen to minimize the error for the given criteria, and K is the number of coefficients. One exponent, one logarithm and one addition are needed to compute $\log(1+e^x)$. However, the number of arithmetic operations can be replaced by one exponent, $(K\text{-}1)$ additions, an $2(K\text{-}1)$d multiplications using equation (9). For x>0, $\log(1+e^x)$ can be computed using the equality $\log(1+e^x)=x+\log(1+e^{-x})$. Similarly, $[\log(1+e^x)]^2$ can be approximated as

$$\left[\log(1+e^x)\right]^2 \approx \sum_{i=2}^{M} b_i e^{ix} \qquad \text{for } x \leq 0 \qquad (10)$$

where b_i's are chosen to minimize the error for the given criteria, $(M-1)$ is the number of coefficients. $\mathrm{erfc}(x)$ can be approximated using

$$\mathrm{erfc}(x) \approx e^{-x^2} \sum_{i=1}^{R} c_i t^i \qquad \text{for } x \geq 0 \qquad (11)$$

where $t=\frac{1}{1+ax}$ and R is the number of coefficients. a, and c_i's are chosen to minimize the error between $\mathrm{erfc}(x)$ and the approximation of $\mathrm{erfc}(x)$ for the given criteria. $\mathrm{erfc}(x)$ can be computed using $\mathrm{erfc}(x)=2-\mathrm{erfc}(-x)$ for $x\leq 0$. For all the functions that were approximated, maximum relative error is minimized, and Parks-McClellan [13] algorithm is used to find the approximations of these functions. Table I shows the maximum relative approximation errors in percentage for 3, 4, 5 and 6 coefficients for the functions $\log(1+e^x)$, $(\log(1+e^x))^2$, and erfc(x).

Table 1. Maximum relative approximation errors in percent

#of coefficients	3	4	5	6
$\log(1+e^x)$	0.283	0.039	0.006	0.0008
$\left[\log(1+e^x)\right]^2$	0.948	0.152	0.024	0.0039
$\mathrm{erfc}(x)$	0.237	0.053	0.009	0.0017

4.1. Fast Computation of Mean and Variance Using Gauss-Hermite Quadrature

The baseline method requires computations of N logarithms, and $(\lfloor N/2 \rfloor+1)$ exponents for computing μ_o and σ_o^2 where N is the number of abscissa. These $(\lfloor N/2 \rfloor+1)$ exponents, and N logarithms can be replaced by only $(\lfloor N/2 \rfloor + 1)$ exponents by approximating the $\log\left(1+e^{\mu_x+\sqrt{2}\sigma_x x}\right)$ using Equation 9. This approximation significantly reduces computational complexity. We call this algorithm as fast version of Gauss-Hermite quadrature (fast version of baseline) method for computing μ_o and σ_o^2 in this paper.

4.2. Fast Computation of Mean and Variance by Approximating the Functions

Gaussian-Quadrature method approximates the integral. However, in this section, we propose to approximate the functions for fast computation of mean and variance. In order to compute μ_o and σ_o^2, we need to compute expected values of $\left[(X\text{-}\mu_x)\log(1+e^X)\right]$, $\log(1+e^x)$, and $(\log(1+e^x))^2$. Approximate values of these expected values can be computed as follows. We assume $\mu_x \leq 0$ for the sake of simplicity.

$$E\left[(X-\mu_x)\log(1+e^X)\right] = \int_{-\infty}^{+\infty} \sqrt{\frac{2}{\pi}}\sigma_x x \frac{\log\left(1+e^{\mu_x+\sqrt{2}\sigma_x x}\right)}{\sqrt{\pi}} e^{-x^2} dx \qquad (12)$$
$$\approx 0.5\sigma_x^2 \left[e^{-0.5\frac{\mu_x^2}{\sigma_x^2}} \left(\sum_{i=1}^{R} c_i t_0^i - \sum_{k=1}^{K} k a_k \sum_{i=1}^{R} c_i t_k^i \right) + \sum_{k=1}^{K} k a_k e^{0.5k^2\sigma_x^2+k\mu_x} \mathrm{erfc}\left(\frac{\mu_x + k\sigma_x^2}{\sqrt{2}\sigma_x} \right) \right]$$

where $t_k^i = 1/\left(1+a\left(\frac{-\mu_x+k\sigma_x^2}{\sqrt{2}\sigma_x}\right)\right)^i$. $\mathrm{erfc}(x)$ can be computed using Equation 11.

$$E\left[\log(1+e^{X})\right]=\int_{-\infty}^{+\infty}\frac{\log\left(1+e^{\mu_x+\sqrt{2}\sigma_x x}\right)}{\sqrt{\pi}}e^{-x^2}dx$$

$$=\int_{\frac{\mu_x}{\sqrt{2}\sigma_x}}^{+\infty}\frac{\log\left(1+e^{\mu_x-\sqrt{2}\sigma_x x}\right)}{\sqrt{\pi}}e^{-x^2}dx+$$

$$\int_{\frac{-\mu_x}{\sqrt{2}\sigma_x}}^{+\infty}\frac{\left(\mu_x+\sqrt{2}\sigma_x x+\log\left(1+e^{-\mu_x-\sqrt{2}\sigma_x x}\right)\right)}{\sqrt{\pi}}e^{-x^2}dx \quad (13)$$

$$\approx 0.5e^{-0.5\frac{\mu_x^2}{\sigma_x^2}}\left[\sigma_x\sqrt{\frac{2}{\pi}}+\mu_x\sum_{i=1}^{R}c_i t_0^i+\sum_{k=1}^{K}a_k\sum_{i=1}^{R}c_i t_k^i\right]$$

$$+0.5\sum_{k=1}^{K}a_k e^{0.5k^2\sigma_x^2+k\mu_x}\operatorname{erfc}\left(\frac{\mu_x+k\sigma_x^2}{\sqrt{2}\sigma_x}\right)$$

$$E\left[\left(\log(1+e^{X})\right)^2\right]=\int_{-\infty}^{+\infty}\frac{\left(\log\left(1+e^{\mu_x+\sqrt{2}\sigma_x x}\right)\right)^2}{\sqrt{\pi}}e^{-x^2}dx$$

$$=\int_{\frac{\mu_x}{\sqrt{2}\sigma_x}}^{+\infty}\frac{\left(\log\left(1+e^{\mu_x-\sqrt{2}\sigma_x x}\right)\right)^2}{\sqrt{\pi}}e^{-x^2}dx+$$

$$\int_{\frac{-\mu_x}{\sqrt{2}\sigma_x}}^{+\infty}\frac{\left(\mu_x+\sqrt{2}\sigma_x x+\log\left(1+e^{-\mu_x-\sqrt{2}\sigma_x x}\right)\right)^2}{\sqrt{\pi}}e^{-x^2}dx \quad (14)$$

$$\approx 0.5e^{-0.5\frac{\mu_x^2}{\sigma_x^2}}\begin{bmatrix}2\mu_x\sum_{k=1}^{K}a_k\sum_{i=1}^{R}c_i t_k^i-2\sigma_x^2\sum_{k=1}^{K}ka_k\sum_{i=1}^{R}c_i t_k^i+\\ \left(\mu_x^2+\sigma_x^2\right)\sum_{i=1}^{R}c_i t_0^i+\frac{\sqrt{2}\sigma_x}{\sqrt{\pi}}\left(\mu_x+2\sum_{k=1}^{K}a_k\right)+\\ \sum_{k=2}^{M}b_k\sum_{i=1}^{R}c_i t_k^i\end{bmatrix}$$

$$+0.5\sum_{k=2}^{M}b_k e^{0.5k^2\sigma_x^2+k\mu_x}\operatorname{erfc}\left(\frac{\mu_x+k\sigma_x^2}{\sqrt{2}\sigma_x}\right)$$

$e^{0.5k^2\sigma_x^2+k\mu_x}\operatorname{erfc}\left(\frac{\mu_x+k\sigma_x^2}{\sqrt{2}\sigma_x}\right)$ can be computed as

$$\begin{cases} e^{-0.5\frac{\mu_x^2}{\sigma_x^2}}\sum_{i=1}^{R}\frac{c_i}{\left(1+ay_k\right)^i} & if\ y_k\geq 0 \\ 2e^{0.5k^2\sigma_x^2+k\mu_x}-e^{-0.5\frac{\mu_x^2}{\sigma_x^2}}\sum_{i=1}^{R}\frac{c_i}{\left(1-ay_k\right)^i} & else \end{cases} \quad (15)$$

where $y_k=\left(\frac{\mu_x+k\sigma_x^2}{\sqrt{2}\sigma_x}\right)$. Computation of logarithm and exponent of a number using a computer takes a long time compared to addition, multiplication and division of numbers. Despite many additions, multiplications and divisions are used, only three exponents $\left(e^{\mu_x},\ e^{0.5\sigma_x^2},\ \text{and}\ e^{-0.5\frac{\mu_x^2}{\sigma_x^2}}\right)$ are used for computing μ_o and σ_o^2 in the method proosed in this section. Therefore, the method proposed in this section could demand less computation time compared to the Gauss-Hermite Quadrature method. We call the method proposed in this section as the second proposed method.

The method proposed in this section has an advantage and a disadvantage over the baseline or the fast version of the baseline method. The advantage is that, the percent relative error in σ_o^2 does not increase as σ_x^2 increases for the given number of coefficients used to compute σ_o^2 and μ_o using the method proposed in this section unlike the baseline or the fast version of baseline method.

The disadvantage is that, there are subtractions in computing σ_o^2 and μ_o using the method proposed unlike the baseline method or fast version of the baseline method. When we subtract one number from the other that are close to each other, there will be loss of significance [14]. When the value of σ_x^2 is small, there will be subtraction of one number from the other that are close to each other. Therefore, the relative error will increase substantially due to the loss of significance, when the value of σ_x^2 is small. As a result, for small values of σ_x^2, we may need to use more coefficients to keep the relative percent error under a prescribed value if we use the method described in this section. However, a few abscissa will be enough for computing σ_o^2 and μ_o for small values of σ_x^2 using the baseline method or fast version of baseline method. Experimental results which discuss these will be given in the next section.

5. EXPERIMENTAL RESULTS

Accuracy for both proposed methods and the baseline method depends on the parameters σ_x^2 and μ_x. Therefore, we must decide on ranges of σ_x^2 and μ_x. We must also decide on the maximum acceptable errors for σ_o^2 and μ_o. In this paper, the speeds of the proposed methods and baseline method were compared for $0 < \sigma_x^2 \leq 1000$, $-100 \leq \mu_x \leq 0$, and the maximum relative error in σ_o^2 less than 1%.

Since we use numerical integration method to compute the parameters, it is not possible to compute the exact values of the parameters. Consequently, we must decide on the error. The percent relative error criterion is used in the experiments. $100\left(\frac{\hat{\sigma}^2-\sigma^2}{\sigma^2}\right)$ gives the percent error for variance where σ^2 is the true variance and $\hat{\sigma}^2$ is the computed variance. However, percent error criterion is not appropriate for the mean since the value of mean could be zero. $100\left(\frac{\hat{\mu}-\mu}{\sigma}\right)$ could be a good criterion for the mean where μ is the true mean and $\hat{\mu}$ is the computed mean. Experimental results showed that when the error criterion for variance is satisfied, the error criterion for mean will also be satisfied. Therefore, we consider to satisfy only the error criterion for variance. After setting these error criteria, we can compare the computational complexity of the proposed methods and the baseline method.

Since the number of additions, subtractions and multiplications depend on the values of μ_x, and σ_x^2, it is not easy to compare computational complexity of the proposed methods and baseline method. Therefore, we executed the baseline algorithm and the proposed algorithms for estimating the parameters for 1000×1000 times on a computer with an intel i7 860 CPU without parallelizing the algorithm, and compared the execution time. To do this, the ranges of μ_x and σ_x^2 were divided into 1000 equally spaced values and for each value of μ_x the algorithm were run for these 1000 different σ_x^2 values.

We run an experiment to compare the execution time of baseline method and fast version of baseline method. Figure 1 shows the percent decrease in execution time for the fast version of baseline algorithm over the baseline algorithm for the number of abscissa from 3 to 190. We set the number of coefficients K as 5 for approximating $\log(1+e^X)$. There are 31.19% and 44.75% decreases in execution time for 3 and 190 coefficients, respectively. The average (over all coefficients) decrease in execution time is 43.98%.

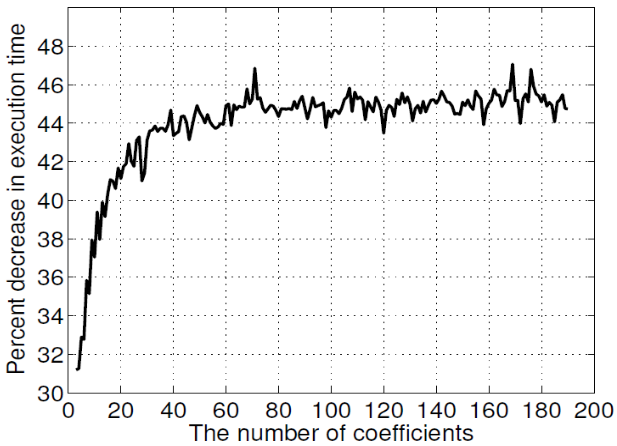

Figure 1. Number of coefficients versus percent decrease in execution time

We run an experiment to analyze the execution time compared to the number of coefficients. Figure 2 shows normalized execution time versus number of abscissa. The normalized execution time increases as the number of abscissa increase as expected since the number of exponents which demand most of execution time increases linearly as the number of abscissa increases.

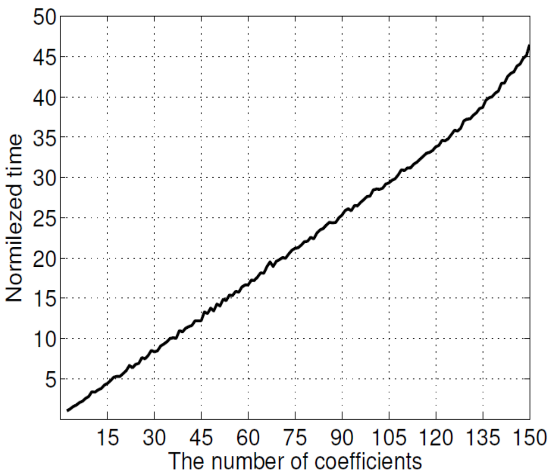

Figure 2. Number of coefficients versus normalized execution time for the fast version of baseline method

We run an experiment to find the maximum value of variance σ_x^2 that makes the maximum relative percent errorin σ_o^2 less than one. The main effects on the error are the values of σ_x^2, and μ_x for both baseline method and fast version of baseline method. We approximate $\log(1+e^X)$ for the fast version of baseline method. Since the approximation error for $\log(1+e^X)$ is very small (less than 0.00567% for K=5) the percent relative errors for both baseline and fast version of baseline method are almost same for the given σ_x^2 value, μ_x value, and number of abscissa. Figure 3 shows number of coefficients versus variance (σ_x^2) that makes the maximum relative percent error in σ_o^2 less than 1 when $-100 \leq \mu_x \leq 0$. Similarly, Figure 4 shows number of coefficients versus variance (σ_x^2) that makes the maximum relative percent error in σ_o^2 less than 1 when $-10 \leq \mu_x \leq 0$. From these figures, we can conclude that both σ_x^2 and μ_x have significant effects on the number of abscissa that keeps the relative percent error under one. The ranges of μ_x are from -100 to 0 for Figure 3 and from -10 to 0 for Figure 4. We can observe from Figure. 3 and Figure 4 that less coefficients are needed to keep the maximum relative percent error in σ_o^2 under one when the range of μ_x is small. We can also conclude from Figure 3 and Figure 4 that the execution time increases as the variance (σ_x^2) increases since more coefficients are needed to keep the relative percent error in σ_o^2 less than 1 for large values of σ_x^2.

Table 2 shows the same information for Figure 3 and Figure 4 in terms of number of abscissas from 2 to 11 in addition to the normalized time for the fast version of baseline method. The first column shows the number of abscissa, second column shows the maximum variance value that keeps the percent error in σ_o^2 under 1 for $-100 \leq \mu_x \leq 0$ for the given number of abscissa. Similarly, the third column shows the maximum variance value that keeps the percent error in σ_o^2 under one for $-10 \leq \mu_x \leq 0$ for the given number of abscissa. The last column shows the normalized execution time for the given number of abscissa.

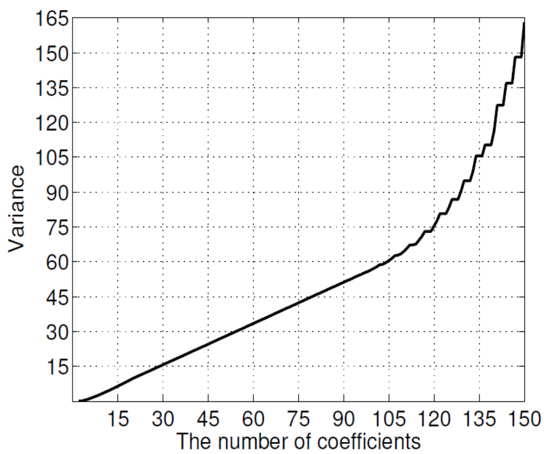

Figure 3. Variance σ_x^2 versus number of coefficients for $(-100 \leq \mu_x \leq 0)$, and relative error inσ_o^2 less than 1% for the fast version of baseline method.

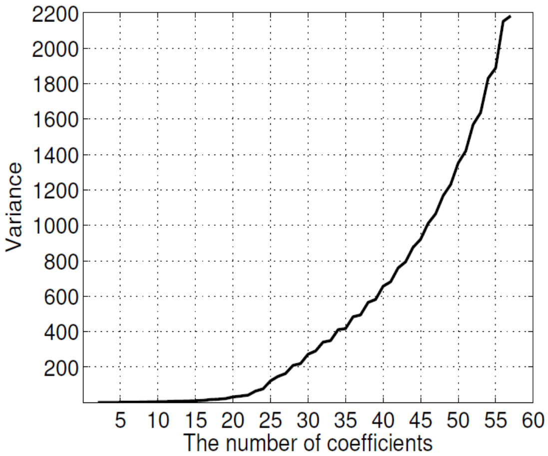

Figure 4. Variance σ_x^2 versus number of coefficients for $(-10 \leq \mu_x \leq 0)$, and relative error in σ_o^2 less than 1% for the fast version of baseline method

We used 3, 4, and 5 coefficients for approximations of $\log(1+e^x)$, $(\log(1+e^x))^2$, and $\mathrm{erfc}(x)$, respectively for computation of σ_o^2 and μ_o using the second proposed method. These coefficients are given in Table III. Finally, we run an experiment to see the speed and accuracy of the second proposed method. We measured the normalized execution time as 1.873 for this method. The good thing about the second proposed method is that the normalized execution time does not increase (1.873 seconds) as the variance σ_x^2 increases unlike the baseline and the fast version of baseline methods. The experimental results showed that the percent error in σ_o^2 is less than 1 when $\sigma_x^2 > 0.325$ and $-100 \leq \mu_x \leq 0$. From these results we realize that the fastest method which keeps the percent error in σ_o^2 less than one is the second proposed method for computing σ_o^2 and μ_o for $-100 \leq \mu_x \leq 0$ and $\sigma_x^2 > 0.325$. The fastest method is the fast version of baseline method for $\sigma_x^2 \leq 0.325$ as seen from Table II. The fast version of baseline method that uses 2, 3, 4, and 5 abscissa will be the fastest method for $\sigma_x^2 \leq 0.008$, $0.008 \leq \sigma_x^2 \leq 0.144$, $0.144 \leq \sigma_x^2 \leq 0.438$, and $0.438 \leq \sigma_x^2 \leq 0.830$, respectively for $-100 \leq \mu_x \leq 0$ as seen from Table II.

Table 2. Number of coefficients versus variances (σ_x^2) and normalized execution time for fast version of baseline method that keeps the relative percent error in σ_o^2 less than one

Functions	Index	Coefficients
$\log(1+e^x)$	1	0.9971742202972404545136
	2	-0.4437795339412708983673
	3	0.1417111754378272969746
$[\log(1+e^x)]^2$	2	0.9984854111176986179999
	3	-0.9510743713797964460355
	4	0.6370018861419275424396
	5	- 0.204687600754972720551
erfc(x)	1	0.3179095096078142779206
	2	0.3202728919600088541841
	3	0.2377829824350161658231
	4	0.2941637083449997192020
	5	-0.1702177063239194154676
	a	0.56353

Table 3. Coefficient values for approximation $\log(1+e^{x})$, $[\log(1+e^{x})]^{2}$, and $\mathrm{erfc}(x)$ which are used for the experiments

# of coef-ficients	Variance (σ_x^2) ($-100 \le \mu_x \le 0$)	Variance (σ_x^2) ($-10 \le \mu_x \le 0$)	Normalized time
2	0.008	0.008	1.0
3	0.144	0.144	1.231
4	0.438	0.438	1.531
5	0.830	0.832	1.723
6	1.282	1.291	2.023
7	1.773	1.819	2.208
8	2.292	2.659	2.554
9	2.835	3.395	2.777
10	3.397	4.199	3.385
11	4.574	5.096	3.331

6. CONCLUSIONS

Two new fast methods were proposed to compute the mean and variance of the logarithm of a random variable which is obtained by adding two log-normally distributed random variables. It is shown that the first proposed method which is called the fast version of baseline method is the fastest method for $\sigma_x^2 \le 0.325$ and $-100 \le \mu_x \le 0$, and the second proposed method is the fastest method for $\sigma_x^2 \ge 0.325$ and $-100 \le \mu_x \le 0$ which keeps the percent errors in σ_o^2 under one. In addition to this, the execution time for the second proposed method does not increase as the variance σ_x^2 increases unlike the baseline and the fast version of baseline method. The future work could be exploring fast algorithms for computing the covariance between the random variables which are logarithm of random variables obtained by adding two log-normally distributed random variables.

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