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An Investigation of the Complexity of the Problems Posed by Prospective Teachers: The Case of Whole Number

***Ahmet IŞIK* [a] *& Reyhan TEKİN SİTRAVA* [b*]**

a Prof. Dr., Gazi University, https://orcid.org/0000-0002-1599-2570

b* Assoc. Prof. Dr., Kırıkkale University, https://orcid.org/0000-0002-1285-2791 reyhantekin@kku.edu.tr

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Abstract

The aim of this case study is to examine the linguistic and mathematical complexity of the problems that prospective primary school teachers posed related to four operations with whole numbers. A Problem Posing Questionnaire was administered to 64 participants, and semi-structured interviews were conducted with 20 of 64 participants. The data were analyzed through descriptive analysis. The findings of the study revealed that many prospective teachers posed problems based on the single-statement problem root without any relation/condition. That is, the linguistic complexity of the problems written by the prospective teachers is in the lowest level, which is regarded as assignment category, regardless of the complexity of the problem situation. Furthermore, if the problem situations did not contain any relationship, many prospective teachers preferred to pose one-step problems. On the other hand, when the information in the problem situation was related to each other, then they were able to pose multi-step problems, which could be regarded as complex problems in terms of mathematically. It can be concluded that although the complexity of the problem situation given to the prospective teachers does not affect the linguistic complexity of the problems they pose; it affects the mathematical complexity.

Keywords: Problem posing, linguistic complexity, mathematical complexity, whole numbers, prospective primary school teachers.

Sınıf Öğretmeni Adaylarının Kurdukları Problemlerin Karmaşıklığının İncelenmesi: Doğal Sayılar Örneği

Öz

Bu durum çalışmasının amacı, sınıf öğretmeni adaylarının doğal sayılarda dört işlemle ilgili kurdukları problemlerin dilsel ve matematiksel karmaşıklığını incelemektir. 64 katılımcıya Problem Kurma Anketi uygulanmış ve 64 katılımcıdan 20'si ile yarı yapılandırılmış görüşmeler yapılmıştır. Veriler betimsel analiz yoluyla analiz edilmiştir. Araştırmanın bulguları, birçok öğretmen adayının herhangi bir ilişki/koşul olmaksızın tek ifadeli problem köküne dayalı problemler kurduklarını ortaya koymuştur. Yani öğretmen adayları tarafından yazılan problemlerin dilsel karmaşıklığı, problem durumunun karmaşıklığına bakılmaksızın, ödev kategorisi olarak kabul edilen en düşük düzeydedir. Ayrıca problem durumları herhangi bir ilişki içermiyorsa birçok öğretmen adayı tek adımlı problemler kurmayı tercih etmiştir. Öte yandan, problem durumundaki bilgiler birbiriyle ilişkili olduğunda matematiksel olarak karmaşık problemler olarak kabul edilebilecek çok adımlı problemler kurabilmişlerdir. Öğretmen adaylarına verilen problem durumunun karmaşıklığı, oluşturdukları problemlerin dilsel karmaşıklığını etkilemese de matematiksel karmaşıklığı etkilemektedir.

Anahtar kelimeler: Problem kurma, dilsel karmaşıklık, matematiksel karmaşıklık, doğal sayılar, ilkokul öğretmen adayları.

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INTRODUCTION

In more than two decades, problem-posing has become central to teaching and learning mathematics (English, 2019; National Council of Teachers of Mathematics [NCTM], 2014). The mathematics educators and researchers argued that knowing and understanding mathematics is strongly related to generating and exploring problems (Cai & Hwang, 2019). However, in most math classes, the problems are presented in textbooks and the students have to solve these problems with the aim of reaching the correct solution. It is claimed that these kinds of mathematics activities are not effective tools to ensure students comprehend mathematics (Crespo & Sinclair, 2008). The general consensus among mathematics educators is that it is vital for teachers and prospective teachers to change the types of activities related to problems in order to provide their students with more efficient learning experiences. In relation to that, Demirci (2018) suggested that the teachers and prospective teachers need to have such experiences including different kinds of activities as mathematics learners. One of the kinds of activities that the prospective teachers and the teachers need to have is the practice of problem posing, which has several benefits in teaching mathematics (Cai et al., 2019; Xu et al., 2019).

Problem posing helps teachers develop students' mathematical understanding, mathematical reasoning, and creativity (Singer et al., 2017). Moreover, it allows the students to explore the relationship among mathematics concepts, changing existing situations to new ones, and checking the solvability and complexity of the problems (English, 2019; Xie & Masingila, 2017). Besides, problem posing helps teachers realize students' misconceptions and their reasons, improve students' problem-solving ability, evaluate students' learning, and develop their content knowledge and pedagogical content knowledge (Chen & Cai, 2019; Lin, 2004; Xu et al., 2020). Due to the several benefits of the practice of problem posing, many countries have included problem-posing activities in their mathematics curriculum (Australian Education Council and Curriculum Corporation, 1991; National Council of Teachers of Mathematics [NCTM], 2000; Ministry of National Education, [MoNE], 2018). In addition to including problem-posing activities in the mathematics curriculum, it is also important to include such activities in teacher education programs to make the prospective teachers pose problems having different levels of complexity, to evaluate the problems posed by their students, and to decide what makes some problems better than others (Chen & Cai, 2020; Lee et al., 2019). Depending on these, it is important to assess the problems posed by prospective teachers, the solvability of the problems, and the level of complexity of them, too.

In order to assess problem-posing ability of prospective teachers, teachers, and students, a variety of research studies were conducted using different kinds of problem-posing classification (Christou et al., 2005; Silver, 1994; Silver & Cai, 1996; Stoyanova & Ellerton, 1996). Among these classifications, Silver emphasized that problem posing is the generation of new problems and takes place before, during, or after problem-solving. Accordingly, pre-solution, within-solution, and post-solution are the classification of problem posing. Different from this categorization, Stoyanova and Ellerton (1996) categorized problem-posing tasks as free problem posing, semi-structured problem posing, and structured problem posing. Grounded on Stoyanova and Ellerton's classification, Christou and his colleagues presented another classification focusing on the cognitive processes of the students. They defined semi-structured problem posing as editing and translating, and structured problem posing as selecting and comprehending. Finally, Silver and Cai (1996) analyzed problems in terms of complexity by categorizing them as assignment, relational, and conditional problems, which constitute the present study's theoretical framework. Although assignment type of problems involves only one statement without having a relationship among the issues, concepts, numbers, etc., relational and conditional problems are composed of relationship or condition. For instance, the problem of how many pencils Katrin has is regarded as an assignment, whereas the problem of how many more pencils Marc has compared to Katrin is considered relational. Lastly, the problem, if Katrin has twice as many pencils as Paul, then how many pencils Katrin has is regarded as conditional. Among these problem types, Mayer et al. (1992) specified that conditional and relational problems are more difficult than assignments for students to solve.

Due to the vital role of problem-posing activities in teaching and learning mathematics, many researchers aimed to investigate the problems posed by students, teachers, and prospective teachers. For instance, Van Harpen and Sririman (2013) aimed at investigating high school students' problem-posing abilities. They concluded that the students posed problems including inadequate information to solve the problem or they posed simple problems. In another study, Ngah et al. (2016) explored students' problem-posing ability in free, semi-structured, and structured problem-posing situations. Based on the data analysis, they concluded that the free problem-posing situations were more challenging tasks compared to semi-structured and structured problem-posing situations. In other words, the problem situations including more restrictions and directions are more likely easier tasks for students. Additionally, some researchers focused on teachers' and prospective teachers' problem-posing abilities. Most of these studies were related to the difficulties that prospective teachers faced while posing problems (e.g. Chapman, 2012; Kilic, 2013; Köken & Gökkurt-Özdemir, 2018), the problem-posing performance of prospective teachers (e.g. Albayrak et al., 2006), and the most effective strategies that can be used to improve the problem-posing skills of prospective teachers and their problem-posing skills (Abu-Elwan, 1999). Moreover, some studies were conducted to reveal the complexity of the problems posed by teachers and prospective teachers. For instance, Isik et al. (2011) identified the problems posed by prospective teachers in relation to verbal and visual representations. As a result of the study, they stated that most of the problems posed by prospective teachers were in the category of assignment rather than in the category of conditional and relational. According to Mayer et al.'s statement, it could be specified that these teachers tended to pose easy and simple problems. Furthermore, the study conducted with prospective teachers by Tekin-Sitrava and Işık (2018) reached a similar conclusion. They stated that the prospective teachers established low-level problems due to having superficial content knowledge, inadequate problem-solving and problem-posing experience, and insufficient creativity ability. However, high-level problems have a vital role in developing students' mathematical understanding, mathematical reasoning, and teaching and learning mathematics (English, 1998; Silver, 1994; Stoyanova, 2003). From this point of view, the complexity of the problems presented during the mathematics lessons gained great importance. In this context, when it is considered that the problems posed by prospective teachers and teachers are used as teaching materials, it would be significant to investigate the level of complexity that the problems posed by prospective teachers and teachers. Owing to the fact that the prospective teachers are the future teachers, it would be significant to explore the problems posed by them from different perspectives both to gain insight about how to guide the students to pose problems and to make recommendations to the teacher education programs. In this sense, the study aims to investigate the problems that prospective primary teachers posed in relation to four operations with whole numbers from the point of the appropriateness to the given problem situation, its solvability, linguistic (verbal) and mathematical complexity. Thus, the present study seeks answers for the following research questions.

1. What is the level of linguistic complexity of the problems posed by prospective primary school teachers in the context of four operations with whole numbers?

2. What is the level of mathematical complexity of the problems posed by prospective primary school teachers in the context of four operations with whole numbers?

METHOD

Research Design

The design utilized in the present study is a case study providing an in-depth analysis and description of a case within a bounded system (Creswell, 2007). Concerning case study design, Yin (2003) proposed a more detailed framework focusing on the number of cases and the number of units of analysis. Of the different types of case study design, the single-case holistic design was employed which is defined as consisting of one single case and one unit of analysis. The single case was prospective primary school teachers and the unit of analysis was the problems that prospective teachers posed.

Context of the Study

The aim of the case study is to identify and choose the most suitable cases with regard to knowledge and experience related to the phenomenon of interest to acquire rich and comprehensive data (Creswell & Clark, 2011). Herein, the study context is the primary teacher education program in which prospective primary school teachers were educated. In this context, the study was conducted within the primary school education (grades 1-4) degree program in a public university in Central Anatolia, Turkey.

The Primary School Education Program, designated by Higher Education Institution (HEI, 2007), is a four-year program that qualifies graduates working as primary school teachers to teach grades 1-4. The program includes content courses (e.g. Mathematics, Physics, and Chemistry), education courses (e.g. Introduction to Educational Sciences, Education Psychology, Teaching Principles and Methods) and mathematics education courses (e.g. Science and Technology Teaching, Mathematics Teaching, Social Studies Teaching). The prospective teachers in this program take content courses and education courses mostly during the first two years of the program and take mathematics education courses during the final two years. The courses assumed to develop prospective teachers' problem-posing abilities are compulsory courses, which are mathematics content courses (Basic Mathematics I-II), mathematics education courses (Mathematics Teaching I-II), and education courses

(School Experience, Teaching Practice I-II). While the topics of Basic Mathematics courses include the basic concepts related to numbers, four operations, algebra, geometry, and data analysis; Mathematics Teaching courses comprise how to teach mathematics belonging to primary school mathematics teaching program. Moreover, within the scope of School Experience and Teaching Practice courses, prospective teachers observe the mathematics lessons in the real classroom environment, plan and prepare the mathematics lessons, and acquire teaching experience under the guidance of mentor teachers. During taking these courses, they become acquainted with the content of problems related to basic four operations, and also posing problems.

Participants

Participants were chosen through the purposive sampling method which allows researchers to choose participants intentionally who are proficient and have well-developed knowledge about the topic of interest (Creswell & Clark, 2011). Accordingly, the participants were selected based on two criteria: information-rich and being experienced in real classrooms. In order to select information-rich participants, the courses that they took were considered. Since the content of the Basic Mathematics and Mathematics Teaching courses cover the formation and structural properties of natural numbers, the acquisitions of arithmetic operations, problem types, problem-solving, and posing strategies, it was important to take these courses before the data collection period. Moreover, it is only possible for prospective teachers to be experienced if they have taken the teaching practice courses. Of these courses, the prospective teachers take the School Experience course in the fall semester of the third year and the Teaching Practice course in the fall semester of their fourth year. Thus, the prospective teachers who are in the 4th year of their teacher education program have taken the school experience course and are taking the Teaching Practice course. Under such circumstances, in order to ensure the prospective teachers are experienced in real classrooms, fourth-grade prospective primary school teachers were eligible. From this point of view, 64 prospective teachers who volunteered to participate in the study were selected. Pseudonyms (i.e., PST1, PST2…PST64) were used for all participants to ensure confidentiality.

Data Collection Tools

In order to investigate the problems posed by prospective primary school teachers related to basic mathematical operations with whole numbers, a problem-posing questionnaire and semi-structured interviews were applied to participants.

A Problem Posing Questionnaire consisting of 2 problem situations was prepared by the researchers with respect to the learning outcomes related to the four operations with whole numbers presented in the 4th-grade primary school mathematics curriculum (MoNE, 2018). Both problem situations in the questionnaire consisted of more than one situation in order to explore the complexity of the problems. The questionnaire, which is presented in Table 1, offers opportunities for prospective teachers to pose problems with linguistic complexity and mathematical complexity.

In order to ensure the validity and reliability of the Problem Posing Questionnaire, firstly, expert opinion was obtained from 2 mathematics educators who are experts in the field. After the necessary corrections were made in line with the opinions of the experts, a pilot study was conducted by applying the Problem-Posing Questionnaire to the prospective teachers who did not participate in the study. As a result of the pilot study, the questionnaire has been finalized.

Table 1. Problem Posing Questionnaire

1) Pose two problems at different levels according to the situation given below.

   A supermarket has 30 kilograms of rice, 50 kilograms of sugar, and 45 kilograms of flour. 6 kilograms of rice, 5 kilograms of sugar, and 3 kilograms of flour are sold every week.

2) Pose three problems using the situation given below.

   Zeynep, Beril and Nil went shopping together. Nil spent 200 Turkish Liras more than Beril. Beril's money is twice the money Zeynep spends. Zeynep spent 75 Turkish Liras.

After administering the questionnaire to 64 prospective teachers, the data was analyzed and the problems having different levels of complexity were determined. Based on this analysis, 20 volunteer prospective teachers were purposefully selected for semi-structured interviews. While some of the 20 prospective teachers were unable to pose an appropriate problem for the learning outcome and the given problem situation, the others posed problems with different levels of complexity. During the interviews, prospective teachers who posed different types of problems were first asked to explain the relationship between the problem and the given problem situation.

Afterwards, the questions such as "What kind of way did you follow while posing the problem?", "What did you think?", "Can you pose another problem with the given problem situation?" were asked to prospective teachers. Also, it was asked prospective teachers, who could not pose an appropriate problem, to read the problem and try to pose a problem. Thus, it was aimed to procure rich and comprehensive data and triangulate the data gathered from the questionnaire through the interviews.

Data Analysis

To attain the aim of the study, the data from the questionnaire and semi-structured interviews were analyzed through descriptive analysis proposed by Strauss and Corbin (1990). They specified that the data gathered from a variety of data sources is summarized and interpreted based on predetermined themes through descriptive analysis method. Moreover, in descriptive analysis method, the data are presented using direct quotations to reflect participants' understanding, views, and beliefs related to the issue of the study. Thus, the model proposed by Silver and Cai (1996) formed the basis of the data analysis. In addition, frequencies are presented to see the number of the problem types posed by the prospective teachers. As a first step of the analysis, the prospective teachers' responses were analyzed in terms of their appropriateness to the problem situation. Accordingly, the prospective teachers' responses were categorized as "appropriate to the given problem situation", "not appropriate to the given problem situation" and "empty". As a second step of the data analysis, the responses which were coded as appropriate to the given problem situation were analyzed with regard to solvability and coded as "solvable" and "unsolvable". The responses containing incomplete information or that cannot be solved with written expressions were examined under the category of unsolvable and the analysis of these responses was not continued. In the third step of the data analysis, the responses coded as solvable were analyzed from the point of linguistic and mathematical complexity according to the model proposed by Silver and Cai (1996). Accordingly, the solvable problems were coded as "assignment", "relational" and "conditional" based on their linguistic content. Lastly, if the solution of the problem necessitated one operation, then it was coded as a "one-step problem" and if the solution necessitated more than one operation, then it was coded as a "multiple-step problem". The data analysis was done by two mathematics educators to ensure the trustworthiness of the study. Although the inter-rater reliability, which was 81%, was acceptable due to over 70% (Miles & Huberman, 1994), the inconsistencies were discussed until a 100% consensus was reached among the coders.

Research Ethics

The approval of the Kırıkkale University Social and Behavioral Sciences Ethics Committee was obtained for ethical compliance with the research procedures.

FINDINGS

In this study, it was aimed to investigate the types of problems that prospective primary teachers posed in relation to four operations with whole numbers. In this sense, the appropriateness to the given problem situation, its solvability, linguistic (verbal) and mathematical complexity are presented on the basis of problem situations in the problem-posing questionnaire.

Findings Related to the First Problem-Posing Situation

In the first problem-posing situation, it was given prospective primary school teachers a problem situation indicating that **"A supermarket has 30 kilograms of rice, 50 kilograms of sugar and 45 kilograms of flour. 6 kilograms of rice, 5 kilograms of sugar and 3 kilograms of flour are sold every week"** and they were asked to pose two problems, coded as 1A and 1B, at different levels suitable for this situation. Based on the data analysis process, the frequencies and percentages obtained from the analysis of the responses written by the prospective teachers are given in Table 2.

Table 2. Frequency analysis of the 1st problem posing situation

	Appropriateness to the given problem situation								
	Appropriate							Not Appropriate	Empty
	Unsolvable	Solvable							
		Assignment		*Relational*		*Conditional*			
		One-Step	Multiple-Step	One-Step	Multiple-Step	One-Step	Multiple-Step		
1A	2 (%3.13)	31 (%48,44)	11 (%17,19)	1 (%1,56)	1 (%1,56)	0	1 (%1,56)	13 (%20,31)	4 (%6,24)
1B	3 (%4,68)	18 (%28,13)	8 (%12,5)	2 (%3,13)	8 (%12,5)	0	4 (%6,25)	17 (%26,56)	4 (%6,25)

As can be seen in Table 2, most of the statements written by prospective primary school teachers are in the category of solvable problems. The statements in this category are categorized linguistically (verbal) as assignments, relational and conditional, and mathematically as one-step and multiple-step problems. According to the results of the analysis made in this direction, the number of problems in the one-step assignment is higher than the others. In other words, most of the problems are routine problems, do not contain any relation or conditional situation, and can be solved with only one basic operation. An example of problems in the one-step assignment category is given below.

The problem of Prospective Teacher 3 (PT 3)

A supermarket has 30 kilograms of rice, 50 kilograms of sugar, and 45 kilograms of flour. 6 kilograms of rice, 5 kilograms of sugar, and 3 kilograms of flour are sold every week. Ahmet sells 5 kg of sugar each week. If Ahmet has 50 kg of sugar, then in how many weeks does Ahmet sell all the sugar?

As seen in the example, PT3's problem is a single-expression problem that can be solved with one step, where there is no relation or condition between the statements. For this reason, the problems of prospective teachers who posed problems in this way were evaluated in the one-step assignment category.

Among the prospective teachers who posed 2 problems, the maximum number of prospective teachers who posed relational problems was 8. The problem posed by PT 29 is presented below as an example of multiple-step relational problem.

The problem of PT 29

A supermarket has 30 kilograms of rice, 50 kilograms of sugar, and 45 kilograms of flour. 6 kilograms of rice, 5 kilograms of sugar, and 3 kilograms of flour are sold every week. After 6 weeks, how much more sugar is sold than the flour?

In the problem posed by PT29, a relationship has been established between the amount of sugar and the amount of flour, and the problem can be solved by performing more than one step. Therefore, the problems similar to PT29's are regarded as in the relational multi-step problems category.

On the other hand, as seen in Table 1, the number of prospective teachers who posed conditional problems for the 1st problem posing situation is quite limited. As an example of conditional problems, the problem written by PT26 is given.

The problem of PT 26

A supermarket has 30 kilograms of rice, 50 kilograms of sugar, and 45 kilograms of flour. 6 kilograms of rice, 5 kilograms of sugar, and 3 kilograms of flour are sold every week. How many kilograms of rice, sugar, and flour should be bought at the supermarket so that all three products are finished at the same time?

Due to the fact that it is asked how many kg of flour, sugar, and rice should be taken from these products to the supermarket on the condition that they finished at the same time, PT26's problem is in the conditional problem category. Also, since the problem can be solved with more than one step, it is coded as a multi-step problem.

In addition, at least 20% of the prospective primary school teachers could not write any problem statement appropriate to the given situation. The problem posed by PT12 is below as an example of the responses evaluated in this category.

The problem of PT12

Since a kilogram of rice is 5 TL, a kilogram of sugar is 7 TL, and a kilogram of flour is 4 TL, how much rice, sugar, and flour are sold in 3 weeks?

Although the problem given above is a problem that can be solved with four operations in whole numbers, 30 kg of rice, 50 kg of sugar, and 45 kg of flour were not used in the problem. However, since prospective primary school teachers had to pose problems using the given problem situation, the problem of PT12 and similar problems were evaluated as not appropriate to the given problem situation.

In addition, some prospective teachers' responses were evaluated in the category of unsolvable. As an example of the responses in this category, the response of PT7 is given below.

The problem of PT7

A supermarket has 30 kilograms of rice. Since 5 kilograms of rice are sold each week, how many kilograms of sugar would be sold in the second week?

PT7 stated that 6 kg of rice is sold every week, but asked how many kg of sugar is sold in the second week. Since it did not give any information about the amount of sugar sold each week, the amount of sugar sold in the 2nd week could not be calculated; therefore, this response was coded as unsolvable.

In the 1st problem-posing situation, the majority of the prospective primary teachers posed problems which could be solved with one-step. More specifically, since most of these problems include only one statement, they lie in the category of assignment.

Findings Related to the Second Problem-Posing Situation

The second problem-posing situation, "**Zeynep, Beril, and Nil went shopping together. Nil spent 200 Turkish Liras more than Beril. Beril's money is twice the money Zeynep spends. Zeynep spent 75 Turkish Liras",** was presented to pose three problems with different levels, labeled as 2A, 2B and 2C. The findings related to the second problem-posing situation are indicated in Table 3.

Table 3. Frequency Analysis of the 2nd Problem Posing Situation

	Appropriateness to the given problem situation								
	Appropriate							Not Appropriate	Empty
	Unsolvable	Solvable							
		Assignment		*Relational*		*Conditional*			
		One-Step	Multiple-Step	One-Step	Multiple-Step	One-Step	Multiple-Step		
1A	1(%1,56)	14 (%21,88)	37 (%57,81)	0	1 (%1,56)	0	2 (%3,13)	5 (7,81)	4 (%6,25)
2B	3(%4,68)	6 (%9,38)	26 (%40,62)	0	15 (%23,43)	0	6 (%9,38)	6 (%9,38)	4 (%6,25)
2C	2(%3,13)	3 (%4,68)	24 (%37,5)	0	13 (%20,31)	0	7 (%10,94)	8 (%12,5)	7 (%10,94)

Table 3 presents that most of the responses of the prospective primary teachers are appropriate to the given problem situation. When the problems appropriate to the given situation are evaluated in terms of solvability, it is seen that almost all of them are solvable problems. However, the majority of such problems are in the assignment category. As an example of the problems in the assignment category, the problem written by PT4 is given below.

The problem of PT4

Zeynep, Beril, and Nil went shopping together. Nil spent 200 Turkish Liras more than Beril. Beril's money is twice the money Zeynep spends. Zeynep spent 75 Turkish Liras. According to this, how much did Nil spend?

Since the problem given above does not contain any relations or conditions, it was regarded as an assignment. However, since more than one step is required to solve the problem, it was handled as a multi-step problem.

Although the number of prospective teachers who posed multi-step relational and conditional problems in the second problem-posing situation is higher than the first problem-posing situation, there are no teachers who posed one-step relational and conditional problems. The problems posed by PT20 as an example of relational problems and by PT50 as an example of conditional problems are given below.

The problem of PT20

Zeynep, Beril, and Nil went shopping together. Nil spent 200 Turkish Liras more than Beril. Beril's money is twice the money Zeynep spends. How much is Zeynep, Beril, and Nil's total money more than Beril's money?

The problem of PT50

Zeynep, Beril, and Nil went shopping together. Nil spent 200 Turkish Liras more than Beril. Beril's money is twice the money Zeynep spends. If Nil had spent 25 TL less than the total money spent by Zeynep and Beril, how much money would three of them have spent?

The problem posed by PT20 is a problem that can be solved with more than one step by establishing a relationship between the total money of Zeynep, Beril and Nil and Beril's money. PT50, on the other hand, put a condition in the problem statement by using the statement "if Nil had spent 25 less than the total money spent by Zeynep and Beril". Also, PT50 posed the problem in such a way that it can be solved with multiple steps.

Apart from these problems, some prospective teachers could not pose a problem appropriate to the given problem situation. As an example to these, the problem that PT9 has posed is given as an example below.

The problem of PT9

Nil and Beril went to shopping. Nil spent 400 TL. If Beril spent 200 TL more than Nil, how much did Nil and Beril spend in total?

Although they were asked to pose a problem appropriate to the situation presented in the problem-posing questionnaire, PT9 ignored this situation and posed a problem related to a different situation by using the same names. For this reason, the problem of PT9 and the problems of prospective teachers who have problems similar to PT9 are considered as not appropriate to the given situation.

On the other hand, less than 5% of the prospective teachers participating in the study posed unsolvable problems. In the following, the problem of PT34 was given as an example.

The problem of PT34

How much money does Zeynep, Nil, and Beril have before going shopping?

Although the problem posed by PT34 is appropriate to the given situation, the information given in case of a problem is related to the money Zeynep, Beril, and Nil spend in shopping. There is no information required to calculate how much money Zeynep, Beril, and Nil have before going shopping. For this reason, this problem has been evaluated as an unsolvable problem. Apart from this, approximately 10% of the prospective teachers could not establish any problem for the 2nd problem situation.

Different from the 1st problem posing situation, most of the prospective teachers posed multi-step problems in the 2nd problem posing situation. However, in terms of linguistic complexity, the majority of prospective teachers posed the problems in the assignment category as in the case of 1st problem posing situation.

DISCUSSION & CONCLUSION

In this study, the types of problems that prospective primary teachers posed in relation to four operations with whole numbers, the solvability of these problems, and their linguistic (verbal) and mathematical complexity were examined.

Based on the analysis of the data, it was concluded that the vast majority of prospective teachers posed solvable verbal problems in accordance with both problem situations. Similar results were presented in the literature which emphasized that most of the teachers and prospective teachers posed verbal problems (Kilic, 2013; Korkmaz & Gür, 2006). The linguistic complexity of the problems posed by the prospective teachers was evaluated based on the classification stated in the study of Silver and Cai in 1996. As a result of this evaluation, it was seen that most of the prospective teachers wrote problems based on the single-statement problem root, without any relation or condition. In other words, the linguistic complexity of the problems written by the prospective teachers is in the lowest level which is regarded as the assignment category, regardless of the complexity of the problem situation. Similarly, in their study, Işık, Işık, and Kar (2011) reached the conclusion that the problems that most of the prospective teachers had posed were in the assignment category. From the point of mathematical complexity, the problems posed by prospective teachers vary according to the given problem situation. More specifically, if the problem situations do not contain any relationship as it is the case of the first problem situation, then the majority of prospective teachers preferred to pose one-step problems. On the other hand, as in the case of the second problem-posing situation, when the information contained in the problem situation is related to each other, then they were able to pose multi-step problems which could be regarded as complex problems in terms of mathematically. In line with these results, it can be reached an outstanding conclusion which is related to the relationship between the complexity of the given problem situation and the linguistic and mathematical complexity of the problems posed by prospective teachers. In other words, it can be concluded that although the complexity of the problem situation given to the prospective teachers does not affect the linguistic complexity of the problems they pose; it affects the mathematical complexity of the problem.

In addition, it was observed that some prospective teachers could not pose any problems in both problem situations, some of them could not set up appropriate problems for the given situation, or some of them posed problems containing data deficiency. One of the reasons for prospective teachers not to be able to pose verbal solvable problems may be their inability to analyze and interpret mathematical expressions in the given problem situation. To be more detailed, it can be said that the prospective teachers were unable to formulate given problem situations into new situations by establishing relationships between mathematical concepts. Similar results were also demonstrated in the literature. For instance, Kılıç (2013) stated that prospective teachers had difficulties in choosing the correct numbers and concepts and establishing relationships between them while posing problems. Moreover, Örnek and Soylu (2017) emphasized that the prospective teachers could pose problems containing the addition of fractions; however, they could not establish a relationship between the given fractions in the problem-posing situation and the result of the problem that they pose. In relation to this, mathematics educators emphasized that analyzing given situations, and exploring and formulating them into new situations are the basis of problem-posing (Abu-Elwan, 1999; Silver, 1994). In addition, it is revealed that the prospective teachers who cannot set up problems in accordance with the given situation or who set up problems that contain data deficiency do not control the problem in terms of solvability after establishing the problems. This result is consistent with the result of Örnek and Soylu’s (2017) study which concluded that the prospective teachers did not check whether the problem they posed could be solved or not. However, English (1998) stated that one of the important features of problem posing is checking the correctness of the problem. In addition, problem posing is highly correlated with the content knowledge that teachers have (Demirci, 2018; Örnek & Soylu, 2021; Van Harpen & Presmeg, 2013). In line with this information, another reason for prospective teachers to pose problems that could not be solved may be their inadequate content knowledge.

One of the important findings of the study is that prospective primary school teachers tend to pose problems based on the single-statement problem root in terms of linguistic complexity. These types of problems are not only important for students to understand the problem sentence, but also easier ones. However, it is important that students not only solve problems that can turn the problem sentence into simple operations, but also solve problems that will enable them to think multi-faceted, explore mathematical situations, and make mathematical reasoning (English, 1998; Silver, 1994). From this point of view, it is necessary to provide opportunities for prospective teachers to pose problems based on relational and conditional problems, as well as posing single-statement problem roots. In this direction, taking into account the gains in the elementary school mathematics teaching program, problem-posing activities can be given more place in 'Mathematics Teaching I' and 'Mathematics Teaching II' courses. In addition, within the scope of these courses, it can be emphasized that problem posing has a very important place in mathematics teaching and learning (English, 1998; MONE, 2018; NCTM, 2000), and the necessity of including problem posing activities in their classes can be emphasized. In addition, it can be suggested that the knowledge and beliefs of the prospective teachers who are not successful in problem-posing activities might be increased, and problem-posing teaching approach might be included more in teacher training programs.

Despite the study’s findings, further research would provide additional details about teachers’ problem-posing abilities and the complexity of the problems that they posed. Since the complexity of the problems posed by prospective primary school teachers are investigated in the frame of one of the basic subjects of primary school mathematics, there is still a need for additional studies aiming to investigate the complexity of the problems in the context of various subjects such as fractions, algebra, etc. Moreover, due to the fact that problem-posing has many benefits for teachers, prospective teachers, and students (Lin, 2004; Stoyanova, 2003), it would be noteworthy to conduct studies aiming at the ways of developing their problem-posing abilities. Moreover, prospective teachers’ difficulties and errors that they confront while posing problems could be explored so that suggestions could be made to overcome these difficulties and errors.

Statements of Publication Ethics

This research was reviewed by the Kirikkale University Social and Humanities Ethics Committee and it was decided that the research was ethically appropriate. Meeting date: 29/11/2017.

Researchers' Contribution Rate

Authors	Introduction	Method	Data Collection	Data Analysis	Results	Conclusion
Ahmet Işık	☒	☒	☒	☒	☒	☒
Reyhan Tekin Sitrava	☒	☒	☐	☒	☒	☒

Conflict of Interest

We confirm that there are no conflicts of interest associated with this research.

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