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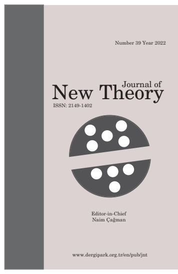

I-Connectedness

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Abstract — In this paper, we introduce a weak form of connectedness with respect to an ideal. We also investigate its relation to connectedness. We examine the I-connectedness property on the new topology introduced by the ideal. In addition, it is revealed under what conditions I-connectedness and connectedness coincide and one differs from another.

Keywords — *Connectedness, I-connectedness, ideal topological space*

Mathematics Subject Classification (2020) — 54A05, 54D05

1. Introduction

Vaidyanathaswamy [1] and Kuratowski [2] introduced the concept of an ideal in a topological space and also introduced a new topology using this concept of an ideal. Later, more detailed investigations were carried out on the ideal and the new topology it determines. An ideal I on a non-empty set X is a collection of subsets of X, which is closed under taking subsets and finite union operations. If (X,τ) is a topological space and I is an ideal on X, then (X,τ,I) is called an ideal topological space. If (X,τ) is a topological space and $A \subseteq X$, then the closure and the interior of A are denoted by $\bar{A}$ and $\overset{\circ}{A}$, respectively. $V(x)$ will denote the open neighborhood system at x, $(V(x) = \{V \in \tau : x \in V\})$, Given a topological space (X,τ) and a subset A of X, the subspace topology on A is defined by $\tau_{|A} = \{A \cap U : U \in \tau\}$.

Previously, some types of connectedness and their details in ideal topological spaces were studied by Ekici and Noiri [3], Sathiyasundari and Renukadevi [4], Modak and Noiri [5], Kılınç [6] and Tyagi, Bhardwaj and Singh [7], respectively. In this study, we have studied a different type of connectedness from earlier existing connectedness of ideal topological spaces. Then, using this new type of connectedness, new component and new locally-connectedness definitions were introduced. It has been shown that some of the basic properties of connectedness are also preserved according to this new type of connectedness.

This article is organized as follows. In the next section some basic notions and properties of ideal and ideal topological space are reviewed. In Section 3, we introduce the concept of I-connectedness. Moreover, we introduce the concept of I-component, locally I-connectedness, and totally I- disconnectedness. We show that not connected space is not I-connected and it has been shown that I-connectedness is preserved under bijective continuous functions. Then, we study some basic properties of them.

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2. Preliminaries

Definition 2.1. [2] Let (X,τ) be a topological space, and I be an ideal on X. The set $A^*(I,\tau) = \{x \in X : A \cap U \notin I \text{ for every } U \in V(x)\}$ is called the local function of A with respect to τ and I. We will write simply A^* for $A^*(I,\tau)$.

Definition 2.2. [8] Let (X,τ) be a topological space, and I be an ideal on X. It well known, the operator $Cl^*(.)$ from $\mathcal{P}(X)$ to $\mathcal{P}(X)$ defined by $Cl^*(A) = A \cup A^*(I)$ for every subset A of X, is a Kuratowski closure operator. Thus, $\{U \subseteq X : Cl^*(X-U) = X-U\}$ is a topology on X. The topology is denoted by $\tau^*(I)$.

When there is no chance for confusion $\tau^*(I)$ is denoted by τ^*. The topology τ^* has as a base $\beta(\tau, I) = \{W - B : W \in \tau \text{ and } B \in I\}$. [8]. It is easy to show that $\tau \subset \tau^*$. Also note that, if $I = \{\emptyset\}$ then $\tau = \tau^*$ and if $I = \mathcal{P}(X)$ then $A^*(\mathcal{P}(X), \tau) = \emptyset$ which implies $\tau^* = \mathcal{P}(X)$. We will call each element of τ^* as a $*$-open set.

Definition 2.3. [9] Let (X,τ,I) be an ideal topological space. For every $A \subset X$, if

$$\forall x \in A, \exists U \in V(x) | U \cap A \in I \Rightarrow A \in I$$

Then the topology τ is compatible with the ideal and denoted by $\tau \sim I$.

Lemma 2.4. [10] Let (X,τ) and (Y,φ) be two topological spaces. If $f : X \to Y$ is a function and I is an ideal on X, then the set $f(I) = \{f(A) : A \in I\}$ is an ideal on Y. Furthermore, If $f : X \to Y$ is a one to one function and J is an ideal on Y, then the set $f^{-1}(J) = \left\{f^{-1}(B) : B \in J\right\}$ is an ideal on X.

3. Main Results

Definition 3.1. Let (X,τ,I) be an ideal topological space. If there exist open sets U and V with $U \neq \emptyset, V \neq \emptyset$, and $U \cap V \in I$, such that $X = U \cup V$, then X is called a not I-connected (I-disconnected) space. If the open sets U and V can not be found to meet the above conditions, the space X is called I-connected..

Theorem 3.2. Every not connected space is a not I-connected.

Proof. Let (X,τ) be a disconnected space. There exist nonempty disjoint open sets U, V in X such that $X = U \cup V$. Since $U \cap V = \emptyset \in I$, then X is not an I-connected space. □

We can see from the following example that the inverse of the above theorem is not always true.

Example 3.3. Let $X = \{a,b,c\}$, $\tau = \{\emptyset, X, \{a\}, \{a,b\}, \{a,c\}\}$, $I = \{\emptyset, \{a\}\}$, $U = \{a,b\}$, and $V = \{a,c\}$. Since $X = U \cup V$ and $U \cap V = \{a\} \in I$, then X is not I-connected. Moreover, since $\emptyset$ and X are the only subsets of X being open and closed sets, then X is a connected space.

Theorem 3.4. [9] Let (X,τ) be a topological space and I be a ideal on X. If $\tau \sim I$, then $\beta(\tau, I)$ is a topology on X.

Theorem 3.5. Let the ideal topological space (X,τ,I) be an I-connected space. If $\tau \sim I$, then the topological space (X,τ^*) is connected.

Proof. Let $\tau \sim I$. Suppose that, (X,τ^*) is not connected. Since (X,τ^*) is not connected, then there are disjoint $*-$open sets $M \neq \emptyset, N \neq \emptyset$ such that $X = M \cup N$. Moreover, since $\tau \sim I$, then $U, V \in \tau$ and $A, B \in I$ such that $M = U - A$ and $N = V - B$. Here,

$$\begin{aligned} M \cap N =& (U - A) \cap (V - B) \\ =& (U \cap A^c) \cap (V \cap B^c) \\ =& (U \cap V) \cap (A^c \cap B^c) \\ =& (U \cap V) \cap (A \cup B)^c \\ =& (U \cap V) - (A \cup B) \end{aligned}$$

$U \cap V \in \tau$ and $A \cup B \in I$. Thus, M and N are disjoint $*$-open sets, $(U \cap V) - (A \cup B) = \emptyset$. That is $(U \cap V) \subset (A \cup B)$. Hence, since I is an ideal on X, $U \cap V \in I$. Consequently, $X = M \cup N$ and $M \subseteq U$, $N \subseteq V$, so it becomes $X = U \cup V$. This shows that X is not I-connected, a contradiction. □

Example 3.6. Let (X, τ) be a space and $A \subseteq X$. We know that $I(A) = \{B \subseteq X : B \subseteq A\}$ is an ideal on X [8]. Let's choose $X = \mathbb{R}$, $\tau = \{\emptyset, X\}$, and for $p \in X$, $I(X - \{p\}) = \{A \subseteq X : p \notin A\}$ specifically. We know $\tau \sim I$. This ideal generates a topology $\tau^* = \{A \subseteq X : p \in A\} \cup \{\emptyset\}$ known as a particular point topology [8]. The space X is I-connected because the only open set of X that is different from the empty set is X, if $U = V = X$ is selected, then $U \cap V = X \notin I$. Moreover, we know particular point topology is connected.

Theorem 3.7. If (X, τ^*) is a connected space and $\tau \cap I = \{\emptyset\}$, then the ideal topological space (X, τ, I) is I-connected.

Proof. Suppose that (X, τ, I) is not I-connected. Then, there are non-empty open subsets U, V with $U \cap V \in I$ such that $X = U \cup V$. Since $\tau \cap I = \{\emptyset\}$ and $U \cap V \in I$, then $U \cap V = \emptyset$. Proceeding from this, the contradiction arises that the space (X, τ^*) is not connected because $U, V \in \tau \subseteq \tau^*$, so our assumption is wrong, that is, the ideal topological space (X, τ, I) is I-connected. □

Theorem 3.8. Let $f : (X, \tau, I) \to (Y, \varphi, f(I))$ be a bijective and continuous function. If X is an $I-$connected space, then Y is a $f(I)-$connected space.

Proof. Let X be an I-connected space. Suppose that Y is not $f(I)-$connected. Then, there are non-empty open subsets U, V with $U \cap V \in f(I)$ such that $Y = U \cup V$. Since the function f is bijective, then $f^{-1}(U)$ and $f^{-1}(V)$ are non-empty open subsets of X. Then, $X = f^{-1}(U \cup V) = f^{-1}(U) \cup f^{-1}(V)$. Simultaneously, since $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V)$, $U \cap V \in f(I)$ and f is bijective, it becomes $f^{-1}(U \cap V) \in f^{-1}(f(I)) = I$. Hence X is not I-connected. This is a contradicition. Consequently, Y is a $f(I)$-connected space. □

Recall, let (X, τ, I) be an ideal topological space and A be a nonempty subset of X. Then, $I_{|A} = \{B \cap A : B \in I\}$ is an ideal on A, and $\left(A, \tau_{|A}, I_{|A}\right)$ is an ideal topological space.

Definition 3.9. Let (X, τ, I) be an ideal topological space and A be a subset of X. If subspace $\left(A, \tau_{|A}, I_{|A}\right)$ is $I_{|A}$-connected, the set A is called an $I_{|A}$-connected set in the ideal topological space (X, τ, I).

Definition 3.10. An I-component A of an ideal topological space (X, τ, I) is a maximal $I-$connected subset of X; that is A is $I-$connected and A is not a proper subset of any $I-$connected subset of X.

Theorem 3.11. Let the space X be equal to the union of non-empty and open subsets of U and V with $U \cap V \in I$. If the set $A \subset X$ is an $I_{|A}-$connected subset in the ideal topological space (X, τ, I), then $A \subset U$ or $A \subset V$.

Proof. Because of the hypothesis, there exist non-empty open subsets U and V of X such that, $X = U \cup V$ and $U \cap V \in I$. Let $A \subset X$ be an $I_{|A}-$connected space. Moreover, let $A \cap U \neq \emptyset$ and $A \cap V \neq \emptyset$. Since U and V are open subsets, then $A \cap U \in \tau_{|A}$ and $A \cap V \in \tau_{|A}$. Besides, since $U \cap V \in I$, then $(A \cap U) \cap (A \cap V) \in I_{|A}$. Since $A \subset X$ and $X = U \cup V$, then $A = A \cap (U \cup V) = (A \cap U) \cup (A \cap V)$ and so A is not $I_{|A}$ connected, a contradiction. Thus either$A \cap U = \emptyset$ or $A \cap V \neq \emptyset$. If $A \cap U = \emptyset$, then $A \subset V$. If $A \cap V = \emptyset$, then $A \subset U$. □

Theorem 3.12. If $\{A_j : j \in J\}$ is a non-empty family of $I_{|A}-$connected sets with $\tau_{|\mathrm{A}} \cap I_{|\mathrm{A}} = \{\emptyset\}$, then $\underset{j}{\cup} A_j$ is $I_{|A}-$connected.

Proof. Let $B = \underset{j}{\cup} A_j$. Suppose that B is not $I_{|A}-$connected. Then, there are non-emty subsets $U, V \in \tau_{|\mathrm{A}}$ with $U \cap V \in I_{|A}$ such that $B = U \cup V$. From Theorem 3.11, we know that, for every j, either $A_j \subset U$ or $A_j \subset V$. Assume that for j, $A_j \subset U$ and for k, $A_k \subset V$. Then, $A_j \cap A_k \subseteq U \cap V$.

By the definition of I, $A_i \cap A_j \in I_{|A}$, which contradicts the fact that $\cap A_j \notin I$. Here, for every j, either $A_j \subset U$ or $A_j \subset V$. Assume that for every j, $A_j \subset U$, then $B = \cup A_j = U$. Thus, $V \subset U$. Since $\tau_{|A} \cap I_{|A} = \{\emptyset\}$, then $V \cap U = V \in I_{|A}$. Hence $V = \emptyset$, which contradicts the fact that $V \neq \emptyset$. If $A_j \subset V$ is chosen, a similar contradiction is obtained. □

Theorem 3.13. Let A be an $I_{|A}-$connected subset of (X, τ, I). If there is a set B such that $A \subset B \subset \overline{A}$, then B is $I_{|B}-$connected.

PROOF. Suppose that B is not $I_{|\mathrm{B}}-$connected. Then, there are non-empty subsets $U, V \in \tau_{|\mathrm{B}}$ with $U \cap V \in I_{|B}$ such that $B = U \cup V$. Since A is an $I_{|A}-$connected subset of B, from Theorem 3.11 we know that either $A \subset U$ or $A \subset V$. It also happens that $A \cap U \in \tau_{|A}$, $A \cap V \in \tau_{|\mathrm{A}}$, $(A\cap U)\cap(A\cap V) = A\cap(U\cap V) \in I_{|A}$, and $A = A\cap(U\cup V) = (A\cap U)\cup(A\cap V)$. Now let's consider three different cases of $A \cap U$ and $A \cap V$.

Case 1. If $A \cap U \neq \emptyset$ and $A \cap V \neq \emptyset$, then A is not $I_{|A}-$connected, it contradicts the fact that A is a $I_{|A}$ connected.

Case 2. In the case of $A\cap U = \emptyset$ and $A\cap V \neq \emptyset$. By the definition of $\overline{A}$, for any $x \in A$, $A\cap W \neq \emptyset$ for every neighborhood W of the point x. Since every point of B is an element of $\overline{A}$, the intersection of A with the neighborhood of any of these points will be different from the empty set. Therefore, if $A \cap U = \emptyset$, then $U = \emptyset$ must be. This is a contradiction. A similar contradiction is obtained if $A \cap U \neq \emptyset$ and $A \cap V = \emptyset$.

Case 3. If $A \cap U = \emptyset$ and $A \cap V = \emptyset$, then A is an empty set. This is a contradiction.
Consequently, our assumption is not true, and the set B is $I_{|B}-$connected. □

Corollary 3.14. Let (X, τ, I) be an ideal topological space. If A is an $I_{|A}$-connected subset of X, then $\bar{A}$ is an $I_{|\bar{A}}$-connected subset of X.

PROOF. Let A be an $I_{|A}$-connected subset of X. From Theorem 3.13, we know every set B such that $A \subset B \subset \bar{A}$ is an $I_{|B}$-connected. Moreover, since $A \subset B$, then $\bar{A} \subset \bar{B}$ and $A \subset B \subset \bar{A} \subset \bar{B}$. Therefore, from Theorem 3.13 since $B \subset \bar{A} \subset \bar{B}$ and B is an $I_{|B}$-connected, then $\bar{A}$ is an $I_{|\bar{A}}$ -connected. □

Theorem 3.15. Let (X, τ, I) be an ideal topological space. If A is an $I_{|A}-$connected subset of X, then $cl^*(A)$ is $I_{cl^*(A)}-$connected.

PROOF. Since $A \subseteq cl^*(A) \subseteq \overline{A}$ in an ideal topological space (X, τ, I), then the proof is obvious from Theorem 3.13. □

Theorem 3.16. Let (X, τ, I) be an ideal topological space and $A \subset X$. If A is an $I_{|A}-$connected and dense subset of X, then X is an I-connected space.

PROOF. Let (X, τ, I) be an ideal topological space, A be an $I_{|A}$-connected, and a dense subset of X. Suppose that X is not I-connected. Then, there exist $U, V \in \tau$ such that $U \neq \emptyset, V \neq \emptyset, U \cap V \in I$, and $X = U \cup V$. Moreover, since A is a dense subset of X, then $\bar{A} = X$. Thus, $W \cap A \neq \emptyset$, for all $W \in \tau$. Besides, since $\tau_{|A} := \{G\cap A : G \in \tau\}$, then $A\cap U \in \tau_{|A}, A\cap V \in \tau_{|A}, A\cap U \neq \emptyset$, and $A\cap V \neq \emptyset$. Furthermore, from $I_{|A} := \{H\cap A : H \in I\}$, since $U\cap V \in I$, then $(A\cap U)\cap(A\cap V) \in I_{|A}$. Additionally, since $A \subset X$ and $X = U\cup V$, then $A \subset U\cup V$ and so $A = A\cap(U\cup V) = (A\cap U)\cup(A\cap V)$. Consequently, since there exist $A \cap U \in \tau_{|A}, A \cap V \in \tau_{|A}$ such that $A \cap U \neq \emptyset, A \cap V \neq \emptyset, A \cap (U \cap V) \in I_{|A}$ and $A = (A \cap U) \cup (A \cap V)$, then A is not $I_{|A}$-connected. Hence, this is a contradiction. Therefore, X is an I-connected space. □

The following theorem will show us the necessary and sufficient conditions for a subspace $A \subset X$ to be $I_{|A}$-connected.

Theorem 3.17. Let A be a subset of an ideal topological space (X, τ, I). Then, A is $I_{|A}-$connected if and only if for every $U, V \in \tau$, $A \subset U \cup V$, $A \cap U \neq \emptyset$, and $A \cap V \neq \emptyset$ such that $A \cap U \cap V \notin I$.

Proof. $\Rightarrow$: Let A be $I_{|A}$-connected, for $U, V \in \tau, A \subset U \cup V, A \cap U \neq \emptyset$ and $A \cap V \neq \emptyset$. Suppose that $A \cap U \cap V \in I$. Therefore, there exist $A \cap U \in \tau_{|A}, A \cap V \in \tau_{|A}$ such that $A \cap U \neq \emptyset, A \cap V \neq \emptyset, A \cap (U \cap V) \in I_{|A}$ and $A = (A \cap U) \cup (A \cap V)$, then A is not $I_{|A}$-connected. Thus, this is a contradiction. Hence, $A \cap U \cap V \notin I$.
$\Leftarrow$: Let for every $U, V \in \tau, A \subset U \cup V, A \cap U \neq \emptyset$, and $A \cap V \neq \emptyset$ such that $A \cap U \cap V \notin I$. Therefore, there exist $A \cap U \in \tau_{|A}, A \cap V \in \tau_{|A}$ such that $A \cap U \neq \emptyset, A \cap V \neq \emptyset, A = A \cap (U \cup V) = (A \cap U) \cup (A \cap V)$, and $(A \cap U) \cup (A \cap V) \notin I$. Then, A is $I_{|A}$-connected. □

Example 3.18. Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$, and $A = \{a, b, c\}$. If $I = \{\emptyset, \{c\}\}$, then A is an $I_{|A}-$connected subset of X. If $I = \{\emptyset, \{a\}\}$, then A is not an $I_{|A}-$connected subset of X.

Theorem 3.19. Let (X, τ_1, I) and (Y, τ_2, J) be ideal topological spaces, f be a homeomorphism from X to a dense subset of Y with $J = f(I)$, that is, Y be a compactification of X. If (X, τ, I) is $I-$connected, then (Y, φ, J) is $J-$connected.

Proof. Let (Y, τ_2, J) be a compactification of (X, τ_1, I). Then, there is a homeomorphism $f : X \to A$ such that $\overline{A} = Y$. Since f is bijective and ideal topological space (X, τ, I) is $I-$connected, then from Theorem 3.8 an ideal topological space (Y, τ_2, J) is $J-$connected. □

Corollary 3.20. There is only one I-component of an I-connected space, and that is the space itself.

Corollary 3.21. Let A be an I-component of (X, τ, I). Then, A is a component of (X, τ).

Theorem 3.22. If A is an I-component of (X, τ, I) and $\tau \sim I$, then A is a component of (X, τ^*).

Proof. Let A be a maximal $I_{|A}-$connected subspace of (X, τ). In other words, the space (A, τ_A) is maximal $I_{|A}-$connected subspace of (X, τ). Since $\tau \sim I$, then from Theorem 3.5, A is a maximal $I_{|A}-$connected subspace of (X, τ^*). □

Theorem 3.23. Every I-component of the ideal topological space (X, τ, I) is closed.

Proof. Let A be an I-component of the ideal topological space (X, τ, I). Since A is $I_{|A}-$connected, then from Corollary 3.14, $\overline{A}$ is $I_{|\overline{A}}-$connected. Since A is a maximal $I_{|A}-$connected subset of X, then $\overline{A} \subseteq A$. Moreover, $A \subseteq \overline{A}$. Thus, $A = \overline{A}$. □

Definition 3.24. Let (X, τ, I) be an ideal topological space. For each pair of distinct points $x, y \in X$, if there is a pair of open neighborhoods U and V of x and y such that $U \cap V \in I$ and $X = U \cup V$, then X is called totally I-disconnected.

Corollary 3.25. Every totally disconnected space is totally I-disconnected.

Theorem 3.26. If an ideal topological space (X, τ, I) is totally I-disconnected the I-components in X are the one-point sets.

Proof. Let (X, τ, I) be a totally I-disconnected space and C be an I-component of this space. Suppose that I-component C contains more than one point. Here, let $x \neq y \in C$. Since the ideal topological space (X, τ, I) is totally I-disconnected, then there exist $U, V \in \tau$ such that $x \in U, y \in V$, $U \cap V = \emptyset$, $X = U \cup V$, and $U \cap V \in I$. Since C is I-component of this space, $C \subset U \cup V$, $C \cap U \neq \emptyset$, $C \cap V \neq \emptyset$ and $C \cap (U \cap V) \subseteq U \cap V \in I$. Then, from Theorem 3.17. C is not $I_{|c}-$connected. Thus, C cannot be an I-component, then our assumption is wrong. Hence, C is the one-point set.

□

Definition 3.27. Let (X, τ, I) be an ideal topological space. For all $V \in \tau$ such that $x \in V$, if there exist an I-connected open subset U such that $x \in U$ and $U \subset V$, then (X, τ, I) is called locally I-connected at x.
An ideal topological space (X, τ, I) is called locally I-connected if it is locally connected at x for every $x \in X$.

Theorem 3.28. Every I-component of a locally I-connected space is an open set.

Proof. Let C be an I-component of a locally I-connected space (X,τ,I) and x be an arbitrary point of C. Since (X,τ,I) is locally I-connected and X is an open neighborhood of x, x belongs to an I-connected open set U of X. Therefore, since C is an I-component, then C is maximal I- connected subset of X, and so $x \in U \subset C$. Hence, C is a neighborhood of each of its points and so C is an open set. □

Theorem 3.29. If an ideal topological space (X,τ,I) is a locally I-connected, then the I-components of every open subspace of (X,τ,I) are open in X.

Proof. Let (X,τ,I) be locally I-connected. Let Y be an open set in X and C be an I-component of Y. Then by the definition of locally I-connectednes for every $x \in C$, there exists an I-connected open set U in Y containing x. Since C is an I-component, then $U \subset C$. Thus, C contains a neighborhood of each of its points in Y and so C is open in Y. Since Y is open in X, thus C is open in X. □

Corollary 3.30. Every I-component of a locally I-connected space is both closed and open.

Theorem 3.31. Let $f:(X,\tau,I)\to(Y,\varphi,f(I))$ be a homeomorphism. If (X,τ,I) is locally I-connected, then $(Y,\varphi,f(I))$ is locally $f(I)-$connected.

Proof. Let $f:(X,\tau,I)\to(Y,\varphi,f(I))$ be a homeomorphism. Let y be a point of $f(X)=Y$ and V be any open neighborhood of y in Y. Also, there exists $x \in X$ such that $y=f(x)$. As f is continuous, then $f^{-1}(V)$ is an open neighborhood of x. Since (X,τ,I) is locally I-connected, then there exist I-connected open $U \in \tau$ such that $x \in U \subset f^{-1}(V)$. Thus,

$$y=f(x)\in f(U)\subset f\left(f^{-1}(V)\right)=V$$

Since f is an open map, then $f(U)$ is an open subset of Y. From Theorem 3.8, $f(U)$ is $f(I)$-connected. Consequently, Y is locally $f(I)$-connected. □

4. Conclusion

In this paper, we defined the concept of I-connectedness in ideal topological spaces. Then, we examined the relationship between connectedness and I-connectedness. We shown that every not connected space is not I-connected and the opposite is not true. We shown that some basic properties of connectedness are valid in I-connectedness. Next, we put forward the definitions of I-component, totally I-disconnectedness, and locally I-connectedness. We revealed some of their basic properties.

The relations of I-connectedness defined here in with other types of connectedness previously defined in ideal topological spaces, can be investigated. In addition, the relationship between I-connectedness and $*$-Hyperconnectedness in ideal topological spaces [11] can also be examined. Furthermore, [12] have defined I-extremally disconnected spaces and have revealed the connection of this connectedness concerning some weak continuity varieties. A similar study can also be conducted on the I-connectedness defined here in.

Author Contributions

The author read and approved the last version of the paper.

Conflicts of Interest

The author declares no conflict of interest.

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