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Three-Dimensional Vibration of an Isotropic Plate Enclosed in a Rigid Body

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ABSTRACT

In this study, vibration of plates embedded in a rigid enclosure has been investigated analytically for the first time in the literature. It is assumed that the isotropic plate is always in contact with outer enclosure. Therefore, the normal displacement at a boundary surface is constrained but tangential displacement at a surface is allowed. The displacement field is assumed in trigonometric function form. This analytical solution is the only available exact solution of three-dimensional isotropic plate. Numerical results were presented for various geometrical parameters. It is believed that the present formulation and the results can be used as a benchmark for the numerical methods where the exact solution is not possible.

Keywords:
Vibration; Three-dimensional isotropic plate; Exact solution; Plate enclosed in a rigid; Trigonometric function form.

INTRODUCTION

Plates are used in the many engineering applications such as civil, mechanical and aerospace. In the service conditions, they are used as dynamic conditions. Therefore, it is an important point to understand the dynamic behaviour of the plate like structures.

Vibration of the isotropic plates were considered in many of the previous studies. They were summarized in the work of Leissa [1-3]. Different theories were developed in order to study vibration of plates. The classical plate theory is the first model developed by Kirchhoff [4]. In this theory, it is assumed that the normal lines before the deformation will be normal to the mid-surface of the plate after deformation. This theory is acceptable for the isotropic thin plates. In order to improve the accuracy the first and higher order shear deformation theories were proposed by some authors (see for example: Mindlin [5], Reissner [6] for the first order shear deformation theory and Reddy [7], Touratier [8], Soldatos [9] theories for the higher order models.). Noor and Burton [10] developed three-dimensional solutions for the free vibration and buckling analysis of multilayered angle-ply composite plates. Vibration of delaminated composite plates has been investigated using three-dimensional theory of linear elasticity in [11]. A finite element method has been developed for vibration analysis of delaminated composite plates and obtained results were compared with the experimental observations in that paper. Liew et al. [12] studied the vibration analysis of thick rectangular plates using a continuum three-dimensional Ritz formulation. In another study [13], static and dynamic analyses of thick laminated plates have been examined using three-dimensional theory of linear elasticity and approximate Ritz method. Chen and Lü [14] investigated the three-dimensional free vibration analysis of cross-ply laminated plates. They used differential quadrature method in solution of the problem. The free vibration analysis of cross-ply laminated rectangular plates with clamped boundaries has been investigated in the framework of the three-dimensional elasticity in [15]. The results obtained from that paper were given for either isotropic or cross-ply laminated plates having different combinations of simply supported and clamped boundaries. Three-dimensional elasticity solutions have been presented for the free vibrations of rectangular plates based on the differential quadrature method in [16]. So and Leissa [17] studied the three-dimensional vibration analysis of thick cicular and annular plates. The flexural thickness-shear, in plane stretching, and torsional modes are included in the analyses in that paper. Liew and Yang [18] investigated the vibrational characteristics of annular plates based on the three-dimensional elasticity theory. Recently, Huang et al. [19] have developed a three-dimensional exact solution for the vibration analysis of functionally graded rectangular plates. The free vibrations of three-dimensional orthotropic rectangular plates have been investigated by Wang et al. [20]. An extended separation of variable method for the free vibration of orthotropic rectangular plates has been presented in that study. In addition to the studies mentioned above, there are some important papers related to mechanical analyses of one-dimensional, two-dimensional and three-dimensional structures [21-25].

It is well known that analytical closed form solutions are possible for all edge simply supported plates in two-dimensional plate theories.. This solution method is known as Navier-type solution method in which trigonometric displacement functions are used in the displacement field in the solution. Levy type solution method is proposed for plates with at least two opposite simply supported edges. In this method, trigonometric functions are used for simply supported boundaries and equations of motion are converted to the ordinary differential equations. Then, they are solved using remaining boundary conditions. Other than these six boundary conditions, the remaining fifteen boundary conditions can not be solved analytically. Three-dimensional vibration of plates is a complicated problem and there is no analytical solution for many of the boundary conditions. Vibration of plates embedded in a rigid enclosure has been investigated analytically in the present study. These kind of dynamic problems can be encountered in machine elements or in some manufacturing processes like forging. This problem has not been investigated previously in the open literature. An isotropic plate is inserted in a rigid enclosure. Isotropic material properties were considered for the elastic plate and it is always in contact with outer enclosure. So, the normal displacement at a boundary surface is constrained but tangential displacement at a surface is allowed. The displacement fields are assumed in trigonometric function form since they satisfy the equations of motion and boundary conditions. Numerical results were presented for various geometrical parameters.

THEORY

The equations of motion of a three-dimensional plate for free vibration are defined as

$$\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xz}}{\partial z}=\rho\frac{\partial^2 u(x,t)}{\partial t^2} \tag{1}$$

$$\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \sigma_y}{\partial y}+\frac{\partial \tau_{yz}}{\partial z}=\rho\frac{\partial^2 v(x,t)}{\partial t^2} \tag{2}$$

$$\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \sigma_z}{\partial z}=\rho\frac{\partial^2 w(x,t)}{\partial t^2} \tag{3}$$

where ρ is the mass density, t is the time , σ_x , σ_y and σ_z are normal stress in x,y and z-axes, τ_{xy}, τ_{yz} and τ_{xz} are shear stress in x-y, y-z and x-z planes, and u ,v ,w are the displacement components in x, y and z directions, respectively. By using small strain assumption, normal and shear strain displacement relations are written as

$$\varepsilon_x=\frac{\partial u}{\partial x},\varepsilon_y=\frac{\partial v}{\partial y},\varepsilon_z=\frac{\partial w}{\partial z},$$
$$\gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x},\gamma_{yz}=\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y},\gamma_{xz}=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x} \tag{4}$$

Here, ε_x, ε_y and ε_z represent the normal strains and γ_{xy}, γ_{yz} and γ_{xz} represent the shear strains. For an isotropic plate, stress-strain relations can be written using Hooke's law as follows

$$\sigma_x=\lambda e+2G\varepsilon_x \tag{5}$$

$$\sigma_y=\lambda e+2G\varepsilon_y \tag{6}$$

$$\sigma_z=\lambda e+2G\varepsilon_z \tag{7}$$

$$\tau_{xy}=\frac{\gamma_{xy}}{G} \tag{8}$$

$$\tau_{yz}=\frac{\gamma_{yz}}{G} \tag{9}$$

$$\tau_{xz}=\frac{\gamma_{xz}}{G} \tag{10}$$

where

$$e=\varepsilon_x+\varepsilon_y+\varepsilon_z \tag{11}$$

$$\lambda=\frac{vE}{(1+v)(1-2v)} \tag{12}$$

$$G=\frac{vE}{2(1+v)} \tag{13}$$

Here, λ denotes the Lamé constant, G denotes the shear modulus, E denotes the Young's modulus and v denotes the Poisson's ratio. Inserting Eqs. (5), (6), (7) and (12) into Eqs. (1-3) leads to

$$(\lambda+G)\frac{\partial e}{\partial x}+G\nabla^2 u(x,t)=\rho\frac{\partial^2 u(x,t)}{\partial t^2} \tag{14}$$

$$(\lambda+G)\frac{\partial e}{\partial y}+G\nabla^2 v(y,t)=\rho\frac{\partial^2 v(y,t)}{\partial t^2} \tag{15}$$

$$(\lambda+G)\frac{\partial e}{\partial z}+G\nabla^2 w(z,t)=\rho\frac{\partial^2 w(z,t)}{\partial t^2} \tag{16}$$

where ∇^2 is the three-dimensional Laplaćian operator which is defined as

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2} \tag{17}$$

The boundary conditions which involved combination of displacements and stresses on any face can be used for the present problem. In this context, the exact vibration solution of Eqs. 14-17 can be assumed as

$$u(x,y,z,t)=\mathrm{A}\,sin(\alpha x)cos(\beta y)cos(\gamma z)sin\omega t \tag{18a}$$

$$v(x,y,z,t) = B\,cos(\alpha x)\,sin(\beta y)\,cos(\gamma z)\,sin\omega t \quad (18b)$$

$$w(x,y,z,t) = C\,cos(\alpha x)\,cos(\beta y)\,sin(\gamma z)\sin\omega t \quad (18c)$$

where

$$\alpha = \frac{m\pi}{a},\ \beta = \frac{n\pi}{b},\ \gamma = \frac{p\pi}{h}. \quad (19)$$

Here, ω is the circular frequency, m, n, p are integers which represent the half wave number, a, b, h are the length components of the isotropic three-dimensional plate about x, y and z-axis, respectively.

These boundary conditions can be considered as "an elastic block is enclosed by a much more rigid outer material or block which is well lubricated contact surface between two bodies" as stated by Leissa [1] (p.316) (Fig. 1).

It should be noted that an initial stress is required for continuous contact between elastic plate and outer enclosure. Although this boundary conditions are not common, it can be considered in a manufacturing process of a closed die forging. A more important point is the present results can be considered as a benchmark solution for the approximate solutions such as finite element method, finite difference method, differential quadrature method, etc.

Then, substituting Eqs. (18) and (19) into the equations of motion (14-16) gives a sixth-order characteristic equation of the present vibration problem of three-dimensional isotropic plate enclosed in a rigid body with the following equation:

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (20)$$

For a non-trivial solution, the determinant of the coefficient matrix in the previous equation (20) must be zero. Thus, it can be obtained the sixth-order frequency determinant. The positive roots of the sixth-order frequency determinant obtained from Eq. (20) give three positive frequencies of isotropic three-dimensional plate. These three positive vibration frequencies represent the first, second and third spectrum frequencies for each half wave number. It should be noted that the first and second spectrum frequencies give same values for the present problem. The elements of the coefficient matrix of Eq. (20) can be obtained in dimensionless form as follows:

$$\begin{aligned}
K_{11} &= \bar{\alpha}^2 - 2\frac{G}{\lambda}\bar{\alpha}^2 - \frac{G}{\lambda}k_1^2\bar{\beta}^2 - \frac{G}{\lambda}k_2^2\bar{\gamma}^2 + \Omega^2\bar{D}, \\
K_{12} &= -k_1\bar{\alpha}\bar{\beta} - \frac{G}{\lambda}k_1\bar{\alpha}\bar{\beta}, \\
K_{13} &= -k_2\bar{\alpha}\bar{\gamma} - \frac{G}{\lambda}k_2\bar{\alpha}\bar{\gamma}, \\
K_{21} &= -k_1\bar{\alpha}\bar{\beta} - \frac{G}{\lambda}k_1\bar{\alpha}\bar{\beta}, \\
K_{22} &= -\bar{\alpha}^2\frac{G}{\lambda} - k_1^2\bar{\beta}^2 - \frac{2G}{\lambda}k_1^2\bar{\beta}^2 - \frac{G}{\lambda}k_2^2\bar{\gamma}^2 + \Omega^2\bar{D}, \quad (21) \\
K_{23} &= -k_1k_2\bar{\beta}\bar{\gamma} - \frac{G}{\lambda}k_1k_2\bar{\beta}\bar{\gamma}, \\
K_{31} &= -k_2\bar{\alpha}\bar{\gamma} - \frac{G}{\lambda}k_2\bar{\alpha}\bar{\gamma}, \\
K_{32} &= --k_1k_2\bar{\beta}\bar{\gamma} - \frac{G}{\lambda}k_1k_2\bar{\beta}\bar{\gamma}, \\
K_{33} &= -\bar{\alpha}^2\frac{G}{\lambda} - k_2^2\bar{\gamma}^2 - \frac{G}{\lambda}k_1^2\bar{\beta}^2 - \frac{2G}{\lambda}k_2^2\bar{\gamma}^2 + \Omega^2\bar{D}.
\end{aligned}$$

where dimensionless terms are

$$\bar{\alpha} = \frac{\alpha}{a},\ \bar{\beta} = \frac{\beta}{b},\ \bar{\gamma} = \frac{\gamma}{y},\ k_1 = \frac{a}{b},\ k_2 = \frac{a}{h},\ \bar{D} = \frac{D}{\lambda h a^2}. \quad (22)$$

and the dimensionless frequency parameter (Ω^2) of three-dimensional isotropic plate is defined as

$$\Omega = \omega a^2\sqrt{\frac{\rho h}{D}},\ \mathrm{D} = \frac{Eh^3}{12(1-\mathrm{v}^2)}. \quad (23)$$

The roots of the characteristic equations obtained from Eq. (20) are

$$\Omega_{mnp,1} = \Omega_{mnp,2} = \frac{\sqrt{\bar{D}G\lambda(\bar{\beta}^2\,\mathrm{k}_1^2 + \bar{\alpha}^2 + \bar{\gamma}^2\,\mathrm{k}_2^2)}}{\bar{D}\lambda} \quad (24a)$$

$$\Omega_{mnp,3} = \frac{\sqrt{\bar{D}\lambda(2G+\lambda)(\bar{\beta}^2\,\mathrm{k}_1^2 + \bar{\alpha}^2 + \bar{\gamma}^2\,\mathrm{k}_2^2)}}{\bar{D}\lambda} \quad (24b)$$

Here, $\Omega_{mnp,1}$, $\Omega_{mnp,2}$ and $\Omega_{mnp,3}$ denote the first, second and third set frequency roots of isotropic three-dimensional plate for each half wave number.

RESULTS AND DISCUSSION

The numerical results of the formulation presented in Section 2 are given for various geometrical parameters. Three sets of frequencies are obtained for a given set of wave numbers (m, n, p). The frequencies with only one wave number in one of the coordinate axes can be obtained by selecting the corresponding half wave integer (m, n, p) as a nonzero integer and zero for other integers in Table 1a. They are Ω_{100}, Ω_{010} and Ω_{001} frequencies. These cases are pure axial deformations along the correspon-

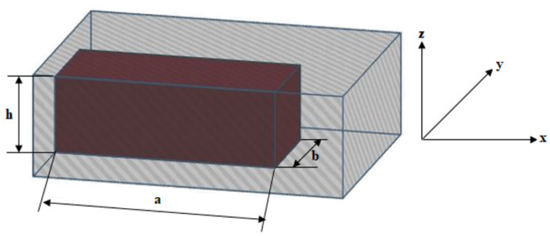

Figure 1. The structure of the model.

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Table 1a. The dimensionless vibration frequencies (Ω) for different mode numbers (Ω_{100},Ω_{010},Ω_{001}).

		Ω_{100}	
a/h	a/b	$\Omega_{100,1}\Omega(_{100,2})$	$\Omega_{100,3}$
5	1	32.1917	60.2252
	2	32.1917	60.2252
	3	32.1917	60.2252
	5	32.1917	60.2252
20	1	128.7669	240.9009
	2	128.7669	240.9009
	3	128.7669	240.9009
	5	128.7669	240.9009
100	1	643.8349	1204.5048
	2	643.8349	1204.5048
	3	643.8349	1204.5048
	5	643.8349	1204.5048

		Ω_{010}	
a/h	a/b	$\Omega_{010,1}\Omega(_{010,2})$	$\Omega_{010,3}$
5	1	32.1917	60.2252
	2	64.3834	120.4504
	3	96.5752	180.6757
	5	160.9587	301.1262
20	1	128.7669	240.9009
	2	257.5339	481.8019
	3	386.3009	722.7028
	5	643.8349	1204.5048
100	1	643.8349	1204.5048
	2	1287.6698	2409.0096
	3	1931.5047	3613.5144
	5	3219.1745	6022.5240

		Ω_{001}	
a/h	a/b	$\Omega_{001,1}\Omega(_{001,2})$	$\Omega_{001,3}$
5	1	160.9587	301.1262
	2	160.9587	301.1262
	3	160.9587	301.1262
	5	160.9587	301.1262
20	1	2575.3396	4818.0192
	2	2575.3396	4818.0192
	3	2575.3396	4818.0192
	5	2575.3396	4818.0192
100	1	64383.4904	120450.4814
	2	64383.4904	120450.4814
	3	64383.4904	120450.4814
	5	64383.4904	120450.4814

Table 1b. The dimensionless vibration frequencies (Ω) for different mode numbers (Ω_{110},Ω_{101},Ω_{011}).

		Ω_{110}	
a/h	a/b	$\Omega_{110,1}\Omega(_{110,2})$	$\Omega_{110,3}$
5	45.5260	85.1713	85.1713
	71.9829	134.6677	134.6677
	101.7992	190.4489	190.4489
	164.1463	307.0896	307.0896
20	182.1040	340.6854	340.6854
	287.9317	538.6709	538.6709
	407.1969	761.7957	761.7957
	656.5853	1228.3587	1228.3587
100	910.5200	1703.4270	1703.4270
	1439.6586	2693.3546	2693.3546
	2035.9847	3808.9786	3808.9786
	3282.9267	6141.7935	6141.7935

		Ω_{101}	
a/h	a/b	$\Omega_{101,1}\Omega(_{101,2})$	$\Omega_{101,3}$
5	1	164.1463	307.0896
	2	164.1463	307.0896
	3	164.1463	307.0896
	5	164.1463	307.0896
20	1	2578.5567	4824.0380
	2	2578.5567	4824.0380
	3	2578.5567	4824.0380
	5	2578.5567	4824.0380
100	1	64386.7095	120456.5037
	2	64386.7095	120456.5037
	3	64386.7095	120456.5037
	5	64386.7095	120456.5037

		Ω_{011}	
a/h	a/b	$\Omega_{011,1}\Omega(_{011,2})$	$\Omega_{011,3}$
5	1	164.1463	307.0896
	2	173.3578	324.3228
	3	187.7085	351.1704
	5	227.6300	425.8567
20	1	2578.5567	4824.0380
	2	2588.1842	4842.0494
	3	2604.1510	4871.9204
	5	2654.5993	4966.3005
100	1	64386.7095	120456.5037
	2	64396.3659	120474.5690
	3	64412.4565	120504.6719
	5	64463.9196	120600.9505

Table 1c. The dimensionless vibration frequencies (Ω) for different mode numbers (Ω_{221},Ω_{211},Ω_{121}).

		Ω_{221}	
a/h	a/b	$\Omega_{221,1}\Omega(_{221,2})$	$\Omega_{221,3}$
5	1	184.9274	345.9676
	2	215.9487	404.0031
	3	259.5381	485.5514
	5	365.6279	684.0272
20	1	2600.9655	4865.9609
	2	2638.9378	4937.0006
	3	2701.0389	5053.1812
	5	2890.8115	5408.2132
100	1	64409.2387	120498.6519
	2	64447.8418	120570.8717
	3	64512.1289	120691.1419
	5	64717.4186	121075.2038

		Ω_{211}	
a/h	a/b	$\Omega_{211,1}\Omega(_{211,2})$	$\Omega_{211,3}$
5	1	176.3214	329.8672
	2	184.9274	345.9676
	3	198.4432	371.2533
	5	236.5600	442.5633
20	1	2591.3855	4848.0383
	2	2600.9655	4865.9609
	3	2616.8542	4895.6860
	5	2667.0622	4989.6166
100	1	64399.5843	120480.5902
	2	64409.2387	120498.6519
	3	64425.3261	120528.7487
	5	64476.7789	120625.0081

		Ω_{121}	
a/h	a/b	$\Omega_{121,1}\Omega(_{121,2})$	$\Omega_{121,3}$
5	1	176.3214	329.8672
	2	208.6263	390.3041
	3	253.4780	474.2140
	5	361.3514	676.0266
20	1	2591.3855	4848.0383
	2	2629.4961	4919.3368
	3	2691.8151	5035.9250
	5	2882.1951	5392.0933
100	1	64399.5843	120480.5902
	2	64438.1932	120552.8208
	3	64502.4899	120673.1090
	5	64707.8102	121057.2281

Table 1d. The dimensionless vibration frequencies (Ω) for different mode numbers (Ω_{331},Ω_{321},Ω_{231}).

		Ω_{331}	
a/h	a/b	$\Omega_{331,1}\Omega(_{331,2})$	$\Omega_{331,3}$
5	1	211.0953	394.9233
	2	269.3354	503.8805
	3	345.2180	645.8437
	5	518.0771	969.2335
20	1	2632.6471	4925.2318
	2	2716.3424	5081.8113
	3	2850.3786	5332.5701
	5	3242.2697	6065.7312
100	1	64441.4095	120558.8380
	2	64528.1907	120721.1907
	3	64672.5672	120991.2944
	5	65132.4214	121851.6028

		Ω_{321}	
a/h	a/b	$\Omega_{321,1}\Omega(_{321,2})$	$\Omega_{321,3}$
5	1	198.4432	371.2533
	2	227.6300	425.8567
	3	269.3354	503.8805
	5	372.6463	697.1575
20	1	2616.8542	4895.6860
	2	2654.5993	4966.3005
	3	2716.3424	5081.8113
	5	2905.1155	5434.9734
100	1	64425.3261	120528.7487
	2	64463.9196	120600.9505
	3	64528.1907	120721.1907
	5	64733.4295	121105.1573

		Ω_{231}	
a/h	a/b	$\Omega_{231,1}\Omega(_{231,2})$	$\Omega_{231,3}$
5	1	198.4432	371.2533
	2	259.5381	485.5514
	3	337.6298	631.6476
	5	513.0519	959.8323
20	1	2616.8542	4895.6860
	2	2701.0389	5053.1812
	3	2835.7985	5305.2933
	5	3229.4594	6041.7654
100	1	64425.3261	120528.7487
	2	64512.1289	120691.1419
	3	64656.5413	120961.3127
	5	65116.5086	121821.8328

ding directions. The frequencies are given in Table 1a for various a/h and a/b ratios for the first three frequencies. The frequencies for Ω_{100} and Ω_{001} are independent from a/h and a/b ratios.

Similar to one-dimensional waves, plane waves can be obtained by setting one of the half wave integer to zero. Therefore, the integers are Ω_{110}, Ω_{101}, Ω_{011} (Table 1b). For example Ω_{110} is a wave in the xy plane with zero flexure (w=0). Plane wave frequencies are higher than one-dimensional frequencies. This fact can be seen from Eq. (24).

Table 2a. The first three dimensionless vibration frequencies (Ω) for *a*/*b*=1.

	First mode	
a/h	$\Omega_{111,1}\Omega(_{111,2})$	$\Omega_{111,3}$
1	11.1515	20.8626
2	31.5413	59.0084
3	64.0607	119.8467
5	167.2732	312.9395
10	650.2413	1216.4902
20	2581.7699	4830.0492
30	5800.9489	10852.5816
50	16102.3096	30124.6629
70	31554.3498	59032.7797
100	64389.9285	120462.5258
	Second mode	
a/h	$\Omega_{222,1}\Omega(_{222,2})$	$\Omega_{222,3}$
1	22.3030	41.7252
2	63.0826	118.0168
3	128.1215	239.6934
5	334.5464	625.8790
10	1300.4827	2432.9804
20	5163.5398	9660.0985
30	11601.8978	21705.1633
50	32204.6193	60249.3259
70	63108.6997	118065.5594
100	128779.8570	240925.0516
	Third mode	
a/h	$\Omega_{333,1}\Omega(_{333,2})$	$\Omega_{333,3}$
1	33.4546	62.5879
2	94.6240	177.0253
3	192.1822	359.5401
5	501.8196	938.8185
10	1950.7241	3649.4706
20	7745.3098	14490.1478
30	17402.8467	32557.7450
50	48306.9290	90373.9889
70	94663.0496	177098.3391
100	193169.7855	361387.5775

Table 2b. The first three dimensionless vibration frequencies (Ω) for *a*/*b*=2.

	First mode	
a/h	$\Omega_{111,1}\Omega(_{111,2})$	$\Omega_{111,3}$
1	15.7706	29.5042
2	38.6301	72.2702
3	72.2702	135.2053
5	176.3214	329.8672
10	659.7344	1234.2501
20	2591.3855	4848.0383
30	5810.5877	10870.6142
50	16111.9604	30142.7179
70	31564.0021	59050.8408
100	64399.5843	120480.5902
	Second mode	
a/h	$\Omega_{222,1}\Omega(_{222,2})$	$\Omega_{222,3}$
1	31.5413	59.0084
2	77.2601	144.5405
3	144.5405	270.4106
5	352.6429	659.7344
10	1319.4689	2468.5003
20	5182.7710	9696.0767
30	11621.1754	21741.2284
50	32223.9209	60285.4358
70	63128.0042	118101.6816
100	128799.1687	240961.1805
	Third mode	
a/h	$\Omega_{333,1}\Omega(_{333,2})$	$\Omega_{333,3}$
1	47.3120	88.5126
2	115.8902	216.8108
3	216.8108	405.6159
5	528.9643	989.6016
10	1979.2033	3702.7504
20	7774.1565	14544.1150
30	17431.7631	32611.8427
50	48335.8813	90428.1537
70	94692.0063	177152.5224
100	193198.7530	361441.7707

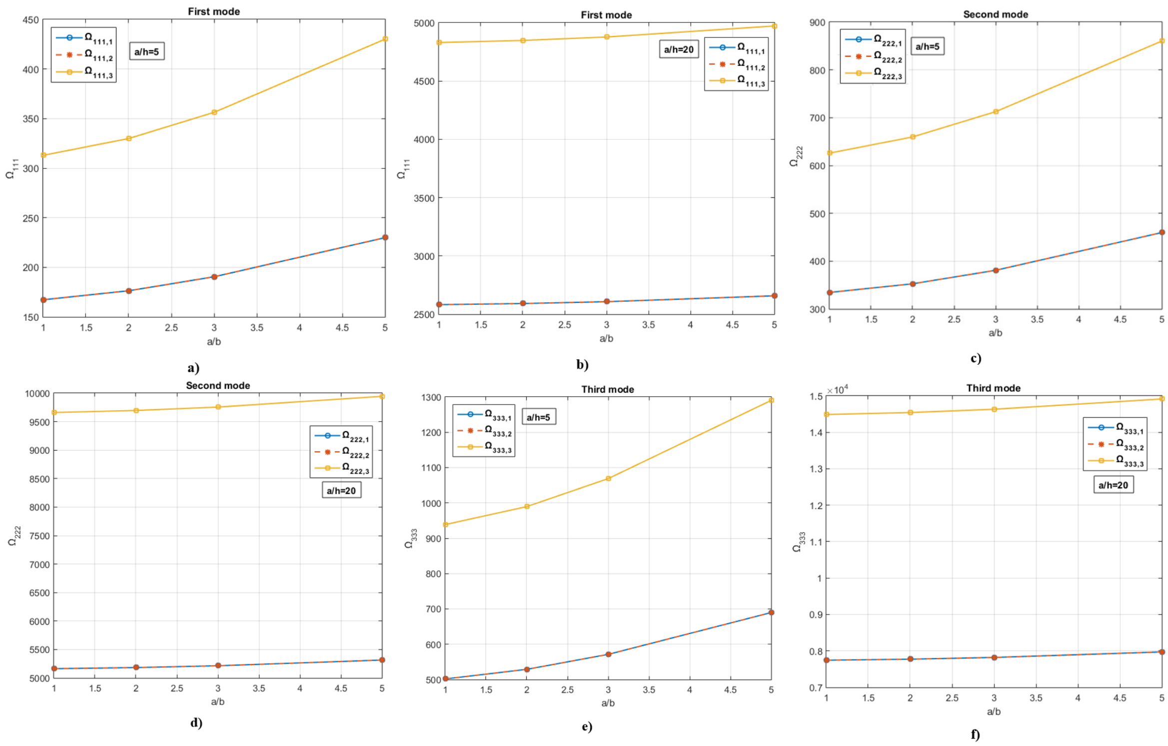

Figure 2. Variation of the first three dimensionless frequencies with *a*/*b*.

More general vibration case can be obtained by setting all of the wave number integers to a nonzero value. The corresponding results for various a/h and a/b ratios for the first three frequencies. The first three frequency parameters of plates are given in Tables 1 and 2 for various modes and a/h and a/b ratios. It is seen that first two sets of frequencies are identical for the all cases considered. This can be seen from Eq. (24). It is interesting to note that third set frequency is 1.87 times of the first set frequency approximately. The frequency parameters are increasing with a/h ratio.

The variation of first three frequencies parameters with *a*/*b* ratios were given in Fig. 2. The frequency parameters of thicker plates are more sensitive to *a*/*b* ratio when compared to thinner plates. It should be also noted that *a*/*b* ratio is also more effective for the higher modes of vibration. Plate outer surfaces are always in contact with the outer body. First two frequencies are in-plane dominated frequencies whereas the third frequency is flexural dominated frequency.

Some mode shapes of vibrating plates are given in Fig. 3. These mode shapes are given for *a*/*h*=5. For the first mode, there is a nodal line at *y*=*b*/2. The highest displacement is observed at *x*=*a*/2. The v-mode shape is obtained by rotating a 90° in the clockwise direction. The first mode shape of w is in a shape of saddle with the symmetry axis located at *y*=-*x*+1.

CONCLUSION

The analytical solution of isotropic three-dimensional plate in a rigid enclosure has been presented in the present study. Dimensionless frequencies of three-dimensional plate have been obtained for various geometrical values. Three sets of frequencies are obtained for each set of wave numbers and the first and second set frequencies become identical. It is obtained that one-dimensional frequencies Ω_{100} and Ω_{001} are independent from *a*/*h* and *a*/*b* ratios. Plane wave frequencies give higher values than one-dimensional frequencies. The frequencies of thicker plates are more sensitive to *a*/*b* ratio when compared to thinner plates. Two groups of frequencies have been observed as in-plane dominated (the first two set frequencies) and flexural dominated (third set frequency).

CONFLICT OF INTEREST

Authors approve that to the best of their knowledge, there is not any conflict of interest or common interest with an institution/organization or a person that may affect the review process of the paper

AUTHOR CONTRIBUTION

Ufuk Gul: Writing - original draft, Visualization, Investigation. Metin Aydogdu: Supervision, Conceptualization, Writing - review & editing.

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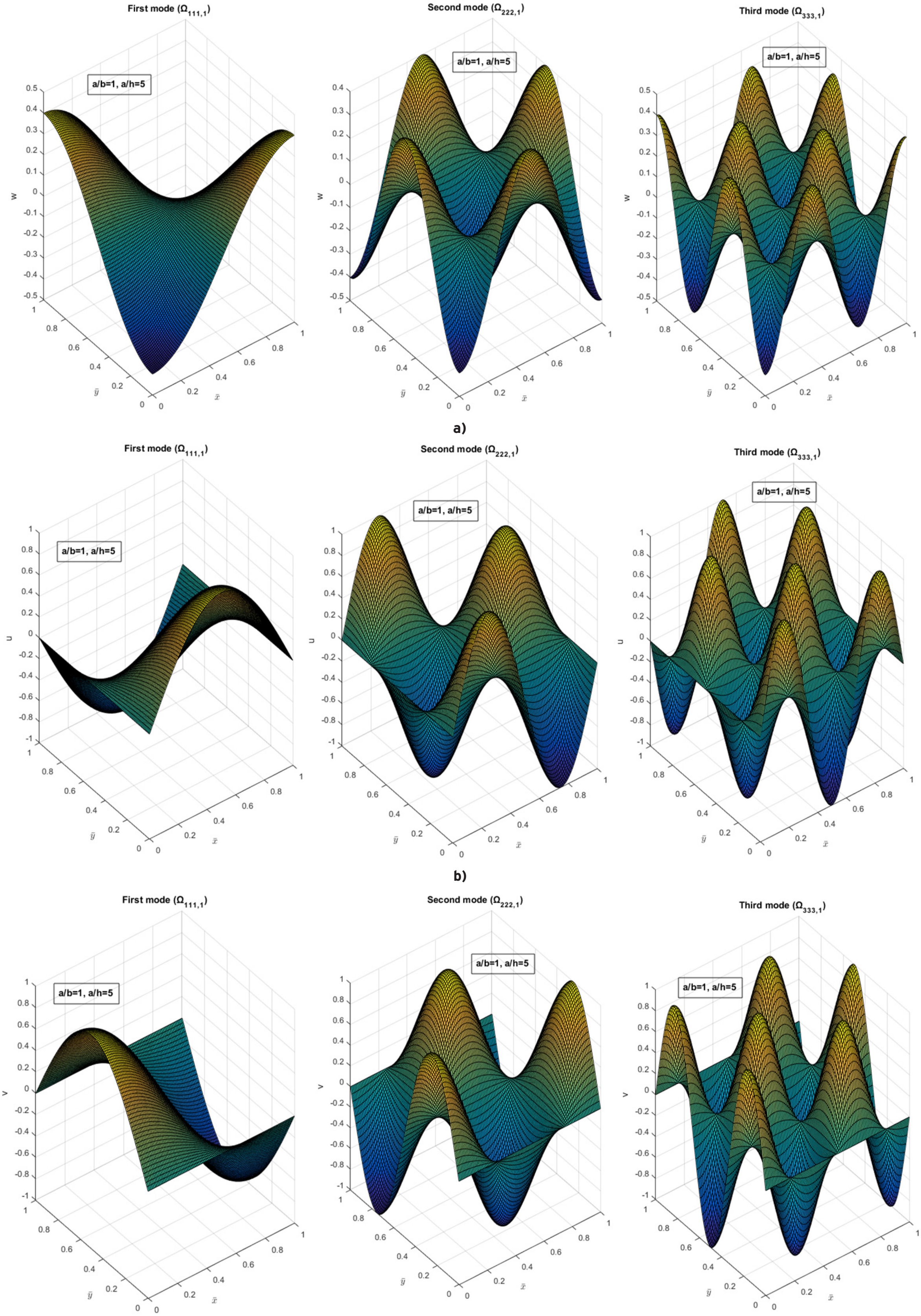

Figure 3. Mode shapes for a/b=1 and a/h=5.

References

1. Leissa AW, Qatu MS. Vibration of Continuous Systems, US: McGraw-Hill Professional, 2011.
2. Fromme JA, Leissa AW. Free vibration of the rectangular parallelepiped. The Journal of the Acoustical Society of America 48 (1970) 290.
3. Leissa AW. The free vibration of rectangular plates. Journal of Sound and Vibration 31(3) (1973) 257-293.
4. Kirchhoff G. Über das Gleichgewicht und die Bewegung einer elastischen Scheibe. Journal für die Reine und Angewandte Mathematik 40 (1850) 51–88.
5. Mindlin RD. Influence of rotatory inertia and shear in flexural motions of isotropic, elastic plates. Journal of Applied Mechanics 18(1) (1951) 31-38.
6. Reissner E. The effect of transverse shear deformation on the bending of elastic plates. Journal of Applied Mechanics 12(2) (1945) 69-77.
7. Reddy JN. A simple higher-order theory for laminated composite plates. Journal of Applied Mechanics 51(4) (1984) 745–752.
8. Touratier M. An efficient standard plate theory. International Journal of Engineering Science 29(8) (1991) 901-916.
9. Soldatos KP. A transverse shear deformation theory for homogeneous monoclinic plates. Acta Mechanica 94(3–4) (1992) 195–220.
10. Noor AK, Burton WS. Three-Dimensional Solutions for the Free Vibrations and Buckling of Thermally Stressed Multilayered Angle-Ply Composite Plates. Journal of Applied Mechanics 59(4) (1992) 868-877.
11. Tenek LH, Henneke EG, Gunzburger MD. Vibration of delaminated composite plates and some applications to non-destructive testing. Composite Structures 23(3) (1993) 253-262.
12. Liew KM, Hung KC, Lim MK. A continuum three-dimensional vibration analysis of thick rectangular plates. International Journal of Solids and Structures 30(24) (1993) 3357-3379.
13. Cheung YK, Kong J. Approximate three-dimensional analysis of rectangular thick laminated plates: Bending, vibration and buckling. Computers & Structures 47(2) (1993) 193-199.
14. Chen WQ, Lü CF. 3D free vibration analysis of cross-ply laminated plates with one pair of opposite edges simply supported. Composite Structures 69(1) (2005) 77-87.
15. Ye JQ. A three-dimensional free vibration analysis of cross-ply laminated rectangular plates with clamped edges. Computer Methods in Applied Mechanics and Engineering 140(3-4) (1997) 383-392.
16. Malik M, Bert CW. Three-dimensional elasticity solutions for free vibrations of rectangular plates by the differential quadrature method. International Journal of Solids and Structures 35(3-4) (1998) 299-318.
17. So J, Leissa AW. Three-dimensional vibrations of thick circular and annular plates. Journal of Sound and Vibration 209(1) (1998) 15-41.
18. Liew KM, Yang B. Elasticity solutions for free vibrations of annular plates from three-dimensional analysis. International Journal of Solids and Structures 37(52) (2000) 7689-7702.
19. Huang W, Xue K, Li Q. Three-Dimensional Solution for the Vibration Analysis of Functionally Graded Rectangular Plate with/without Cutouts Subject to General Boundary Conditions. Materials 14(22) (2021) 7088.
20. Wang Z, Xing Y, Li G. Closed-form solutions for the free vibrations of three-dimensional orthotropic rectangular plates. International Journal of Mechanical Sciences 199 (2021) 106398.
21. Liew KM, Hung KC, Lim MK. Three-dimensional vibration of rectangular plates: Effects of thickness and edge constraints. Journal of Sound and Vibration 182(5) (1995) 709-727.
22. Uzun B, Yaylı MÖ. Nonlocal vibration analysis of Ti-6Al-4V/ZrO2 functionally graded nanobeam on elastic matrix. Arabian Journal of Geosciences 13 (2020) 155.
23. Uzun B, Kafkas U, Yaylı MÖ. Stability analysis of restrained nanotubes placed in electromagnetic field. Microsystem Technologies 26 (2020) 3725-3736.
24. Yaylı MÖ, Uzun B, Deliktaş B. Buckling analysis of restrained nanobeams using strain gradient elasticity. Waves in Random and Complex Media (2021) https://doi.org/10.1080/17455030.2020.1871112.
25. Alazwari MA, Zenkour AM. A Quasi-3D Refined Theory for the Vibration of Functionally Graded Plates Resting on Visco-Winkler-Pasternak Foundations. Mathematics 10(5) (2022) 716.