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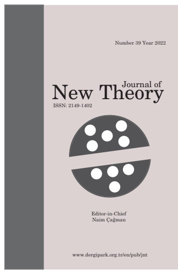

On the Multiplicative (Generalised) (α, α)-Derivations of Semiprime Rings

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Research Article

Abstract — The algebraic properties and identities of a semiprime ring are investigated with the help of the multiplicative (generalised)-(α, α)-reverse derivation on the non-empty ideal of the semiprime ring.

Keywords — *Semiprime ring, derivation, reverse derivation, generalised derivation, generalised reverse derivation*

Mathematics Subject Classification (2020) — 16N60, 16W25

1. Introduction

Many authors have investigated the relationship between the commutativity of a ring and the act of derivations(reverse derivation, (α, β)-reverse derivation, multiplicative reverse derivation, multiplicative (generalised) - (α, β)-reverse derivation etc.) defined on ring. Herstein was the first to introduce the concept of reverse derivation [1]. He shows that if R is a prime ring, and d is a nonzero reverse derivation of R, then R is a commutative integral domain, and d is a derivation. Firstly, Samman and Alyamani extended the result of Herstein to semiprime rings and investigated some more properties of reverse derivations in [2]. Asma and Bano inquired into some identities involving multiplicative (generalised) reverse derivation and demonstrated some theorems in which we characterise these mappings in [3]. Sandhu and Kumar investigate some properties of multiplicative reverse derivations on prime rings in [4]. Tiwari et al. described multiplicative (generalised) reverse derivation in [5]. The paper, as mentioned earlier, substantiated the commutativity of semiprime rings getting a multiplicative (generalised) reverse derivation satisfying some identities. Alhaidary and Majeed [6] proved commutativity of prime ring admitting a multiplicative (generalised) (α, β) reverse derivation such that α and β are automorphism on the prime ring, satisfying some identities. Further, they investigate some more properties of multiplicative (generalised)-(α, β)-reverse derivation of prime rings on square closed Lie ideals in [7]. The present paper study is directly motivated by the studies mentioned earlier and the work of Ulutaş and Gölbaşı [8]. We aim to investigate some identities with multiplicative (generalised) - (α, α)- reverse derivation on a nonzero ideal of a semiprime ring. Thus, we proved the following theorem:

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Theorem 1.1. Let R be a semiprime ring, α is an anti-epimorphism of R, $I \subsetneq Ker\alpha$ is a nonzero ideal of R and F is a nonzero multiplicative (generalised)-(α,α)-reverse derivation with the map d of R. If one of the following conditions holds,

i. $F([x,y]) = 0$,

ii. $F(xoy) = 0$,

iii. $F([x,y]) = \pm\alpha([x,y])$,

iv. $F(xoy) = \pm\alpha(xoy)$,

v. $F([x,y]) = \pm\alpha(xoy)$,

vi. $F(xoy) = \pm\alpha([x,y])$,

vii. $F([x,y]) = \pm\alpha([F(x),y])$,

viii. $F(xoy) = \pm\alpha(F(x)ox)$,

ix. $F(xy) = F(x)F(y)$,

x. $F(xy) + F(x)F(y) = 0$,

for all $x,y \in I$, then $[\alpha(x), d(x)] = 0$ for all $x \in I$.

2. Preliminaries

If the ring R satisfies the condition, $a = 0$ while $aRa = (0)$ for $a \in R$, it is called a semiprime ring. An additive mapping $d : R \to R$ is called (α,α) −derivation if $d(xy) = d(x)\alpha(y) + \alpha(x)d(y)$ holds for all $x,y \in R$, where α is automorphism of R. An additive mapping $d : R \to R$ is a reverse derivation, if $d(xy) = d(y)x + yd(x)$ for all $x,y \in R$. Let $d : R \to R$ be a map. If for all $x,y \in R$, $d(xy) = d(y)\alpha(x) + \alpha(y)d(x)$ such that α is an anti-epimorphism of R, then d is called multiplicative-(α,α)-reverse derivation. Let $F : R \to R$ be a mapping and d be a multiplicative (α,α)-reverse derivation . If for all $x,y \in R$,

$$F(xy) = F(y)\alpha(x) + \alpha(y)d(x)$$

then F is a called multiplicative (generalised)-(α,α)-reverse derivation associated with d. Hence the concept of multiplicative (generalised)-(α,α)-reverse derivation involves the concept of multiplicative (α,α)-reverse derivation and multiplicative generalised reverse derivation. Below, a multiplicative (generalised)-(α,α)-reverse derivation which is not multiplicative (generalised)-(α,α)-derivation and multiplicative (generalised)-(α,α)-derivation, which is not multiplicative (generalised)-(α,α)-reverse derivation examples, are given, respectively.

Example 2.1. Let $(R,+,.)$ be a commutative ring, and $(S,\oplus,\odot)$ be a noncommutative ring. Now let's consider operation $\circledast : S\times S \to S$, $a \circledast b = b \odot a$. With these operations $(S,\oplus,\circledast)$ called opposite ring and it is shown S^{op}.

$\alpha : S^{op} \to S^{op}$ is identity mapping, $d : S^{op} \to S^{op}$ is a multiplicative (α,α)-derivation, and $F : S^{op} \to S^{op}$ is a multiplicative (generalised)-(α,α)- derivation of R associated with a nonzero mapping d of R. Define the maps $\mu,\phi,\varphi : R\times S^{op} \to R\times S^{op}$ as follows: $\mu(a,x) = (a,F(x))$, $\phi(a,x) = (a,d(x))$ and $\varphi(a,x) = (a,\alpha(x))$. φ is an anti-homomorphism of R, and ϕ is multiplicative (φ,φ)-reverse derivation.

Then it is straightforward to verify that μ is a multiplicative (generalised)-(φ,φ)-reverse derivation associated with ϕ, but μ is not a multiplicative (generalised)-(φ,φ)-derivation of R.

Example 2.2. Now, define the maps $\mu,\phi,\varphi : R\times S^{op} \to R\times S^{op}$ as follows: $\mu(a,x) = (F(a),x)$, $\phi(a,x) = (d(a),x)$ and $\varphi(a,x) = (\alpha(a),x)$. ϕ is multiplicative (φ,φ)-derivation, and φ is an anti-homomorphism of R.

It is easy to see that μ is a multiplicative (generalised)-(φ,φ)-derivation if there exists a mapping ϕ, but μ is not a multiplicative (generalised)-(φ,φ)- reverse derivation of R.

3. Main Results

As of now on, R refers a semiprime ring, α is an anti-epimorphism of R, I is a nonzero ideal of R such that $I \subsetneq Ker(\alpha)$ and F is a nonzero multiplicative (generalised)-(α, α)-reverse derivation with the map d of R unless otherwise mentioned.

Lemma 3.1. d is a multiplicative -(α, α)-reverse derivation, that is, $d(xy) = d(y)\alpha(x) + \alpha(y)d(x)$ for all $x, y \in R$.

PROOF. By our assumption, we have

$$F(xz) = F(z)\alpha(x) + \alpha(z)d(x) \text{ for all } x, z \in R. \tag{1}$$

We put $x = xy$, $y \in R$ in (1), and since α is an anti-epimorphism of R,

$$F((xy)z) = F(z)\alpha(y)\alpha(x) + \alpha(z)d(xy) \text{ for all } x, y, z \in R. \tag{2}$$

Since $(xy)z = x(yz)$ and F is a multiplicative (generalised)-(α, α)-reverse derivation associated with the map d, that is

$$F(x(yz)) = F(z)\alpha(y)\alpha(x) + \alpha(z)d(y)\alpha(x) + \alpha(yz)d(x) \text{ for all } x, y, z \in R. \tag{3}$$

Subtracting (3) from (2), we obtain

$$\alpha(z)(d(xy) - d(y)\alpha(x) - \alpha(y)d(x)) = 0 \text{ for all } x, y, z \in R.$$

That is,

$$\alpha(R)(d(xy) - d(y)\alpha(x) - \alpha(y)d(x)) = 0 \text{ for all } x, y \in R.$$

Since α is an anti-epimorphism of R, $\alpha(R) = R$ and hence from above we have

$$R(d(xy) - d(y)\alpha(x) - \alpha(y)d(x)) = 0 \text{ for all } x, y \in R. \tag{4}$$

Left multipliying (4) by $d(xy) - d(y)\alpha(x) - \alpha(y)d(x)$, we get

$d(xy) - d(y)\alpha(x) - \alpha(y)d(x)R(d(xy) - d(y)\alpha(x) - \alpha(y)d(x)) = 0$ for all $x, y \in R$. Since R is a semiprime ring, we have

$$d(xy) - d(y)\alpha(x) - \alpha(y)d(x) = 0 \text{ for all } x, y \in R.$$

□

Lemma 3.2. $F(0) = 0$.

PROOF. From the definition of F, we get

$$F(xz) = F(z)\alpha(x) + \alpha(z)d(x) \text{ for all } x, z \in R. \tag{5}$$

Taking $x = 0$ and $z = 0$ in (5), one can obtain

$$F(0) = F(0)\alpha(0) + \alpha(0)d(0). \tag{6}$$

Since α is an additive map of R, it gives us $F(0) = 0$. □

Theorem 3.3. If $F([x, y]) = 0$ for all $x, y \in I$, then $[\alpha(x), d(x)] = 0$ for all $x \in I$.

PROOF. From our assumption,

$$F([x, y]) = 0 \text{ for all } x, y \in I. \tag{7}$$

If we write yx instead of x in (7), we get

$$0 = F([yx, y]) = F(y[x, y] + [y, y]x) \text{ for all } x, y \in I. \tag{8}$$

Besides, since F is a nonzero multiplicative (generalised)-(α, α)-reverse derivation with the map d, we have

$$F([x, y])\alpha(y) + \alpha([x, y])d(y) = 0 \text{ for all } x, y \in I. \tag{9}$$

When editing the last equation, we obtained

$$\alpha([x, y])d(y) = 0 \text{ for all } x, y \in I. \tag{10}$$

That is,

$$[\alpha(x), \alpha(y)]d(y) = 0 \text{ for all } x, y \in I. \tag{11}$$

Since α is an anti-epimorphism of R,

$$[z, \alpha(y)]d(y) = 0 \text{ for all } y \in I, z \in J, \tag{12}$$

where $J = \alpha(I)$ a nonzero ideal of R. We put $z = rz$, where $z \in J$, $r \in R$ in (12),

$$[r, \alpha(y)]zd(y) = 0 \text{ for all } y \in I, z \in J, r \in R. \tag{13}$$

We put $z = z\alpha(y)$ in (13),

$$[r, \alpha(y)]z\alpha(y)d(y) = 0 \text{ for all } y \in I, z \in J, r \in R. \tag{14}$$

Right multipliying (13) by $\alpha(y)$,

$$[r, \alpha(y)]zd(y)\alpha(y) = 0 \text{ for all } y \in I, z \in J, r \in R. \tag{15}$$

Subtracting (15) from (14),

$$[r, \alpha(y)]z[d(y), \alpha(y)] = 0 \text{ for all } y \in I, z \in J, r \in R. \tag{16}$$

Replacing r by $d(y)$ in (16),

$$[d(y), \alpha(y)]z[d(y), \alpha(y)] = 0 \text{ for all } y \in I, z \in J, r \in R,$$

where $J = \alpha(I)$ a semiprime ring, we get the required result. □

Theorem 3.4. If $F(xoy)=0$ for all $x,y\in I$, then $[\alpha(x),d(x)]=0$ for all $x\in I$.

PROOF. From the hypothesis,

$$F(xoy)=0 \text{ for all } x,y\in I. \tag{17}$$

Substituting y by yx in (17),

$$\alpha(xoy)d(x)=0 \text{ for all } x,y\in I. \tag{18}$$

Replacing y by zy, $z\in I$ in (18), and using α is an anti-epimorphism of R, we get

$$[\alpha(x),w]td(x)=0 \text{ for all } x\in I, w,t\in J. \tag{19}$$

Replacing w by $wd(x)$ in (19),

$$w[\alpha(x),d(x)]td(x)=0 \text{ for all } x\in I, w,t\in J. \tag{20}$$

Right multiplying (20) by $\alpha(x)$,

$$w[\alpha(x),d(x)]td(x)\alpha(x)=0 \text{ for all } x\in I, w,t\in J. \tag{21}$$

We put $t=t\alpha(x)$ in (20),

$$w[\alpha(x),d(x)]t\alpha(x)d(x)=0 \text{ for all } x\in I, w,t\in J. \tag{22}$$

Subtracting (22) from (21),

$$w[\alpha(x),d(x)]t[\alpha(x),d(x)]=0 \text{ for all } x\in I, w,t\in J. \tag{23}$$

Replacing t by w in (23),

$$w[\alpha(x),d(x)]tw[\alpha(x),d(x)]=0 \text{ for all } x\in I, w,t\in J,$$

where $J=\alpha(I)$ is semiprime ring, we get the required result. □

Theorem 3.5. If $F([x,y])=\pm\alpha([x,y])$ for all $x,y\in I$, then $[\alpha(x),d(x)]=0$ for all $x\in I$.

PROOF. By our assumption,

$$F([x,y])=\pm\alpha([x,y]) \text{ for all } x,y\in I. \tag{24}$$

Replacing x with yx in (24),

$$\alpha([x,y])d(x)=0 \text{ for all } x,y\in I.$$

Using the same arguments after (10) in the proof of Theorem 3.3, the desired result is obtained. □

Theorem 3.6. If $F(xoy)=\pm\alpha(xoy)$ for all $x,y\in I$, then $[\alpha(x),d(x)]=0$ for all $x\in I$.

PROOF. From the assumption,

$$F(xoy) = \pm\alpha(xoy) \text{ for all } x, y \in I. \tag{25}$$

Substituting x by yx in (25),

$$\alpha(xoy)d(y) = 0 \text{ for all } x, y \in I.$$

Since the last case is the same as the equation (10) and using the similar argument as used in the Theorem 3.4, the desired result is obtained. □

Theorem 3.7. If $F([x, y]) = \pm\alpha(xoy)$ for all $x, y \in I$, then $[\alpha(x), d(x)] = 0$ for all $x \in I$.

PROOF. By the assumption,

$$F([x, y]) = \pm\alpha(xoy) \; for \; all \; x, y \in I. \tag{26}$$

Replacing x with yx in (26),

$$\alpha([x, y])d(y) = 0 \text{ for all } x, y \in I.$$

The last expression is the same as the relation (10) and hence using the similar argument as used in Theorem 3.3, we get the required result. □

Theorem 3.8. If $F(xoy) = \pm\alpha([x, y])$ for all $x, y \in I$, then $[\alpha(x), d(x)] = 0$ for all $x \in I$.

PROOF. Substituting yx instead of x in hypothesis,

$$\alpha(xoy)d(y) = 0 \text{ for all } x, y \in I.$$

Since the last expression is the same as the equation (18), the desired result is obtained by the following similar steps in the Theorem 3.4, □

Theorem 3.9. If $F([x, y]) = \pm\alpha([F(x), y])$ for all $x, y \in I$, then $[\alpha(x), d(x)] = 0$ for all $x \in I$.

PROOF. By the supposition, we have

$$F([x, y]) = \pm\alpha([F(x), y]) \; for \; all \; x, y \in I. \tag{27}$$

We put $y = xy$ in (27),

$$\alpha([x, y])d(x) \; = \pm\alpha(y)\alpha([F(x), x]) \; for \; all \; x, y \in I. \tag{28}$$

Replacing x in place of y in (27),

$$\pm\, \alpha([F(x), x]) = 0 \; for \; all \; x \in I. \tag{29}$$

Appliying (29), (28) yields that

$$\alpha([x, y])d(x) = 0 \text{ for all } x, y \in I.$$

The equation is same as the equation (10) in Theorem 3.3, thus we proceed in the same way as in Theorem 3.3 and we get the required result. □

Theorem 3.10. If $F(xoy) = \pm\alpha(F(x)ox)$ for all $x, y \in I$, then $[\alpha(x), d(x)] = 0$ for all $x \in I$.

PROOF. By the hypothesis,

$$F(xoy) = \pm\alpha(F(x)ox) \ for \ all \ x, y \in I. \tag{30}$$

Replacing y with xy in (30),

$$\alpha(xoy)d(x) = \pm\alpha(y)\alpha([F(x), x]) \ for \ all \ x, y \in I. \tag{31}$$

Substituting yr, $r \in R$ for y in (31) and using this equation,

$$\alpha([x, r])\alpha(y)d(x) = 0 \ for \ all \ x, y \in I, r \in R. \tag{32}$$

Seeing α is an anti-epimorphism of R,

$$[\alpha(x), r]zd(x) = 0 \text{ for all } x \in I, z \in J, r \in R.$$

The last expression is the same as the relation (13) and hence using the similar argument as used in Theorem 3.3, we get the required result. □

Theorem 3.11. If $F(xy) = F(x)F(y)$ for all $x, y \in I$, then $[\alpha(x), d(x)] = 0$ for all $x \in I$.

PROOF. By the hypothesis,

$$F(xy) = F(x)F(y) \text{ for all } x, y \in I. \tag{33}$$

Then replacing y with xy in (33),

$$F(x(xy)) = F(xy)\alpha(x) + F(x)\alpha(y)d(x) \text{ for all } x, y \in I. \tag{34}$$

Since F is a nonzero multiplicative generalised-(α, α)-reverse derivation associated with a nonzero mapping d of R, it follows that

$$F(x(xy)) = F(xy)\alpha(x) + \alpha(xy)d(x) \text{ for all } x, y \in I. \tag{35}$$

Subtracting (35) from (34),

$$(\alpha(xy) - F(x)\alpha(y))d(x) = 0 \text{ for all } x, y \in I. \tag{36}$$

Substituting yr, $r \in R$ for y in (36) and since α is an anti-epimorphism of R,

$$(r\alpha(xy) - F(x)r\alpha(y))d(x) = 0 \text{ for all } x, y \in I, r \in R. \tag{37}$$

Replacing r with $F(z)$, where $z \in I$, in (37),

$$(F(z)\alpha(xy) - F(x)F(z)\alpha(y))d(x) = 0 \text{ for all } x, y, z \in I. \tag{38}$$

Left multiplying (36) by $F(z)$ and then subracting from (38),

$$(F(zx) - F(xz))\alpha(y)d(x) = 0 \text{ for all } x, y, z \in I. \tag{39}$$

Replacing xz in place of z in (39),

$$\alpha([z, x])d(x)\alpha(y)d(x) = 0 \text{ for all } x, y, z \in I. \tag{40}$$

Since for $r \in R$, $[z, x]r \in I$, we put $y = [z, x]r$ in (40) and α is an anti-epimorphism of R,

$$\alpha([z, x])d(x)R\alpha([z, x])d(x) = 0 \text{ for all } x, z \in I. \tag{41}$$

Since R is a semiprime ring,

$$\alpha([z, x])d(x) = 0 \text{ for all } x, z \in I. \tag{42}$$

Replacing z with zr, where $r \in R$, in (42),

$$[\alpha(r), \alpha(x)]\alpha(z)d(x) = 0 \text{ for all } x, z \in I, r \in R. \tag{43}$$

Right multiplying (43) by $\alpha(x)$,

$$[\alpha(r), \alpha(x)]\alpha(z)d(x)\alpha(x) = 0 \text{ for all } x, z \in I, r \in R. \tag{44}$$

Replacing xz in place of z in (43) and then subracting from (44),

$$[\alpha(r), \alpha(x)]\alpha(z)[d(x), \alpha(x)] = 0 \text{ for all } x, z \in I, r \in R. \tag{45}$$

Since α is an anti-epimorphism of R, $\alpha(R) = R$ and hence from above,

$$[r, \alpha(x)]\alpha(I)[d(x), \alpha(x)] = 0 \text{ for all } x \in I, r \in R. \tag{46}$$

We put $r = d(x)$ in (46) and since α is an anti-epimorphism of R,

$$[d(x), \alpha(x)]\alpha(I)[d(x), \alpha(x)] = 0 \text{ for all } x \in I. \tag{47}$$

Since $\alpha(I)$ is a semiprime ring,

$$[d(x), \alpha(x)] = 0 \text{ for all } x \in I.$$

□

Theorem 3.12. If $F(xy) + F(x)F(y) = 0$ for all $x, y \in I$, then $[\alpha(x), d(x)] = 0$ for all $x \in I$.

Proof. If F is a nonzero multiplicative generalised-(α, α)-reverse derivation associated with a nonzero map d, then $-F$ is a nonzero multiplicative generalised-(α, α)-reverse derivation associated with a nonzero map $-d$. We get the results by replacing F with $-F$ and d with $-d$ in Theorem 3.11. □

Theorem 3.13. If $F(xy) = F(y)F(x)$ for all $x, y \in I$, then $\alpha(I)d(I) = 0$ and $[F(y), \alpha(y)] = 0$ for all $y \in I$.

PROOF. We have

$$F(xy) = F(y)F(x) \text{ for all } x, y \in I. \tag{48}$$

Then replacing x with xz in (48), where $z \in I$,

$$F((xz)y) = F(zy)\alpha(x) + F(y)\alpha(z)d(x) \text{ for all } x, y, z \in I. \tag{49}$$

Since F is a nonzero multiplicative generalised-(α, α)-reverse derivation associated with a nonzero map d of R,

$$F(x(zy)) = F(zy)\alpha(x) + \alpha(y)\alpha(z)d(x) \text{ for all } x, y, z \in I. \tag{50}$$

Subtracting (49) from (50),

$$(F(y) - \alpha(y))\alpha(z)d(x) = 0 \text{ for all } x, y, z \in I. \tag{51}$$

Substituting z by zr, $r \in R$ in (51) and since α is an anti-epimorphism of R,

$$(F(y) - \alpha(y))R\alpha(z)d(x) = 0 \text{ for all } x, y, z \in I. \tag{52}$$

We put $x = y$ in (52),

$$(F(y) - \alpha(y))R\alpha(z)d(y) = 0 \text{ for all } y, z \in I. \tag{53}$$

Left multiplying (53) by $F(z)$, that is

$$F(z)(F(y) - \alpha(y))R\alpha(z)d(y) = 0 \text{ for all } x, y, z \in I. \tag{54}$$

Again, from (48) we can write $F(z)\alpha(y) + \alpha(z)d(y) = F(z)F(y)$ for all $x, y, z \in I$, that is

$$F(z)(F(y) - \alpha(y)) = \alpha(z)d(y) \ for\ all\ x, y, z \in I. \tag{55}$$

Let's substitute (55) in (54), we get $F(z)(F(y) - \alpha(y))RF(z)(F(y) - \alpha(y)) = 0$ for all $y, z \in I$. Moreover $\alpha(z)d(y)R\alpha(z)d(y) = 0$ for all $x, z \in I$ where R is semiprime ring, we conclude that $\alpha(z)d(y) = 0$ $for\ all\ x, z \in I$, that is $\alpha(I)d(I) = 0$ and $F(z)(F(y) - \alpha(y)) = 0$ for all $y, z \in I$. Thus we have

$$F(xy) = F(y)\alpha(x) \text{ for all } x, y \in I.$$

Now putting $z = yz$ and $y = y^2$ in $F(z)(F(y) - \alpha(y)) = 0$ for all $y, z \in I$,

$$F(z)\alpha(y)(F(y) - \alpha(y)) = 0 \ for\ all\ y, z \in I. \tag{56}$$

and

$$F(z)(F(y)\alpha(y) - \alpha(y)^2) = 0 \ for\ all\ y, z \in I. \tag{57}$$

Subtracting (57) from (56),

$$F(z)[F(y), \alpha(y)] = 0 \; for \; all \; y, z \in I. \tag{58}$$

We put $z = xz$ in (58),

$$F(z)\alpha(x)[F(y), \alpha(y)] = 0 \text{ for all } x, y, z \in I. \tag{59}$$

Then replacing z with z^2 in (59) and since α is an anti-epimorphism of R,

$$F(z)\alpha(z)\alpha(x)[F(y), \alpha(y)] = 0 \text{ for all } x, y, z \in I. \tag{60}$$

We put $x = xz$ in (59), we obtain

$$F(z)\alpha(z)\alpha(x)[F(y), \alpha(y)] = 0 \text{ for all } x, y, z \in I. \tag{61}$$

Left multiplying (59) by $\alpha(z)$, that is

$$\alpha(z)F(z)\alpha(x)[F(y), \alpha(y)] = 0 \text{ for all } x, y, z \in I. \tag{62}$$

Subtracting (61) from (62), we obtain

$$[F(z), \alpha(z)]\alpha(x)[F(y), \alpha(y)] = 0 \text{ for all } x, y, z \in I. \tag{63}$$

Then replacing x with xr, $r \in R$ in (63), and since α is an anti-epimorphism of R,

$$[F(z), \alpha(z)]R\alpha(x)[F(y), \alpha(y)] = 0 \text{ for all } x, y, z \in I. \tag{64}$$

Left multiplying (64) by $\alpha(x)$, we obtain

$$\alpha(x)[F(z), \alpha(z)]R\alpha(x)[F(y), \alpha(y)] = 0 \text{ for all } x, y, z \in I. \tag{65}$$

We put $y = z$ in (65), for all $x, y, z \in I$

$$\alpha(x)[F(z), \alpha(z)]R\alpha(x)[F(z), \alpha(z)] = 0 \text{ for all } x, z \in I,$$

is obtained.Since F and $\alpha(I)$ is semiprime ring, we conclude that $[F(y), \alpha(y)] = 0$ for all $y \in I$. □

Theorem 3.14. If $F(xy) + F(y)F(x) = 0$ for all $x, y \in I$, then $\alpha(I)d(I) = 0$ and $[F(y), \alpha(y)] = 0$ for all $y \in I$.

PROOF. If F is a nonzero multiplicative generalised-(α, α)-reverse derivation associated with a nonzero map d, then $-F$ is a nonzero multiplicative (generalised)-(α, α)-reverse derivation associated with a nonzero map d. Thus replacing F with $-F$ and d with $-d$ in Theorem 3.13. □

4. Conclusions

We have shown some properties of a nonzero ideal of a semiprime ring with multiplicative (generalised) (α, α)-reverse derivation. Moreover, when R is a semiprime ring, α is an anti-epimorphism of R, I is a nonzero ideal of R such that $I \subsetneq Ker(\alpha)$ and $F : R \to R$ is a nonzero multiplicative (generalised) (α, α)-reverse derivation, we investigated the commutativity of semiprime rings. Also, we give examples for each multiplicative (generalised) (α, α)-reverse derivation and generalised (α, α)-reverse derivation. Furthermore, we adapt some well-known results in reverse derivation to (α, α)-reverse derivation. The commutativity of a ring can be investigated in the sense of this article and the articles in [9–13].

Author Contributions

This study was derived from the master's thesis of the Handan Karahan. Neşet Aydın posed the problem and supervised this work's findings. Handan Karahan and Neşet Aydın wrote the manuscript in consultation with Didem Yeşil. Didem Yeşil reviewed and edited the manuscript. They all read and approved the last version of the paper.

Conflicts of Interest

The authors declare no conflict of interest.

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