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Capital budgeting of independent projects with budget limitation under intuitionistic fuzziness

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Abstract

Companies must make the right investment moves at the right time to sustain their existence and growth. For this reason, it is a critical decision for companies to distribute the existing capital to which projects and to what extent. Investment alternatives can be classified into two groups as dependent and independent. In dependent investment projects, the decision of whether to invest in a project directly affects the investment decision of the other project. In independent projects, it is necessary to consider the budget constraint in accordance with the reality of life. No matter how big the companies mentioned are, it is impossible to talk about an endless source of investment. For this reason, present value analysis, which is one of the most common methods used when evaluating independent project investments, is discussed under intuitionistic fuzziness in this study. It is aimed to achieve the most realistic results by reflecting the ambiguity and vagueness in the thoughts of the decision makers. In addition, this study demonstrates the integrability of fuzzy sets into engineering economics analysis methods. With this study, the cash flow data has been fuzzified and alternatives with both equal and different expected lives have been formulated to provide maximum returns under a certain budget limit. In addition, the responses of different parameters to changes were measured with intuitionistic fuzzy sensitivity analysis.

1. Introduction

For-profit companies operating in competitive market economies must make a constant effort to maintain their presence in the market and increase their market value. Increasing international competition in the globalization process rapidly changes all markets and causes companies to turn from national markets to international markets. In an increasingly competitive environment, the demand structure and customer preferences are changing rapidly. In such an economic environment, companies will always have to invest to increase their profitability and guaranty their continuity in the market. Investment is needed to create new production opportunities and increase existing ones, or to adapt to changes in the demand structure and customer choices. Companies must invest to avoid the risks posed by technological developments, take advantage of the opportunities, or to modernize. Otherwise, it will not be possible for companies to continue their existence in the market and increase their market values. If a company that is very successful today cannot make the necessary investments on time, it will have to leave the market after a while.

Planning is as important as the investment itself in making investments. However, investment plans differ according to whether the investments are large or small, short-term, or long-term. While the plans for small

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investments can be made by middle-level managers, since the investment is usually less than one year and has a small monetary volume, the plans for large investments are made by the senior management. Plans for large investments are made in more detail than small investments, since the investment usually lasts longer than one year and has larger cash flows in monetary terms and significantly affects the value and the continuity of the company. For this reason, the process of planning expenditures on assets whose returns are expected to exceed one year is characterized as a separate concept by the definition of capital budgeting. The word capital in this term refers to fixed assets in production, while the word budget refers to a certain period in the future.

Capital budgeting decisions must be taken with a great care, as they are so important to companies since the projects require large amounts of cash outflows and will be realized in the long run. The fact that the project covers a long period causes the decision makers to lose their flexibility. Because a significant part of the resources will be connected to this project for a long time. The success of the project depends on the recovery of the spent funds. Failures that may occur in a poorly designed project may lead to partial or complete failure of the expected income. In addition, allocating more resources than necessary both prevents the efficient use of resources and imposes a great burden on the company.

No matter how strong and large financial funds the enterprises have, they do not have the opportunity to finance and realize all alternative investment projects at the same time. For this reason, businesses must make a choice between investment offers that compete in terms of using their limited resources, rank them according to their importance and give up some investments for a while. In practice, businesses use different methods to evaluate investment projects. Depending on the method to be used in the selection of the investment project, different preferences may be encountered. For this reason, businesses must adopt a consistent method or see the results and decide according to each method to make a choice between investment proposals. Fixed asset investments can be classified as economically independent investments and economically dependent investments according to the effects on other investment projects.

Mutually exclusive projects are projects that cannot be realized together, but where only one project can be realized. If there are two projects as A and B, and these two projects cannot be realized, but one of these two projects can be realized, the projects A and B are alternative or mutually exclusive projects. When independent investments are considered mutually, it is assumed that there is an equal chance in these alternative projects. However, this may not be the case in practice. If there is equal chance of mutually exclusive investments, projects are evaluated within equal time periods. For this, reliable methods such as net present value (NPV) and internal rate of return are used. For example, if the firm adopts a long-lived project, it only looks at the NPV or internal rate of return; At the end of this period, the possibilities offered by alternative investments, which are short-lived, are ignored.

Dependent investments are when the expenditures required by an investment project and the cash inflows expected from the investment project are affected by the acceptance or rejection of other investment projects. Dependent investments occur when investment offers are mutually accepted. If investment decisions are thus interrelated, groups of consolidated units continue to depend on a single investment venture. In this case, mutual investments are accepted provided that the NPV is greater than zero. If the initial capital commitment amounts in the consolidated units are different, the profitability indexes on the investments are compared.

The subjective opinions of the decision maker in capital budgeting methods make them vague and ambiguous. To overcome the impreciseness in the decision makers' personal evaluations, fuzzy sets theory can be affectively applied in these methods. The concept of ordinary fuzzy set, developed by Zadeh (1965), has been accepted as a successful tool to overcome vagueness and ambiguity and has been successfully applied in various fields. The concept of ordinary fuzzy set has been developed based on the inadequacy of classical sets expressed with binary membership function in real world problems, complex systems involving human judgments and thoughts. The membership degree, which forms the basis of fuzzy sets, proposes to express the attributes with membership degree functions. The membership degree, which takes the value of 0 or 1 in classical sets, can take all values in the range of [0,1] in fuzzy sets. Ordinary fuzzy sets proposed by Zadeh have been frequently developed and extended by researchers in recent years. Intuitionistic fuzzy set (IFS) theory, which is the most widely applied in the literature and in many fields, was developed by Atanassov (1986). Studies have shown that IFSs are more effective in overcoming uncertainty than ordinary fuzzy set theory (Xu, 2007).

In Zadeh's ordinary fuzzy set theory, elements are represented by membership degree defined only in the range of 0-1, while in Atanassov's IFS theory, in addition to membership degree, non-membership degree is also defined. In ordinary fuzzy set theory, both membership and non-membership degrees can get values between 0 and 1. From this point of view, the sum of membership degree and non-membership degree is limited by 1. However, in IFS theory, the sum of these parameters does not require to be equal to 1. Atanassov defined a third parameter, called the degree of hesitation, to complete this sum to 1. In the literature, IFSs are used in various application areas

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recently such as evaluating subjective workload (Can, 2018) and green supplier selection with an application in the machine manufacturing sector (Zaralı, 2021).

The original contribution of this study to the literature is for the first time to evaluate capital budgeting problem under IFSs and to develop original equations in this field. The paper proposes capital budgeting for equal life projects and with different lives under IF environment. Also, a sensitivity analysis of the proposed approaches is presented. The proposed models can be easily extended by employing the other recent fuzzy set extensions such as picture fuzzy sets or spherical fuzzy sets. Through this study, it is aimed to prove the modelability and functionality of different engineering economics applications under fuzzy logic. By taking into account the uncertainties in economic decisions, the applicability of analyzes that reflect real life problems more accurately has been proven. In addition, this IF capital budgeting method developed can be successfully applied in many different case studies and evaluating real life problems.

Fuzzy logic has been integrated with capital budgeting analyses in various studies in the literature. Sergi et al. (2022) extended capital budgeting techniques using interval-valued Fermatean fuzzy sets and Sergi and Sari (2021) extended with single-valued Fermatean fuzzy sets. Sampaio Filho et al. (2018) presented a unified solution in fuzzy capital budgeting. de Souza Sampaio Filho et al. (2018) developed modified methods of capital budgeting under vagueness based on fuzzy numbers and interval arithmetic. Schneider and Kuchta (2018) introduced fuzzy capital budgeting for step type fuzzy interval projects. Silva et al. (2018) applied fuzzy goal programming to the process of capital budget in an economic environment under uncertainty. Etemadi et al. (2018) presented a goal programming capital budgeting model under uncertainty in construction industry using fuzzy analytic hierarchy process. Liu et al. (2017) showed fuzzy capital budgeting for project abandonment. Ucal Sari and Kahraman (2015) presented interval type-2 fuzzy capital budgeting. To the best knowledge of the authors, capital budgeting under budget limit has never been evaluated with fuzzy sets in the literature before.

The remaining sections of the paper are given as follows. Section 2 presents the preliminaries of IFSs. Section 3 presents the proposed intuitionistic fuzzy capital budgeting method for independent projects with budget limitation for both equal and unequal life projects, including a sensitivity analysis. Section 5 includes an illustrative application of the proposed approach on an aesthetic and beauty center's capital budgeting decision. Section 6 presents the conclusions and recommendations for future study.

2. Preliminieries: intuitionistic fuzzy sets

The membership degree of an element in the fuzzy sets can get a value between 0 and 1. Since there may be some hesitation degree, the non-membership degree may not be equal to 1 minus the membership degree. Thus, a generalization of ordinary fuzzy sets was proposed by Atanassov (1986) as intuitionistic fuzzy sets (IFS) which also considers the hesitation degree and defines it as 1 minus the sum of membership and non-membership degrees.

Definition 1. Let $X \neq \emptyset$ be a given set. An IFS in X is an object A given by

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)\rangle; x \epsilon X\}, \tag{1}$$

where $\mu_{\tilde{A}}: X \to [0,1]$ and $v_{\tilde{A}}: X \to [0,1]$ satisfy the condition

$$0 \leq \mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \leq 1, \tag{2}$$

for every $x \epsilon X$. Hesitancy is equal to "1-$\left(\mu_{\tilde{A}}(x) + v_{\tilde{A}}(x)\right)$"

Definition 2. An IF number $\tilde{A}$ is defined as follows:

An IF subset of the real line

Normal, i.e., there is any $x_0 \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x_0) = 1 \; (so \; v_{\tilde{A}}(x_0) = 0)$

A convex set for the membership function $\mu_{\tilde{A}}(x)$, i.e.,

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq min\left(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\right) \quad \forall x_1, x_2 \in \mathbb{R}, \lambda \in [0,1] \tag{3}$$

A concave set for the non-membership function $v_{\tilde{A}}(x)$, $i.e.$,

$$v_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \leq max\left(v_{\tilde{A}}(x_1), v_{\tilde{A}}(x_2)\right) \; \forall x_1, x_2 \in \mathbb{R}, \lambda \in [0,1]. \tag{4}$$

Definition 3. Suppose $\tilde{X} = (\mu_x, v_x)$ and $\tilde{Y} = (\mu_y, v_y)$ be two IFSs. Some basic mathematical operations are defined as follows [2].

$$\tilde{X} \oplus \tilde{Y} = (\mu_x + \mu_y - \mu_x\mu_y, v_x v_y) \tag{5}$$

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$$\tilde{X} \otimes \tilde{Y} = (\mu_x \mu_y, v_x + v_y - v_x v_y) \quad (6)$$

$$\alpha \tilde{X} = (1 - (1 - \mu_x)^\alpha, v_x^\alpha) \quad (7)$$

$$\tilde{X}^\alpha = (\mu_x^\alpha, 1 - (1 - v_x)^\alpha) \quad (8)$$

Definition 4. Suppose $\tilde{X} = (\mu_x, v_x)$ is an IFN. Intuitionistic Fuzzy Weighted Geometric Operator (IFWG) and Intuitionistic Fuzzy Weighted Arithmetic Operator (IFWA) with respect to, $w_i = (w_1, w_2 \ldots\ldots, w_n)$; $w_i \in [0,1]$; $\sum_{i=1}^{n} w_i = 1$, is defined as follows.

$$IFWG(\tilde{X}_1, \ldots\ldots, \tilde{X}_n) = \left\{ \prod_{i=1}^{n} \mu_{x_i}^{w_i}, 1 - \prod_{i=1}^{n} (1 - v_{x_i})^{w_i} \right\} \quad (9)$$

$$IFWA(\tilde{X}_1, \ldots\ldots, \tilde{X}_n) = \left\{ 1 - \prod_{i=1}^{n} (1 - \mu_{x_i})^{w_i}, \prod_{i=1}^{n} v_{x_i}^{w_i} \right\} \quad (10)$$

Definition 5. To defuzzify IFNs, the following score function is given in Equation 11.

$$Sc = Score(\tilde{X}) = \mu_{\tilde{X}} - v_{\tilde{X}} \quad (11)$$

3. Independent projects with budget limitation

This section will present the proposed IF capital budgeting approach for independent projects under budget limitation. The approach will be given separately for equal and unequal life projects. Finally, a sensitivity analysis will be shown.

3.1 Equal-life projects

First list all mutually exclusive bundles such as one project at a time, two at a time, etc. to choose among the projects with equal lives when the investment budget limit is l. The total investment of each feasible bundle cannot exceed this limit l and a bundle as "do-nothing (DN)" should be added. 2^m is the total number of bundles for m number of projects. Then, each bundle's present value (PV) should be calculated with the given MARR. The bundle that gives the highest PV should be selected. The steps of capital budgeting using IF-PV analysis for equal-life projects are as follows.

Aggregate the first cost (FC), annual benefit (AB), annual cost (AC), and salvage value (SV) values for each year by using Equations (12-15), respectively.

$$\widetilde{NCF}_{agg,FC} = \begin{pmatrix} w_1 FC_{j1} + w_2 FC_{j2} + w_3 FC_{j3}; \\ 1 - \prod_{e=1}^{3} (1 - \mu_{je})^{w_e}, \prod_{e=1}^{3} v_{je}{}^{w_e} \end{pmatrix} \text{ for j=0} \quad (12)$$

$$\widetilde{NCF}_{agg,AB} = \begin{pmatrix} w_1 AB_{j1} + w_2 AB_{j2} + w_3 AB_{j3}; \\ 1 - \prod_{e=1}^{3} (1 - \mu_{je})^{w_e}, \prod_{e=1}^{3} v_{je}{}^{w_e} \end{pmatrix} \text{ for j=1,…, } n. \quad (13)$$

$$\widetilde{NCF}_{agg,AC} = \begin{pmatrix} w_1 AC_{j1} + w_2 AC_{j2} + w_3 AC_{j3}; \\ 1 - \prod_{e=1}^{3} (1 - \mu_{je})^{w_e}, \prod_{e=1}^{3} v_{je}{}^{w_e} \end{pmatrix} \text{ for j=1,…, } n. \quad (14)$$

$$\widetilde{NCF}_{agg,SV} = \begin{pmatrix} w_1 SV_{j1} + w_2 SV_{j2} + w_3 SV_{j3}; \\ 1 - \prod_{e=1}^{3} (1 - \mu_{je})^{w_e}, \prod_{e=1}^{3} v_{je}{}^{w_e} \end{pmatrix} \text{ for j=1,…, } n. \quad (15)$$

where w_e is the weights of the experts depending on their experience levels.

List all mutually exclusive bundles with a total initial investment limited by l.

Sum the net cash flows $\widetilde{NCF}_{jt}$ for all projects in each bundle j ($j = 1,2, \ldots, 2^m$) and each year t $\left(t = 1,2, \ldots, n_j\right)$. Refer to the initial investment of bundle j at time $t = 0$ as $\widetilde{NCF}_{j0}$.

Calculate the $\widetilde{PV}_j$ for each bundle at the MARR (Equation 16).

$$\widetilde{PV}_j = \sum_{t=1}^{t=n_j} \widetilde{NCF}_{jt}(P/F, i, t) - \widetilde{NCF}_{j0} \quad (16)$$

Select the bundle with the highest PV_j.

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3.2 Unequal-life projects

Generally, in real case scenarios, independent projects do not have the same expected life. If the PV method is used to solve capital budgeting problems, it is assumed that each project will last for the longest lasting project. In addition, any positive net cash flow reinvestments are assumed to be in MARR from the moment they occur to the end of the longest-lived project.

Assume two independent projects have the cash flows in Table 1 and their useful lives are different. The least common multiple (LCM) of their lives is n_{LCM}.

Table 1. Two independent projects

	Alternative A	Alternative B
First Cost	$(\$\mathrm{FC}_A;\, \mu_{\mathrm{FC}_A}, v_{\mathrm{FC}_A})$	$(\$\mathrm{FC}_B;\, \mu_{\mathrm{FC}_B}, v_{\mathrm{FC}_B})$
Uniform Annual Benefit	$(\$\mathrm{UAB}_A;\, \mu_{UAB_A}, v_{UAB_A})$	$(\$\mathrm{UAB}_B;\, \mu_{UAB_B}, v_{UAB_B})$
Uniform Annual Cost	$(\$\mathrm{UAC}_A;\, \mu_{UAC_A}, v_{UAC_A})$	$(\$\mathrm{UAC}_B;\, \mu_{UAC_B}, v_{UAC_B})$
Useful Life	n_A	n_B
Salvage Value	$(\$\mathrm{SV}_A;\, \mu_{SV_A}, v_{SV_A})$	$(\$\mathrm{SV}_B;\, \mu_{SV_B}, v_{SV_B})$

Considering the analysis period based on the LCM lives the following table can be prepared. It is assumed that the useful life of n_B is double of the useful life of n_A.

$n_{LCM} = LCM(n_A, n_B) = n_B$

Table 2. Cash flow of the independent projects along LCM years

Year	Alternative A	Alternative B
0	$(-\$\mathrm{FC}_A;\, \mu_{\mathrm{FC}_A}, v_{\mathrm{FC}_A})$	$(-\$\mathrm{FC}_B;\, \mu_{\mathrm{FC}_B}, v_{\mathrm{FC}_B})$
1 to n_A	$(\$\mathrm{UAB}_A;\, \mu_{UAB_A}, v_{UAB_A})$ – $(\$\mathrm{UAC}_A;\, \mu_{UAC_A}, v_{UAC_A})$	$(\$\mathrm{UAB}_B;\, \mu_{UAB_B}, v_{UAB_B})$ – $(\$\mathrm{UAC}_B;\, \mu_{UAC_B}, v_{UAC_B})$
n_A	$(-\$\mathrm{FC}_A;\, \mu_{\mathrm{FC}_A}, v_{\mathrm{FC}_A}) + (\$\mathrm{SV}_A;\, \mu_{SV_A}, v_{SV_A})$	-
$(n_A + 1)$ to n_B	$(\$\mathrm{UAB}_A;\, \mu_{UAB_A}, v_{UAB_A})$ – $(\$\mathrm{UAC}_A;\, \mu_{UAC_A}, v_{UAC_A})$	$(\$\mathrm{UAB}_B;\, \mu_{UAB_B}, v_{UAB_B})$ – $(\$\mathrm{UAC}_B;\, \mu_{UAC_B}, v_{UAC_B})$
$n_B = n_{LCM}$	$(\$\mathrm{SV}_A;\, \mu_{SV_A}, v_{SV_A})$	$(\$\mathrm{SV}_B;\, \mu_{SV_B}, v_{SV_B})$

After the cash flows along the LCM years for all the alternatives are built, the same procedure in the previous section is applied.

3.3 Sensitivity surface approach

Sensitivity analysis allows us to see the effects of possible changes in investment parameters on the investment decision before making the investment. Thus, we determine how sensitive our decision is to the possible changes in investment parameters.

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Table 3. Most likely parameter values of investment project

Parameter	Most Likely Estimates by Experts	Aggregated Estimate
$\widetilde{FC}$	$(FC_1;\mu_{FC_1},v_{FC_1}),(FC_2;\mu_{FC_2},v_{FC_2}),(FC_3;\mu_{FC_3},v_{FC_3})$	$\left(FC_{agg};\mu_{FC_{agg}},v_{FC_{agg}}\right)$
$\widetilde{AB}$	$(AB_1;\mu_{AB_1},v_{AB_1}),(AB_2;\mu_{AB_2},v_{AB_2}),(AB_3;\mu_{AB_3},v_{AB_3})$	$\left(AB_{agg};\mu_{AB_{agg}},v_{AB_{agg}}\right)$
$\widetilde{AC}$	$(AC_1;\mu_{AC_1},v_{AC_1}),(AC_2;\mu_{AC_2},v_{AC_2}),(AC_3;\mu_{AC_3},v_{AC_3})$	$\left(AC_{agg};\mu_{AC_{agg}},v_{AC_{agg}}\right)$
$\widetilde{SV}$	$(SV_1;\mu_{SV_1},v_{SV_1}),(SV_2;\mu_{SV_2},v_{SV_2}),(SV_3;\mu_{SV_3},v_{SV_3})$	$\left(SV_{agg};\mu_{SV_{agg}},v_{SV_{agg}}\right)$

Let's assume that the parameters that the experts are not sure about their estimated values are AB and AC. Then the PV equation for sensitivity analysis can be written as in Equation (17).

$$\widetilde{NPV} = -\widetilde{FC}_{agg} + \widetilde{AB}_{agg}(1+x)\left(\frac{P}{A},i,n\right) - \widetilde{AC}_{agg}(1+y)(P/A,\ i,n) + \widetilde{SV}_{agg}(P/F,i\,n) \quad (17)$$

To have an acceptable investment project it should be NPV ≥ 0. Then, we obtain an inequality as follows.

$$\mathcal{B} + \mathcal{F}x + \mathcal{R}y \geq 0 \quad (18)$$

When $x = 0$ in Eq. (18) $y = h$ and when $y = 0$ in Eq. (18) $x = \ell$.

Then the function in Eq. (18) can be illustrated as in Figure 1.

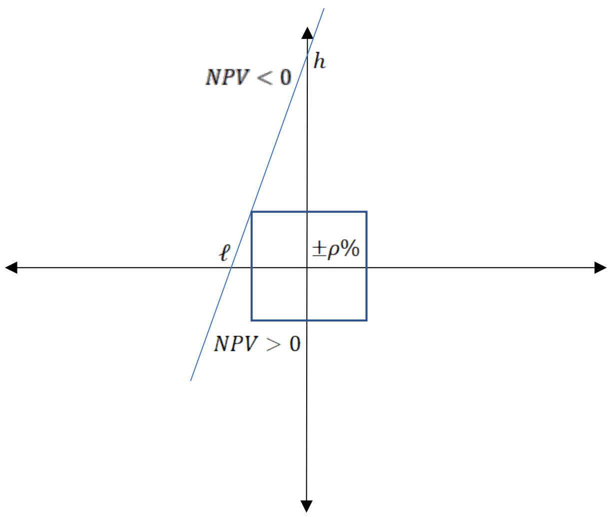

Figure 1. Sensitivity graph

Figure 1 illustrates that the considered project is acceptable up to $\pm\rho\%$ deviations in AB and AC simultaneously. Larger deviations than $\pm\rho\%$ may cause the project to give a negative PV.

4. Application

A business serving as an aesthetic and beauty center in Istanbul aims to attract more customers by providing digitalization in one of their manual processes. There are three alternative devices (A1, A2, and A3) that can be purchased for this purpose. The investment costs, annual costs and benefits of the alternatives, and the salvage values of the devices at the end of their 3-year useful life are evaluated by three experts under intuitionistic fuzziness and given in Table 4 below. Experts' weights are taken as 0.4, 0.4, and 0.2, respectively depending on their experience levels. The uncertainty in the initial cost is because the cost of the device to be purchased in foreign currency will vary depending on the current exchange rate and the personal predictions of the experts. 12%

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return is expected. The total budget allocated by the aesthetic and beauty center for the initial investment of the device is $30,000.

Table 4. IF cash parameters

Investment Alternatives	Experts	First Cost	Annual Benefit	Annual Cost	Salvage Value
A1	E1	($15,000; 0.85, 0.10)	($14,000; 0.85, 0.10)	($3,000; 0.85, 0.10)	($1,500; 0.85, 0.10)
	E2	($18,000; 0.90, 0.10)	($13,000; 0.85, 0.15)	($4,000; 0.90, 0.10)	($2,500; 0.90, 0.10)
	E3	($17,500; 0.80, 0.20)	($12,500; 0.95, 0.05)	($3,500; 0.80, 0.20)	($2,000; 0.80, 0.20)
A2	E1	($20,000; 0.85, 0.10)	($15,000; 0.85, 0.15)	($6,000; 0.85, 0.10)	($3,500; 0.85, 0.10)
	E2	($18,000; 0.85, 0.15)	($13,000; 0.95, 0.00)	($5,000; 0.85, 0.15)	($2,500; 0.85, 0.15)
	E3	($18,000; 0.95, 0.05)	($14,000; 0.95, 0.05)	($5,500; 0.95, 0.05)	($3,000; 0.95, 0.05)
A3	E1	($16,000; 0.85, 0.15)	($14,500; 0.85, 0.10)	($2,500; 0.85, 0.15)	($4,000; 0.85, 0.15)
	E2	($15,000; 0.95, 0.00)	($15,500; 0.90, 0.10)	($3,000; 0.95, 0.00)	($3,000; 0.95, 0.00)
	E3	($14,000; 0.95, 0.05)	($14,000; 0.80, 0.20)	($4,500; 0.95, 0.05)	($3,000; 0.95, 0.05)

There are $2^3 = 8$ possible bundles as presented in Table 5.

Table 5. Possible bundles and their first costs

Bundle *j*	Alternatives Included	First Cost
1	A1	$-12,480.56
2	A2	$-14,591.88
3	A3	$-14,003.39
4	A1-A2	$-27,072.44
5	A1-A3	$-26,483.95
6	A2-A3	$-28,595.26
7	A1-A2-A3	$-41,075.82
8	DN	$0

Except for bundle 7, since all of them are below the initial investment budget, the decision will be made by looking at their PVs. To calculate the PV of the alternatives, first the cash parameters are aggregated by using Equations (12-15) and the results are given in Table 6.

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Table 6. Aggregated parameters

Investment Alternatives	Aggregated FC	Aggregated AB	Aggregated AC	Aggregated SV
A1	($16,639.99; 0.865, 0.115)	($13,286.50; 0.880, 0.102)	($3,471.25; 0.865, 0.115)	($1,949.03; 0.865, 0.115)
A2	($18,774.81; 0.880,0.102)	($13,971.38; 0.922, 0.000)	($5,481.77; 0.880, 0.102)	($2,966.38; 0.880, 0.102)
A3	($15,181.34; 0.922, 0.000)	($14,787.87; 0.865, 0.115)	($3,024.59; 0.922, 0.000)	($3,365.87; 0.922, 0.000)

Then the aggregated IF parameters are defuzzied by using the score function in Equation (11).

Table 7. Defuzzified parameters

Investment Alternatives	Deffuzzified FC	Deffuzzified AB	Deffuzzified AC	Deffuzzified SV
A1	$12,480.56	$10,326.34	$2,603.56	$1,461.84
A2	$14,591.88	$12,887.31	$4,260.46	$2,305.49
A3	$14,003.39	$11,091.40	$2,789.91	$3,104.70

Next, the PVs of the parameters are calculated by in Equations (12-15), respectively.

Table 8. PVs of the parameters

Investment Alternatives	FC	AB	AC	SV
A1	$12,480.56	$24,802.11	$6,253.31	$1,040.51
A2	$14,591.88	$30,953.15	$10,232.91	$1,641.00
A3	$14,003.39	$26,639.68	$6,700.89	$2,209.86

Finally, the total PVs of the bundles are calculated as in Table 9.

Table 9. Bundles and their PVs.

Bundle *j*	Alternatives Included	PV
1	A1	$7,108.76
2	A2	$7,769.37
3	A3	$8,145.27
4	A1-A2	$14,878.12
5	A1-A3	$15,254.03
6	A2-A3	$15,914.64
8	DN	0

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Since the bundle with the highest PV is 6 among the seven alternatives whose initial investment cost is below the budget limit, this alternative should be selected.

4.1 Sensitivity analysis of the given problem

In the following, the same problem will be solved for different expected lives of the alternatives. Suppose that the expected life of A1 is 2 years, expected life of A2 is 3 years, and expected life of A3 is 6 years. The possible bundles and their first costs will be the same as in Table 5, which means there will be seven possible bundles except Bunde 7. So, it is needed to calculate the PVs of the bundles along the LCM of lives. First, the cash flow of the three alternatives along LCM of lives, which is 6 years, are given as in Table 10.

Table 10. Cash flow of alternatives projects along LCM of lives

n	A1	A2	A3
0	$12,480.56	$14,591.88	$14,003.39
1	$10,326.34 -$2,603.56	$12,887.31 -$4,260.46	$11,091.40 -$2,789.91
2	$10,326.34 -$2,603.56 -$12,480.56 +$1,461.84	$12,887.31 -$4,260.46	$11,091.40 -$2,789.91
3	$10,326.34 -$2,603.56	$12,887.31 -$4,260.46 -$14,591.88 +$2,305.49	$11,091.40 -$2,789.91
4	$10,326.34 -$2,603.56 -$12,480.56 +$1,461.84	$12,887.31 -$4,260.46	$11,091.40 -$2,789.91
5	$10,326.34 -$2,603.56	$12,887.31 -$4,260.46	$11,091.40 -$2,789.91
6	$10,326.34 -$2,603.56 +$1,461.84	$12,887.31 -$4,260.46 +$2,305.49	$11,091.40 -$2,789.91 +$3,104.70

Then the PVs of the projects are calculated as in Table 11. NPVs are given in the last row.

Table 11. Cash flow of alternatives projects along LCM of lives

n	A1	A2	A3
0	$12,480.56	$14,591.88	$14,003.39
1	$6,895.34	$7,702.54	$7,412.04
2	$-2,627.50	$6,877.27	$6,617.90
3	$5,496.92	$-2,604.79	$5,908.84
4	$-2,094.63	$5,482.52	$5,275.75
5	$4,382.11	$4,895.11	$4,710.49
6	$4,653.21	$5,538.66	$5,778.73
NPV	$29,186.02	$42,483.20	$49,707.13

Table 12 shows the bundles with their total PVs.

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Table 12. Bundles and their PVs

Bundle j	Alternatives Included	PV
1	A1	\$29,186.02
2	A2	\$42,483.20
3	A3	\$49,707.13
4	A1-A2	\$71,669.21
5	A1-A3	\$78,893.15
6	A2-A3	\$92,190.33
8	DN	0

Since the bundle with the highest NPV is 6 among the seven alternatives whose initial investment cost is below the budget limit, this alternative should be selected.

In the following a sensitivity analysis approach will be presented for the first alternative (A1) with 3 years of expected life. The parameters that the three experts are not sure about their predicted values are $\widetilde{AB}$ and $\widetilde{AC}$. For these two parameters a sensitivity surface approach analysis will be applied, and the riskless area of this investment will be determined.

First, the NPV equation is set as follows.

$NPV = (16{,}639.99;\ 0.865, 0.115) + (13{,}286.50;\ 0.880, 0.102)(1 + x)(P/A, 12\%, 3) + (-3{,}471.25;\ 0.865, 0.115)(1 + y)(P/A, 12\%, 3) + (1{,}949.03;\ 0.865, 0.115)(P/F, 12\%, 3)$

The IF parameter values in the above NPV equation are defuzzified by using Equation (11) as follows.

$= -12{,}480.56 + 24{,}802.11(1 + x) - 6{,}253.31(1 + y) + 1{,}040.51$

$= 7{,}108.76 + 24{,}802.11x - 6{,}253.31y$

The investment will be acceptable if $7{,}108.76 + 24{,}802.11x - 6{,}253.31y \geq 0$. From this inequality, we can obtain Figure 2 by substituting $x = 0$ and $y = 0$ and finding the corresponding y and x values, respectively. For $x = 0$, $y = 1.137$ and for $y = 0$, $x = -0.287$ are obtained.

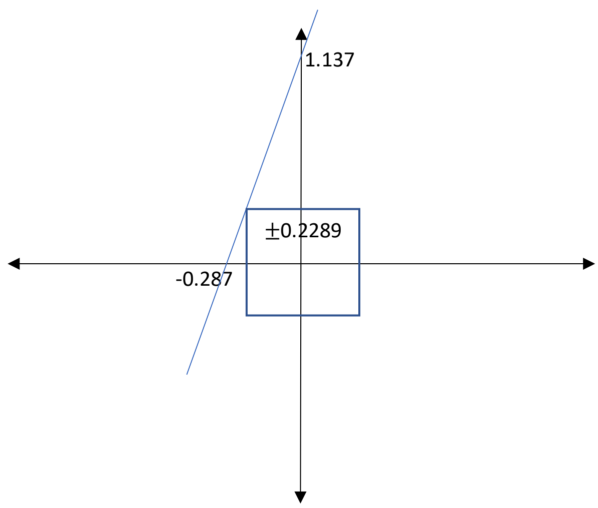

Figure 2. Sensitivity graph

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Based on Figure 2 we can state that the simultaneous variations in AB and AC up to 23% make no risk for the investment. If the possible variation in the market conditions is at most 20%, for instance, there should be no hesitation to make the investment.

5. Conclusion

In today's world, it is almost impossible to talk about companies with endless resources. Regardless of the size of companies, their capital is limited as well as resources such as manpower, raw materials, etc. Capital budgeting is the process of decision making about capital investment in long-term assets. Since the amounts invested in long-term assets are often large, the research and decision process take a long time and requires many upfront transactions. Different methods are used to make decisions in the capital budgeting process. It is an important issue whether the time factor is considered in the methods used. For this reason, these methods can be divided into two groups as ignoring the time factor (static) and taking it into account (dynamic). Present value analysis is one of the most frequently used dynamic methods. Although there are many studies in the literature on this subject, no study has been found that deals with independent projects under budget constraints. With the new equations proposed in this study, a new perspective has been brought to capital budgeting under fuzziness. While the study included the evaluation of projects with both equal and different lives, the effect of the changes in the parameters on the result was examined with the sensitivity analysis.

With this study, it has been proven why the evaluation of engineering economics problems under blur will give more realistic results. Especially in long-term investments, investment parameters contain considerable uncertainty. As useful life increases in inflationary markets, values such as uniform annual benefit, uniform annual cost, and salvage value become uncertain. In addition, the first cost parameter is highly dependent on environmental factors such as exchange rate changes and inflation. Again, in long-term investments, the useful life may vary significantly over time compared to the initial estimate. In this study, the uncertainties in all these parameters were handled under IFSs and reflected in a realistic way.

The limitation of this study is that it does not take into consideration of various non-financial aspects of the projects which play an important role in successful and profitable implementation and possible modifications in capital budgeting decisions in time are not considered since it is hard to locate the market for capital goods.

In future studies, it is recommended to consider the equations proposed in this study with different fuzzy set extensions such as picture fuzzy sets, spherical fuzzy sets, or Fermatean fuzzy sets and present a comparison analysis. Also, it is recommended to expend the analysis with infinite life projects.

Conflicts of Interest

The author declared that there is no conflict of interest.

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