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SEPARATION AXIOMS ON NEAR SOFT TOPOLOGICAL SPACES

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Abstract. Near soft sets are a very successful mathematical model that has been used in order to express the decision-making process for uncertainty in a more ideal way, especially in recent years. The purpose of this paper is to contribute to the theoretical studies on near soft topological spaces. In addition, it presents basic concepts and constructs that will form the basis for a near theoretical set-up of near soft topological spaces. These concepts and structures include sub near soft set, near soft subspaces of a near soft topological space and near soft T_i-spaces for $0 \leq i \leq 4$. The important aspects of the paper are discussed, especially by examining the definitions and properties given.

1. Introduction

In recent days, mathematical modeling for uncertain data has become an increasingly important topic in various research fields. Therefore, many researchers have worked on mathematical modeling to describe uncertainty. Rough set theory [8], based on the equivalence relations to describe uncertainty, was proposed by Pawlak in 1982. In addition to this theory; theories such as probability theory, fuzzy set theory [1], rough set theory [8] and interval mathematics theory [9, 10] is considered as a very successful tool to describe uncertainty. But each of them has its own inherent difficulties. In 1999, Molodtsov [2] introduced the new concept of soft sets to deal with the challenges of existing methods of uncertainty and established the fundamental results of this theory in solving many practical problems in economics, social science, medical science, etc. Studies on soft sets are progressing rapidly in recent years [13, 14, 15, 16, 17, 18, 19]. In 2010, Feng et al. [11] described the concept of the soft rough set using soft set and rough set. In [11, 12], basic properties of soft rough approximations were presented and supported by some illustrative examples. In the following years, the definition of the near soft sets, which managed to attract the attention of the researchers with its structure similar to soft rough sets, was introduced by Tasbozan et al.[3]. This set theory can be

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given to obtaining the lower and upper approximations of a soft set in nearness approximation space (NAS). Moreover, the structure of near soft topology was given in the near soft set theory by [3] and Tasbozan et al.[3] and Özkan [4] examined some properties of the near soft topological spaces.

In this study, some new concepts for near soft topological spaces are given and near soft separation axioms are examined in detail. In addition, many examples are provided to make the study more understandable.

2. Preliminaries

In this section, we recall some basic notions in near soft sets and near soft topological spaces and detailed information on all rotations used can be found at [4, 3, 5, 6, 7].

Let O be an initial universe set and $\mathcal{F}$ be a set of parameters with respect to O unless otherwise specified and $A, B \subseteq \mathcal{F}$. Let $N_r(B)(X)$ be a family of neighborhoods of a set X and $X \subseteq O$.

Definition 2.1. [2] A soft set over O is characterized by a pair F_B, where $B \subseteq \mathcal{F}$ and F is a mapping given by $F : B \to P(O)$.

Definition 2.2. [4] Let $NAS = (O, \mathcal{F}, \sim_{Br}, N_r, V_{N_r})$ be a nearness approximation space and $\sigma = F_B$ be a soft set over O. The lower and upper near approximation of $\sigma = F_B$ with respect to NAS are denoted by $N_{r^*}(\sigma) = F_{B^*}$ and $N_r^*(\sigma) = F_B^*$, which are soft sets over with the set-valued mappings given by

$$F_*(\phi) = N_{r^*}(F(\phi)) = \bigcup_{a \in O} \{\bar{a}_{B_r} \subseteq F(\phi)\},$$

$$F^*(\phi) = N_r^*(F(\phi)) = \bigcup_{a \in O} \{\bar{a}_{B_r} \cap F(\phi) \neq \emptyset\},$$

where all $\phi \in B$. The operators N_{r^*} and N_r^* are called the lower and upper near approximation operators on soft sets, respectively. If $BN_{N_r(B)}(X) \geq 0$, then the soft set σ is called a near soft set.

Example 2.3. Let $O = \{a_1, a_2, a_3, a_4, a_5\}$, $B = \{\phi_1, \phi_2\} \subseteq \mathcal{F} = \{\phi_1, \phi_2, \phi_3\}$ denote a set of perceptual objects and a set of parameters, respectively. Sample values of the ϕ_i, $1 \leq i \leq 3$ functions are shown in Table 1.

Table 1.

	a_1	a_2	a_3	a_4	a_5
ϕ_1	2	1	3	2	3
ϕ_2	1	0	2	3	2
ϕ_3	1	1	0	2	0

Let $\sigma = F_B$, $B = \{\phi_1, \phi_2\}$ be a soft set defined by

$$F_B = \{(\phi_1, \{a_1, a_2\}), (\phi_2, \{a_2, a_3, a_4\})\}$$

Then for $r = 1$,

$$\overline{a_1}_{\phi_1} = \{a_1, a_4\}, \quad \overline{a_2}_{\phi_1} = \{a_2\}, \quad \overline{a_4}_{\phi_1} = \{a_3, a_5\}$$

$$\overline{a_1}_{\phi_2} = \{a_1\}, \quad \overline{a_2}_{\phi_2} = \{a_2\}, \quad \overline{a_3}_{\phi_2} = \{a_3, a_5\}, \quad \overline{a_4}_{\phi_2} = \{a_4\}$$

$N_*(\sigma) = \{(\phi_1, \{a_2\}), (\phi_2, \{a_2, a_4\})\}$ and $N^*(\sigma) = \{(\phi_1, \{a_1, a_2, a_4\}), (\phi_2, \{a_2, a_3, a_4, a_5\})\}$. $BN_N(\sigma) \geq 0$ and then σ is a near soft set.

Now for r = 2;

$$\overline{a_1}_{\phi_1,\phi_2} = \{a_1\}, \quad \overline{a_2}_{\phi_1,\phi_2} = \{a_2\}, \quad \overline{a_3}_{\phi_1,\phi_2} = \{a_3, a_5\}, \quad \overline{a_4}_{\phi_1,\phi_2} = \{a_4\}$$

$N_*(\sigma) = \{(\phi_1, \{a_1, a_2\}), (\phi_2, \{a_2, a_4\})\}$ and $N^*(\sigma) = \{(\phi_1, \{a_1, a_2\}), (\phi_2, \{a_2, a_3, a_4, a_5\})\}$. $BN_N(\sigma) \geq 0$ and then σ is a near soft set.

We consider only near soft sets F_B over a universe O in which all the parameter sets B are the same. The set of all near soft sets over O is denoted by $NSS(O_B)$.

Definition 2.4. [3] Let O be an initial universe set and $\mathcal{F}$ be a universe set of parameters. Let F_A and G_B be near soft sets over a common universe set O and $A, B \subset \mathcal{F}$.

(i) The extended intersection of F_A and G_B over O is the near soft set H_C , where $C = A \cup B$, and for all $\phi \in C$,

$$H(\phi) = \begin{cases} F(\phi) & \text{if } e \in A - B \\ G(\phi) & \text{if } e \in B - A \\ F(\phi) \cap G(\phi) & \text{if } e \in A \cap B \end{cases}$$

We write $F_A \cap G_B = H_C$.

(ii) The restricted intersection of F_A and G_B is the near soft set H_C , where $C = A \cap B$, and $H(\phi) = F(\phi) \cap G(\phi)$ for all $\phi \in C$. We write $F_A \cap_R G_B = H_C$.

(iii) The extended union of F_A and G_B is the near soft set H_C , where $C = A \cup B$, and for all $\phi \in C$,

$$H(\phi) = \begin{cases} F(\phi) & \text{if } e \in A - B \\ G(\phi) & \text{if } e \in B - A \\ F(\phi) \cup G(\phi) & \text{if } e \in A \cap B \end{cases}$$

We write $F_A \cup G_B = H_C$.

(iv) The restricted union of F_A and G_B is the near soft set H_C , where $C = A \cap B$, and $H(\phi) = F(\phi) \cup G(\phi)$ for all $\phi \in C$. We write $F_A \cup_R G_B = H_C$.

Definition 2.5. [3] Let $F_B \in NSS(O_B)$. Then F_B is called:
(i) a null near soft set, denoted by $\emptyset_B$, if $F(\phi) = \emptyset$ for all $\phi \in B$.
(ii) a whole near soft set, denoted by O_B, if $F(\phi) = O$ for all $\phi \in B$.

Definition 2.6. [3] Let $F_B \in NSS(O_B)$. Then F_B^c is said to be the relative complement of F_B; where $F^c(\phi) = O - F(\phi)$ for all $\phi \in B$.

Definition 2.7. [3] Let $F_B, G_B \in NSS(O_B)$. Then F_B is a near soft subset of G_B, denoted by $F_B \subseteq G_B$, if $N_*(F_B) \subseteq N_*(G_B)$ for all $\phi \in B$, i.e. $N_*(F(\phi), B) \subseteq N_*(G(\phi), B)$ for all $\phi \in B$.

F_A is called a near soft superset of G_B; denoted by $F_A \supseteq G_B$, if G_B is a near soft subset of F_A.

Definition 2.8. [4] Let $F_B, G_B \in NSS(O_B)$. If F_B and G_B near soft sets are subsets of each other, then they are called equal, denoted by $F_B = G_B$.

Definition 2.9. [3] Let $\sigma = F_B$ be a near soft set over O_B, τ be the collection of near soft subsets of σ, and $B \subseteq \mathcal{F}$ be the nonempty set of parameters; then τ is called a near soft topology on O_B if τ satisfies the following axioms:

(i) $\emptyset_B, O_B \in \tau$ where $\emptyset(\phi) = \emptyset$ and $F(\phi) = F$, for all $\phi \in B$.
(ii) Finite intersections of near soft sets in τ belong to τ.
(iii) Arbitrary unions of near soft sets in τ belong to τ.

The pair (O, τ) is called a near soft topological space. We write nsts instead of near soft topological space.

Example 2.10. Let $O = \{a_1, a_2, a_3, a_4, a_5\}$, $B = \{\phi_1, \phi_2\} \subseteq \mathcal{F} = \{\phi_1, \phi_2, \phi_3\}$ and the tabular representation of O_B is given by Table 1. Then
$\sigma = F_B = \{(\phi_1, \{a_1, a_2\}), (\phi_2, \{a_2, a_3, a_4\})\}$,
$F_B^1 = \{(\phi_1, \emptyset), (\phi_2, \{a_2\})\}$,
$F_B^2 = \{(\phi_1, \{a_1\}), (\phi_2, \{a_2, a_3\})\}$.
$\tau = \{\emptyset_B, F_B, F_B^1, F_B^2\}$. Then τ is a near soft topology on O_B.

Definition 2.11. [4] Let (O, τ) be a nsts over O_B , then the members of τ are said to be near soft open sets O_B. Also, a near soft set F_B over O_B is called a near soft closed set in O_B, if its relative complement F_B^c belongs to τ.

Definition 2.12. [3] Let (O, B, τ) be a nsts on O_B . Then:
(i) $\emptyset_B^c$, O_B^c are near soft closed sets over O_B.
(ii) Arbitrary intersections of the near soft closed sets are near soft closeds over O_B.
(iii) Finite unions of the near soft closed sets are near soft closed sets over O_B.

Definition 2.13. [4] Let (O, B, τ) be a nsts and $F_B \in NSS(O_B)$. Then:
(i) The set $\cap\{G_B \supseteq F_B : G_B\ is\ a\ near\ soft\ closed\ set\ of\ O_B\}$ is called the near soft closure of F_B in O_B, denoted by $cl_n(F_B)$.
(ii) The set $\cup\{G_B \subseteq F_B : G_B\ is\ a\ near\ soft\ open\ set\ of\ O_B\}$ is called the near soft interior of F_B in O_B, denoted by $int_n(F_B)$.

Corollary 1. [4] Let (O, B, τ) be a nsts and $F_B, G_B \in NSS(O_B)$. Then the following hold:
(i) F_B is near soft closed iff $F_B = cl_n(F_B)$.
(ii) G_B is near soft open iff $G_B = int_n(G_B)$.

Definition 2.14. [4] Let $F_B \in NSS(O_B)$. If for the element $\phi \in B$, $F(\phi) \neq \emptyset$ and $F(\phi') = \emptyset$ for all $\phi' \in B - \{\phi\}$, then F_B is called a near soft point in O_B, denoted by ϕ_F^N.

Definition 2.15. [4] Let (O, B, τ) be a nsts and $G_B \in NSS(O_B)$. The near soft point $\phi_F^N \in O_B$ is called a near soft interior point of a near soft set G_B if there exists a near soft open set H_B such that $\phi_F^N \in H_B \subseteq G_B$.

It can be easily stated here; every near soft point $\phi_F^N \in G_B$ is a near soft interior point.

Definition 2.16. [4] Let $F_B, G_B \in NSS(O_B)$. The near soft point ϕ_F^N is said to be in the near soft set G_B, denoted by $\phi_F^N \in G_B$, if for the element $\phi \in B$ and $F(\phi) \subseteq G(\phi)$.

Definition 2.17. [4] Let (O, B, τ) be a nsts and $G_B \in NSS(O_B)$. If there exists a near soft open set H_B such that $\phi_F^N \in H_B \subseteq G_B$, then G_B is called a near soft neighborhood (written near soft nbd) of the near soft point $\phi_F^N \in O_B$.

The near soft nbd system of near soft point ϕ_F^N, which is denoted by $N_\tau(\phi_F^N)$, is the set of all its near soft nbd.

Definition 2.18. [4] Let (O, B, τ) be a nsts and $G_B \in NSS(O_B)$. If there exists a near soft open set H_B such that $F_B \subseteq H_B \subseteq G_B$, then G_B is called a near soft nbd of the near soft set F_B.

Theorem 2.19. [4] *Let (O, B, τ) be a nsts and $G_B \in NSS(O_B)$. The neighborhood system $N_\tau(\phi_F^N)$ at ϕ_F^N in (O, B, τ) has the following properties:*
(i) If $G_B \in N_\tau(\phi_F^N)$, then $\phi_F^N \in G_B$,
(ii) If $G_B \in N_\tau(\phi_F^N)$ and $G_B \subseteq H_B$, then $H_B \in N_\tau(\phi_F^N)$,
(iii) If $G_B, H_B \in N_\tau(\phi_F^N)$, then $G_B \cap H_B \in N_\tau(\phi_F^N)$,
(iv) If $G_B \in N_\tau(\phi_F^N)$, then there is $M_B \in N_\tau(\phi_F^N)$ such that $G_B \in N_\tau(\phi_H^{'})$, for each $\phi_H^{'} \in M_B$.

Definition 2.20. [4] Let $NSS(O_A)$ and $NSS(V_B)$ be the families of all near soft sets over O and V , respectively. The mapping f is called a near soft mapping from O to V; denoted by $f : NSS(O_A) \to NSS(V_B)$, where $u : O \to V$ and $p : A \to B$ are two mappings.

(i) Let F_A be a near soft set in $NSS(O_A)$. Then for all $\omega \in B$ the image of F_A under f, written as $f(F_A) = (f(F), p(A))$, is a near soft set in $NSS(V_B)$ defined as follows:

$$f(F)(\omega) = \begin{cases} \bigcup_{\phi \in p^{-1}(\omega) \cap A} u(F(\phi)), & p^{-1}(\omega) \cap A \neq \emptyset \\ \emptyset, & \text{otherwise.} \end{cases}$$

(ii) Let G_B be a near soft set in $NSS(V_B)$. Then for all $\phi \in A$ the near soft inverse image of G_B under f, written as $f^{-1}(G_B) = (f^{-1}(G), p^{-1}(B))$, is a near soft set in $NSS(O_A)$ defined as follows:

$$f^{-1}(G)(\phi) = \begin{cases} u^{-1}(G(p(\phi))), & p(\phi) \in B \\ \emptyset, & \text{otherwise.} \end{cases}$$

Definition 2.21. [4] Let (O, A, τ) and (V, B, τ^*) be two nstss. Let u be a mapping from O to V and p be a mapping from A to B. Let f be a mapping from $NSS(O_A)$ to $NSS(V_B)$ and $\phi_F^N \in O_A$. Then:
(i) f is near soft continuous at $\phi_F^N \in O_A$ if for each $G_B \in N_{\tau^*}(f(\phi_F^N))$, there exists a $H_A \in N_\tau(\phi_F^N)$ such that $f(H_A) \subseteq G_B$.
(ii). f is near soft continuous on O_A if f is near soft continuous at each near soft point in O_A.

Theorem 2.22. [4] *Let (O, A, τ) and (V, B, τ^*) be two nstss and $f : O \to V$. f is near soft continuous on O_A if and only if the inverse image of a near soft open (closed) set in V_B is a near soft open (closed) set in O_A.*

Theorem 2.23. [4] *Let (O, A, τ) and (V, B, τ^*) be two nstss and $\phi_F^N \in O_A$. For a function $f : NSS(O_A) \to NSS(V_B)$, the following are equivalent:*
(i). f is near soft continuous at ϕ_F^N;

(ii) $\forall G_B \in N_{\tau^*}(f(\phi_F^N))$, *and there exists a* $H_A \in N_\tau(\phi_F^N)$ *such that* $H_A \subseteq f^{-1}(G_B)$*;*
(iii) For each $G_B \in N_{\tau^*}(f(\phi_F^N))$, $f^{-1}(G_B) \in N_\tau(\phi_F^N)$.

Theorem 2.24. [4] *Let* (O, A, τ) *and* (V, B, τ^*) *be two nstss. For a function* $f : NSS(O_A) \to NSS(V_B)$*, the following are equivalent:*
(i) f *is near soft continuous;*
(ii) $\forall F_A \in NSS(O_A)$*, and the inverse image of every near soft nbd of* $f(F_A)$ *is a near soft nbd of* F_A*;*
(iii) $\forall F_A \in NSS(O_A)$ *and for each near soft nbd* H_B *of* $f(F_A)$*, there is a near soft nbd* G_A *of* F_A *such that* $f(G_A) \subseteq H_B$.

Definition 2.25. [4] Let (O, A, τ) and (V, B, τ^*) be two nstss and $f : O \to V$. f be a near soft mapping on O_A: If the image of each near soft open (resp. near soft closed) set in O_A is a near soft open (resp. near soft closed) set in V_B, then f is called a near soft open (resp. near soft closed) function.

3. Some new concepts for near soft topological spaces

Definition 3.1. The difference H_B of two near soft sets F_B and G_B over O, denoted by $F_B \setminus G_B$, is defined as $H(\phi) = F(\phi) \setminus G(\phi)$ for all $\phi \in B$.

Definition 3.2. Let $F_B \in NSS(O_B)$ and V be a non-empty subset of O. Then the sub near soft set of F_B over V_B denoted by ${}^V F_B$, is defined as follows

$$^V F(\phi) = V \cap F(\phi), \quad \forall \phi \in B$$

In other words ${}^V F_B = V_B \cap F_B$.

Definition 3.3. Let O be an initial universe set, B be the set of parameters and $\tau = \{\emptyset_B, O_B\}$. Then τ is called the near soft indiscrete topology on O_B and (O, B, τ) is said to be a near soft indiscrete space over O_B.

Definition 3.4. Let O be an initial universe set, B be the set of parameters and let τ be the collection of all near soft sets which can be defined over O_B. Then τ is called the near soft discrete topology on O_B and (O, B, τ) is said to be a near soft discrete space over O_B.

Definition 3.5. Let (O, B, τ) be a nsts over O_B and V be a non-empty subset of O. Then $\tau_V = \{{}^V F_B : F_B \in \tau\}$ is said to be the near soft relative topology on V_B and (V, B, τ_V) is called a near soft subspace of (O, B, τ).

We can easily verify that τ_V is, in fact, a near soft topology on V_B.

Proposition 1. Let (V, B, τ_V) be a near soft subspace of a nsts (O, B, τ) and F_B be a near soft open set in V_B. If $V_B \in \tau$ then $F_B \in \tau$.

Proof. Let F_B be a near soft open set in V_B, then there exists a near soft open set G_B in O_B such that $F_B = V_B \cap G_B$. Now, if $V_B \in \tau$ then $V_B \cap G_B \in \tau$ by the third axiom of the definition of a near soft topological space and hence $F_B \in \tau$. □

Proposition 2. Let (O, B, τ) and (O, B, ϑ) be two nsts over the same universe O_B, then $(O, B, \tau \cap \vartheta)$ is a nsts over O_B.

Proof. *(i)* $\emptyset_B$, O_B belong to $\tau \cap \vartheta$.
(ii) Let $\{F_B^i : i \in I\}$ be a family of near soft sets in $\tau \cap \vartheta$. Then $F_B^i \in \tau$ and $F_B^i \in \vartheta$ for all $i \in I$, so $\cup_{i \in I} F_B^i \in \tau$ and $\cup_{i \in I} F_B^i \in \vartheta$. Thus $\cup_{i \in I} F_B^i \in \tau \cap \vartheta$.

(*iii*) Let $F_B, G_B \in \tau \cap \vartheta$. Then $F_B, G_B \in \tau$ and $F_B, G_B \in \vartheta$. Since $F_B \cap G_B \in \tau$ and $F_B \cap G_B \in \vartheta$, so $F_B \cap G_B \in \tau \cap \vartheta$. □

Remark 3.6. The union of two nsts on O_B may not be a nsts on O_B.

Example 3.7. Let $O = \{a_1, a_2, a_3, a_4, a_5\}$, $B = \{\phi_1, \phi_2\} \subseteq \mathcal{F} = \{\phi_1, \phi_2, \phi_3\}$ and the tabular representation of O_B is given in Table 1. Then, $\tau = \{\emptyset_B, F_B, F_B^1, F_B^2\}$ and $\vartheta = \{\emptyset_B, F_B, F_B^3, F_B^4\}$ be two nsts defined on O_B where F_B, F_B^1, F_B^2, F_B^3 and F_B^4 are near soft sets over O_B, defined as follows

$$F_B = \{(\phi_1, \{a_1, a_2\}), (\phi_2, \{a_2, a_3, a_4\})\},$$
$$F_B^1 = \{(\phi_1, \emptyset), (\phi_2, \{a_2\})\},$$
$$F_B^2 = \{(\phi_1, \{a_1\}), (\phi_2, \{a_2, a_3\})\},$$
$$F_B^3 = \{(\phi_1, \{a_2\}), (\phi_2, \{a_4\})\},$$
$$F_B^4 = \{(\phi_1, \{a_1\}), (\phi_2, \{a_2\})\}.$$

Now, we define

$$\varphi = \tau \cup \vartheta = \{\emptyset_B, F_B, F_B^1, F_B^2, F_B^3, F_B^4\}$$

If we take $F_B^1 \cap F_B^3 = G_B$, then

$$G(\phi_1) = F^1(\phi_1) \cup F^3(\phi_1) = \emptyset \cup \{a_2\} = \{a_2\}$$

and

$$G(\phi_2) = F^1(\phi_2) \cup F^3(\phi_2) = \{a_4\} \cup \{a_2\} = \{a_2, a_4\}$$

but $G_B \notin \varphi$. Thus φ is not a nsts on O_B.

4. Near soft separation axioms

Definition 4.1. Two near soft sets G_B, H_B in $NSS(O_B)$ are said to be near soft disjoint, written $G_B \cap H_B = \emptyset_B$, if $G(\phi) \cap H(\phi) = \emptyset$, for all $\phi \in B$.

Definition 4.2. Two near soft points ϕ_G^N, ϕ_H^N in O_B are distinct, written $\phi_G^N \neq \phi_H^N$, if there corresponding near soft sets G_B and H_B are disjoint.

Definition 4.3. Let (O, B, τ) be a nsts over O_B and $\phi_G^N, \phi_H^N \in O_B$ such that $\phi_G^N \neq \phi_H^N$. Then;

(*i*) If there exist at least one near soft open set F_B^1 or F_B^2 such that $\phi_G^N \in F_B^1$, $\phi_H^N \notin F_B^1$ or $\phi_H^N \in F_B^2$, $\phi_G^N \notin F_B^2$, then (O, B, τ) is called a near soft T_0-space.

(*ii*) If there exist near soft open sets F_B^1 and F_B^2 such that $\phi_G^N \in F_B^1$, $\phi_H^N \notin F_B^1$ and $\phi_H^N \in F_B^2$, $\phi_G^N \notin F_B^2$, then (O, B, τ) is called a near soft T_1-space.

(*iii*) If there exist near soft open sets F_B^1 and F_B^2 such that $\phi_G^N \in F_B^1$, $\phi_H^N \in F_B^2$ and $F_B^1 \cap F_B^2 = \emptyset_B$, then (O, B, τ) is called a near soft T_2-space.

Proposition 3. Every near soft T_1-space is a near soft T_0-space.

Proof. Straightforward. □

The following example shows that the near soft T_0-space may not be a near soft T_1-space.

Example 4.4. Let $O = \{a_1, a_2, a_3, a_4, a_5\}$, $B = \{\phi_1\} \subseteq \mathcal{F} = \{\phi_1, \phi_2, \phi_3\}$ and the tabular representation of O_B is given in Table 1. Then, $\tau = \{\emptyset_B, F_B, F_B^1\}$ where $F_B = \{(\phi_1, \{a_2, a_4\})\}$ and $F_B^1 = \{(\phi_1, \{a_4\})\}$. Then (O, B, τ) is a near soft T_0-space but not near soft T_1-space. Since $\phi_G^N = (\phi_1, \{a_4\})$, $\phi_H^N = (\phi_1, \{a_2\})$ are two near soft points ($\phi_G^N \neq \phi_H^N$) and the only open near soft set which containing ϕ_H^N is F_B also contains ϕ_G^N. Hence (O, B, τ) is not a near soft T_1-space. On the other hand it is a near soft T_0-space since for each two near soft points ϕ_G^N, ϕ_H^N, $\phi_G^N \neq \phi_H^N$ and near soft open set F_B^1 ($\phi_G^N \in F_B^1$ but $\phi_H^N \notin F_B^1$).

Proposition 4. Every near soft T_2-space is a near soft T_1-space.

Proof. If (O, B, τ) is a near soft T_2-space, then by definition of near soft T_2-space, for $\phi_G^N, \phi_H^N \in O_B$, $\phi_G^N \neq \phi_H^N$, there exist near soft open sets F_B^1 and F_B^2 in O_B such that $\phi_G^N \in F_B^1$, $\phi_H^N \in F_B^2$ and $F_B^1 \cap F_B^2 = \emptyset_B$. Since $F_B^1 \cap F_B^2 = \emptyset_B$, $\phi_G^N \notin F_B^2$ and $\phi_H^N \notin F_B^1$. Thus it follows that (O, B, τ) is a near soft T_1-space. □

The following example shows that the near soft T_1-space may not be a near soft T_2-space.

Example 4.5. Let $O = \{a_1, a_2, a_3, a_4, a_5\}$, $B = \{\phi_1, \phi_2\} \subseteq \mathcal{F} = \{\phi_1, \phi_2, \phi_3\}$ and the tabular representation of O_B is given in Table 1. Also, let $\tau = \{\emptyset_B, F_B, F_B^1, F_B^2, F_B^3\}$ where $F_B = \{(\phi_1, \{a_2, a_4\}), (\phi_2, \{a_2, a_4\})\}$, $F_B^1 = \{(\phi_1, \{a_2\})\}$, $F_B^2 = \{(\phi_2, \{a_4\})\}$ and $F_B^3 = \{(\phi_1, \{a_2\}), (\phi_2, \{a_4\})\}$. Then (O, B, τ) is a near soft topological space over O_B. There are two pairs of distinct near soft points namely, $\phi_{G_1}^N = (\phi_1, \{a_4\})$, $\phi_{H_1}^N = (\phi_2, \{a_2\})$ and $\phi_{G_2}^N = (\phi_2, \{a_2\})$, $\phi_{H_2}^N = (\phi_1, \{a_4\})$. Then for the near soft pair $\phi_{G_1}^N \neq \phi_{H_1}^N$ of points, there does not exist near soft disjoint near soft open sets M_B and N_B such that $\phi_{G_1}^N \in M_B$, $\phi_{H_1}^N \notin N_B$ and $\phi_{H_1}^N \in N_B$, $\phi_{G_1}^N \notin N_B$. Thus (O, B, τ) is not a near soft T_2-space. Clearly (O, B, τ) is a near soft T_1-space and hence a near soft T_0-space.

Theorem 4.6. *Let (O, B, τ) be a nsts. Then each near soft point is near soft closed if and only if (O, B, τ) is a near soft T_1-space.*

Proof. Suppose near soft points $\phi_F^N = F_B$, $\phi_G^N = G_B$ are near soft closed and $\phi_F^N \neq \phi_G^N$. Then F_B^c and G_B^c are near soft open in (O, B, τ). Then by definition F_B^c, where $F^c(\phi) = O - F(\phi)$ and G_B^c, where $G^c(\phi) = O - G(\phi)$. Since $F(\phi) \cap G(\phi) = \emptyset$. This implies $F(\phi) \subseteq G(\phi) = G^c(\phi)$, for all $\phi \in B$. This implies $\phi_F^N = F_B \in G_B^c$. Similarly $\phi_G^N = G_B \in F_B^c$. Thus, we have $\phi_F^N \in G_B^c$, $\phi_G^N \notin G_B^c$ and $\phi_F^N \notin F_B^c$, $\phi_G^N \in F_B^c$. This proves that (O, B, τ) is a near soft T_1-space.

Conversely, let (O, B, τ) is a near soft T_1-space. To prove that $\phi_F^N = F_B \in O_B$ is near soft closed, we show that F_B^c is near soft open in (O, B, τ). Let $\phi_G^N = G_B \in F_B^c$. Then $\phi_F^N \notin \phi_G^N$. Since (O, B, τ) is a near soft T_1-space, there exists a near soft open set H_B such that $\phi_G^N \in H_B$ and $\phi_F^N \notin H_B$. Thus $\phi_G^N \in H_B \subseteq F_B^c$ and hence $\bigcup_{\phi_G^N} \{H_B, \phi_G^N \in F_B^c\} = F_B^c$. This proves that F_B^c is near soft open in (O, B, τ), that is $\phi_F^N = F_B$ is near soft closed in (O, B, τ). □

Proposition 5. Let (O, B, τ) be a nsts over O and V be a nonempty subset of O. If (O, B, τ) is a near soft T_0-space, then (V, B, τ_V) is a near soft T_0-space.

Proof. Let $\phi_G^N, \phi_H^N \in V_B$ be such that $\phi_G^N \neq \phi_H^N$. Then $\phi_G^N, \phi_H^N \in O_B$. Since (O, B, τ) is a near soft T_0-space, therefore there exist near soft open sets F_B and

G_B in (O,B,τ) such that $\phi_G^N \in F_B$ and $\phi_H^N \notin F_B$ or $\phi_H^N \in G_B$ and $\phi_G^N \notin G_B$. Therefore $\phi_G^N \in V_B \cap F_B =^V F_B$. Similarly it can be proved that if $\phi_H^N \in G_B$ and $\phi_G^N \notin G_B$, then $\phi_H^N \in^V G_B$ and $\phi_G^N \notin^V G_B$. Thus (V,B,τ_V) is a near soft T_0-space. □

Proposition 6. Let (O,B,τ) be a nsts and V be a nonempty subset of O. If (O,B,τ) is a near soft T_1-space, then (V,B,τ_V) is a near soft T_1-space.

Proof. The proof is similar to the proof of Proposition 5. □

Proposition 7. Let (O,B,τ) be a nsts over O and V be a nonempty subset of O. If (O,B,τ) is a near soft T_2-space, then (V,B,τ_V) is a near soft T_2-space and (O,B,τ) is a T_2-space, for each $\phi \in B$.

Proof. Let $\phi_G^N, \phi_H^N \in V_B$ be such that $\phi_G^N \neq \phi_H^N$. Then $\phi_G^N, \phi_H^N \in O_B$. Since (O,B,τ) is a near soft T_0-space, therefore there exist near soft open sets F_B^1 and F_B^2 such that $\phi_G^N \in F_B^1$, $\phi_H^N \in F_B^2$ and $F_B^1 \cap F_B^1 = \emptyset_B$. Thus $\phi_G^N \in V_B \cap F_B^1 =^V F_B^1$, $\phi_H^N \in V_B \cap F_B^2 =^V F_B^2$, and ${}^V F_B^1 \cap^V F_B^2 = \emptyset_B$. This proves that (V,B,τ_V) is a near soft T_2-space. □

Definition 4.7. Let (O,B,τ) be a nsts, G_B a near soft closed set in (O,B,τ) and $\phi_F^N \in O_B$ such that $\phi_F^N \notin G_B$. If there exist near soft open sets F_B^1 and F_B^2 such that $\phi_F^N \in F_B^1$, $G_B \subseteq F_B^2$ and $F_B^1 \cap F_B^2 = \emptyset_B$, then (O,B,τ) is called a near soft regular space.

In the following theorem, we give the characterizations of near soft regular spaces.

Theorem 4.8. *Let (O,B,τ) be a near soft regular space over O_B. Then every near soft subspace of (O,B,τ) is near soft regular.*

Proof. Let (V,B,τ_V) be a near soft subspace of a near soft regular space (O,B,τ). Suppose H_B is a near soft closed set in (V,B,τ_V) and $\phi_F^N \in V_B$ such that $\phi_F^N \notin H_B$. Then $H_B = G_B \cap V_B$, where G_B is near soft closed in (O,B,τ). Then $\phi_F^N \notin G_B$. Since (O,B,τ) is near soft regular, there exist disjoint near soft open sets F_B^1, F_B^2 in (O,B,τ) such that $\phi_F^N \in F_1$, $G_B \subseteq F_B^2$. Clearly $\phi_F^N \in F_1 \cap V_B =^V F_B^1$ and $H_B \subseteq F_B^2 \cap V_B =^V F_B^2$ such that ${}^V F_B^1 \cap^V F_B^2 = \emptyset_B$. This proves that (V,B,τ_V) is a near soft regular subspace of (O,B,τ). □

Definition 4.9. Let (O,B,τ) be a nsts. Then (O,B,τ) is said to be a near soft T_3-space, if it is a near soft regular and a near soft T_1-space.

Remark 4.10. A near soft T_3-space may not be a near soft T_2-space.

Proposition 8. Let (O,B,τ) be a nsts over O_B and V be a nonempty subset of O. If (O,B,τ) is a near soft T_3-space then (V,B,τ_V) is a near soft T_3-space.

Proof. It is open from Proposition 6 and Theorem 4.8. □

Definition 4.11. Let (O,B,τ) be a nsts, F_B and G_B near soft closed sets over O_B such that $F_B \cap G_B = \emptyset_B$. If there exist near soft open sets F_B^1 and F_B^2 such that $F_B \subseteq F_B^1$, $G_B \subseteq F_B^2$ and $F_B \cap G_B = \emptyset_B$, then (O,B,τ) is called a near soft normal space.

Definition 4.12. Let (O,B,τ) be a nsts. Then (O,B,τ) is said to be a near soft T_4-space, if it is near soft normal and a near soft T_1-space.

Theorem 4.13. *A near soft topological space (O, B, τ) is near soft normal if and only if for any near soft closed set F_B and near soft open set G_B such that $F_B \subseteq G_B$, there exists at least one near soft open set H_B containing F_B such that*

$$F_B \subseteq H_B \subseteq cl_n(H_B) \subseteq G_B.$$

Proof. Suppose that (O, B, τ) is a near soft normal space and F_B is any near soft closed subset of (O, B, τ) and G_B a near soft open set such that $F_B \subseteq G_B$. Then G_B^c is near soft closed and $F_B \cap G_B^c = \emptyset_B$. So by supposition, there are near soft open sets H_B and K_B such that $F_B \subseteq H_B$, $G_B^c \subseteq K_B$ and $H_B \cap K_B = \emptyset_B$. Since $H_B \cap K_B = \emptyset_B$, $H_B \subset K_B^c$. But K_B^c is near soft closed, so that $F_B \subseteq H_B \subseteq cl_n(H_B) \subseteq K_B^c \subseteq G_B$. Hence $F_B \subseteq H_B \subseteq cl_n(H_B) \subseteq G_B$.

Conversely, suppose that for every near soft closed set F_B and a near soft open set G_B such that $F_B \subseteq G_B$, there is a near soft open set H_B such that $F_B \subseteq H_B \subseteq cl_n(H_B) \subseteq G_B$. Let F_B^1, F_B^2 be any two disjoint near soft closed sets. Then $F_B^1 \subseteq (F_B^2)^c$, where $(F_B^2)^c$ is near soft open. Hence there is a near soft open set H_B such that $F_B \subseteq H_B \subseteq cl_n(H_B) \subseteq (F_B^2)^c$. But then $F_B^2 \subseteq (cl_n(H_B))^c$ and $H_B \cap (cl_n(H_B))^c \neq \emptyset_B$. Hence $F_B^1 \subseteq H_B$, $F_B^2 \subseteq (cl_n(H_B))^c$ with $H_B \cap (cl_n(H_B))^c \neq \emptyset_B$. Hence (O, B, τ) is near soft normal. This completes the proof. □

Proposition 9. Let (V, B, τ_V) be a near soft subspace of a nsts (O, B, τ) and F_B be a near soft open (closed) in (V, B, τ_V). If V_B is near soft open (closed) in (O, B, τ), then F_B is near soft open (closed) in (O, B, τ).

Proof. Straightforward. □

Theorem 4.14. *A near soft closed subspace of a near soft normal space is near soft normal.*

Proof. Let (V, B, τ_V) be near soft subspace of near soft normal space (O, B, τ) such that $V_B \in \tau^c$. Let F_B^1, F_B^2 be two disjoint near soft closed subsets of (V, B, τ_V). Then there exists near soft closed sets F_B, G_B in (O, B, τ) such that $F_B^1 = V_B \cap F_B$ and $F_B^2 = V_B \cap G_B$. Since V_B is near soft closed in (O, B, τ), therefore F_B^1, F_B^2 are disjoint near soft closed in (O, B, τ). Then (O, B, τ) is near soft normal implies that there exist near soft open sets F_B^3, F_B^4 in (O, B, τ) such that $F_B^1 \subseteq F_B^3$, $F_B^2 \subseteq F_B^4$ and $F_B^3 \cap F_B^4 = \emptyset_B$. But then $F_B^1 \subseteq V_B \cap F_B^3$, $F_B^2 \subseteq V_B \cap F_B^4$, where $V_B \cap F_B^3$, $V_B \cap F_B^4$ are near soft disjoint near soft open subsets of (V, B, τ_V). This proves that (V, B, τ_V) is near soft normal. □

Theorem 4.15. *Every near soft closed subspace of a near soft T_4-space is a near soft T_4-space.*

Proof. It is open from Proposition 6 and Theorem 4.14. □

5. Conclusions

Thanks to this study, some missing structures and concepts for near soft topological spaces are introduced to the literature. In this way, we think that it can contribute to the efficiency of future studies for near soft topological spaces.

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