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Investigation of educational mathematics mobile applications (EMMAs) with multi-criteria decision-making methods: A TOPSIS algorithm implementation

Yusuf Can [a], Mehmet Akif Aksoy [b] Esra Aksoy [c *], Serkan Narlı [d]

[a] Dokuz Eylül University, Türkiye
[b] Dokuz Eylül University, Türkiye
[c] 19 Mayıs Middle School (Ministry of Education) , Türkiye
[d] Dokuz Eylül University, Türkiye

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Highlights

- Multi-Criteria Decision-Making in Mathematics Education
- TOPSIS Algorithm in Ranking Mobile Apps
- Desmos: Graphing Calculator

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Abstract

The number and variety of educational mathematics mobile applications (EMMAs) make it difficult to select mobile applications for mathematics learning and teaching. Therefore, in this study, multi-criteria decision-making (MCDM) techniques, which are effectively used in a wide variety of disciplines, were applied to choose among alternative applications according to specified criteria. In this context, it was aimed to determine which of the 13 considered EMMAs that work on Android-based tools and were proposed by experts according to certain features were most effective with the help of the TOPSIS algorithm, one of the popular MCDM methods. The results obtained from an evaluation using 10 criteria (4 evaluator-independent, 6 evaluator-dependent) were analysed with MATLAB. As a result, the Desmos: Graphing Calculator application was found to rank first among the 13 EMMAs in order of importance. Considering the results obtained, it can be said that the use of MCDM techniques in such decision problems can facilitate the work of decision-makers.

1. Introduction

With the development of technology and artificial intelligence, the variety and number of mobile devices and applications are increasing rapidly. These tools and applications are used effectively in many fields such as education, health, industry, economy, and banking. It can be said that mobile tools and applications are generally used to increase efficiency and productivity in the areas in which they are developed. Due to the COVID-19 pandemic, which has affected the whole world in the last few years, mobile devices and applications that are frequently used around the world, and especially in the field of education, have played increasingly important roles in ensuring the continuity of educational processes. Existing tools and applications have been further developed and new ones have been produced to increase productivity along with ensuring continuity in education. It has been determined by meta-analysis studies that the use of mobile

* Corresponding author. 19 Mayıs Middle School (Ministry of Education), Türkiye.
e-mail address: aksoyesra.math@gmail.com
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applications in mathematics education makes learning more enjoyable and understandable (e.g., Güler, Bütüner, Danişman, & Gürsoy, 2021). Since mobile applications are portable and provide easy access to many platforms, especially in internet environments, they have made physical boundaries unimportant in mathematics teaching and have allowed educational environments and boundaries to be expanded. In this context, which of the thousands of available applications are most effective in mathematics teaching has become a subject that requires research. In this study, since the problem is a decision-making problem, 13 educational mobile mathematics applications (EMMAs) were evaluated with a multi-criteria decision-making (MCDM) model in terms of 13 criteria proposed by experts. The TOPSIS algorithm ("Technique for Order Preference by Similarity to Ideal Solution") as a popular MCDM technique was used in this process with the aim of determining which of the 13 considered applications was most effective. In the next section, literature studies on the use of mobile applications in education, MCDM methods, and the TOPSIS algorithm are reviewed.

2. Literature Review

2.1. Mobile Applications

In recent years, mobile devices have attracted increasing attention from researchers as they enable learning to expand beyond the constraints of space and time. Features of mobile devices such as portability, usability, access to the internet, and wide acceptance among young people and others have made mobile devices new tools that can advance the boundaries of mathematics teaching and learning beyond classroom walls (Borba, Askar, Engelbrecht, Gadanidis, Llinares, & Aguilar, 2016). As more and more students have mobile devices, researchers and educators have begun to look for educational applications that will facilitate the teaching and learning processes.

The multiple functions of mobile phones, such as mobility, interaction, voice communication, taking pictures or recording audio and video files, measuring time, and transmitting information, facilitate authentic learning based on real-life events and expand the range of possible mathematical activities (Daher & Baya'a, 2012). Mobile learning also supports individualized learning by allowing students to work at their own pace (Cheon, Lee, Crooks, & Song, 2012). It allows students to collaborate and share their ideas (Hamidi & Chavoshi 2018). There are many studies showing that mobile learning positively affects learning performance (e.g., Elfeky & Masadeh, 2016; Jeno, Vandvik, Eliassen, & Grytnes, 2019; Hwang and Wu, 2014; Martin & Ertzberger, 2013). In addition to increasing the rate of academic success, it can be said that it positively affects students' learning motivation (Hwang & Wu, 2014), attitudes (Hwang & Chang, 2011; Martin & Ertzberger, 2013; Riconscente, 2013), interests (Kyriakides, Meletiou-Mavrotheris, & Prodromou, 2016), self-efficacy (Shank & Cotten, 2014; Hung, Huang, & Hwang, 2014), communication skills and creativity (Lai & Hwang, 2014), and problem-solving skills (Lai & Hwang, 2014; Saedi, Taghizade, & Hatami, 2018).

Research has also shown that mobile learning has positive effects on students' cognitive characteristics in areas including mathematics achievement (e.g., Hung et al., 2014; Kyriakides et al., 2016; Riconscente, 2013) and problem-solving skills (Haydon, Hawkins, Denune, Kimener, McCoy, & Basham 2012; Saedi et al., 2018). One of the benefits of using mobile technologies in mathematics teaching and learning is that they support the learning of mathematics conceptually with features such as different representations and they can minimize problems with students' abstract or unconventional perceptions of mathematics terminology (Baker, Ashill, Amer, & Diab, 2018; Kluge & Dolonen, 2015). Franklin and Peng (2008) also reported that mobile technologies can be beneficial in terms of enabling students to learn difficult concepts through visualization.

The literature findings about the positive effects of mobile technologies on the cognitive and affective characteristics of students show that teachers can integrate this innovative approach into their teaching, making it more interesting and providing uninterrupted learning by carrying the teaching outside the classroom. However, a teacher who will integrate mobile learning into course materials should be able to choose the most appropriate application for the lessons by first considering the mathematical content and pedagogical approaches. Dubé, Kacmaz, Wen, Alam, and Xu (2020) examined 90 Apple applications related to mathematics, aiming to see whether the information provided in application stores helps parents and educators find quality educational applications. Among their conclusions, they cited a lack of transparency and meaningful information. They suggested that the Apple App Store should explain how it selects the "best" apps and that developers should provide criteria for the quality of education in their app descriptions.

In the literature, some studies have presented evaluation scales consisting of criteria that will help teachers choose the most suitable mobile applications for learning mathematics. For example, Namukasa, Gadanidis, Sarina, Scucuglia, and Aryee (2016) shared an assessment tool for teachers to use in evaluating practices. With that tool, they reviewed mobile applications from among Apple applications suitable for primary and secondary schools and focused on the learning of negative integers as a mathematics content area. This assessment tool had four dimensions: (a) the nature of the curriculum addressed in practice; (b) the degree of actions and interactions provided by the app as a learning tool; (c) the level of interaction and various options available to the user; and (d) the quality of design features and graphics in the app. Using these dimensions, the researchers rated applications on a three-level scale. They could not identify a top-ranking application for all dimensions, but a few applications were top-ranked for some selected dimensions. In another similar study, Harrison and Lee (2018) developed an assessment tool consisting of 16 criteria to evaluate the effectiveness of iPad apps. The criteria were grouped into the four categories of Interactions, Content Quality, Feedback and Support, and Usability. They selected 7 interactive mathematics applications that focused on learning algebraic content, applied their newly developed assessment tool to them, and demonstrated the use of the tool for these examples.

It can be said that the two studies mentioned above mainly focused on developing rating scales to select useful mobile applications. In addition, in some studies, applications have been evaluated using various criteria without focusing on the development of assessment tools. For instance, Larkin (2015) categorized 142 math mobile apps as conceptual (in-depth understanding of the meaning of math) and procedural (following a sequence of steps to solve a math problem) or declarative (knowledge from memory). He stated that most of the mobile applications he reviewed emphasized types of declarative or procedural information that encouraged rote learning. He classified only three of the 40 considered applications intended to support learning mathematics as outstanding. In the study of Kay and Kwak (2018), 20 primary school students evaluated mathematics applications from four different categories (practice-based, constructive, productive, and game-based) for 5 weeks. While conducting the evaluations, three criteria (perceived learning value, usability, and participation) were taken into consideration. Pre- and post-test results showed that the students' knowledge of basic addition and subtraction increased significantly after using the math apps. The results showed clear differences between the four applications studied. The researchers stated that game-based applications received high scores for all criteria.

The diversity of mobile applications, which makes it difficult to compare them with each other, is the biggest difficulty in choosing a mobile learning application (Başaran & Haruna, 2017). Although evaluation criteria were proposed in the studies above, it can be said that the applications did not receive high scores for all of the criteria or most of them had similar scores and it was difficult to choose the most effective application. The fact that the selection process is difficult despite the existence of evaluation scales shows that there is a need for more effective techniques that can be used in such situations involving multiple criteria. In this context, MCDM techniques are used in a wide variety of disciplines to choose among alternative applications according to determined criteria and these are recognized as effective approaches (e.g., Volaric, Brajkovic, & Sjekavica, 2014). For this reason, MCDM models were used in the present study.

2.2. Multi-Criteria Decision Making (MCDM)

MCDM is a special branch of operation research that deals with decision problems (Zeleny, 2005) and has been a rapidly growing field for decision problems since the 1970s (Pohekar & Ramachandran, 2004). As the name suggests, MCDM is a decision-making tool that allows the most preferred alternative to be selected with precision in environments where many criteria are applied simultaneously (Mendoza & Prabhub, 2000). The MCDM approach is used in various decision-making areas and in solving complex problems. This approach requires decision-makers to determine the relative weights of the performance of each alternative for each criterion and the evaluation criteria for all objectives and to make a quantitative and qualitative evaluation (Mahdavi et al., 2008).

MCDM methods offer decision-makers the opportunity to overcome the challenges of the decision-making process in situations where there are multiple goals. Decision-makers may need to choose from among many measurable or unmeasurable criteria. Goals can often be in conflict with each other for decision-makers when there are many criteria. The solution is to have a systematic structure that will support the decision-maker while remaining loyal to the preferences of the decision-makers. In decision-making problems, different decision-making groups are included in the process, different criteria are added to the model with different perspectives of the groups, and results are sought by consensus. On the other hand, MCDM methods offer a systematic approach that combines risk level, assessment, and uncertainty by bringing together different perspectives of stakeholders (Linkov et al., 2006).

There are various MCDM methods related to the selection, elimination, and ranking of alternatives in the literature. For example, AHP (“Analytical Hierarchy Process”), TOPSIS (“Technique for Order Preference by Similarity to Ideal Solution”), PROMETHEE (“Preference Ranking Organization Method for Enrichment Evaluation”), ELECTRE (“Elimination Et Choix Traduisant la Realité”), DEMATEL (“Decision-Making Trial and Evaluation Laboratory”), and VIKOR (“VIseKriterijumsa Optimizacija I Kompromisno Resenje”) are the most commonly used methods for MCDM. The method to be used is selected according to the condition of whether or not the criteria are affected by each other and the structure of the problem. While DEMATEL is used when the criteria are affected by each other, AHP or VIKOR can be preferred if the selection is made when the criteria do not affect each other. TOPSIS or PROMETHEE is used when ranking among alternatives is required. TOPSIS is preferred to PROMETHEE in cases where a large number of qualitative and quantitative criteria are present. In cases where there are many criteria with few alternatives and there are both quantitative and qualitative criteria, ELECTRE is generally preferred. Different MCDM techniques can be used for the same problem. The most important criteria when choosing a method are that it facilitates the work of the decision-maker and the calculation cost is minimal. In this context, the TOPSIS method was preferred in this study because it includes both qualitative and quantitative variables, the number of alternatives is relatively high, and TOPSIS offers ease of operation and practical use.

2.3. TOPSIS (“Technique for Order Preference by Similarity to Ideal Solution”)

The TOPSIS method, which is one of the MCDM methods developed by Hwang and Yoon (1981), ranks possible alternatives. This method, which allows alternatives to be ranked by taking into account their relative closeness to the best solution (positive-ideal solution), is proposed based on the idea of the shortest distance to the positive-ideal solution and the longest distance to the negative-ideal solution. The solution process of the TOPSIS application consists of the following steps (Ezhilarasan & Vijayalakshmi, 2020): (a) creating the decision matrix, (b) creating the normalized decision matrix, (c) creating the normalized weighted decision matrix, (d) determining the positive- and negative-ideal solutions, (e) measuring the distances to the positive- and negative-ideal solution sets, (f) calculating the relative closeness to the ideal solution, and (g) ranking the importance.

The TOPSIS method, which is used in many fields such as education, health, industry, production, and communication, has become a method applied by many researchers due to its ease of use. In the literature, there are some studies in which the TOPSIS method was used in the selection of mobile applications within the scope of various educational areas or skills. For instance, Uslu, Gür, Eren, and Özcan (2020) determined six criteria affecting mobile application selection according to a literature review and expert opinions, and they selected five applications related to different fields, including education, to evaluate. They evaluated those applications according to the determined criteria with three different methods, including TOPSIS, and decided on the most appropriate mobile application. Zhao, Muthu, and Shakeel (2021) evaluated mobile applications for learning English according to their effectiveness for students in terms of listening, speaking, reading, and writing. In that study, six mobile applications were evaluated with the TOPSIS method according to 17 criteria and the most effective application was determined. Similarly, among applications for learning English, Ibrahim et al. (2019) evaluated mobile applications according to 17 subcriteria based on three main criteria and proposed an evaluation and comparison decision matrix based on MCDM in terms of listening, speaking, reading, and writing skills. Volaric et al. (2014), who examined multimedia applications in general without focusing on only mobile applications, integrated the fuzzy analytic hierarchy process (FAHP) and TOPSIS methods to select an appropriate learning application according to the revised Bloom taxonomy framework and used the TOPSIS method to determine the final ranking of multimedia applications. They reported that the integration of these methods enabled the teacher to efficiently select multimedia applications that were more suitable for learning.

In addition to these studies, there are some studies in which the TOPSIS algorithm was used in the selection of mobile applications for learning mathematics. In one of these studies, Başaran and Haruna (2017) applied the FAHP and TOPSIS methods together, and the five mobile mathematics applications with the highest market scores were evaluated. After the determined criteria were weighted with FAHP, they were ranked with TOPSIS and it was seen that the results obtained were different from the Google Play Store user ratings. In another study, Başaran and Homsi (2022) aimed to create a guiding model for selecting the most appropriate mathematics application. For this purpose, six applications with the highest market scores were evaluated by two experts according to 40 different criteria. Fuzzy TOPSIS was applied to the obtained results.

It can be said that the above-mentioned studies, in which the TOPSIS method was applied, undertook ranking and selection processes with small numbers of mobile applications, such as five or six. The present study therefore aimed to determine which of a relatively higher number of EMMAs (13) is most effective by using the TOPSIS algorithm. Since the results of such studies are affected by the values of the decision-maker's preferences with different criteria and varieties of applications, it can be said that a ranking created considering more applications and experts will contribute to the literature in terms of comparisons and the creation of common values.

3. Methodology

In this study, the following steps were undertaken: selection of the EMMAs to be evaluated (determination of alternatives), determination of evaluation criteria, evaluation of the selected EMMAs by experts according to the determined criteria, and application of the TOPSIS method to the evaluation results.

3.1. Selection of EMMAs to be ranked (identification of alternatives)

The results of the worldwide market shares of mobile operating systems (2012-2022) show that phones with Android operating systems outnumber phones with iOS operating systems (Laricchia, 2022). Accordingly, this study was carried out with apps that can be run with Android-based tools. As a result of research conducted by the researchers in the Google Play Store, it was seen that there are numerous apps available in the category of mathematics education. Therefore, the EMMAs to be evaluated and ranked were selected by experts considering certain characteristics in January 2022. Consequently, 13 applications were selected by four mathematics teachers and two mobile application development experts, taking into account the following features: (a) number of downloads (500 B+), (b) score on Google Play (above 4.0 out of 5), (c) able to be downloaded for free, (d) having been updated in the last 2 years, and (e) being suitable for mathematics learning for middle or high school students.

Trigonometry Unit Circle: This app provides convenience in the visual understanding and calculation of sine, cosine, tangent, cotangent, secant, and cosecant functions, degrees, and radians, as well as defining functions and value tables. Information and applications of concepts related to trigonometry such as symmetry, periodicity, sums and differences of angles, double angles, half angles, sum and difference functions, multiplication of functions, and degree reduction formulas are available. Many activities related to trigonometry gains are suitable for educational use.

Microsoft Math Problem Solver: This program allows users to learn step-by-step solutions to mathematical problems. The Microsoft Math Problem Solver app provides help with a variety of problems, including arithmetic, algebra, trigonometry, calculus, statistics, and more, using an advanced artificial intelligence-powered math problem solver.

Desmos: Graphing Calculator: This app makes it easy to graph functions and tabular data, calculate unknowns in equations, and explore transformation geometry. To create a new graph, the desired expression is written in the algebra window. While the desired expression is being written, the calculator immediately draws the graph on a graph grid. It has some basic functions and graphics available.

Math Master: This math game was developed to help children learn math and help adults train their brains. It covers primary and secondary education mathematics. It is intended to teach mathematics through games and entertainment. There are 97 math topics in the app. The individual develops a character in the game up to the level of professor by giving correct answers to questions, and in this way, it is aimed to learn mathematics in a fun way.

Geometryx: This simple calculator uses trigonometric functions, the Pythagorean theorem, and the Thales theorem. It makes it possible to calculate some dimensions of basic shapes and complex shapes in two or three dimensions. It shows what kinds of changes happen to shapes when some dimensions of the selected shapes are changed.

Euclidea: This app was designed for learning, exploring, and having fun with Euclidean axioms. Its main aim is learning based on fun. The task is to solve interesting problems by constructing geometric structures with a plane and compass, and whoever designs the simplest solutions in the least number of moves wins the highest points.

Math Duel: This app was designed to be used in preschool or primary school education, allowing children to play math games and learn easily. It is suitable for multiple players thanks to the split-screen option. The aim is to answer faster than the opponent and collect points.

GeoGebra: This dynamic mathematical mobile app combines geometry, algebra, and analysis. GeoGebra can be defined as a computer algebra system due to its symbolic and visualization features such as direct input of equations and coordinates or algebraic definitions of functions. It is also defined as dynamic geometry software because it includes concepts such as points, line segments, lines, and conic sections and provides the dynamic relations between these concepts.

GeoGebra 3D Graphing Calculator: This app was designed for creating solids, spheres, planes, plane sections, and many more three-dimensional objects. It provides the opportunity to solve linear geometry problems and draw graphs of functions and parametric surfaces of the form $z = f(x,y)$. It is an augmented reality-compatible application.

Pythagorea: This application aims to help those who are new to geometry understand the important ideas and features of Euclidean geometry. It is a mobile application that facilitates the learning of basic geometric concepts in puzzle style while students have fun and satisfy their curiosity.

Math Game: This app was designed for those who want to improve themselves in terms of basic math operations. It contains tests that involve four operations and exponential and square root expressions. The tests should be completed within the given time.

Mathematics-Fractions, Geometry: This app contains tests at every grade level from elementary school to high school, from easy to difficult, on almost every mathematics subject. In the app, which does not aim to teach by entertaining, students can see where they made mistakes and can increase their ranks in the ranking system in the application by winning trophies.

Mental Math Master: This app aims to increase arithmetic skills and concentration. Starting from the most basic four arithmetic operations and asking students to provide correct answers to questions in the given time, more complex arithmetic operations (e.g., logarithms) are reached in the following levels.

3.2. EMMA evaluation criteria

For the criteria to be used in the evaluation of these mobile applications, the mobile application evaluation criteria in the literature were first examined. As a result, 10 criteria (4 evaluator-independent, 6 evaluator-dependent) to be used in the evaluation of EMMAs were determined in line with the criteria in the literature and the opinions of four mathematics teachers and two mobile application development experts. The diagram of the decision problem for the EMMAs regarding the evaluation process is given in Figure 1. Explanations of the criteria are given and the evaluation process is summarized below.

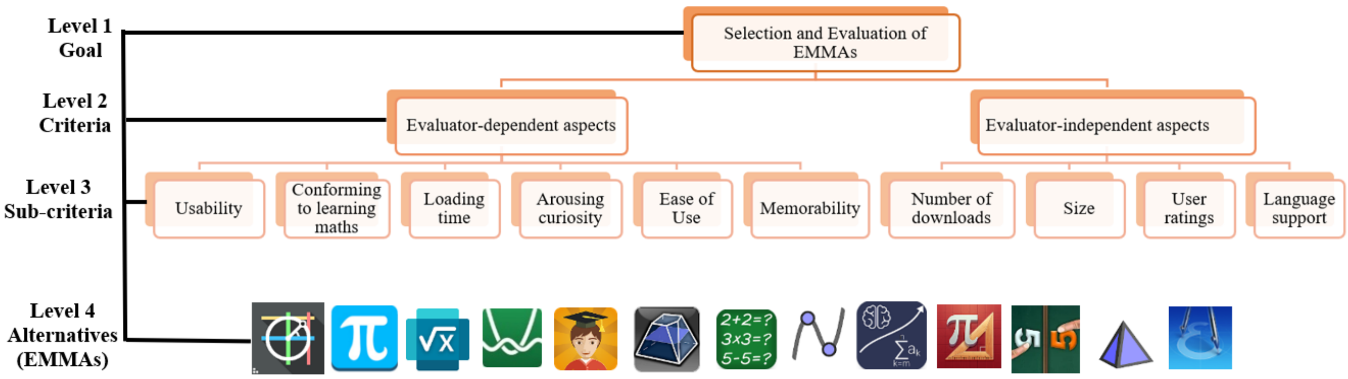

Figure 1. Diagram of the decision problem for EMMAs

The evaluator-independent criteria were as follows:

- Number of downloads: The number of downloads of the app given by Google Play Store.
- User ratings: The app's market score in Google Play Store (average of scores given to the application by users in Google Play Store).
- Language support: Existence of language options for the country where the app is to be used (Turkish language support was checked since this study was conducted in Turkey).
- Size: Space (MB) that the app occupies on a phone.

The evaluator-dependent criteria were as follows:

- Ease of use: Factors such as a simple menu/interface, clear instructions in the app, easy-to-find navigation buttons, clear font, harmony of text and images, proportional use of font size, and careful colour selections.
- Usability: A measure of how well the user can use the application for the intended purpose (ability of the app to meet the expectations and needs of users).
- Conforming to learning maths: The app's coverage of mathematics learning outcomes in terms of quantity and variety.
- Arousing curiosity: The state of curiosity and excitement that the application generates in the user.
- Loading time: The amount of time for user mode to initialize and the speed of use of the app.
- Memorability: The app content's support for memorability (e.g., the app's appeal to multiple senses).

The user ratings and numbers of downloads of the apps were taken directly from the Google Play Store. Some of the evaluator-independent criteria required different scoring approaches according to their type. For example, the space that an application will occupy on a phone was considered in MB, and since larger size is a negative situation for these calculations, the scoring was calculated using the formula of 1/Dimension. Apps were scored as 0 (absent) or 1 (present) for Turkish language support. Evaluator-dependent criteria were evaluated by giving scores in the range of 1-5 (lowest: 1, highest: 5).

For the evaluator-dependent criteria, the EMMAs were evaluated by four mathematics teachers who gave them scores in the range of 1 (lowest) to 5 (highest). The final scores for the EMMAs were decided with joint decisions by the teachers and a decision matrix for evaluation was created. The results obtained from all these evaluations were analysed using the TOPSIS algorithm in the MATLAB program (R2020a version).

3.3. Findings and Discussions

To solve the EMMA decision problem, first the alternatives (EMMAs) were coded as A_1, A_2, A_3, …, A_{13} (Table 1), and the criteria were coded as C_1, C_2, C_3, …, C_{10} (Table 2).

Table 1.

Alternatives and their codes

Codes	Alternatives (EMMAs)
A_1	GeoGebra
A_2	GeoGebra 3D Graphing Calculator
A_3	Trigonometry Unit Circle
A_4	Desmos: Graphing Calculator
A_5	Microsoft Math Problem Solver
A_6	Math Duel
A_7	Geometryx
A_8	Math Master
A_9	Euclidea
A_{10}	Pythagorea
A_{11}	Mathematics-Fractions, Geometry
A_{12}	Math Game
A_{13}	Mental Math Master

Table 2.

Criteria and their codes

Codes	Criteria
C_1	Conforming to learning maths
C_2	Usability
C_3	Ease of use
C_4	Arousing curiosity
C_5	Loading time
C_6	Memorability
C_7	Number of downloads
C_8	Size (Mb)
C_9	User ratings
C_{10}	Language support

The decision matrix obtained as a result of the evaluation of the alternatives in terms of the determined criteria is shown in Table 3.

Table 3.

The decision matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A1	5	4	3	3	4	5	10	0.071	4.4	1
A2	5	3	3	4	5	3	1	0.047	4.4	1
A3	5	5	4	4	4	5	0.5	0.277	4.7	0
A4	5	5	4	3	5	5	5	0.303	4.8	1
A5	3	5	5	5	5	4	10	0.035	4.2	1
A6	2	5	5	3	5	3	5	0.028	4.0	1
A7	3	2	2	2	3	4	1	0.108	4.6	1
A8	2	4	4	2	2	2	1	0.066	4.5	0
A9	2	3	2	5	2	3	1	0.050	4.6	0
A10	3	3	2	5	3	3	1	0.149	4.7	1
A11	5	4	4	4	5	2	1	0.036	4.7	1
A12	4	4	5	2	5	1	10	0.303	4.2	1
A13	4	3	2	3	3	2	1	0.438	4.3	0

After the decision matrix was created, the following steps were applied according to the solution steps of the MATLAB program and the analysis was carried out:

1. A normalized decision matrix was obtained from the decision matrix (Table 4).

Table 4.

Normalized decision matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A1	0.3571	0.2801	0.2281	0.2294	0.2715	0.4003	0.5291	0.1001	0.2731	0.3333
A2	0.3571	0.2100	0.2281	0.3059	0.3394	0.2402	0.0529	0.0662	0.2731	0.3333
A3	0.3571	0.3501	0.3041	0.3059	0.2715	0.4003	0.0265	0.3904	0.2917	0
A4	0.3571	0.3501	0.3041	0.2294	0.3394	0.4003	0.2645	0.4270	0.2979	0.3333
A5	0.2143	0.3501	0.3801	0.3824	0.3394	0.3203	0.5291	0.0493	0.2607	0.3333
A6	0.1429	0.3501	0.3801	0.2294	0.3394	0.2402	0.2645	0.0395	0.2421	0.3333
A7	0.2143	0.1400	0.1521	0.1529	0.2037	0.3203	0.0529	0.1522	0.2855	0.3333
A8	0.1429	0.2801	0.3041	0.1529	0.1358	0.1601	0.0529	0.0930	0.2793	0
A9	0.1429	0.2100	0.1521	0.3824	0.1358	0.2402	0.0529	0.0705	0.2855	0
A10	0.2143	0.2100	0.1521	0.3824	0.2037	0.2402	0.0529	0.2100	0.2917	0.3333
A11	0.3571	0.2801	0.3041	0.3059	0.3394	0.1601	0.0529	0.0507	0.2917	0.3333
A12	0.2857	0.2801	0.3801	0.1529	0.3394	0.0801	0.5291	0.4270	0.2607	0.3333
A13	0.2857	0.2100	0.1521	0.2294	0.2037	0.1601	0.0529	0.6173	0.2669	0

2. The normalized weighted decision matrix (Table 5) was created by using the normalized decision matrix. At this stage, no technique was used for the weighting process and the criteria were accepted as equally weighted.

Table 5.

Normalized weighted decision matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
A1	0.0357	0.028	0.0228	0.0229	0.0272	0.040	0.0529	0.0100	0.0273	0.0333
A2	0.0357	0.021	0.0228	0.0306	0.0339	0.024	0.0053	0.0066	0.0273	0.0333
A3	0.0357	0.035	0.0304	0.0306	0.0272	0.040	0.0026	0.0390	0.0292	0
A4	0.0357	0.035	0.0304	0.0229	0.0339	0.040	0.0265	0.0427	0.0298	0.0333
A5	0.0214	0.035	0.0380	0.0382	0.0339	0.032	0.0529	0.0049	0.0261	0.0333

A_6	0.0143	0.035	0.0380	0.0229	0.0339	0.024	0.0265	0.0039	0.0242	0.0333
A_7	0.0214	0.014	0.0152	0.0153	0.0204	0.032	0.0053	0.0152	0.0286	0.0333
A_8	0.0143	0.028	0.0304	0.0153	0.0136	0.016	0.0053	0.0093	0.0279	0
A_9	0.0143	0.021	0.0152	0.0382	0.0136	0.024	0.0053	0.0070	0.0286	0
A_{10}	0.0214	0.021	0.0152	0.0382	0.0204	0.024	0.0053	0.0210	0.0292	0.0333
A_{11}	0.0357	0.028	0.0304	0.0306	0.0339	0.016	0.0053	0.0051	0.0292	0.0333
A_{12}	0.0286	0.028	0.0380	0.0153	0.0339	0.008	0.0529	0.0427	0.0261	0.0333
A_{13}	0.0286	0.021	0.0152	0.0229	0.0204	0.016	0.0053	0.0617	0.0267	0

3. The positive-ideal solution and negative-ideal solution were obtained for each criterion (Table 6).

Table 6.

Positive- and negative-ideal solutions

	C_1	**C_2**	**C_3**	**C_4**	**C_5**	**C_6**	**C_7**	**C_8**	**C_9**	**C_{10}**
A^+	0.0143	0.014	0.0152	0.0153	0.0136	0.008	0.0026	0.0039	0.0242	0
A^-	0.0357	0.035	0.0380	0.0382	0.0339	0.040	0.0529	0.0617	0.0298	0.0333

4. After obtaining the ideal solutions, the deviations of each criterion from this ideal solution were calculated and the obtained values are shown in Table 7. These deviations were calculated with the standard Euclidean distance function.

Table 7.

Deviations from the positive- and negative-ideal solutions

	S^+	**S^-**
A_1	0.0569	0.0753
A_2	0.0778	0.0510
A_3	0.0657	0.0619
A_4	0.0368	0.0764
A_5	0.0592	0.0786
A_6	0.0708	0.0580
A_7	0.0798	0.0440
A_8	0.0906	0.0233
A_9	0.0906	0.0294
A_{10}	0.0727	0.0486
A_{11}	0.0789	0.0524
A_{12}	0.0450	0.0805
A_{13}	0.0718	0.0614

5. After calculating the deviations from the ideal solutions, the relative closeness of the alternatives (C) was calculated, and the order of importance was determined according to this value. The ranking is shown in Table 8.

Table 8.

Ranking of importance

Codes	**Alternatives (EMMAs)**	**C**	**Rank**
A_1	GeoGebra	0.5693	4
A_2	GeoGebra 3D Graphing Calculator	0.3962	10
A_3	Trigonometry Unit Circle	0.4852	5
A_4	Desmos: Graphing Calculator	0.6751	1
A_5	Microsoft Math Problem Solver	0.5702	3

A_6	Math Duel	0.4503	7
A_7	Geometryx	0.3555	11
A_8	Math Master	0.2042	13
A_9	Euclidea	0.2452	12
A_{10}	Pythagorea	0.4006	8
A_{11}	Mathematics-Fractions, Geometry	0.3991	9
A_{12}	Math Game	0.6412	2
A_{13}	Mental Math Master	0.4612	6

According to Table 8, Desmos: Graphing Calculator ranked first among 13 EMMAs in order of importance. This application was followed by Math Game and Microsoft Math Problem Solver. Considering the user ratings of these three applications in the Google Play Store, Desmos: Graphing Calculator ranks first among them, while Math Game and Microsoft Math Problem Solver are at the bottom of the list. Similarly, the last three apps in the ranking obtained here are Math Master, Euclidea, and Geometryx. When the market scores of these applications are examined, it is seen that they are among the top seven applications. In this context, it can be said that these findings support conclusions in the literature that market score should not be the only selection criterion in choosing an application (Dubé et al., 2020; Başaran & Homsi, 2022). Because, it has been found that the ranking made with TOPSIS based on the evaluation scores of the teachers is different from the ranking of the applications according to the market scores. These results supports the view that information provided in app stores cannot help parents and educators to find quality apps (Dubé et al., 2020).

The reason why Desmos: Graphing Calculator ranks first among 13 EMMAs in order of importance may be that it offers a high level of interaction that provides more conceptual learning compared to other applications. For this reason, it can be thought that teachers give higher points to this app compared to other apps. However, the Geogebra application also offers similar content. Despite this, it can be said that the reason why it is ranked 4th is a result that needs to be investigated. Similarly, Larkin (2015) classified mobile applications as conceptual, procedural and declarative. And he emphasized that the last two encourage rote learning. Although the 13 EMMAs in this study included applications from these three categories, it was seen that the application selected in order of importance was conceptual. Similarly, Kay and Kwak (2018) categorized applications as practical, productive, game-based and constructive. In this study, although the 13 EMMAs include all kinds of applications, it can be said that the most effective application is in the productive category. In addition, Kay and Kwak (2018) stated that game-based applications scored higher. This is in line with the finding of Math Games, which is a game-based application was second in importance among EMMAs in this study.

Unlike some studies in literature (e.g., Namukasa et al., 2016, Harrison & Lee, 2018), in this study, the 13 EMMAs were not selected from a specific math subject or learning area. However, it can be thought that teachers may also need evaluation and selection recommendations for specific subject areas that students have difficulty in learning. Therefore, it can be suggested that this study should also be carried out for EMMUs selected for various mathematics subjects.

The findings in this study include practices evaluated by mathematics teachers. Although it is important to evaluate and select applications, the main sources that show how these applications can contribute to learning mathematics will be the studies with students. In the study of Kay and Kwak (2018), the effect of mobile applications on students' learning was also observed. However, there was no observation on students' learning in this study. Choosing apps is just one step towards effective math learning. In addition, how these applications are used in the teaching process is one of the factors that affect the contribution of the applications to education.

In some studies, tools have been developed to help evaluate applications (eg, Harrison & Lee, 2018). Such assessment tools provide direction for what teachers can look for when examining mobile applications. However, it can be said that it is more difficult to choose a limited number of applications after the evaluation. In this context, it can be thought that this study reveals the importance of MCDM techniques in order to make a choice after the evaluations made.

4. Conclusion and Suggestions

In this study, which aimed to determine the most effective EMMA with the help of the TOPSIS algorithm, the results obtained from the evaluations of 13 alternatives in terms of 10 criteria by four experts showed that some apps' ranks were parallel to user rating ranks in the Google Play Store while some apps' ranks differed. As a result, although apps that are planned to be downloaded are often evaluated by looking at user ratings that reflect customer satisfaction in general, deciding whether an application will be beneficial for mathematics education may not be plausible based on those ratings. However, it is known that increases in the number of evaluation criteria make it more difficult for a user to make a decision. Thus, choosing the most suitable mobile application, the numbers of which are increasing day by day, is steadily becoming a more difficult decision problem. In this context, the use of techniques such as MCDM in decision problems facilitates the work of decision-makers. It can be suggested that similar studies be carried out to disseminate such approaches in educational environments, which will help teachers, parents, and students make important and complex decisions, including decisions about the most appropriate mobile applications for mathematics education.

The fact that all of the applications in the top three in the ranking of this study provide language support shows that this is a feature sought by mathematics teachers, who constituted the experts of this study. In this context, according to the location, it can be said that providing multilingual support in these applications is an important factor in user preferences.

It will contribute to both the identification and the development of useful applications for mathematics education if more applications are evaluated by experts (teachers) and users (students and teachers) according to various criteria. In terms of MCDM, these EMMAs can be re-analysed with different MCDM methods by increasing the number of alternatives and adding different criteria, and weighting them with an appropriate method according to the structure of the criteria and the effects of the criteria on each other.

References

Baker, J., Ashill, N., Amer, N., & Diab, E. (2018). The internet dilemma: An exploratory study of luxury firms' usage of internet-based technologies. *Journal of Retailing and Consumer Services*, *41*, 37-47.

Başaran, S., & El Homsi, F. (2022). Mobile Mathematics Learning Application Selection using Fuzzy TOPSIS. *International Journal of Advanced Computer Science and Applications*, *13*(2).

Başaran, S., & Haruna, Y. (2017). Integrating FAHP and TOPSIS to evaluate mobile learning applications for mathematics. *Procedia Computer Science*, *120*, 91-98.

Borba, M. C., Askar, P., Engelbrecht, J., Gadanidis, G., Llinares, S., & Aguilar, M. S. (2016). Blended learning, e-learning and mobile learning in mathematics education. *ZDM*, *48*(5), 589-610.

Cheon, J., Lee, S., Crooks, S. M., & Song, J. (2012). An investigation of mobile learning readiness in higher education based on the theory of planned behavior. *Computers & Education*, *59*(3), 1054-1064.

Daher, W., & Baya'a, N. (2012). Characteristics of middle school students learning actions in outdoor mathematical activities with the cellular phone. *Teaching Mathematics and its Applications: An International Journal of the IMA*, *31*(3), 133-152.

Dubé, A. K., Kacmaz, G., Wen, R., Alam, S. S., & Xu, C. (2020). Identifying quality educational apps: Lessons from 'top' mathematics apps in the Apple App store. *Education and Information Technologies*, *25*(6), 5389-5404.

Elfeky, A. I. M., & Masadeh, T. S. Y. (2016). The Effect of Mobile Learning on Students' Achievement and Conversational Skills. *International Journal of higher education*, *5*(3), 20-31.

Ezhilarasan, N., & Vijayalakshmi, C. (2020). Optimization of Fuzzy programming with TOPSIS Algorithm. *Procedia Computer Science*, *172*, 473-479.

Franklin, T., & Peng, L. W. (2008). Mobile math: Math educators and students engage in mobile learning. *Journal of Computing in Higher Education*, *20*(2), 69-80.

Güler, M., Bütüner, S. Ö., Danişman, Ş., & Gürsoy, K. (2021). A meta-analysis of the impact of mobile learning on mathematics achievement. *Education and Information Technologies*, 1-21.

Hamidi, H., & Chavoshi, A. (2018). Analysis of the essential factors for the adoption of mobile learning in higher education: A case study of students of the University of Technology. *Telematics and Informatics*, *35*(4), 1053-1070.

Harrison, T. R., & Lee, H. S. (2018). iPads in the mathematics classroom: Developing criteria for selecting appropriate learning apps. *International Journal of Education in Mathematics, Science and Technology*, *6*(2), 155-172.

Haydon, T., Hawkins, R., Denune, H., Kimener, L., McCoy, D., & Basham, J. (2012). A comparison of iPads and worksheets on math skills of high school students with emotional disturbance. *Behavioral Disorders*, *37*(4), 232-243.

Hung, C. M., Huang, I., & Hwang, G. J. (2014). Effects of digital game-based learning on students' self-efficacy, motivation, anxiety, and achievements in learning mathematics. *Journal of Computers in Education*, *1*(2), 151-166.

Hwang, G. J., & Chang, H. F. (2011). A formative assessment-based mobile learning approach to improving the learning attitudes and achievements of students. *Computers & Education*, *56*(4), 1023-1031.

Hwang, C.L. & Yoon, K. (1981). *Multiple Attribute Decision Making: Methods and Applications.* Verlag Berlin Heidelberg New York: Springer.

Hwang, G. J., & Wu, P. H. (2014). Applications, impacts and trends of mobile technology-enhanced learning: a review of 2008–2012 publications in selected SSCI journals. *International Journal of Mobile Learning and Organisation*, *8*(2), 83-95.

Ibrahim, N. K., Hammed, H., Zaidan, A. A., Zaidan, B. B., Albahri, O. S., Alsalem, M. A., Mohammed, R. T., Jasim, A. N., Shareef, A. H., Jalood, N. S., Baqer, M. J., Nidhal, S., Almahdi, E. M., & Alaa, M. (2019). Multi-criteria evaluation and benchmarking for young learners' English language mobile applications in terms of LSRW skills. *IEEE Access*, *7*, 146620-146651.

Jeno, L. M., Vandvik, V., Eliassen, S., & Grytnes, J. A. (2019). Testing the novelty effect of an m-learning tool on internalization and achievement: A Self-Determination Theory approach. *Computers & Education*, *128*, 398-413.

Kay, R., & Kwak, J. Y. (2018). Comparing types of mathematics apps used in primary school classrooms: an exploratory analysis. *Journal of Computers in Education*, *5*(3), 349-371.

Kluge, A., & Dolonen, J. (2015). Using mobile games in the classroom. *Mobile Learning and Mathematics*, 106-121.

Kyriakides, A. O., Meletiou-Mavrotheris, M., & Prodromou, T. (2016). Mobile technologies in the service of students' learning of mathematics: the example of game application ALEX in the context of a primary school in Cyprus. *Mathematics Education Research Journal*, *28*(1), 53-78

Lai, C. L., & Hwang, G. J. (2014). Effects of mobile learning time on students' conception of collaboration, communication, complex problem–solving, meta–cognitive awareness and creativity. *International Journal of Mobile Learning and Organisation*, *8*(3-4), 276-291.

Laricchia, F. (2022, July 18). Market share of mobile operating systems worldwide 2012-2022. Retrieved July 25, 2022, from https://www.statista.com/statistics/272698/global-market-share-held-by-mobile-operating-systems-since-2009/

Larkin, K. (2015). An app! An app! My kingdom for an app: An 18-month quest to determine whether apps support mathematical knowledge building. In *Digital games and mathematics learning* (pp. 251–276). Dordrecht, Netherlands: Springer.

Linkov, I., Satterstrom, F.K., Kiker, G., Seager, T.P, Bridges, T., Gardner, K.H., Rogers, S.H., Belluck, D.A. ve Meyer., A. (2006). Multicriteria Decision Analysis: A Comprehensive Decision Approach for Management of Contaminated Sediments. *Risk Analysis.* 26: 61-78.

Mahdavi, I., Amiri, N.M., Heidarzade, A. ve Nourifar, R. (2008). Designing a model of fuzzy TOPSIS in multiple criteria decision making. *Applied Mathematics and Computation.* 1-11.

Martin, F., & Ertzberger, J. (2013). Here and now mobile learning: An experimental study on the use of mobile technology. *Computers & Education*, *68*, 76-85.

Mendoza, G.A. & Prabhub, R. (2000). Multiple criteria decision making approaches to assessing forest sustainability using criteria and indicators: a case study. *Forest Ecology and Management.* 131: 107-126.

Namukasa, I. K., Gadanidis, G., Sarina, V., Scucuglia, S., & Aryee, K. (2016). Selection of apps for teaching difficult mathematics topics: An instrument to evaluate touch-screen tablet and smartphone mathematics apps. In *International perspectives on teaching and learning mathematics with virtual manipulatives* (pp. 275-300). Springer, Cham.

Pohekar, S.D. & Ramachandran, M. (2004). Application of multi-criteria decision making to sustainable energy planning-A review. *Renewable and Sustainable Energy Reviews. 8*: 365–381.

Riconscente, M. M. (2013). Results from a controlled study of the iPad fractions game Motion Math. *Games and Culture*, *8*(4), 186-214.

Saedi, N., Taghizade, A., & Hatami, J. (2018). The Effect of Mobile Learning Applications on Students' High-level Cognitive Skills. *Interdisciplinary Journal of Virtual Learning in Medical Sciences*, *9*(4).

Shank, D. B., & Cotten, S. R. (2014). Does technology empower urban youth? The relationship of technology use to self-efficacy. *Computers & Education*, *70*, 184-193.

Uslu, B., Gür, Ş., Eren, T., & Özcan, E. (2020). Mobil uygulama seçiminde etkili olan kriterlerin belirlenmesi ve örnek uygulama. *İstanbul İktisat Dergisi*, *70*(1), 113-139.

Volaric, T., Brajkovic, E., & Sjekavica, T. (2014). Integration of FAHP and TOPSIS methods for the selection of appropriate multimedia application for learning and teaching. *International journal of mathematical models and methods in applied sciences*, *8*, 224-232.

Zeleny, M. (1974). A Consept of Compromise Solutions and The Method of The Displaced İdeal. *Comput Operat Res*. 1: 479-496.

Zeleny, M. (2005). *Human Systems Management: Integrating Knowledge, Management and Systems*. World Scientific.

Zhao, C., Muthu, B., & Shakeel, P. M. (2021). Multi-Objective Heuristic Decision Making and Benchmarking for Mobile Applications in English Language Learning. *Transactions on Asian and Low-Resource Language Information Processing*, *20*(5), 1-16