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Alpha-Stable Autoregressive Modeling of Chua's Circuit in the Presence of Heavy-Tailed Noise

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ABSTRACT This study presents alpha-stable autoregressive (AR) modeling of the dynamics of Chua's circuit in the presence of heavy-tailed noise. The parameters of the AR time series are estimated using the covariation-based Yule-Walker method, and the parameters of alpha-stable distributed residuals are calculated using the regression type method. Visual depictions of the calculated parameters of the AR model and alpha-stable distributions of residuals are presented. The medians of the estimated parameters of the AR model and alpha-stable distributions parameters of residuals are presented for heavy-tailed noise with various stability index parameters. Thus, the impulsive behavior of Chua's circuit can be modeled as alpha-stable AR time series, and the model can provide an alternative approach to describe the chaotic systems driven by heavy-tailed noise.

KEYWORDS
Alpha-stable distribution
Chua's circuit
Autoregressive model
Yule-Walker equations

INTRODUCTION

There has been an increasing interest in stochastic processes based on heavy-tailed distributions for real-world data modeling. It is well known that stochastic fluctuations are inevitable due to various uncertainties or unpredictable factors in the real-world systems. Understanding the effect of fluctuations on the chaotic dynamics is also of fundamental interest. The importance of additive noise in chaotic attractors is considered in (Argyris *et al.* 1998). The effect of stochastic excitations which have asymmetric distributions on chaotic dynamics is analyzed in (Yilmaz *et al.* 2018) by considering the generalized Chua's circuit driven by skew-Gaussian distributed noise. However, Gaussian distribution cannot be applied for modeling data across multiple application areas for which real-world data exhibit significant peaks.

Some examples that might have heavy-tailed behavior include tracking highly maneuvering objects (Gan and Godsill 2020; Gan *et al.* 2021), interference in IoT networks (Clavier *et al.* 2021), financial data (McCulloch 1996; Maleki *et al.* 2020; Janczura *et al.* 2011; Wesselhöfft 2021), chaotic systems (Savaci and Yilmaz 2015; Contreras-Reyes 2021), frequency fluctuations in the power grid (Schäfer *et al.* 2018; Anvari *et al.* 2020), the dose distributions for proton breast treatment (Van den Heuvel *et al.* 2015), proton pencil beams for cancer therapy (Van den Heuvel *et al.* 2018), climate dynamics (Ditlevsen 1999; Broszkiewicz-Suwaj and Wyłomańska 2021). Therefore, alpha-stable (α-stable) distributions are more suitable for modeling such impulsive behavior (Nolan 2003). Alpha-stable distributions require four parameters: skewness parameter (β), scale parameter (σ), location parameter (μ), and stability index (α), which is responsible for the heavy-tailedness of the distribution.

To model the real-world data based on heavy-tailed time series, the α-stable autoregressive (AR) model is proposed in (Gallagher 2001), and generalized Yule-Walker equations are used to estimate the parameters of the α-stable AR process. The use of α-stable distributions in multivariate processes is presented in (Pai and Ravishanker 2010) and the approach is illustrated on time series of daily average temperatures.

The α-stable distribution with $\alpha = 2$ corresponds to the Gaussian distribution. Since stable distributions have an infinite variance for $\alpha < 2$, autocorrelation is not defined for heavy-tailed random sequences. Therefore, other measures of dependence, such as autocovariation are needed for consideration in an infinite variance system. A new autocovariation estimator for α-stable AR processes is introduced in (Gallagher 2001), in which the real-world data set is considered as the time series of sea surface temperatures.

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A modified method of Yule-Walker is presented in (Kruczek *et al.* 2017) to estimate the parameters of the stable periodic autoregressive (PAR) model. This method obtains the PAR model for electricity market data describing the hourly volume of upregulating bid prices in Norway. The classical one-dimensional α-stable AR model is generalized to the multidimensional case in (Grzesiek *et al.* 2021). The method is applied to a real data set which contains daily prices of KGHM and copper.

This paper considers a stochastic nonlinear electronic circuit, specifically the Chua's circuit with α-stable noise. Chua's circuit is a nonlinear chaotic circuit, and the presence of heavy-tailed noise makes the circuit more unpredictable and complex. Our study focuses on applying the α-stable autoregressive (AR) model to characterize the impulsive behavior of the Chua's circuit, and thus aims to provide a better-linearized way to analyze the dynamics of the states of stochastic Chua's circuit.

The paper is structured as follows. In the first part, Chua's circuit in the presence of heavy-tailed noise is presented, and its α-stable AR model is proposed. The next part gives the modified Yule-Walker equations for α-stable AR models based on the autocovariation estimator. In the last part, the dynamical behaviors of the system are obtained by using the Euler-Maruyama method, and the estimation method presented is applied to the simulated data.

CHUA'S CIRCUIT IN THE PRESENCE OF ALPHA-STABLE NOISE

The set of differential equations representing the dynamics of dimensionless Chua's circuit in the presence of heavy-tailed noise is given as follows (Suykens and Huang 1997):

$$\begin{aligned} dx &= a[y-h(x)]dt + dL_\alpha(t) \\ dy &= (x-y+z)dt \\ dz &= -bydt. \end{aligned} \tag{1}$$

with the bifurcation parameters a, b and the piecewise-linear function $h(x)$:

$$h(x) = m_1 x + 0.5(m_0 - m_1)(|x+1| - |x-1|) \tag{2}$$

and $dL_\alpha(t)$ is α-stable random variable $\sim S_\alpha(\beta, \sigma, \mu)$ with the stability index $\alpha \in (0, 2]$, the skewness parameter $\beta \in [-1, 1]$, the scale parameter $\sigma \in \mathbb{R}_+$ and the location parameter $\mu \in \mathbb{R}$ (Samorodnitsky and Taqqu 1994; Nikias and Shao 1995).

The characteristic function of an α-stable random variable is given as (Samorodnitsky and Taqqu 1994; Nikias and Shao 1995)

$$\varphi(w) = \begin{cases} exp\left\{-|\gamma w|^\alpha [1 - i\beta sign(w)\tan(\frac{\pi\alpha}{2})] + i\mu w\right\} & \text{for} \quad \alpha \neq 1 \\ exp\left\{-|\gamma w|[1 + i\beta sign(w)\frac{2}{\pi} log(|w|)] + i\mu w\right\} & \text{for} \quad \alpha = 1 \end{cases} \tag{3}$$

where *sign(w)* is signum function.

Due to the lack of analytical expression for α-stable density functions, the numerical approximation of the corresponding density function $f(y; \alpha, \beta, \sigma, \mu)$ of an α-stable random variable can be evaluated by the inverse Fourier transform of the characteristic function given in (3) as:

$$f(y; \alpha, \beta, \sigma, \mu) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-jwy} \varphi(w) dw. \tag{4}$$

When $\beta = 0$, the distribution is symmetric around μ. The impulsiveness of the distribution increases with the decreasing stability index α, which makes the tails of the corresponding distributions heavier. Gaussian distribution ($\alpha = 2$ and $\beta = 0$), Cauchy distribution ($\alpha = 1$ and $\beta = 0$), and Lévy distribution ($\alpha = 0.5$ and $\beta = 1$) are the exceptional cases of the α-stable distributions.

In this paper, steady states of the dynamical behaviors of the system (1) are proposed to model as an α-stable third-order AR process given as

$$\begin{aligned} x(t) &= \sum_{i=1}^{3} \phi_{1,i} x(t-i) + \xi_1(t) \\ y(t) &= \sum_{i=1}^{3} \phi_{2,i} y(t-i) + \xi_2(t) \\ z(t) &= \sum_{i=1}^{3} \phi_{3,i} z(t-i) + \xi_3(t) \end{aligned} \tag{5}$$

where $\phi_{j,i}$ is the AR parameter and $\xi_i(t)$ is the sequence of i.i.d. symmetric alpha-stable (SαS) random variables for $i, j = 1, 2, 3$.

ESTIMATION METHOD FOR ALPHA-STABLE AR MODELS

The autoregressive model parameters are commonly estimated using the Yule-Walker method based on the autocorrelation function (ACF) (Brockwell and Davis 2002). Since ACF is not defined for α-stable random variables, the modified Yule-Walker method is introduced based on the autocovariation function in (Gallagher 2001), and the parameters of α-stable AR models are found using the modified Yule-Walker method. The procedure of the method is described in the following part:

Let X_t be an autoregressive process of order p which satisfies the following equation:

$$X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2} - \cdots \phi_p X_{t-p} = \xi_t \tag{6}$$

where the sequence $\{\xi_t\}$ is an i.i.d. SαS random variables with $\alpha > 1$.

Multiplying (6) by vector $\mathbf{S} = [S_{t-1}, S_{t-2}, \ldots, S_{t-p}]'$ where $S_t = sign(X_t)$ and taking the expectation, the system consisting of p number of equations is obtained as follows:

$$\begin{aligned} \mathbb{E}X_t S_{t-1} &- \sum_{i=1}^{p} \phi_i \mathbb{E}X_{t-i} S_{t-1} = \mathbb{E}\xi_t \\ \mathbb{E}X_t S_{t-2} &- \sum_{i=1}^{p} \phi_i \mathbb{E}X_{t-i} S_{t-2} = \mathbb{E}\xi_t \\ &\vdots \\ \mathbb{E}X_t S_{t-p} &- \sum_{i=1}^{p} \phi_i \mathbb{E}X_{t-i} S_{t-p} = \mathbb{E}\xi_t \end{aligned} \tag{7}$$

Then dividing the equations respectively by $\mathbb{E}|X_{t-1}|, \mathbb{E}|X_{t-2}|, \ldots, \mathbb{E}|X_{t-p}|$ the following system is obtained:

$$\begin{aligned} \frac{\mathbb{E}X_t S_{t-1}}{\mathbb{E}|X_{t-1}|} &- \sum_{i=1}^{p} \phi_i \frac{\mathbb{E}X_{t-i} S_{t-1}}{\mathbb{E}|X_{t-1}|} = 0 \\ \frac{\mathbb{E}X_t S_{t-2}}{\mathbb{E}|X_{t-2}|} &- \sum_{i=1}^{p} \phi_i \frac{\mathbb{E}X_{t-i} S_{t-2}}{\mathbb{E}|X_{t-2}|} = 0 \\ &\vdots \\ \frac{\mathbb{E}X_t S_{t-p}}{\mathbb{E}|X_{t-p}|} &- \sum_{i=1}^{p} \phi_i \frac{\mathbb{E}X_{t-i} S_{t-p}}{\mathbb{E}|X_{t-p}|} = 0 \end{aligned} \tag{8}$$

in which $\mathbb{E}\xi_t = 0$ since ξ_t has SαS distribution with $\alpha > 1$.

By using the normalized autocovariation (NCV) for stationary SαS process $\{X_t\}$ for lag k proposed in (Gallagher 2001), the matrix form of (8) can be written as follows:

$$\lambda = \Lambda\Phi \tag{9}$$

where λ and ϕ are vectors with the length of p, and they are defined as

$$\begin{aligned} \lambda &= [NCV(X_t, X_{t-1}), \ldots, NCV(X_t, X_{t-p})]' \\ \Phi &= [\phi_1, \ldots, \phi_p]' \end{aligned} \tag{10}$$

in which

$$NCV(X_t, X_{t-k}) = \frac{\mathbb{E}X_t sign(X_{t-k})}{\mathbb{E}|X_{t-k}|} \tag{11}$$

The Λ is the $p \times p$ matrix, and its elements are described by:

$$\Lambda(i,j) = NCV(X_t, X_{t-i+j}) \quad \text{for i, j=1, \ldots,p.} \tag{12}$$

The values of the model parameters Φ can be estimated using the sample autocovariation estimator $\widehat{NCV}$ based on p-th moment. The sample estimator of the normalized autocovariation $\widehat{NCV}$ for $\{X(t)\}$ proposed in (Gallagher 2001) is given by:

$$\widehat{NCV}(X_t, X_{t-k}) = \frac{\sum_{t=l}^{r} x_t sign(x_{t-k})}{\sum_{t=1}^{N} |x_t|} \tag{13}$$

where $x_1, x_2, \ldots, x_N$ is a vector set denotes the realization of the random variable $X(t)$, N is the trajectory size, $l = \max(1, 1+k)$, and $r = \min(N, N+k)$.

If the matrix Λ is nonsingular, then the estimators for AR parameters $\hat{\Phi}$ can be written as:

$$\hat{\Phi} = \hat{\Lambda}^{-1}\hat{\lambda} \tag{14}$$

Since the residuals of the model are thought to be a sample of i.i.d. SαS random variables, having estimated the parameters of the AR(p) model, the distribution of the residuals is analyzed using the Kolmogorov-Smirnov (KS) test.

SIMULATION RESULTS

The bifurcation parameters of (1) are fixed as $a = 9$, $b = 14.28$, and the parameters of the piecewise-linear function (2) are chosen as $m_0 = -1/7, m_1 = 2/7$. Using the Euler-Maruyama method given in (Janicki and Weron 1993; Platen 1999) with the step size $\tau = 0.01$, the system of (1) is solved numerically as

$$\mathbf{X}(t_i) = \mathbf{X}(t_{i-1}) + \mathbf{F}(\mathbf{X}(t_{i-1}))\tau + \Delta L^{\tau}_{\alpha,i} \tag{15}$$

where $\tau = t_i - t_{i-1}$. An increment of the α-stable Lévy process is an α-stable random variable generated in (Janicki and Weron 1993) and is defined by

$$L^{\tau}_{\alpha,i} = L_\alpha([t_{i-1}, t_i]) \sim S_\alpha(\beta, \sigma, \mu). \tag{16}$$

In the first case, Chua's circuit is regarded in the presence of α-stable noise with $\alpha = 1.6$, $\beta = 0$, $\sigma = 4.217 \times 10^{-5}$ and $\mu = 0$ and a time series consisting of 10^5 data is obtained for each state variable.

The parameters $\phi_{j,i}$ of the AR(3) model in (5) are estimated using the sample autocovariation estimator based on the p-th moment proposed in (Gallagher 2001). Afterward, it is assumed that

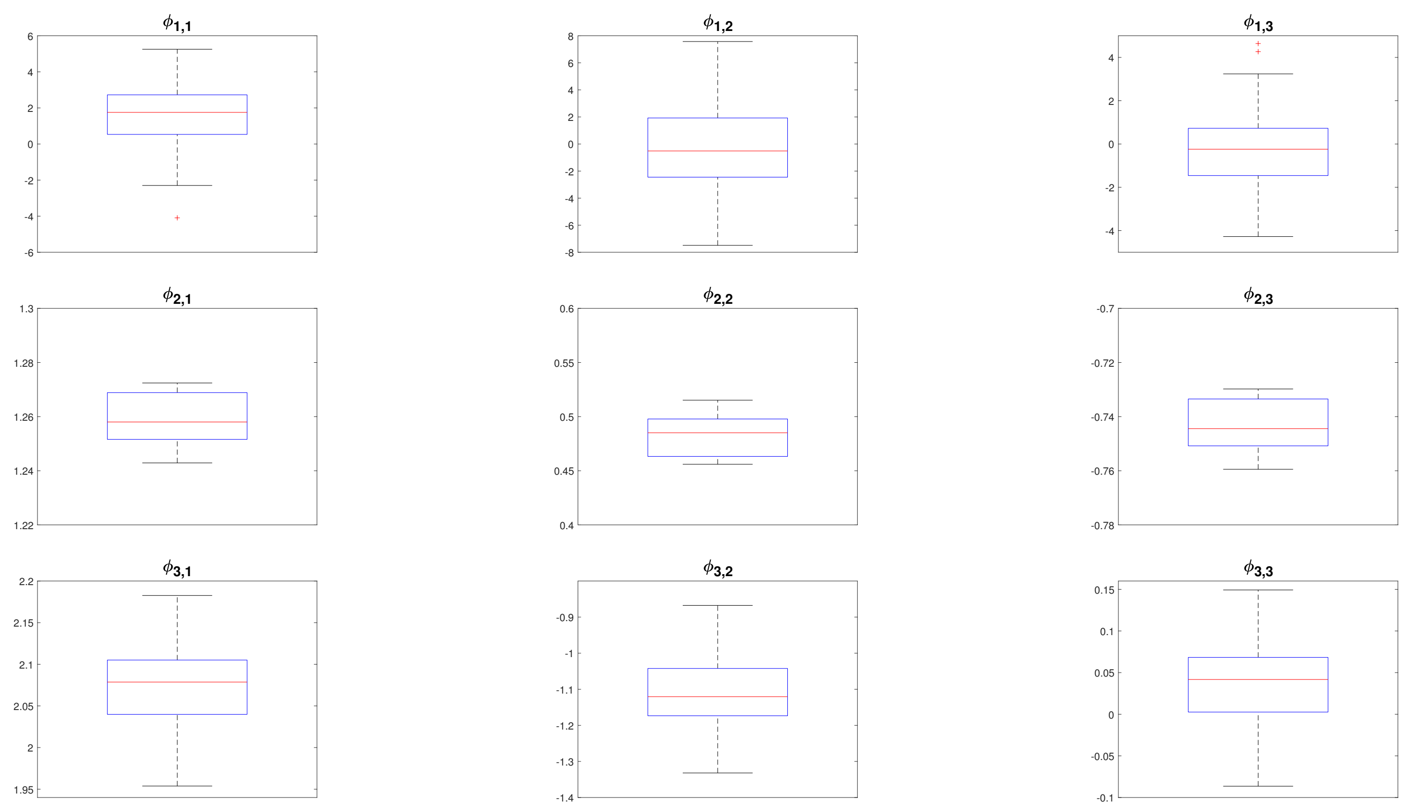
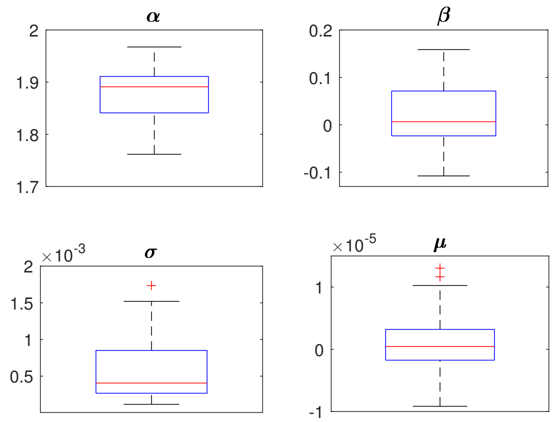
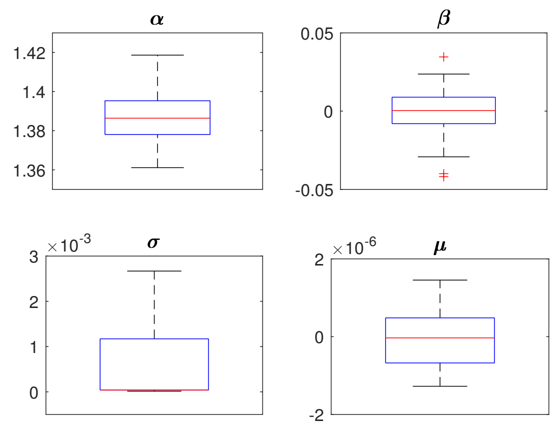


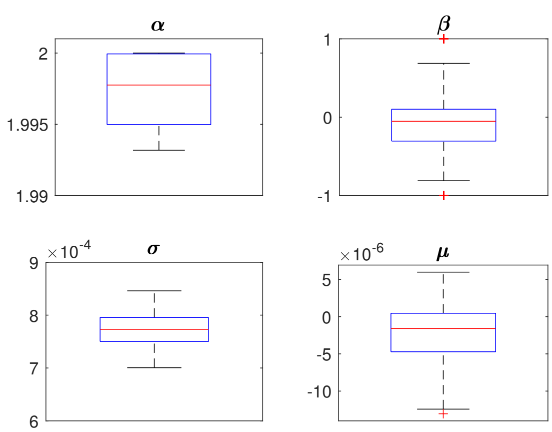
Figure 1 Visual depictions of the calculated parameters of AR(3) model (5) for the system (1) in the presence of α-stable noise with $\alpha = 1.6, \beta = 0, \sigma = 4.217 \times 10^{-5}, \mu = 0$ and $N = 10^5$. Simulations were carried out 100 times using the Monte Carlo method.



(a) Visual depictions of the calculated parameters of α-stable distribution residuals $\xi_1(t)$.



(b) Visual depictions of the calculated parameters of α-stable distribution residuals $\xi_2(t)$.



(c) Visual depictions of the calculated parameters of α-stable distribution residuals $\xi_3(t)$.

Figure 2 Each box visually represents the estimated parameter value of the α-stable AR(3) model in (5). Simulations were performed 100 times using the Monte Carlo method.

the noise series ξ_1, ξ_2 and ξ_3 for each state are the representatives of independent α-stable distributed random variables. These residuals are analyzed using the KS test to confirm that they are drawn from the α-stable distribution. By utilizing the KS test, it is obtained that the hypothesis of α-stable distribution for univariate samples and $\xi_1(t)$, $\xi_2(t)$, and $\xi_3(t)$ cannot be rejected at the significance level of 0.05. Then, the α-stable distribution is fitted to the residual time series of each state, and the parameters of the corresponding α-stable distribution for ξ_1, ξ_2 and ξ_3 are estimated using the regression type method. This procedure is performed 100 times using the Monte Carlo simulations, and the boxplots of the estimated parameters are created.

Visual depictions of the calculated parameters of AR(3) model and residuals distributions are presented in Figure 1 and Figure 2, respectively. Each box visually represents the estimated parameter value of the α-stable AR model. The red line indicates the sample median on each box, and the bottom and top edges of the box denoted by blue lines indicate the first and third quartiles, respectively. The black lines represent the most extreme data points, and an outlier is plotted using the red '+' marker symbol. For the presence of α-stable noise with the parameters $\alpha = 1.6$, $\beta = 0$, $\sigma = 4.217 \times 10^{-5}$ and $\mu = 0$, the medians of parameters of the α-stable AR model (5) which corresponds to the red line on each box shown in Figure 1 are obtained as:

$$\begin{aligned} &\phi_{1,1} = 1.7599, \quad \phi_{1,2} = -0.5207, \quad \phi_{1,3} = -0.2393, \\ &\phi_{2,1} = 1.2615, \quad \phi_{2,2} = 0.4779, \quad \phi_{2,3} = -0.7408, \\ &\phi_{3,1} = 2.1048, \quad \phi_{3,2} = -1.1734, \quad \phi_{3,3} = 0.0688 \end{aligned} \tag{17}$$

The medians of α-stable distribution parameters $(\hat{\alpha}, \hat{\beta}, \hat{\sigma}, \hat{\mu})$ for residual series ξ_1, ξ_2 and ξ_3 are estimated $(1.8882, -0.004, 0.3666 \times 10^{-3}, 0.105 \times 10^{-6})$, $(1.3865, 0.0019, 0.4 \times 10^{-4}, 0.0898 \times 10^{-6})$ and $(1.9983, -0.0074, 0.7492 \times 10^{-3}, -0.1028 \times 10^{-5})$, respectively, as shown in Figure 2.

In the second case, Chua's circuit is considered in the presence of α-stable noise with different impulsive behaviors. The stability index α ranges from 1.1 to 1.9, and the estimation method is applied for each α value. The medians of the estimated parameters of the AR(3) model are obtained as shown in Table 1. After the analysis of residuals, the medians of the parameters of α-stable distribution for the residual series $\xi_1(t)$, $\xi_2(t)$ and $\xi_3(t)$ are obtained as given in Table 2, 3, and 4, respectively.

As seen in Table 2, the estimated stability index $\hat{\alpha}$ for the series ξ_1 is in the range of 1.898 to 1.932, and the estimated value of the scale parameter $\hat{\sigma}$ decreases as the Chua's circuit is driven by noise with heavier tails. It is also seen in Table 3 that the estimated stability index $\hat{\alpha}$ for the residual series ξ_2 is around 1.38, which implies the residual series ξ_2 is impulsive. On the other hand, the estimated stability index $\hat{\alpha}$ for the residual series ξ_3 is around 1.998, as seen in Table 4, which implies that the corresponding distribution is the Gaussian. The phase portrait of the system (5) with the parameters in (17) is obtained as shown in Figure 3. Figure 3 shows that the attractors reconstructed from the time series (5) characterize the double scroll observed in Chua's circuit.

Moreover, the largest Lyapunov exponents of both systems (1) and (5) are determined numerically. Largest Lyapunov exponents from the time series are estimated using the algorithm presented in (Wolf *et al.* 1985). Figure 4 presents the time evolution of the largest Lyapunov exponents. The blue line shows the value of the Lyapunov exponent obtained from the system of (1) (blue line) and the red line shows the Lyapunov exponents obtained from the simulated data of the proposed system of (5).

Table 1 The medians of the estimated parameters of AR(3) in (5).

α	$\phi_{1,1}$	$\phi_{1,2}$	$\phi_{1,3}$	$\phi_{2,1}$	$\phi_{2,2}$	$\phi_{2,3}$	$\phi_{3,1}$	$\phi_{3,2}$	$\phi_{3,3}$
1.1	2.9705	-2.9362	0.9657	1.2614	0.4781	-0.7409	2.0907	-1.1461	0.0554
1.2	2.8887	-2.7765	0.8877	1.2656	0.4696	-0.7366	2.0754	-1.1158	0.0402
1.3	2.6506	-2.3026	0.6520	1.2603	0.4803	-0.7420	2.0944	-1.1528	0.0583
1.4	2.2825	-1.5650	0.2824	1.2647	0.4714	-0.7375	2.0925	-1.1492	0.0566
1.5	1.8815	-0.7660	-0.1155	1.2627	0.4755	-0.7395	2.0948	-1.1529	0.0580
1.6	1.7599	-0.5207	-0.2393	1.2615	0.4779	-0.7408	2.1048	-1.1734	0.0688
1.7	1.5509	-0.1055	-0.4454	1.2594	0.4821	-0.7408	2.1190	-1.2024	0.0829
1.8	1.4834	0.0029	-0.4863	1.2631	0.4747	-0.7391	2.1361	-1.2375	0.1013
1.9	1.6056	-0.2196	-0.3861	1.2621	0.4767	-0.7401	2.1397	-1.2451	0.1053

Table 2 The medians of the estimated parameters of α-stable distribution for the residual series $\xi_1(t)$ in (5).

α	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$ $(\times 10^{-3})$	$\hat{\mu}$ $(\times 10^{-6})$
1.1	1.8987	-0.0019	0.0858	-0.1476
1.2	1.8985	-0.0123	0.1203	-0.1750
1.3	1.8839	-0.0049	0.1678	-02307
1.4	1.9004	0.0076	0.2892	-0.2303
1.5	1.8924	0.0054	0.3360	0.3027
1.6	1.8882	-0.0040	0.3666	0.1050
1.7	1.8967	0.0009	0.4142	0.5840
1.8	1.9037	-0.0086	0.4641	0.3841
1.9	1.9320	0.0018	0.5109	0.1495

Table 3 The medians of the parameters of α-stable distribution for the residual series $\xi_2(t)$ in (5).

α	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$ $(\times 10^{-4})$	$\hat{\mu}$ $(\times 10^{-6})$
1.1	1.3850	-0.0023	0.3954	-0.0955
1.2	1.3834	0.036	0.4048	0.3872
1.3	1.3836	-0.0005	0.4015	0.0148
1.4	1.3863	-0.0002	0.3985	0.2201
1.5	1.3854	-0.0012	0.4042	0.0455
1.6	1.3865	0.0019	0.4000	0.0898
1.7	1.3898	-0.0044	0.3989	-0.2566
1.8	1.3879	0.0016	0.3963	0.0507
1.9	1.3996	-0.0034	0.4137	-0.1121

Table 4 The medians of the parameters of α-stable distribution for the residual series $\xi_3(t)$ in (5).

α	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$ $(\times 10^{-3})$	$\hat{\mu}$ $(\times 10^{-5})$
1.1	1.9976	-0.0091	0.7518	0.0349
1.2	1.9984	0.0093	0.7561	-0.0897
1.3	1.9981	-0.0028	0.7493	-0.0293
1.4	1.9987	0.0112	0.7514	-0.0735
1.5	1.9986	0.0342	0.7560	-0.1394
1.6	1.9983	-0.0074	0.7492	-0.1028
1.7	1.9994	-0.0169	0.7436	-0.0556
1.8	1.9982	-0.0534	0.7329	-0.0061
1.9	1.9984	-0.0184	0.7257	-0.0408

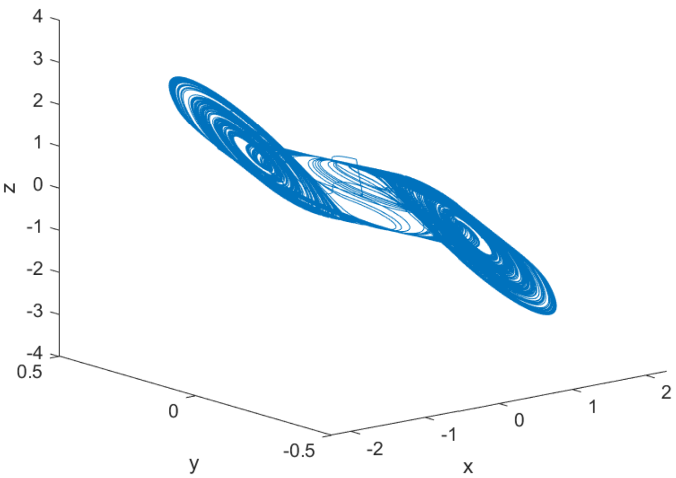


Figure 3 3D phase portrait of system (5) in the presence of α-stable noise with $\alpha = 1.6$, $\beta = 0$, $\sigma = 4.217 \times 10^{-5}$ and $\mu = 0$. The corresponding estimated parameters of (5) are given in Table 1-4.

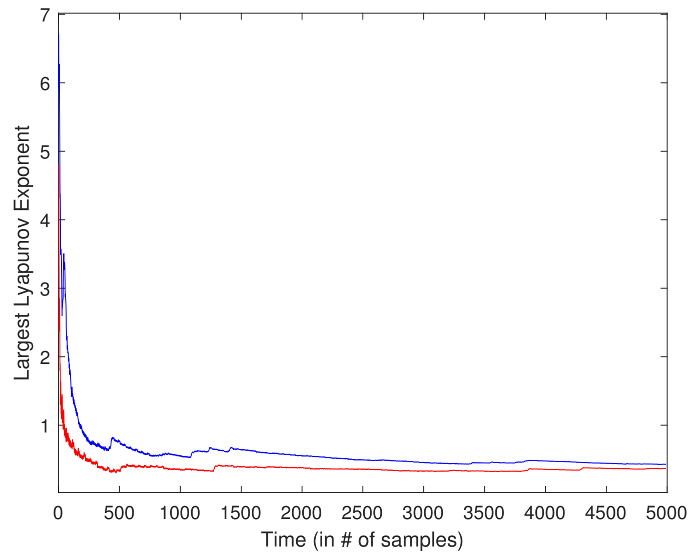


Figure 4 The largest Lyapunov exponents obtained from the system of (1) (blue line) and the system of (5) (red line).

CONCLUSION

In this study, the states of Chua's circuit in the presence of α-stable noise have been modeled as α-stable autoregressive processes. The AR model parameters have been estimated by using the modified Yule-Walker equations and calculating the autocovariation function based on the p-th moment. By estimating the model parameters, the α-stable distribution is fitted to the residual time series of each state, and the parameters of the α-stable distribution have been obtained using the regression type method.

The structure of the double scroll has been observed using the estimated parameters and it has been shown that the model fits very well on simulated data. Chua's circuit has also been considered in the presence of α-stable noise with various stability index α, and the corresponding α-stable AR models have been obtained. Such models will provide new insights into studying nonlinear dynamics in chaotic systems involving stochastic noises. However, further researchs could be considered by using more complex models such as the trivariate vector autoregressive fractional integrated moving average (VARFIMA) model (Contreras-Reyes 2022) instead of simple univariate AR processes. VARFIMA models are considered adaptive estimation methods and also defined for α-stable distributions (Pai and Ravishanker 2010).

Conflicts of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Availability of data and material

Not applicable.

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