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New General Inequalities For Exponential Type Convex Function

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Abstract. In this paper, we introduce the concept of an exponential type convex function. We establish new integral inequalities of the Hermite-Hadamard type by using the Power-Mean and Hölder inequalities. Additionally, we give definitions of the Riemann-Liouville fractional integrals. We use these Riemann-Liouville fractional integrals to establish a new integral inequalities for exponential type convex function.

1. Introduction

The mathematical branches heavily rely on mathematical inequalities. Numerous scientists investigated the characteristics of convexity and came up with several integral inequalities (see references [3]-[8]). Hermite-Hadamard inequality is among the most well-known integral inequalities for convex functions. This double integral inequality is stated as follows:

Let $f : I \to \mathbb{R}$ be a convex function. Then the following inequalities hold:

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}$$

for all $a, b \in I$ with $a < b$.

Convex functions take a significant place in the Mathematical Inequalities. Many researchers have carried out studies on different definitions of convex functions. Previous studies have focused on convexity types such as $s-$convex, $m-$convex, $(\alpha, m)-$convex and quasi-convex (see references [9]-[12]). However, recent studies have found that many new types of convexity have been obtained. One of these new types of convexity is exponential type convex functions. A new definition is given as follows:

Definition 1.1. *[2] A nonnegative function $f : I \to \mathbb{R}$ is called exponential type convex function if, for every $a, b \in I$ and $k \in [0, 1]$,*

$$f(ka + (1-k)b) \leq \left(e^{k} - 1\right) f(a) + \left(e^{1-k} - 1\right) f(b)$$

The class of all exponential type convex functions on interval I is indicated by $EXPC(I)$.

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In [2], Kadakal and İşcan have defined exponential type convex functions and obtained new inequalities related to this definition as follows:

Theorem 1.2. *Let $f : [a, b] \to \mathbb{R}$ be an exponential type convex function. If $a < b$ and $f \in L[a, b]$, then the following Hermite-Hadamard type inequalities hold:*

$$\frac{1}{2\left(\sqrt{e}-1\right)} f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a} \int_a^b f(x)dx \leq (e-2)\left[f(a)+f(b)\right]$$

Theorem 1.3. *Let $f : I \to \mathbb{R}$ be a differentiable function on I°, $a, b \in I^\circ$ with $a < b$, and assume that $f' \in L[a, b]$. If $\left|f'\right|$ is an exponential type convex function on $[a, b]$, then the inequality*

$$\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx\right| \leq (b-a)\left(4\sqrt{e}-e-\frac{7}{2}\right) A\left(\left|f'(a)\right|, \left|f'(b)\right|\right)$$

holds for $k \in [0, 1]$, where $A(u, v)$ is the arithmetic mean of u and v.

Theorem 1.4. *Let $f : I \to \mathbb{R}$ be a differentiable function on I°, $a, b \in I^\circ$ with $a < b$, $q > 1$ and assume that $f' \in L[a, b]$. If $\left|f'\right|^q$ is an exponential type convex function on $[a, b]$, then the inequality*

$$\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx\right| \leq \frac{(b-a)}{2}\left[2(e-2)\right]^{\frac{1}{q}} \left(\frac{1}{p+1}\right)^{\frac{1}{p}} A^{\frac{1}{q}}\left(\left|f'(a)\right|^q, \left|f'(b)\right|^q\right)$$

holds for $k \in [0, 1]$, where $\frac{1}{p} + \frac{1}{q} = 1$ and $A(u, v)$ is the arithmetic mean of u and v.

Theorem 1.5. *Let $f : I \to \mathbb{R}$ be a differentiable function on I°, $a, b \in I^\circ$ with $a < b$, $q \geq 1$ and assume that $f' \in L[a, b]$. If $\left|f'\right|^q$ is an exponential type convex function on $[a, b]$, then the inequality*

$$\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx\right| \leq \frac{(b-a)}{2^{2-\frac{1}{q}}}\left[2\left(4\sqrt{e}-e-\frac{7}{2}\right)\right]^{\frac{1}{q}} A^{\frac{1}{q}}\left(\left|f'(a)\right|^q, \left|f'(b)\right|^q\right)$$

holds for $k \in [0, 1]$, where $A(u, v)$ is the arithmetic mean of u and v.

In [1], Alomari *et al.* proved the following result connected with the right part of Hermite-Hadamard Inequality:

Lemma 1.6. *Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be an absolutely continuous function on I°, where $a, b \in I$ with $a < b$. Then the following equality holds:*

$$\frac{f(a)+rf(b)}{r+1} - \frac{1}{b-a}\int_a^b f(x)dx = \frac{b-a}{r+1}\int_0^1 \left[(r+1)t-1\right] f'(tb+(1-t)a)\,dt.$$

for every fixed $r \in [0, 1]$.

As a result of the research exists, we investigate new inequalities that are connected to right hand side of Hermite-Hadamard integral inequalities for some exponential type convex functions utilizing the Hölder inequality, properties of modulus, power mean inequality, and elementary calculations.

The aim of this paper is to establish some Hermite-Hadamard type inequalities for exponential type convex functions. In order to obtain our results, we utilized Lemma 1.6 and Lemma 3.2.

2. Hermite-Hadamard Inequality For Exponential Type Convex Functions

Theorem 2.1. *Let $f : I \to \mathbb{R}$ be a differentiable function on I^o, $a, b \in I^o$ with $a < b$ and assume that $f' \in L[a,b]$. If $|f'|$ is an exponential type convex function on $[a,b]$, then the inequality*

$$\left|\frac{f(a)+rf(b)}{r+1}-\frac{1}{b-a}\int_a^b f(x)dx\right| \leq \frac{b-a}{r+1}\left[\left((2r+2)e^{\frac{1}{r+1}}-\frac{3r^2+6r+2re+2e+5}{2r+2}\right)\left|f'(b)\right| + \left((2r+2)e^{\frac{r}{r+1}}-\frac{5r^2+6r+2er^2+2er+3}{2r+2}\right)\left|f'(a)\right|\right]$$

holds for every fixed $r \in [0,1]$.

Proof. Using Lemma 1.6 and the exponential type convexity of $|f'|$, it follows that

$$\begin{aligned}
&\left|\frac{f(a)+rf(b)}{r+1}-\frac{1}{b-a}\int_a^b f(x)dx\right| \\
&\leq \frac{b-a}{r+1}\int_0^1 |(r+1)t-1|\left|f'(tb+(1-t)a)\right|dt \\
&\leq \frac{b-a}{r+1}\int_0^1 |(r+1)t-1|\left[\left(e^t-1\right)\left|f'(b)\right|+\left(e^{1-t}-1\right)\left|f'(a)\right|\right]dt.
\end{aligned}$$

Hence

$$\begin{aligned}
&\left|\frac{f(a)+rf(b)}{r+1}-\frac{1}{b-a}\int_a^b f(x)dx\right| \\
&\leq \frac{b-a}{r+1}\left(\int_0^{\frac{1}{r+1}} (1-(r+1)t)\left[\left(e^t-1\right)\left|f'(b)\right|+\left(e^{1-t}-1\right)\left|f'(a)\right|\right]dt\right. \\
&\quad \left.+\int_{\frac{1}{r+1}}^1 ((r+1)t-1)\left[\left(e^t-1\right)\left|f'(b)\right|+\left(e^{1-t}-1\right)\left|f'(a)\right|\right]dt\right) \\
&= \frac{b-a}{r+1}\left[\left((2r+2)e^{\frac{1}{r+1}}-\frac{3r^2+6r+2re+2e+5}{2r+2}\right)\left|f'(b)\right|\right. \\
&\quad \left.+\left((2r+2)e^{\frac{r}{r+1}}-\frac{2r^2e+2re+5r^2+6r+3}{2r+2}\right)\left|f'(a)\right|\right]
\end{aligned}$$

which completes the proof. □

Remark 2.2. *Under the assumptions of Theorem 2.1 with $r = 1$, we get the conclusion of Theorem 1.3.*

Corollary 2.3. *Under the assumptions of Theorem 2.1 with $r = 0$, we obtain*

$$\left|f(a)-\frac{1}{b-a}\int_a^b f(x)dx\right| \leq \frac{b-a}{2}\left[\left|f'(a)\right|+(2e-5)\left|f'(b)\right|\right].$$

Theorem 2.4. *Let $f : I \to \mathbb{R}$ be a differentiable function on I^o, $a, b \in I^o$ with $a < b$, $q > 1$ assume that $f' \in L[a, b]$. If $|f'|^q$ is an exponential type convex function on $[a, b]$, then the inequality*

$$\left| \frac{f(a) + rf(b)}{r+1} - \frac{1}{b-a} \int_a^b f(x)dx \right| \leq \frac{b-a}{r+1} \left[\frac{r^{p+1}+1}{(r+1)(p+1)} \right]^{\frac{1}{p}} (2(e-2))^{\frac{1}{q}} A^{\frac{1}{q}} \left(|f'(a)|^q, |f'(b)|^q \right)$$

holds for every fixed $r \in [0, 1]$ and $\frac{1}{p} + \frac{1}{q} = 1$.

Proof. From Lemma 1.6 and using Hölder inequality with properties of modulus, we have

$$\begin{aligned} & \left| \frac{f(a) + rf(b)}{r+1} - \frac{1}{b-a} \int_a^b f(x)dx \right| \\ \leq\ & \frac{b-a}{r+1} \int_0^1 |(r+1)t - 1| \left| f'(tb + (1-t)a) \right| dt \\ \leq\ & \frac{b-a}{r+1} \left(\int_0^1 |(r+1)t - 1|^p dt \right)^{\frac{1}{p}} \left(\int_0^1 \left| f'(tb + (1-t)a) \right|^q dt \right)^{\frac{1}{q}} \\ \leq\ & \frac{b-a}{r+1} \left(\int_0^{\frac{1}{r+1}} (1 - (r+1)t)^p dt + \int_{\frac{1}{r+1}}^1 ((r+1)t - 1)^p dt \right)^{\frac{1}{p}} \left(\int_0^1 \left| f'(tb + (1-t)a) \right|^q dt \right)^{\frac{1}{q}}. \end{aligned}$$

Since $|f'|^q$ is exponential type convex function on $[a, b]$, we get

$$\begin{aligned} \left| \frac{f(a) + rf(b)}{r+1} - \frac{1}{b-a} \int_a^b f(x)dx \right| \leq\ & \frac{b-a}{r+1} \left(\int_0^{\frac{1}{r+1}} (1 - (r+1)t)^p dt + \int_{\frac{1}{r+1}}^1 ((r+1)t - 1)^p dt \right)^{\frac{1}{p}} \\ & \times \left(|f'(b)|^q \int_0^1 \left(e^t - 1 \right) dt + |f'(a)|^q \int_0^1 \left(e^{1-t} - 1 \right) dt \right)^{\frac{1}{q}} \\ =\ & \frac{b-a}{r+1} \left[\frac{r^{p+1}+1}{(r+1)(p+1)} \right]^{\frac{1}{p}} (2(e-2))^{\frac{1}{q}} A^{\frac{1}{q}} \left(|f'(a)|^q, |f'(b)|^q \right) \end{aligned}$$

which completes the proof. □

Remark 2.5. *Under the assumptions of Theorem 2.4 with $r = 1$, we get the conclusion of Theorem 1.4.*

Corollary 2.6. *Under the assumptions of Theorem 2.4 with $r = 0$, we obtain*

$$\left| f(a) - \frac{1}{b-a} \int_a^b f(x)dx \right| \leq \frac{b-a}{(p+1)^{\frac{1}{p}}} (2(e-2))^{\frac{1}{q}} A^{\frac{1}{q}} \left(|f'(a)|^q, |f'(b)|^q \right).$$

Theorem 2.7. *Let $f : I \to \mathbb{R}$ be a differentiable function on I^o, $a, b \in I^o$ with $a < b$, $q \geq 1$ and assume that $f' \in L[a, b]$. If $|f'|^q$ is an exponential type convex function on $[a, b]$, then the following inequality holds:*

$$\begin{aligned}&\left|\frac{f(a)+rf(b)}{r+1}-\frac{1}{b-a}\int_a^b f(x)dx\right|\\ \leq\ & \frac{b-a}{r+1}\left(\frac{r^2+1}{2r+2}\right)^{1-\frac{1}{q}}\left[\left(e^{\frac{1}{r+1}}(2r+2)-\frac{3r^2+6r+2re+2e+5}{2r+2}\right)\left|f'(b)\right|^q\right.\\ &\left.+\left(e^{\frac{r}{r+1}}(2r+2)-\frac{5r^2+6r+2er^2+2er+3}{2r+2}\right)\left|f'(a)\right|^q\right]^{\frac{1}{q}}.\end{aligned}$$

Proof. From Lemma 1.6 and using the well known power mean inequality, we have

$$\begin{aligned}&\left|\frac{f(a)+rf(b)}{r+1}-\frac{1}{b-a}\int_a^b f(x)dx\right|\\ \leq\ & \frac{b-a}{r+1}\int_0^1 |(r+1)t-1|\left|f'(ta+(1-t)b)\right|dt\\ \leq\ & \frac{b-a}{r+1}\left(\int_0^1 |(r+1)t-1|dt\right)^{1-\frac{1}{q}}\left(\int_0^1 |(r+1)t-1|\left|f'(ta+(1-t)b)\right|^q dt\right)^{\frac{1}{q}}.\end{aligned}$$

On the other hand, we obtain

$$\int_0^1 |(r+1)t-1|dt=\int_0^{\frac{1}{r+1}}[1-(r+1)t]dt+\int_{\frac{1}{r+1}}^1[(r+1)t-1]dt=\frac{r^2+1}{2r+2}.$$

Since $|f'|^q$ is exponential type convex function on $[a, b]$, we obtain

$$\begin{aligned}&\left|\frac{f(a)+rf(b)}{r+1}-\frac{1}{b-a}\int_a^b f(x)dx\right|\\ \leq\ & \frac{b-a}{r+1}\left(\frac{r^2+1}{2r+2}\right)^{1-\frac{1}{q}}\left(\int_0^1 |(r+1)t-1|\left[\left(e^t-1\right)\left|f'(a)\right|^q+\left(e^{1-t}-1\right)\left|f'(b)\right|^q\right]dt\right)^{\frac{1}{q}}\\ =\ & \frac{b-a}{r+1}\left(\frac{r^2+1}{2r+2}\right)^{1-\frac{1}{q}}\\ &\times\left[\left(e^{\frac{1}{r+1}}(2r+2)-\frac{3r^2+6r+2re+2e+5}{2r+2}\right)\left|f'(b)\right|^q\right.\\ &\left.+\left(e^{\frac{r}{r+1}}(2r+2)-\frac{5r^2+6r+2er^2+2er+3}{2r+2}\right)\left|f'(a)\right|^q\right]^{\frac{1}{q}},\end{aligned}$$

which is required. □

Remark 2.8. *Under the assumptions of Theorem 2.7 with $r = 1$, we get the conclusion of Theorem 1.5.*

Corollary 2.9. *Under the assumptions of Theorem 2.7 with $q = 1$, we obtain the Theorem 2.1.*

Corollary 2.10. *Under the assumptions of Theorem 2.7 with $r = 0$, we have*

$$\left| f(a) - \frac{1}{b-a}\int_a^b f(x)dx \right| \leq \frac{b-a}{2}\left[\left|f'(a)\right|^q + (2e-5)\left|f'(b)\right|^q\right]^{\frac{1}{q}}.$$

3. Hermite-Hadamard Inequalities for Fractional Integrals

Now we give some necessary definitions and mathematical preliminaries of fractional calculus theory which are used throughout this paper.

Definition 3.1. *[13] Let $f \in L_1[a,b]$. The Riemann-Liouville integrals $J_{a^+}^{\alpha}f$ and $J_{b^-}^{\alpha}f$ of order $\alpha > 0$ with $a \geq 0$ are defined by*

$$J_{a^+}^{\alpha}f(x) = \frac{1}{\Gamma(\alpha)}\int_a^x (x-t)^{\alpha-1}f(t)dt, \quad x > a$$

and

$$J_{b^-}^{\alpha}f(x) = \frac{1}{\Gamma(\alpha)}\int_x^b (t-x)^{\alpha-1}f(t)dt, \quad x < b$$

respectively where $\Gamma(\alpha) = \int_0^{\infty} e^{-u}u^{\alpha-1}du$. Here is $J_{a^+}^{0}f(x) = J_{b^-}^{0}f(x) = f(x)$.

In the case of $\alpha = 1$, the fractional integral reduces to the classical integral.

In [14], Özdemir *et al.* proved the following result for fractional integrals. Also, different results have been obtained for different values of r.

Lemma 3.2. *Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I with $a < r$, $a, r \in I$. If $f' \in L[a,r]$, then the following equality for fractional integrals holds:*

$$\begin{aligned} &\frac{f(a)+f(r)}{2} - \frac{\Gamma(\alpha+1)}{2(r-a)^{\alpha}}\left[J_{r^-}^{\alpha}f(a) + J_{a^+}^{\alpha}f(r)\right] \\ &= \frac{r-a}{2}\int_0^1 \left[(1-t)^{\alpha} - t^{\alpha}\right] f'(r+(a-r)t)dt. \end{aligned}$$

Theorem 3.3. *Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I with $a < r$, $a, r \in I$ and $f' \in L[a,r]$. If $\left|f'\right|^q$ is an exponential type convex function on $[a,b]$, then the following inequality for fractional integrals holds:*

$$\begin{aligned} &\left|\frac{f(a)+f(r)}{2} - \frac{\Gamma(\alpha+1)}{2(r-a)^{\alpha}}\left[J_{r^-}^{\alpha}f(a) + J_{a^+}^{\alpha}f(r)\right]\right| \\ &\leq \frac{r-a}{2(\alpha p+1)^{\frac{1}{p}}}\left(2(e-2)\right)^{\frac{1}{q}} A^{\frac{1}{q}}\left(\left|f'(a)\right|^q, \left|f'(b)\right|^q\right). \end{aligned}$$

Proof. From Lemma 3.2 and using Hölder inequality with properties of modulus, we obtain

$$\begin{aligned} &\left|\frac{f(a)+f(r)}{2} - \frac{\Gamma(\alpha+1)}{2(r-a)^{\alpha}}\left[J_{r^-}^{\alpha}f(a) + J_{a^+}^{\alpha}f(r)\right]\right| \\ &\leq \frac{r-a}{2}\int_0^1 \left|(1-t)^{\alpha} - t^{\alpha}\right|\left|f'(r+(a-r)t)\right|dt \\ &\leq \frac{r-a}{2}\left(\int_0^1 \left|(1-t)^{\alpha} - t^{\alpha}\right|^p dt\right)^{\frac{1}{p}}\left(\int_0^1 \left|f'(r+(a-r)t)\right|^q dt\right)^{\frac{1}{q}}. \end{aligned}$$

We know that for $\alpha \in [0,1]$ and $\forall t_1, t_2 \in [0,1]$,

$$\left|t_1^\alpha - t_2^\alpha\right| \le |t_1 - t_2|^\alpha ,$$

therefore

$$\begin{aligned}
\int_0^1 \left|(1-t)^\alpha - t^\alpha\right|^p dt &\le \int_0^1 |1-2t|^{\alpha p} dt \\
&= \int_0^{\frac{1}{2}} [1-2t]^{\alpha p} dt + \int_{\frac{1}{2}}^1 [2t-1]^{\alpha p} dt \\
&= \frac{1}{\alpha p + 1}.
\end{aligned}$$

Also, $\left|f'\right|^q$ is exponential type convex function on $[a,b]$,we have

$$\begin{aligned}
\left|f'(r+(a-r)t)\right|^q &= \left|f'(ta+(1-t)r)\right|^q \\
&\le (e^t - 1)\left|f'(a)\right|^q + (e^{1-t}-1)\left|f'(r)\right|^q , \ t \in (0,1).
\end{aligned}$$

and consequently

$$\begin{aligned}
&\left|\frac{f(a)+f(r)}{2} - \frac{\Gamma(\alpha+1)}{2(r-a)^\alpha}\left[J_{r^-}^\alpha f(a) + J_{a^+}^\alpha f(r)\right]\right| \\
\le\ & \frac{r-a}{2(\alpha p+1)^{\frac{1}{p}}} \left(2\left(e-2\right)\right)^{\frac{1}{q}} A^{\frac{1}{q}} \left(\left|f'\left(a\right)\right|^q, \left|f'\left(b\right)\right|^q\right)
\end{aligned}$$

which completes the proof. □

Remark 3.4. *If in Theorem 3.3, we choose $\alpha = 1$ and $r = b$, then we obtain Theorem 1.4.*

4. Conclusion

In this paper, we obtained new general integral inequalities for exponential type convex functions. We proved the Hermite-Hadamard type integral inequalities and obtained new theorems with the Hölder inequality. With this definition, many new integral inequalities can be obtained. Also, by using Hölder-İşcan (see reference [15]) inequality and different Lemmas, new results can be obtained for exponential type convex functions.

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