International Journal of Thermodynamics (IJoT)
ISSN 1301-9724 / e-ISSN 2146-1511
https://dergipark.org.tr/en/pub/ijot

Vol. 26 (No. 2), 046-055, 2023
doi: 10.5541/ijot.1170364
Published online: June 1, 2023

Research Article

Thermally-Induced Stresses in a Pre-Buckling State of a Circular Plate within the Fractional-Order Framework

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Received 2 September 2022, Revised 17 January 2023, Accepted 30 January 2023

Abstract

This paper considers a transient thermoelastic problem in an isotropic homogeneous elastic thin circular plate with clamped edges subjected to thermal load within the fractional-order theory framework. The prescribed ramp-type surface temperature is on the plate's top face, while the bottom face is kept at zero. The three-dimensional heat conduction equation is solved using a Laplace transformation and the classical solution method. The Gaver–Stehfest approach was used to invert Laplace domain outcomes. The thermal moment is derived based on temperature change, and its bending stresses are obtained using the resultant moment and resultant forces per unit length. The results are illustrated by numerical calculations considering the material to be an Aluminum-like medium, and corresponding graphs are plotted.

Keywords: *Fractional-order derivatives; fractional calculus; non-Fourier heat conduction; circular plate; thermal deflection; thermal deflection thermal stress; integral transform.*

1. Introduction

Fractional calculus has been applied in many disciplines in recent times, such as electromagnetism, control engineering, signal processing, chemistry, astrophysics, quantum mechanics, nuclear physics, quantum field theory, etc. A few highly cited books explaining the principle of fractional calculus and several alternate definitions of fractional derivatives have been Oldham and Spanier [1], Miller and Ross [2], Samko et al. [3], Podlubny [4], Hilfer [5] and Harmann [6]. Many analytical studies concerning thermoelasticity within the fractional-order theory framework have also been reported, summarized in the trailing portion. Povstenko [7-11] focused on heat conduction with time and space fractional derivatives and obtained the theory of thermal stresses using the Laplace transform and a direct approach. Youssef [12-18] studied the fractional-order generalized thermoelasticity using the Laplace transform and different techniques. Ezzat and El-Karamany [19-22] constructed a few mathematical models of time-fractional order in the context of the generalized theory of thermoelasticity, thermo-piezoelasticity, and thermo-viscoelasticity. Sherief and Abd El-Latief [23-25] developed thermoelasticity methods using fractional calculus and applied them to a one-dimensional thermal shock problem for a half-space.

Recently, Sur and Kanoria [26] developed a new theory of two-temperature generalized thermoelasticity in the context of heat conduction with fractional orders using the unified parameters in the form of a vector-matrix differential equation. Similarly, Bhattacharya and Kanoria [27] obtained the solution of the two-temperature thermoelastic-diffusion interaction inside a spherical shell in fractional order generalized thermoelasticity using a direct approach. Zenkour and Abouelregal [28] determined the conductive and thermodynamic temperature for an infinite isotropic elastic body with a spherical cavity using Caputo's time-fractional derivative. Bachher [29] discussed the deformation due to periodic heat sources in a temperature-dependent porous material with a time-fractional heat conduction law. Santra et al. [30] employed eigenvalue approaches for obtaining a half-space solution within fractional order generalized thermoelasticity (Green–Lindsay) theory. Gupta and Das [31] used the eigenvalue approach to get a general solution scheme for the deformation of an unbounded transversely isotropic medium within fractional order generalized thermoelasticity with an instantaneous heat source. Bachher and Sarkar [32] investigated the theory of generalized thermoelasticity based on the heat conduction equation with the Caputo time-fractional derivative to study the magneto-thermoelastic response of a homogeneous isotropic two-dimensional rotating elastic half-space solid with an eigenvalue approach technique. Abbas [33] obtained the temperature, displacement, and stresses due to thermal shock loading on the inner surface cavity in an infinite medium with a cylindrical cavity in the fractional-order generalized thermoelasticity theory using an eigenvalue approach.

Similarly, Abbas [34] studied the effect of fractional order derivative on a two-dimensional problem due to thermal shock with weak, normal and strong conductivity in Green and Naghdi of type III model (GN III model) using fractional-order derivative with eigenvalues approach. Lata [35] investigated the thermomechanical interactions in the fractional theory for a thick circular using the Hankel transform technique and a direct method without potential functions. Mittal and Kulkarni [36] proposed a fractional heat conduction model to investigate the thermal variations within the bounded spherical region in the context of the generalized two-temperature theory of fractional thermoelasticity. Bhoyar et al. [37] performed the thermoelastic analysis of an isotropic homogeneous multi-stacked elliptical in the context of the time-fractional derivative using a quasi-orthogonality relationship by modifying Vodicka's method and the Laplace transformation. Using von Mises' yield criterion, Haskul [38,39] has developed analytical solutions for the stresses and displacements of a functionally graded, cylindrically curved beam subjected to a radial heat load. Haskul et al. [40,41] studied the elastic stress response of a thick-walled, cylindrically curved panel subjected to a radial temperature gradient under the assumption of generalized plane strain according to both Tresca and von Mises yield criteria.

According to the review of the relevant literature, only a limited number of research initiatives have explored the thermal stress analysis in an isotropic homogeneous elastic thin circular plate with clamped edges that has been subjected to temperature or mechanical load. Therefore, the purpose of this study was to illustrate the thermoelastic induced stress analysis of a thin circular plate in a pre-buckling condition assumption utilizing the classical approach. The plate was also subjected to a ramp-type sectional heat supply inside the fractional-order framework. The novelty of this work lies in the fact that it employs a Laplace transformation and a classical solution method, both of which have not been carried out by any other researcher up to this point. This enables the authors to determine the influence of different fractional orders on a three-dimensional heat conduction equation.

This article is organized as follows: Section 2 presents the prerequisites of the time-fractional equation. Section 3 presents the problem's mathematical statement within the fractional-order theory. In Section 4, solutions of fractional-order equations are expressed in terms of the Bessels function in the Laplace domain. Section 5 is devoted to estimating solutions of numerical inversion of the Laplace transform, and its convergence is discussed. The numerical results, discussion, and remarks are put forward in Section 6. Finally, conclusions are drawn in Section 7.

2. Prerequisites Of Time Fractional Equation

The investigation of strains brought on by the temperature field that is derived from the parabolic heat conduction equation is the focus of the classical theory of thermoelasticity. The Fourier law serves as the foundation for the traditional idea of heat conduction.

$$q = -k\,\mathrm{grad}\,T, \tag{1}$$

which establishing a connection between the temperature gradient T and the heat flux vector q. In non-classical theories, the Fourier law and the equation for heat conduction are supplanted by more generic equations. In the theory of heat conduction proposed by Gurtin and Pipkin [42], the Fourier law was generalized to time-non-local dependence between the heat flux vector and the temperature gradient, which resulted in an integrodifferential heat conduction equation. This was accomplished by applying the Fourier law to the relationship between the temperature gradient and the heat flux vector. Chen and Gurtin are the ones that came up with the thermoelasticity theory and based it on this equation [43]. The constitutive equation for the heat flux that was proposed by Cattaneo [44,45] and Vernotte [46] can also be rewritten in a non-local form with the 'short-tail' exponential time-non-local kernel. This kernel allows for the equation to be rewritten in a non-local form. The concept of generalized thermoelasticity was first proposed by Kaliski [47] and Lord and Shulman [48], who based their work on the findings of Cattaneo and Vernotte. Within the framework of Green and Naghdi's [49], proposed theory of heat conduction, the relevant generalization of the Fourier law is discussed.

$$q = -k\int_0^t \mathrm{grad}\,T(\tau)\,d\tau, \tag{2}$$

with k being the thermal conductivity. This can lead to getting the wave equation for temperature as well as thermoelasticity with memory

$$\frac{\partial T}{\partial t} = \kappa\int_0^t K(t-\tau)\,\Delta T(\tau)\,d\tau, \tag{3}$$

with κ being the thermal diffusivity and $K(t-\tau)$ is memory kernel. The time-non-local dependences between the heat flux vector and the temperature gradient, along with the "long-tail" power kernel, expressed in terms of fractional integrals and derivatives, and the time-fractional heat conduction equation [7-11] that results is given as

$$\frac{\partial^\alpha T}{\partial t^\alpha} = \kappa\,\Delta T,\ 0 < \alpha \le 2. \tag{4}$$

The uncoupled theory is the only one that has been taken into account by us. There is no consideration given to the impact that deformation has on the thermal state of a solid. The findings of Eq. (4) are presented for obtaining the thermally-induced stresses in a pre-buckling state of a thin circular plate in terms of heat conduction and thermoelasticity.

3. Formulation Of The Fractional Heat Conduction

Consider a circular plate of thickness h occupying the space $D: 0 < r \le a,\ 0 \le \theta \le \theta_0 < 2\pi,\ -h/2 \le z \le h/2,$ in the cylindrical coordinate system (r,θ,z), as shown in Figure 1. The plate is kept at zero initial temperature. The annular region $D_1 : r_0 < r < a,$ of the upper face is subjected to temperature distribution as follows

$$\begin{aligned} T\big|_{z=h/2} &= \frac{T_0}{t_0}[H(r-r_0)-H(r-a)]t \ \text{ for } 0 \le t \le t_0, \\ &= T_0\ [H(r-r_0)-H(r-a)] \ \text{ for } t \ge t_0. \end{aligned} \tag{5}$$

in which $f(r) = H(r-r_0)-H(r-a)$ is the difference in the Heaviside function, t is time, T_0 defines the reference temperature distribution, which does not produce stress or strain in the plate, and t_0 is a fixed ramp parameter value, respectively.

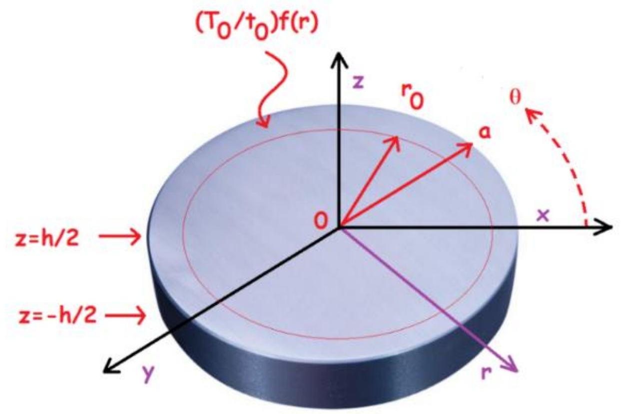

Figure 1. The geometry of a circular plate.

3.1 Transient Heat Conduction Formulation:

The heat conduction equation is

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\kappa}\frac{\partial^\alpha T}{\partial t^\alpha}, \tag{6}$$

subjected to the boundary and initial conditions

$$T\big|_{r=0}\neq\infty, \left.\frac{\partial T}{\partial r}\right|_{r=a}=0, \tag{7}$$

$$T\big|_{\theta=0}=T\big|_{\theta=\theta_0}=0, \tag{8}$$

$$T\big|_{z=-h/2}=0, T\big|_{z=h/2}=\begin{cases}(T_0/t_0)\,f(r)t & \text{for } 0\le t\le t_0\\ T_0\,f(r) & \text{for } t\ge t_0,\end{cases} \tag{9}$$

$$T\big|_{t=0}=0, \left.\frac{\partial T}{\partial t}\right|_{t=0}=0, \tag{10}$$

in which

$$f(r)=\begin{cases}1, \text{ for } r_0<r<a\\ 0, \text{ for } 0<r<r_0\end{cases}, \tag{11}$$

with $T=T(r,\theta,z,t)$ is the temperature distribution at a point (r,θ,z) at a time t, $\kappa=k/c_v\rho$ is the coefficient of thermal diffusivity, c_v is the calorific capacity, ρ is the material density, k is the coefficient of thermal conductivity, and $\partial^\alpha T/\partial t^\alpha$ is the Caputo fractional derivative, respectively.

The temperature change from the initial temperature is given

$$\tau=T-T_i. \tag{12}$$

The Caputo fractional derivative [50,51] in Eq. (5) as

$$\frac{d^\alpha f(t)}{dt^\alpha}=\begin{cases}\dfrac{1}{\Gamma(n-\alpha)}\displaystyle\int_0^t (t-\tau)^{n-\alpha-1}\frac{d^n f(\tau)}{d\tau^n}\,d\tau, & n-1<\alpha<n,\\[2ex] \dfrac{d^n f(\tau)}{dt^n}, & \alpha=n,\end{cases} \tag{13}$$

with the following Laplace transform rule

$$L\left\{\frac{d^\alpha f(t)}{dt^\alpha}\right\}=s^\alpha L\{f(t)\}-\sum_{k=0}^{n-1} f^{(k)}(0^+)\,s^{\alpha-1-k}, \tag{14}$$

in which s is the transform parameter, $n-1<\alpha<n$, $n\in N=\{1,2.....\}$.

3.2 Thermally-Induced Bending Stress Formulation

The stress components [52] along the neutral plane

$$\begin{aligned}
\sigma_{rr}&=\frac{E}{1-\upsilon^2}\left\{\frac{\partial u_r}{\partial r}+\upsilon\left(\frac{u_r}{r}+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}\right)\right.\\
&\quad\left.-z\left[\frac{\partial^2 w}{\partial r^2}+\upsilon\left(\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)\right]\right\}-\frac{E\alpha_t\tau}{1-\upsilon},\\
\sigma_{\theta\theta}&=\frac{E}{1-\upsilon^2}\left\{\upsilon\frac{\partial u_r}{\partial r}+\left(\frac{u_r}{r}+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}\right)\right.\\
&\quad\left.-z\left[\upsilon\frac{\partial^2 w}{\partial r^2}+\left(\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)\right]\right\}-\frac{E\alpha_t\tau}{1-\upsilon},\\
\sigma_{r\theta}&=\frac{E}{2(1+\upsilon)}\left\{\frac{1}{r}\frac{\partial u_r}{\partial \theta}+r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right)-2z\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial w}{\partial \theta}\right)\right\},
\end{aligned} \tag{15}$$

in which u_r, u_θ and w are displacement functions along r, θ, and z directions at the neutral plane of the thin circular plate with its thickness, E is Young's modulus, α_t is the coefficient of thermal expansion in the thickness direction, and υ represent Poisson's ratio, respectively.

The resultant forces $(N_{ij}, i,j=r,\theta)$ and the resultant moments $(M_{ij}, i,j=r,\theta)$ per unit length of the plate is written as [52]

$$\begin{aligned}
N_r&=\int_{-h/2}^{h/2}\sigma_{rr}\,dz,\ N_\theta=\int_{-h/2}^{h/2}\sigma_{\theta\theta}\,dz,\\
N_{r\theta}&=\int_{-h/2}^{h/2}\sigma_{r\theta}\,dz, M_r=\int_{-h/2}^{h/2}z\,\sigma_{rr}\,dz,\\
M_\theta&=\int_{-h/2}^{h/2}z\,\sigma_{\theta\theta}\,dz,\ M_{r\theta}=-\int_{-h/2}^{h/2}z\,\sigma_{r\theta}\,dz.
\end{aligned} \tag{16}$$

Substituting Eqs. (15) into (16), one obtains the resultant forces as

$$\begin{aligned}
N_r&=\frac{Eh}{1-\upsilon^2}\left[\frac{\partial u_r}{\partial r}+\upsilon\left(\frac{u_r}{r}+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}\right)\right]-\frac{1}{1-\upsilon}N_T,\\
N_\theta&=\frac{Eh}{1-\upsilon^2}\left[\upsilon\frac{\partial u_r}{\partial r}+\left(\frac{u_r}{r}+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}\right)\right]-\frac{1}{1-\upsilon}N_T,\\
N_{r\theta}&=\frac{Eh}{2(1+\upsilon)}\left[\frac{1}{r}\frac{\partial u_r}{\partial \theta}+r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right)\right],
\end{aligned} \tag{17}$$

and the resultant moments [52] as

$$M_r = -D\left[\frac{\partial^2 w}{\partial r^2} + \upsilon\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)\right] - \frac{1}{1-\upsilon}M_T,$$
$$M_\theta = -D\left[\upsilon\frac{\partial^2 w}{\partial r^2} + \left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)\right] - \frac{1}{1-\upsilon}M_T, \quad (18)$$
$$M_{r\theta} = (1-\upsilon)D\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial w}{\partial \theta}\right).$$

in which the flexure rigidity is given as $D = E\ell^3/12(1-\upsilon^2)$, and the thermally induced resultant force N_T and the thermally induced resultant moment M_T are defined by [53]

$$N_T = \alpha_t E\int_{-h/2}^{h/2}\tau(r,\theta,z,t)dz,$$
$$M_T = \alpha_t E\int_{-h/2}^{h/2} z\,\tau(r,\theta,z,t)dz. \quad (19)$$

Now taking into account Eq. (18), the differential equation for deflection is given as [53]

$$\nabla^2\nabla^2 w = -\frac{1}{(1-\upsilon)D}\nabla^2 M_T, \quad (20)$$

subjected to the boundary conditions for the thermally induced bending of the plate are [54]

$$w\big|_{r=a} = 0, \frac{\partial w}{\partial r}\bigg|_{r=a} = 0, \quad (21)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}. \quad (22)$$

In order to evaluate $N_{ij}\,(i, j = r,\theta)$, one can introduce [54] a stress function $F(r,\theta,t)$ as

$$N_r = \frac{1}{r}\frac{\partial F}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F}{\partial \theta^2},$$
$$N_\theta = \frac{\partial^2 F}{\partial r^2}, N_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial F}{\partial \theta}\right). \quad (23)$$

Now taking into account Eq. (23) and the strain components, one obtains a relation [54]

$$\nabla^2\nabla^2 F = -\nabla^2 N_T. \quad (24)$$

The thermal bending stress components [54] taken as

$$\sigma_{rr} = \frac{1}{h}N_r + \frac{12z}{h^3}M_r + \frac{1}{1-\upsilon}\left(\frac{1}{h}N_T + \frac{12z}{h^3}M_T - \alpha E\tau\right),$$
$$\sigma_{\theta\theta} = \frac{1}{h}N_\theta + \frac{12z}{h^3}M_\theta + \frac{1}{1-\upsilon}\left(\frac{1}{h}N_T + \frac{12z}{h^3}M_T - \alpha E\tau\right), \quad (25)$$
$$\sigma_{r\theta} = \frac{1}{h}N_{r\theta} - \frac{12z}{h^3}M_{r\theta}.$$

The Eqs. (5) to (25) constitute the mathematical formulation of the problem.

4. Solution For The Problem

4.1 Transient Heat Conduction Analysis

Applying the Laplace transform $\bar{f}(s) = \int_0^\infty \exp(-st)f(t)dt$ to the Eqs. (6)-(10), one obtains

$$\frac{\partial^2\bar{T}}{\partial r^2} + \frac{1}{r}\frac{\partial\bar{T}}{\partial r} + \frac{1}{r^2}\frac{\partial^2\bar{T}}{\partial\theta^2} + \frac{\partial^2\bar{T}}{\partial z^2} = \frac{s^\alpha}{\kappa}\bar{T}, \quad (26)$$

$$\bar{T}\big|_{r=0} \neq \infty, \frac{\partial\bar{T}}{\partial r}\bigg|_{r=a} = 0, \quad (27)$$

$$\bar{T}\big|_{\theta=0} = \bar{T}\big|_{\theta=\theta_0} = 0, \quad (28)$$

$$\bar{T}\big|_{z=-h/2} = 0, \bar{T}\big|_{z=h/2} = \frac{T_0}{t_0}\left(\frac{1-e^{-st_0}}{s^2}\right)f(r)\sin m\theta, \quad (29)$$

in which f_t be a given function defined for all $t \geq 0$, $\bar{f}(s)$ is the transformed generating function of the determining function $f(t)$, $\bar{T}$ is the transformed function of T, and s is the transformed Laplace parameter, respectively.

Now, we assume the temperature distribution is given by

$$\bar{T}(r,\theta,z,s) = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty}A_n\sin m\theta\, J_0(\alpha_n r)\sinh\left\{\gamma_n\left(z+\frac{h}{2}\right)\right\}. \quad (30)$$

The boundary conditions of Eq. (28), taking into account Eq. (27), are self-evidently satisfied in Eq. (26). The first equation of boundary condition (29) at $z = h/2$ is satisfied by Eq. (26). From the second equation of boundary condition (27) on $r = a$, we have $\alpha_n\,(n = 0,1,2...\infty)$ as the roots of the transcendental equation $J_1(\alpha_n a) = 0$.

To satisfy the second equation of boundary condition (29), assume the Fourier-Bessel series

$$f(r) = \sum_{n=1}^{\infty}B_n J_0(\alpha_n r). \quad (31)$$

Then from the theory of the Bessel function

$$B_n\int_0^a r[J_0(\alpha_n r)]^2 dr = \int_0^a f(r)\,rJ_0(\alpha_n r)dr. \quad (32)$$

Using Eqs. (11) into (32), one obtains

$$B_n = \frac{-2r_0 J_1(\alpha_n r_0)}{\alpha_n a^2[J_0(\alpha_n a)]^2}. \quad (33)$$

Substituting Eqs. (30) into (26), one obtains

$$\gamma_n^2 = (s^\alpha/\kappa) + \beta_{mn}^2,\ \beta_{mn}^2 = (m/a^2) + \alpha_n^2. \quad (34)$$

Substituting Eq. (33) into the boundary condition (29) for $z = h/2$, one obtains

$$A_n = \frac{-2T_0 r_0 J_1(\alpha_n r_0)}{\alpha_n a^2 t_0 [J_0(\alpha_n a)]^2 \sinh\{\gamma_n h\}} \left(\frac{1-e^{-st_0}}{s^2} \right). \tag{35}$$

By replacing the values of Eqs. (35) into (30), one obtains

$$\begin{aligned} \bar{T}(r,\theta,z,s) &= \frac{2T_0}{a^2 t_0} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \left(\frac{-r_0 J_1(\alpha_n r_0)}{\alpha_n [J_0(\alpha_n a)]^2 \sinh\{\gamma_n h\}} \right) \\ &\times J_0(\alpha_n r) \sin m\theta \\ &\times \left\langle \left(\frac{1-e^{-st_0}}{s^2} \right) \sinh\left[\left(\frac{s^\alpha}{\kappa} + \beta_{mn}^2 \right)^{1/2} \left(z + \frac{h}{2} \right) \right] \right\rangle. \end{aligned} \tag{36}$$

At $t=0$, initial temperature condition $T_i = 0$, then Eq. (12) in the Laplace domain is shown as

$$\bar{\tau} = \bar{\tau}(r,\theta,z,s) = \bar{T}(r,\theta,z,s) \, . \tag{37}$$

4.2 Thermoelastic Solution

Using Eq. (36) into transformed Eq. (19), the resultant force in the Laplace domain as

$$\begin{aligned} \bar{N}_T &= \frac{2\alpha_t E T_0}{a^2 t_0} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \left(\frac{-r_0 J_1(\alpha_n r_0)}{\alpha_n [J_0(\alpha_n a)]^2 \sinh\{\gamma_n h\}} \right) \\ &\times J_0(\alpha_n r) \sin m\theta \, \phi_{mn}(s), \end{aligned} \tag{38}$$

in which

$$\begin{aligned} \phi_{mn}(s) &= \left(\frac{1-e^{-st_0}}{s^2} \right) \left(\frac{s^\alpha}{\kappa} + \beta_{mn}^2 \right)^{-1/2} \\ &\times \left\{ -1 + \cosh\left[\left(\frac{s^\alpha}{\kappa} + \beta_{mn}^2 \right)^{1/2} h \right] \right\}, \end{aligned} \tag{39}$$

and the resultant moment in the Laplace domain

$$\begin{aligned} \bar{M}_T &= \frac{\alpha_t E T_0}{a^2 t_0} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \left(\frac{-r_0 J_1(\alpha_n r_0)}{\alpha_n [J_0(\alpha_n a)]^2 \sinh\{\gamma_n h\}} \right) \\ &\times J_0(\alpha_n r) \sin m\theta \, \varphi_{mn}(s), \end{aligned} \tag{40}$$

in which

$$\begin{aligned} \varphi_{mn}(s) &= \left(\frac{1-e^{-st_0}}{s^2} \right) \left(\frac{s^\alpha}{\kappa} + \beta_{mn}^2 \right)^{-1} \left\{ \left[\left(\frac{s^\alpha}{\kappa} + \beta_{mn}^2 \right)^{1/2} h \right] \right. \\ &\times \left\{ 1 + \cosh\left[\left(\frac{s^\alpha}{\kappa} + \beta_{mn}^2 \right)^{1/2} h \right] \right\} \\ &\left. - 2\sinh\left[\left(\frac{s^\alpha}{\kappa} + \beta_{mn}^2 \right)^{1/2} h \right] \right\}. \end{aligned} \tag{41}$$

As a solution to transformed Eq. (20) satisfying Eq. (21), we assume $\bar{w}(r,\theta,s)$ as

$$\begin{aligned} \bar{w} &= \frac{\alpha_t E T_0}{a^2 t_0} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} C_n \left(\frac{-r_0 J_1(\alpha_n r_0)}{\alpha_n [J_0(\alpha_n a)]^2 \sinh\{\gamma_n h\}} \right) \varphi_{mn}(s) \\ &\times [J_0(\alpha_n r) - J_0(\alpha_n a)] \sin m\theta. \end{aligned} \tag{42}$$

From Eqs. (20), (35) and (42), and integrating with respect to r from limits 0 to a, one obtains $C_n = a/[(1-\upsilon)D\alpha_n^2]$. Using Eqs. (19) and (21), the resultant moments as

$$\begin{aligned} \bar{M}_r &= \frac{\alpha_t E T_0}{a^2 t_0} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \left(\frac{-r_0 J_1(\alpha_n r_0)}{\alpha_n [J_0(\alpha_n a)]^2 \sinh\{\gamma_n h\}} \right) \\ &\times \left\langle \frac{C_n D}{r^2} \left\{ -m^2 \upsilon J_0(\alpha_n a) + [(\alpha_n r)^2 + m^2 \upsilon] J_0(\alpha_n r) \right. \right. \\ &\left. \left. - (\alpha_n r)(2+\upsilon) J_1(\alpha_n r) \right\} - \frac{J_0(\alpha_n r)}{1-\upsilon} \right\rangle \sin m\theta \, \varphi_{mn}(s), \end{aligned} \tag{43}$$

$$\begin{aligned} \bar{M}_\theta &= \frac{\alpha_t E T_0}{a^2 t_0} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \left(\frac{-r_0 J_1(\alpha_n r_0)}{\alpha_n [J_0(\alpha_n a)]^2 \sinh\{\gamma_n h\}} \right) \\ &\times \left\langle \frac{C_n D}{r^2} \left\{ m^2 J_0(\alpha_n a) - [(\alpha_n r)^2 \upsilon + m^2] J_0(\alpha_n r) \right. \right. \\ &\left. \left. + (\alpha_n r)(1+2\upsilon) J_1(\alpha_n r) \right\} - \frac{J_0(\alpha_n r)}{1-\upsilon} \right\rangle \sin m\theta \, \varphi_{mn}(s), \end{aligned} \tag{44}$$

$$\begin{aligned} \bar{M}_{r\theta} &= (1-\upsilon) \frac{\alpha_t E T_0}{a^2 t_0} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \left(\frac{-r_0 J_1(\alpha_n r_0)}{\alpha_n [J_0(\alpha_n a)]^2 \sinh\{\gamma_n h\}} \right) \\ &\times \left\langle \frac{C_n D}{r^2} m [J_0(\alpha_n a) - J_0(\alpha_n r) + (\alpha_n r) J_1(\alpha_n r)] \right\rangle \\ &\times \cos m\theta \, \varphi_{mn}(s). \end{aligned} \tag{45}$$

Now we assume $\bar{F}(r,\theta,t)$ such that it satisfies Eq. (24), as

$$\begin{aligned} \bar{F}(r,\theta,t) &= \frac{2\alpha_t E T_0}{a^2 t_0} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} D_n \left(\frac{-r_0 J_1(\alpha_n r_0)}{\alpha_n [J_0(\alpha_n a)]^2 \sinh\{\gamma_n h\}} \right) \\ &\times J_0(\alpha_n r) \sin m\theta \, \phi_{mn}(s), \end{aligned} \tag{46}$$

in which the constant can be obtained using transformed Eq. (24) and integrating with respect to r from limits 0 to a, as

$$D_n = a^2 / \{3m^2 \alpha_n^2 [J_0(\alpha_n a) - (\pi/2) J_0(\alpha_n a) H_1(\alpha_n a)]\}, \tag{47}$$

where $H_1(\,)$ denotes the Struve function of the first kind. Substituting Eq. (46) into transformed Eq. (23), one gets the resultant forces as

$$\begin{aligned} \bar{N}_r &= \frac{2\alpha_t E T_0}{a^2 t_0} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \frac{D_n}{r^2} \left(\frac{-r_0 J_1(\alpha_n r_0)}{\alpha_n [J_0(\alpha_n a)]^2 \sinh\{\gamma_n h\}} \right) \\ &\times \{-m^2 J_0(\alpha_n r) + (\alpha_n r) J_1(\alpha_n r)\} \sin m\theta \, \phi_{mn}(s), \end{aligned} \tag{48}$$

$$\begin{aligned} \bar{N}_\theta &= \frac{2\alpha_t E T_0}{a^2 t_0} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} D_n \left(\frac{-r_0 J_1(\alpha_n r_0)}{\alpha_n [J_0(\alpha_n a)]^2 \sinh\{\gamma_n h\}} \right) \\ &\times \{-m^2 J_0(\alpha_n r)\} \sin m\theta \, \phi_{mn}(s), \end{aligned} \tag{49}$$

$$\bar{N}_{r\theta} = \frac{2\alpha_t E T_0}{a^2 t_0}\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\frac{mD_n}{r^2}\left(\frac{-r_0 J_1(\alpha_n r_0)}{\alpha_n [J_0(\alpha_n a)]^2 \sinh\{\gamma_n h\}}\right) \times\{(\alpha_n r)J_1(\alpha_n r) - J_0(\alpha_n r)\}\cos m\theta\, \phi_{mn}(s). \tag{50}$$

Now using Eqs. (36)-(40), (43)-(45) and (48)-(50) in (25), one obtains the expressions for thermal bending stresses $\bar{\sigma}_{rr}, \bar{\sigma}_{\theta\theta}$ and $\bar{\sigma}_{r\theta}$, and they are rather lengthy. Subsequently, the same has been omitted here for the sake of brevity but has been considered the stress equations during graphical discussion using MATHEMATICA software.

5. Inversion Of Laplace Transforms

Now to obtain the original solution of Eqs. (36)-(50), Laplace inversion theorem is to be used, which can be written as

$$f(t) = \frac{1}{2\pi i}\lim_{\beta\to\infty}\int_{\gamma-i\beta}^{\gamma+i\beta}\exp(st)\,\bar{f}(s)ds, \tag{51}$$

along any line $R(s)=\gamma \ge c$ converges to a function $f(t)$ which is independent of γ and whose Laplace transform is $\bar{f}(s)$, $R(s) < c$. The direct integration of Eq. (36) is usually complicated and not analytically feasible in certain situations. The inverse of the Laplace transform is thus obtained by using the Gaver–Stehfast algorithm [55-57], with the aim to approximate $f(t)$ by a sequence of functions as

$$f(t) \approx f_n(t) = \left[\frac{1}{t}\text{In}(2)\right]\sum_{n=1}^{L} a_n F\left[\frac{n}{t}\text{In}(2)\right],\ n\ge 1,\ t>0, \tag{52}$$

$$a_n = (-1)^{n+L/2}\sum_{k=[(n+1)/2]}^{\min(n,L/2)}\frac{k^{L/2}(2k)!}{(L/2-k)!k!(k-1)!(n-k)!(2k-n)!}. \quad n\ge 1,\ 1\le L\le n \tag{53}$$

where $F[.]$ is the Laplace transform of $f(t)$ and coefficients a_n depend only on the number of expansion terms n. It is also noted that the approximations $f_n(t)$ converge to $f(t)$ [58] if a function f is continuous at t and of bounded variation in a neighbourhood of t.

6. Numerical Results, Discussion, And Remarks

The numerical computations have been carried out for an Aluminum plate with thermo-mechanical properties, which is shown in Table 1.

Table 1. Thermo-mechanical properties: Aluminum.

Dimension	Value
Modulus of Elasticity, E	70 GPa
Poisson's ratio	0.35
Thermal Expansion Coefficient, α_t	$23\times10^{-6}/^0$C
Thermal diffusivity, κ	84.18×10^{-6} m^{2}s^{-1}
Thermal conductivity, k	204.2 Wm^{-1}K^{-1}

The 3.831, 7.015, 10.173, 13.323, 16.470, 19.615, 22.760, 25.903, 29.046, 32.189, 35.332, 38.474, 41.617, 44.759, 47.901, are the positive and real roots of the transcendental equation $J_1(\alpha_n a) = 0$. The physical parameter for the sector plate as $a=1$, $h=0.08$, $t_0=0.8$ and T_0=150. For the interest of simplicity, we introduce the following dimensionless values

$$\bar{r} = r/a,\ \bar{z} = [z-(-h/2)]/a,\ \bar{\tau} = \kappa t/a^2,\ \Theta = \tau/T_0,$$
$$\bar{w} = w/\alpha_t T_0 a, \bar{\sigma}_{ij} = \sigma_{ij}/E\alpha_t T_0\ (i,j=r,\theta), \tag{54}$$
$$\bar{F} = F/E\alpha_t T_0 a^2, \bar{N}_{ij} = N_{ij}/Ea^3, \bar{M}_{ij} = M_{ij}/Ea^3$$

Figures (2)-(4) illustrate the numerical results for the circular plate's temperature distribution, using the thermal boundary conditions subjected to ramp-type sectional thermal load on the upper face while keeping the lower face at zero temperature.

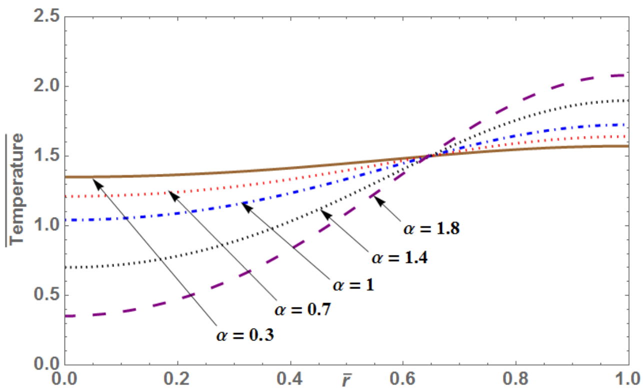

Figure 2. Temperatures distribution along the radial axis at different values of α.

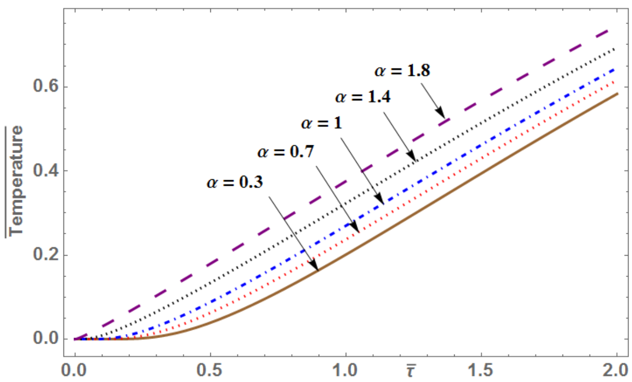

Figure 3. Temperatures distribution as time proceeds at various values of α.

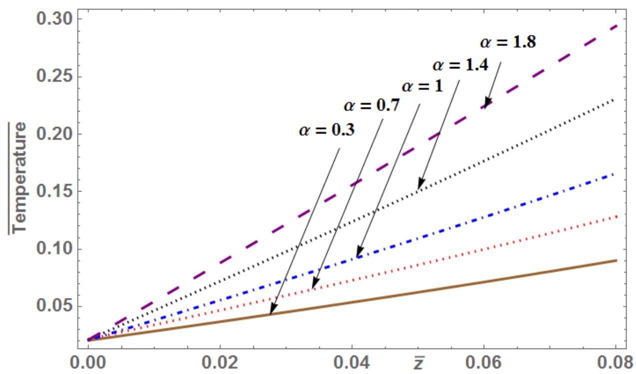

Figure 4. Temperatures distribution along thickness direction at various values of α.

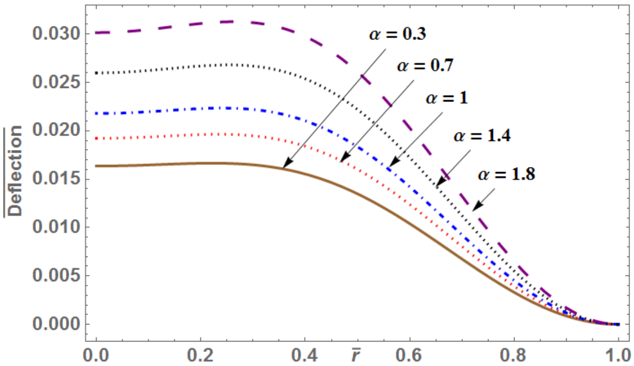

Figure 5. Thermal deflection along the radial axis at different values of α.

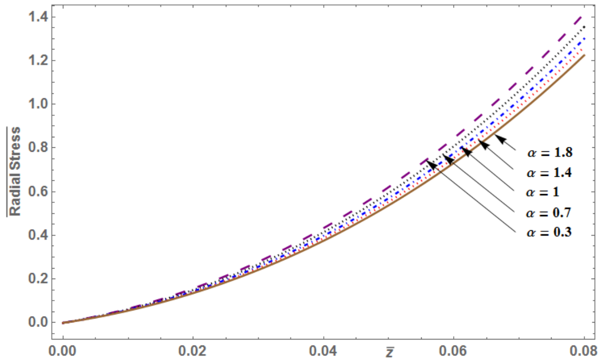

Figure 9. Variation in radial stress along thickness direction at different values of α.

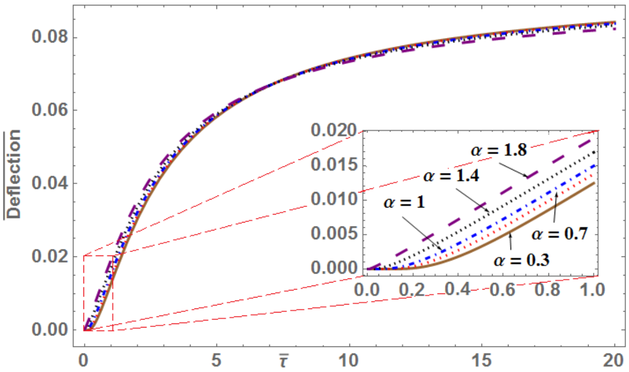

Figure 6. Deflection distribution along the time at various values of α.

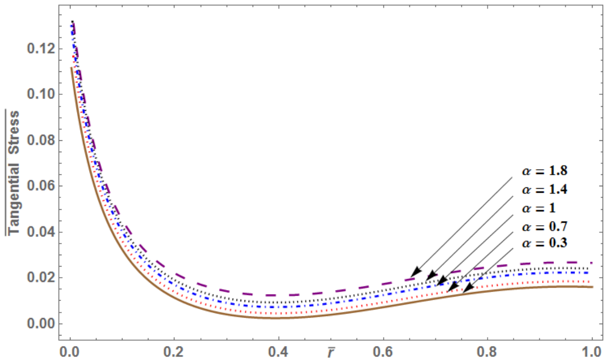

Figure 10. Thermal tangential stress along the radial axis at different values of α.

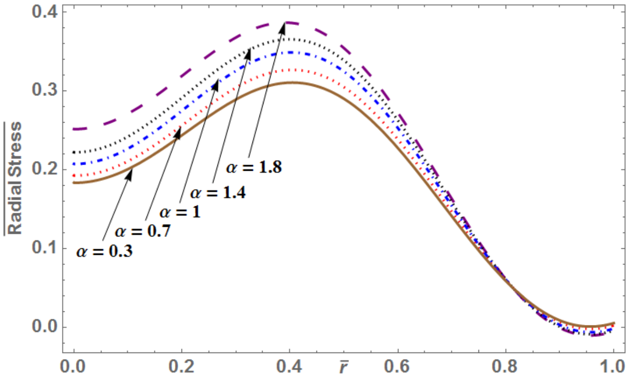

Figure 7. Thermal radial stress along the radial axis at different values of α.

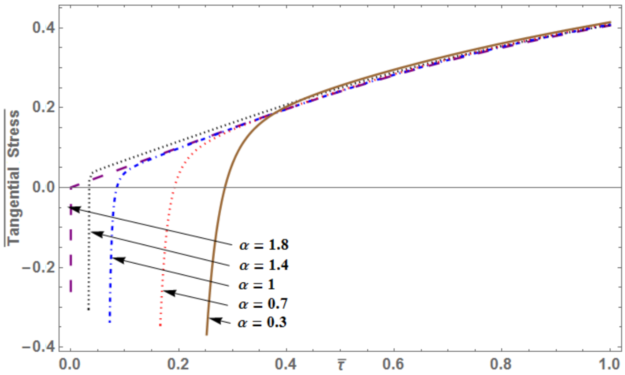

Figure 11. Variation in tangential stress along the time at different values of α.

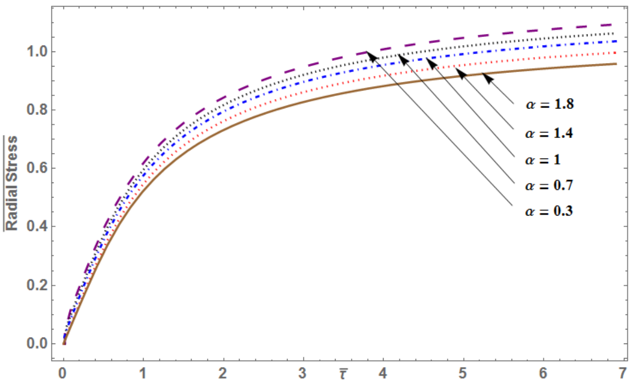

Figure 8. Variation in radial stress along the time at different values of α.

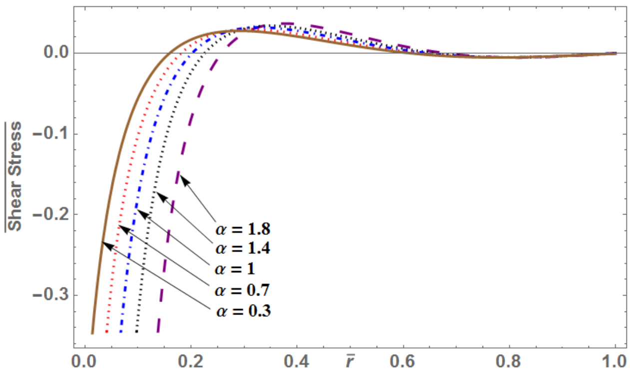

Figure 12. Dimensionless thermal shear stress along the radial axis at different values of α.

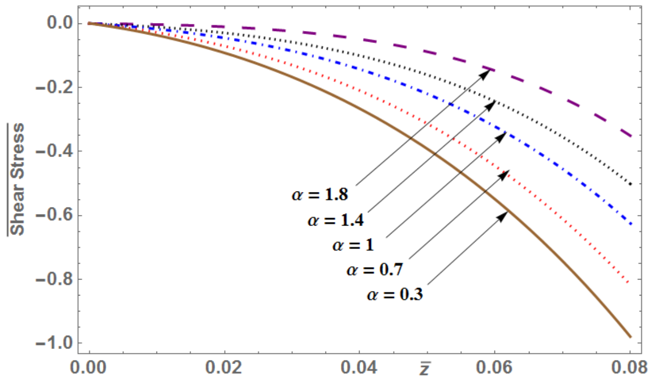

Figure 13. Variation in shear stress along with the thickness at different values of α.

Fig. 2 shows the variation in dimensionless temperature approaches to a maximum value at the extremum of the outer edge, which is impacted by the concentric ramp-type thermal load and drops at $\bar{r}_0 = 0.65$.It is depicted that considerable compressive stress occurs on the heated inner side, and tensile stress occurs on the outer edge along the radial direction. It is also noted that irrespective of the sectional heat source range, the temperature distribution characteristic is the same. As shown in Fig. 3, the temperature is initially zero at $\bar{\tau} = 0$, which approaches a maximum as time proceeds. It is observed that as $\bar{\tau} \to \infty$, the transient temperature distribution attains a constant value. Figure 4 shows the temperature distribution along the thickness direction for different values of α, which is maximum towards the outer edge due to energized heat supply. Figure 5 shows the variation in dimensionless thermal deflection is maximum at the mid of the plate, and the variation decreases remarkably approaching the plate's brink for different α, thus satisfying boundary conditions (21). The deflection is highest at the centre part due to the external energy supply. Figure 6 shows the variation in dimensionless thermal deflection as time proceeds to increase linearly and attain a pre-buckling state's constant value. Figures 7 to 13 show the variations of the circular plate's dimensionless thermal bending stresses subjected to thermal loading. Figure 7 shows that the maximum value of tensile stresses occurs in the middle surface along the radial direction, and then the compressive stress acts towards the end. For $\bar{\sigma}_{rr}$ in Figure 8, as time proceeds, the stress distribution gradually increases due to the accumulation of energy due to sectional heat supply and as $\bar{\tau} \to \infty$ the stress gets fixed towards a stable state. Figure 9 shows the variation in dimensionless radial stress distribution across thickness directions in which it is observed that high tensile stress occurs on the outer edge and attains the maximum stress value. Figure 10 shows the variation in the distribution of dimensionless tangential stress across the radial direction, indicating that tensile forces are high on the middle part and may be due to the available thermal energy. It is learned that the stresses move along the radial direction; the compressive forces are more dominant compared to tensile stresses. Figure 11 shows that the tangential stresses are more of a compressive type, and later on, they increase linearly as time proceeds till it attains maximum and stable tensile stress. Figure 12 depicts the variation in dimensionless shear stress distribution across thickness directions in which it is observed that high tensile stress occurs on the outer edge and attains the maximum stress value. Figure 13 shows the dimensionless thermal shear stress along the thickness direction at different values of α. It is observed that the tensile stresses are maximum at the first part of thickness which is later overlaid by the compressive stress at the end along the thickness direction.

7. Conclusion

This article develops the classical techniques model to analyze the transient thermoelastic in a homogeneous elastic circular plate undergoing ramp-type heating on the concentric region within the fractional-order theory framework. The results obtained while carrying out research are described as follows:

- The value of dimensionless temperature approaches a maximum value at the end boundaries along the radial direction, which may be due to available energy in the form of a sectional heat supply. The overlapping of the curve occurs at the inner rim of the concentric ramp-type thermal load. Similarly, the temperature distribution along the thickness direction is more linear due to the plate's thinness.
- The value of dimensionless thermal deflection is maximum at the mid of the plate. The variation decreases remarkably approaching the plate's brink due to the accumulated energy supplied by the impacted thermal load.
- The value of radial stress has a maximum value at the mid of the plate along the radial direction due to the accumulation of energy and sectional heat supply.
- The value of tangential stress indicates that tensile forces are high in the middle part due to the available thermal energy.
- The value of shear stress shows that the tensile stresses are maximum at the first part of thickness which is later overlaid by the compressive stress at the end along the thickness direction.

Finally, as opposed to the instantaneous response predicted by the generalized theory of thermoelasticity, the fractional theory, currently under consideration, predicts a delayed response to physical stimuli, which can be observed in nature. This supports the motivation for this direction of research.

In this problem, we have used the thin circular plate with clamped edges by applying the time fractional order theory of thermoelasticity. We avoided using conventional potential functions in favour of taking a more direct approach to solving the problem. By doing this, the well-known difficulties that are involved with finding solutions via potential functions can be avoided. The material's conductivity is directly proportional to the measured fractional order parameter. By taking into account, the time-fractional derivative in the field equations, the equation system presented in this article can be beneficial when applied to the study of the thermal characteristics of a variety of entities in the context of real-world engineering challenges.

Acknowledgements:

The author(s) would like to extend utmost gratitude and indebtedness to the Reviewers and Editors for their suggestions.

Nomenclature

Definitions

α_t linear coefficient of thermal expansion ($/^0C$)

κ thermal diffusivity ($m^2 s^{-1}$)

k thermal conductivity (W/m.K)
Greek symbols
μ Lame's constants (GPa),
ν Poisson's ratio,
ρ density (kg/m3),
σ_{ij} components of the stress tensor.

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