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Perturbatıon Solution for a Cracked Euler-Bernoulli Beam

Lütfi Emir SAKMAN*[1]

Abstract

The natural frequencies and mode shapes of an Euler-Bernoulli beam with a rectangular cross-section, which has a surface crack, is investigated. The crack is modeled as a change (sudden or gradual) in the cross-section of the beam, and a modified perturbation approach is used assuming that the crack geometry is much smaller than the beam cross section. Computations of natural frequencies and mode shapes were carried out for various crack shapes and compared with a range of experiments and finite element analyses. It is concluded that the suggested modified perturbation approach gives reliable results with minimal effort for eigenfrequencies of cracked beams. Furthermore, as a new feature, the present perturbation method includes the shape of the crack in eigenfrequency computations and in principle, can work for any type of disturbance on the surface including a small bump for example.

Keywords: Perturbation, surface crack, Euler-Bernoulli beam

1. INTRODUCTION

Identification and characterization of cracks in engineering structures is an important problem from both theoretical and technical point of view. One type of problem deals with determination of vibration characteristics of the structure with a crack. Another problem might be the characterization of the crack from measured vibration characteristics of the structure; this is usually called an inverse problem. The practical solution of the inverse problem normally involves solving the forward problem for a wide range of crack types. Therefore, the ability to solve the vibration characteristics of a cracked structure is an important step in diagnosing structures by means of eigenfrequency measurements.

Most studies dealing with cracked structures models the crack as a change in the elastic characteristics of the structure, and utilize some type of numerical method. The present study aims to model the detailed geometry of the crack therefore shedding light on the solution of the inverse problem. Furthermore, the perturbation theory approach will be used making the solution analytical. However, since the Euler-Bernoulli beam theory depends little on the actual shape of the cross-section (i.e., it

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only comes into consideration in the form of cross-sectional second area moment), the straightforward perturbation theory does not yield satisfactory results. In the present study, a scale factor depending on computed base and perturbation eigenvalues is introduced in front of the perturbed solution. The reasoning for such an approach will be discussed in the next section, nevertheless this method gave good agreement with a wide range of experiments.

One of the earlier studies, Adams et al., demonstrated a vibration method for non-destructive testing of structures [1]. Rubio treats the crack by means of a torsional spring and solves the inverse problem [2] while Morassi treats the crack as a point discontinuity and derives general results about the sensitivity of eigenfrequencies [3]. Shen and Pier studied the convergence of Galerkin approach for beams with symmetric cracks [4]. Papaeconomou and Dimarogonas describe a transfer matrix model for a cracked prismatic bar [5]. Chondros and Dimarogonas used a variational formulation to analyze lumped and continuous cracks [6]. Khiem and Toan used a modification of the Rayleigh quotient method for detection of an unknown number of multiple cracks on beams [7]. Saez et al. performed damage detection by solving the inverse problem [8]. Chaudhari and Maiti used a rotational spring to represent the crack and worked on solving the inverse problem based on the measurement of natural frequencies [9]. He and Lin use an acoustic system for contact-type cracks [10]. Nejad et al. worked on analytical estimation of natural frequencies and mode shapes of a beam having two cracks [11]. Mazanoglu et al. modified the energy-based method [12] presented by Yang et al to solve the vibration of non-uniform Euler–Bernoulli beams with multiple cracks by defining the crack as a spring [13]. Open edge cracks were investigated by Aydin again modeling the crack as a spring [14]. Caddemi and Morassi proposed a justification of the rotational elastic spring model of an open crack in a beam [15]. Finite element method is very popular for investigation of free vibration analysis of cracked beams [16-19]. Cracks in reinforced concrete structures is another popular research topic using Euler-Bernoulli Beam theory [20, 21]. Timoshenko Beam model was also used [22, 23]. Beams with cracks on elastic foundations were considered by Hasan [24]. Gudmundson gives general results using more general elasticity approaches and derives frequency in terms of strain energies of cracked and uncracked structures [25].

hen and Pier assumes a stress distribution around the crack and derives equations of motion in a detailed study [26]. Cooley et al. again models the crack as a torsional spring with an extra factor in the spring constant derived from linear fracture mechanics [27]. Chu and Shen uses a piece-wise linear spring model [28]. A study on cracked thin-walled beam under torsion is Dang et al. using stress concentration factor; this can be easily extended to eigenfrequency analysis [29].

The studies mentioned above mostly do not consider the shape of the crack, but model it as a spring or a change in the local elastic properties. In this study the crack is expressed as a change in the beam cross-section as explained in Section 2. This allows the determination of vibration characteristics depending on the shape of the crack. For the present, our interest is the validation of the modified perturbation computations by a wide range of experiments.

2. BASIC DEFINITIONS AND THEOREMS

Considering an Euler-Bernoulli beam of length L and cross-sectional area A(x), the governing equation for the vibrations is

$$-\frac{\partial^2}{\partial x^2}\left(E\,I(x)\frac{\partial^2 y}{\partial x^2}\right) = m(x)\frac{\partial^2 y}{\partial t^2} \tag{1}$$

$$A(x) = b(x)\, h(x) \tag{2a}$$

$$I(x) = \frac{1}{12}\, b(x) h(x)^3 \tag{2b}$$

$$m(x) = \rho\, b(x)\, h(x) \tag{2c}$$

where ρ is the density of the beam material. Both ends of the beam are assumed to be fixed for demonstrating the solution procedure,

$$y = \frac{\partial y}{\partial x} = 0 \;\; \text{at} \;\; x = 0, L \tag{3}$$

But solutions for other boundary conditions will also be given including

Fixed – fixed
Free – free
Simply supported – simply supported
Cantilever

The non-damaged beam has uniform cross section with constant width and height b_0 and h_0 respectively. The crack is modeled as a change in the beam cross-sectional dimensions in the form

$$b(x) = b_0 + \varepsilon\, f(x) \tag{4a}$$

$$h(x) = h_0 + \varepsilon\, g(x) \tag{4b}$$

where ε is a small non-dimensional perturbation parameter and the functions f(x) and g(x) determine the shape of the crack; these are completely general at this point. Substituting Eq. (4) into (2) and (1), the vibration equation becomes, ignoring higher order terms,

$$-\frac{\partial^2}{\partial x^2}\left(\frac{E}{12}\, (b_0 h_0^3 + \varepsilon\, (3\, b_0 h_0^2\, g(x) + h_0^3\, f(x))\, \frac{\partial^2 y}{\partial x^2}\right) = \rho\, (b_0 h_0 + \varepsilon\, (b_0\, g(x) + h_0\, f(x))\, \frac{\partial^2 y}{\partial t^2} \tag{5}$$

The governing equation is nondimensionalized with the following definitions, starred symbols showing non-dimensional variables,

$$x^* = \frac{x}{L} \tag{6a}$$

$$t^* = \frac{t}{\sqrt{\rho \frac{12\, L^4}{E\, h_0^2}}} \tag{6b}$$

$$y^* = \frac{y}{h_0} \tag{6c}$$

$$G^*(x^*) = \frac{g(x)}{h_0} \tag{6d}$$

$$F^*(x^*) = \frac{f(x)}{b_0} \tag{6e}$$

The non-dimensional vibration equation becomes, omitting stars after this point,

$$\frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 y}{\partial t^2} = -\varepsilon\left[\frac{\partial^2}{\partial x^2}\left(\big(3\, G(x) + F(x)\big)\frac{\partial^2 y}{\partial x^2}\right) + \big(G(x) + F(x)\big)\, \frac{\partial^2 y}{\partial t^2}\right] \tag{7}$$

and the boundary conditions

$$y = \frac{\partial y}{\partial x} = 0 \quad \text{at} \quad x = 0, 1 \tag{8}$$

Assuming a separated solution of the form

$$y(x,t) = u(x)\, p(t) \tag{9}$$

leads to

$$\ddot{p} + \lambda^4 p = 0 \tag{10a}$$

$$u'''' - \lambda^4\, u + \varepsilon\, \psi(u, \lambda) = 0 \tag{10b}$$

Prime and dot denote differentiation with respect to x and t, respectively, and we defined

$$\psi(u, \lambda) = \Big(\big(3\, G(x) + F(x)\big)u''\Big)'' - \lambda^4\, u\, \big(G(x) + F(x)\big) \tag{11}$$

Solution of Eq. (10b) is assumed to be in the form of a perturbation series for both the mode shape and the eigenvalue

$$u(x) = u_0(x) + \varepsilon\, u_1(x) + \cdots \tag{12a}$$

$$\lambda = \lambda_0 + \varepsilon\, \lambda_1 + \cdots \tag{12b}$$

Substituting, the zero and first order problems become

$$u_0'''' - \lambda_0^4 u_0 = 0 \tag{13a}$$

$$u_0(0) = u_0'(0) = u_0(1) = u_0'(1) = 0 \tag{13b}$$

and

$$u_1'''' - \lambda_0^4 u_1 = 4\,\lambda_0^3 \lambda_1 u_0 - \psi(u_0, \lambda_0) \tag{14a}$$

$$u_1(0) = u_1'(0) = u_1(1) = u_1'(1) = 0 \tag{14b}$$

The solution of the zeroth order problem is

$$\cosh \lambda_0^{(n)} \cos \lambda_0^{(n)} = 1\,,\ n = 1,2,3, \ldots \tag{15a}$$

$$u_0^{(n)} = \frac{\cos \lambda_0^{(n)} - \cosh \lambda_0^{(n)}}{\sin \lambda_0^{(n)} - \sinh \lambda_0^{(n)}} \left(\cos \lambda_0^{(n)} x - \cosh \lambda_0^{(n)} x\right) + \sin \lambda_0^{(n)} x - \sinh \lambda_0^{(n)} x \tag{15b}$$

The solution of the first order problem, (14a) can be written by variation of constants as

$$u_1 = C_1(x)\, e^{\lambda_0 x} + C_2(x)\, e^{-\lambda_0 x} + C_3(x)\ \cos \lambda_0 x + C_4(x)\ \sin \lambda_0 x \tag{16}$$

Where

$$C_1{}'(x) = \frac{e^{-\lambda_0 x}\left[4\,\lambda_0^3 \lambda_1 u_0 - \psi(u_0, \lambda_0)\right]}{4\lambda_0^3} \tag{17a}$$

$$C_2{}'(x) = -\frac{e^{\lambda_0 x}\left[4\,\lambda_0^3 \lambda_1 u_0 - \psi(u_0, \lambda_0)\right]}{4\lambda_0^3} \tag{17b}$$

$$C_3{}'(x) = \frac{\sin \lambda_0 x\left[4\,\lambda_0^3 \lambda_1 u_0 - \psi(u_0, \lambda_0)\right]}{2\lambda_0^3} \tag{17c}$$

$$C_4{}'(x) = -\frac{\cos \lambda_0 x\left[4\,\lambda_0^3 \lambda_1 u_0 - \psi(u_0, \lambda_0)\right]}{2\lambda_0^3} \tag{17d}$$

Integrating Eqs. (17) and substituting in (16) gives the general solution in the form

$$u_1 = K_1\, e^{\lambda_0 x} + K_2\, e^{-\lambda_0 x} + K_3\ \cos \lambda_0 x + K_4\ \sin \lambda_0 x + \int_0^x [\sinh \lambda_0 (x - \xi) - \sin \lambda_0 (x - \xi)] \varphi(\xi) d\xi \tag{18}$$

where we defined

$$\varphi(x) = \frac{4\,\lambda_0^3 \lambda_1 u_0 - \psi(u_0, \lambda_0)}{2\lambda_0^3} \tag{19}$$

for brevity. K's are arbitrary constants found by applying the boundary conditions Eq. (14b); the result is the system of equations

$$K_1 + K_2 + K_3 = 0 \tag{20a}$$

$$K_1 - K_2 - K_4 = 0 \tag{20b}$$

$$K_1 e^{\lambda_0} + K_2 e^{-\lambda_0} + K_3 \cos \lambda_0 + K_4 \sin \lambda_0 = A \tag{20c}$$

$$K_1 e^{\lambda_0} - K_2 e^{-\lambda_0} - K_3 \sin \lambda_0 + K_4 \cos \lambda_0 = B \tag{20d}$$

where

$$A = -\int_0^1 [\sinh \lambda_0 (1 - \xi) - \sin \lambda_0 (1 - \xi)] \varphi(\xi)\, d\xi \tag{21a}$$

$$B = -\int_0^1 [\cosh \lambda_0 (1 - \xi) - \cos \lambda_0 (1 - \xi)] \varphi(\xi)\, d\xi \tag{21b}$$

The homogeneous part of the linear system Eqs. (20) is the same as the eigenvalue equation for the zeroth order problem. Therefore, for the system Eqs. (20) to have a solution, the consistency condition.

$$\left(e^{\lambda_0} + \sin \lambda_0 - \cos \lambda_0\right) A = \left(e^{\lambda_0} - \sin \lambda_0 - \cos \lambda_0\right) B \tag{22}$$

should be satisfied. Eq. (22) is the eigenvalue equation for λ_1 (which is inside, Eq. (19), through A and B). Once λ_1 is solved, the constants are found from Eqs. (20).

The eigenfrequencies of the cracked beam is computed from Eq. (12b),

$$\lambda = \lambda_0 + \varepsilon\, \lambda_1 + \cdots \tag{23a}$$

When computations were carried out, the change in eigenfrequency between cracked and uncracked beams, i.e., ε λ_1, was found to be consistently too small compared to experimental values. This is to be expected since beam theory basically averages the elasticity problem over the cross-section of the beam and the details of the cross section does not matter as long as second area moment remains the same. However, there was a structure to the difference in frequency between computed and experimental and/or finite element values which suggested that the change could be fixed once and for all cracked beams by introducing a factor in the perturbation term.

To this end, it is proposed that the perturbation in the eigenvalue is to be enlarged by a factor depending on base and perturbed eigenvalues in the form

$$C\left(\frac{\lambda_0}{\lambda_1}\right)^p$$

i.e. instead of Eq. (23a) we assume

$$\lambda = \lambda_0 + \varepsilon\ C\left(\frac{\lambda_0}{\lambda_1}\right)^p\ \lambda_1 + \cdots \tag{23b}$$

where C and p are constants to be determined by matching (23b) to two experimental measurements. Afterwards, if the method is successful, Eq.(23b) should give good results for other experiments.

The logic of this procedure might be questioned; however, it should be noted that virtually all studies on cracked beams still use beam theory away from the crack, and model the crack by modifying the cracked element either as a different spring constant or some factor depending on well-known fracture theory concepts as alluded to in the introduction. This can be considered as an application of the Saint Venant principle that the local changes in geometry should not affect the global results too much. Therefore, what is done here should be seen in the same light, i.e., it lacks mathematical rigor since beam theory cannot really treat cracks; nevertheless, its value should be evaluated by its possible success. Another advantage of the present method is that it naturally involves the shape of the crack in the computations.

As will be seen in the next section, the proposed method indeed gives good results compared to roughly about 100 experiments (counting higher modes, other boundary conditions and other material constant) once the constants in Eq. (23b) are fixed by comparing to only two experimental results. These were chosen since they gave the closest results to the straightforward perturbation expansion before introducing the factor in Eq. (23b). The result is

$$\lambda = \lambda_0 + \varepsilon\ \sqrt{\frac{2\,\lambda_0}{\lambda_1}}\ \lambda_1 + \cdots \tag{23c}$$

The solution procedure is the same for the other types of boundary conditions. We present the results.

Free – Free:

$$A = -\int_0^1 [\sinh\lambda_0(1-\xi) + \sin\lambda_0(1-\xi)]\varphi(\xi)\,d\xi \tag{24a}$$

$$B = -\int_0^1 [\cosh\lambda_0(1-\xi) + \cos\lambda_0(1-\xi)]\varphi(\xi)\,d\xi \tag{24b}$$

$$(e^{\lambda_0} + \sin\lambda_0 - \cos\lambda_0)A = (e^{\lambda_0} - \sin\lambda_0 - \cos\lambda_0)B \tag{25}$$

Cantilever:

$$A = -\int_0^1 [\sinh\lambda_0(1-\xi) + \sin\lambda_0(1-\xi)]\varphi(\xi)\,d\xi \tag{26a}$$

$$B = -\int_0^1 [\cosh\lambda_0(1-\xi) + \cos\lambda_0(1-\xi)]\varphi(\xi)\,d\xi \tag{26b}$$

$$(e^{\lambda_0} + \sin\lambda_0 - \cos\lambda_0)A = (e^{\lambda_0} + \sin\lambda_0 + \cos\lambda_0)B \tag{27}$$

Simply supported – Simply supported:

$$A = -\int_0^1 [\sinh\lambda_0(1-\xi) - \sin\lambda_0(1-\xi)]\varphi(\xi)\,d\xi \tag{28a}$$

$$B = -\int_0^1 [\cosh\lambda_0(1-\xi) + \cos\lambda_0(1-\xi)]\varphi(\xi)\,d\xi \tag{28b}$$

$$(e^{\lambda_0} + \sin\lambda_0 - \cos\lambda_0)A = (e^{\lambda_0} - \sin\lambda_0 - \cos\lambda_0)B \tag{29}$$

3. RESULTS

The eigenfrequencies for cantilever, fixed – fixed, free – free, and simply supported –

simply supported boundary conditions will be presented and compared with computed or measured values in Ref. [30-36].

Cantilever beam:
Tables 1 – 3 shows comparisons with Refs. [30-32]

Table 1 Comparison with experiment (Ref. [30]) for cantilever boundary conditions.

Crack (mm)		Method	Natural Frequency		
Pos	Depth		Mode 1	Mode 2	Mode 3
10	2	Experiment	182.7	1149.4	3242.9
		F.E.M	182.7	1149.2	3234.1
		Lee and Chung	182.6	1148.0	3222.1
		Analytic	182.3	1144.8	3210.9
	6	Experiment	163.9	1073.4	3097.3
		F.E.M	166.9	1083.9	3108.0
		Lee and Chung	161.9	1048.7	3010.3
		Analytic	176.9	1114.9	3138.1
	10	Experiment	129.8	980.6	2954.2
		F.E.M	136.9	996.1	3970.8
		Lee and Chung	109.0	814.5	3533.9
		Analytic	171.6	1085.5	3066.2
80	2	Experiment	184.0	1160.0	3245.0
		F.E.M	184.0	1159.8	3244.9
		Lee and Chung	184	1159.2	3229.7
		Analytic	183.2	1156.7	3225.3
	6	Experiment	174.7	1155.3	3134.8
		F.E.M	181.8	1102.9	3250.7
		Lee and Chung	175.2	1153.9	3082.5
		Analytic	179.6	1150.6	3180.9
	10	Experiment	153.5	1145.1	2934.3
		F.E.M	158.4	1147.6	2974.5
		Lee and Chung	156.0	1143.3	2764.7
		Analytic	176.1	1144.4	3136.8
140	2	Experiment	184.7	1153.1	3258.1
		F.E.M	184.7	1153.2	3257.2
		Lee and Chung	184.7	1151.9	3246.4
		Analytic	184.1	1150.0	3242.3
	6	Experiment	181.2	1092.9	3250.1
		F.E.M	181.8	1102.9	3250.7
		Lee and Chung	181.6	1086.3	3236.6
		Analytic	182.1	1130.3	3231.4
	10	Experiment	171.5	971.5	3233.6
		F.E.M	174.0	997.7	3238.0
		Lee and Chung	175.3	941.3	3216.8
		Analytic	180.2	1110.9	3220.5
200	2	Experiment	185.0	1155.0	3238.6
		F.E.M	185.0	1155.0	3238.4
		Lee and Chung	185.0	1154.1	3223.2
		Analytic	184.3	1150.2	3216.6
	6	Experiment	184.3	1106.3	3082.9
		F.E.M	184.4	1114.8	3107.3
		Lee and Chung	184.4	1106.5	3020.5
		Analytic	182.7	1131.1	3154.9
	10	Experiment	182.2	1025.0	2819.6
		F.E.M	182.7	1016.5	2871.0
		Lee and Chung	183.2	1004.8	2567.6
		Analytic	181.2	1112.1	3093.8

The comparisons are shown in Tables 1 through 7; analytic column denotes the results found in the present study. All the dimensions in the tables are millimeters.

In this case, analytic solution gives good results for different crack depths and positions, compared to the experimental results.

Table 2 Comparison with experiment (Ref. [31]) for cantilever boundary conditions.

Crack		Method	Natural Frequencies (Hz)		
Location (mm)	Depth (mm)		Mode 1	Mode 2	Mode 3
No Crack			68.00	432.75	1195.00
133	1	Exp	68.00	432.50	1193.50
		Anal	67.68	425.54	1188.33
	2	Exp	67.75	431.50	1190.00
		Anal	67.27	424.37	1181.86
	3	Exp	67.50	430.50	1185.50
		Anal	66.87	423.21	1175.41
	4	Exp	67.25	429.00	1169.50
		Anal	66.47	422.05	1168.98
	5	Exp	66.00	427.25	1156.25
		Anal	66.07	420.88	1162.57
No Crack			67.25	427.25	1184.00
240	1	Exp	67.25	426.50	1183.00
		Anal	67.98	423.91	1188.88
	2	Exp	67.25	424.50	1180.00
		Anal	67.88	421.13	1182.96
	3	Exp	67.25	422.00	1175.25
		Anal	67.78	418.35	1177.06
	4	Exp	67.25	418.25	1169.25
		Anal	67.68	415.59	1171.17
	5	Exp	67.00	411.25	1161.00
		Anal	67.58	412.83	1165.30

Table 2 and Table 3 presents comparisons of experimental and analytic results for different crack depths and locations. Again, analytic results are close to experimental results.

Table 3 Comparison with experiment (Ref. [32]) for cantilever boundary conditions.

Normalized crack location	Crack Depth Ratio	Method	Natural Frequency Mode 1 (Hz)	Natural Frequency Mode 2 (Hz)
No Crack	No Crack	Exp	13.45	84.30
0.1	0.5	Exp	12.80	82.89
		Analytic	12.82	82.57
0.2	0.5	Exp	13.02	84.30
		Analytic	12.96	84.95
0.3	0.5	Exp	13.15	83.30
		Analytic	13.10	85.40
0.4	0.5	Exp	13.28	82.87
		Analytic	13.26	83.11
.5	0.5	Exp	13.37	81.82
		Analytic	13.43	82.17
0.6	0.5	Exp	13.40	81.93
		Analytic	13.58	82.10
0.7	0.5	Exp	13.43	82.50
		Analytic	13.39	82.75

Both ends fixed:

Tables 4 and 5 presents comparisons with Refs. [33, 34]. In this tables, results shows that, analytic solutions are suitable for different boundary conditions. Here, analytic solution have effective results against FEM results.

Table 4 Comparison with experiment (Ref. [33]) for both ends fixed boundary conditions.

Crack (mm)		Method	Natural Frequency		
Position	Depth		Mode 1	Mode 2	Mode3
255	3	Exp	83.984	235.792	462.712
		F.E.M	74.710	223.760	426.146
		Analytic	84.897	234.894	456.869

Table 5 Comparison with experiment (Ref. [34]) for both ends fixed boundary conditions.

Crack Loc. (mm)	Crack Depth Ratio (%)	Method	Mode 1	Mode 2	Mode 3
45	10	No Crack	431.7	1190.0	2333.0
		Exp	431.4	1189.9	2331.6
		Analytic	429.2	1184.7	2316.6
			Mode 4	Mode 5	Mode 6
		Exp	3856.0	5761.0	8046.0
		Analytic	3855.2	5759.1	8041.7

Both ends free:

Table 6 Comparison with experiment (Ref. [35]) for both ends free boundary conditions.

Crack (mm)		Method	Natural Frequency			
Pos	Depth		Mode1	Mode 2	Mode3	Mode4
430	4	Exp	74.688	205.625	405.62	666.250
		Reduced EI	74.670	205.190	405.64	669.335
		Lee and Chung	74.938	206.262	405.97	670.550
		Sinha et al.	74.406	204.183	405.36	668.429
		Analytic	74.136	204.043	403.88	665.973
430	8	Exp	74.063	202.500	404.68	662.813
		Reduced EI	74.004	202.630	404.90	666.527
		Lee and Chung	74.224	203.458	405.23	667.615
		Sinha et al.	73.628	201.283	404.55	665.356
		Analytic	73.108	200.899	401.56	660.479
430	12	Exp	72.813	197.188	403.12	655.938
		Reduced EI	72.284	198.278	403.67	661.720
		Lee and Chung	72.634	197.764	403.77	661.635
		Sinha et al.	72.958	198.928	403.91	662.874
		Analytic	72.088	197.780	399.25	650.123

Table 6 presents comparisons with Ref [35]. Analytic solution compares with different solution procedures and experimental results.

Table 7 presents comparisons with Ref [36]. Also to fix the constants in Eq. (23b), two cases from Ref. [36], crack position 200 mm, and depths of cracks 1 and 3 mm in Table 7, were chosen to be matched to the computed eigenfrequencies.

In all cases the perturbation solution (the present study) obtains good results compared with experiment, in some cases even surpassing the FEM computations (Ref. [30], [33]). It is also interesting to note that the perturbation solution seems to remain valid for quite deep cracks.

Both ends simply supported:

Table 7 Comparison with experiment (Ref. [36]) for both ends simply supported boundary conditions.

Crack (mm)		Method	Natural Frequency	
Position	Depth		Mode 1	Mode 2
80	1	Experiment	151.20	599.80
		Yoon et al.	150.48	601.18
		Analytic	149.80	597.26
	3	Experiment	151.00	588.20
		Yoon et al.	149.61	590.21
		Analytic	148.22	587.10
	5	Experiment	147.80	568.50
		Yoon et al.	145.95	556.08
		Analytic	146.66	577.02
120	1	Experiment	150.50	604.20
		Yoon et al.	150.38	601.16
		Analytic	149.50	597.26
	3	Experiment	149.80	594.20
		Yoon et al.	148.75	588.50
		Analytic	147.34	587.10
	5	Experiment	143.80	564.50
		Yoon et al.	142.16	543.42
		Analytic	145.19	577.02
200	1	Experiment	149.20	599.80
		Yoon et al.	150.27	602.37
		Analytic	149.25	602.36
	3	Experiment	146.80	597.80
		Yoon et al.	147.81	602.37
		Analytic	146.58	602.34
	5	Experiment	139.20	593.00
		Yoon et al.	138.32	602.37
		Analytic	143.93	602.32

Normally, the perturbation solution is expected to be more accurate for very small crack depths compared to beam cross-section; but as mentioned, after “fixing” the theoretical results by the factor introduced in Eq. (23c), good agreement with the experimental values is obtained for crack depths from very small cracks up to cracks with a depth of half of the beam height. Another point to note is, the cracks in experiments were produced by cutting the beam to a certain depth, thus resulting in a crack of rectangular shape. The depth and width of these rectangles were input in to the function g(x), Eq.(4b). The present method should be able to handle other crack shapes, e.g., triangular, circular, etc., bymodifying the function g(x). But no such experiments were available for us to compare.

4. CONCLUSION

A modified perturbation solution was presented to compute the mode shapes and eigenvalues of cracked Euler-Bernoulli beams. The method is general enough and allows computation of mode shapes and eigenvalues for specific crack geometries. In conclusion, the idea that the results of perturbation computations in the nondimensionalised (thus, universal), cracked beam vibration equation can be modified to agree with a wide set of experimental results has been verified [10]

Nomenclature

A (x)	area of cross section
b (x), h (x)	width and height of beam cross section
b_0, h_0	width and height of non-cracked beam cross section
I (x)	second area moment
m (x)	mass per unit lenght
E	modulus of elasticity
ρ	density
L	length of the beam
ε	perturbation parameter
f (x), g (x)	crack shape functions
y (x, t)	vertical displacement
u (x)	mode shape
p (t)	separated time function
λ	eigenvalue

[ ]*	non-dimensional [ ]
C, p	Parameters in modification factor

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The Declaration of Conflict of Interest/Common Interest

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The Declaration of Ethics Committee Approval

This study does not require ethics committee permission or any special permission.

The Declaration of Research and Publication Ethics

During the writing process of our study, the information of which is given above, international scientific, ethical and citation rules have been followed, no falsification has been made on the data collected, and Sakarya University Journal of Science and its editorial board have no responsibility for any ethical violations that may be encountered. I undertake that I have full responsibility that this study has not been evaluated in any academic environment other than Sakarya University Journal of Science.

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