International Journal of Engineering & Applied Sciences (IJEAS)
Vol.8, Issue 4(2016) 30-39

Fourth Order Compact Finite Difference Method for Solving One Dimensional Wave Equation

Gemechis File Duressa[1*], Tesfaye Aga Bullo[2] and Gashu Gadisa Kiltu[3]

[1,2,3]Department of Mathematics, Jimma University, Jimma, P. O. Box 378, ETHIOPIA
**Email:* gammeef@yahoo.com

Received date: November 2016
Accepted date: December 2016

Abstract

This paper introduces the fourth order compact finite difference method for solving the numerical solution of one-dimensional wave equations. The convergence of the method for the problem under consideration had been investigated. To validate the applicability of the method on the proposed equation, two model examples have been solved for different values of mesh sizes. The numerical results in terms of point wise absolute errors presented in tables and graphs show that the present method approximates the exact solution very well.

Keywords: Wave equation, Compact finite difference method, Consistence.

1. Introduction

Partial differential equations are equations that involve unknown functions of several variables and their partial derivatives. Wave equation is a hyperbolic second order linear partial differential equation which describes the nature of waves occurring in various physical phenomena. Initial value problems of hyperbolic type occurring in different fields like: sound wave, elastic, vibrations, fluid dynamics etc [1]. In physics, propagation of sound, light and water waves is modeled by hyperbolic partial differential equations. The efficient and accurate numerical techniques for the wave equations are of fundamental importance for the simulation of time dependent acoustic, electromagnetic or elastic wave phenomena [2].
The development of numerical techniques for the solution of the hyperbolic nonlocal boundary value problems has been an important research topic in many branches of science and engineering [3]. There are many papers that deal with the numerical solution of wave equations. Recently, exponential B-spline collocation method for the numerical solution of one-space dimensional nonlinear wave equation with strong stability preserving time integration [3], numerical solution based on shifted Legendre tau technique for solving one-dimensional wave equation with an integral condition [4], numerical solution of one-dimensional heat and wave equation by non-polynomial quintic spline method [5] and a Galerkin based finite element model has been developed to solve linear second order one dimensional inhomogeneous wave equation numerically with accuracy of the developed scheme has been analyzed by comparing the numerical solution with exact solution given by the authors in [2]. In this paper, we introduce fourth order compact finite difference method (CFDM) for solving homogeneous and non-homogeneous one dimensional wave equations.

2. Description of the Method

Consider the one dimensional wave equation of the form:

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial t^2} = q(x,t), \quad 0 < x < l, \quad 0 < t < T \tag{1}$$

subject to the initial conditions:

$$u(x,0) = f(x) \text{ and } \frac{\partial u}{\partial t}(x,0) = g(x) \tag{2}$$

and with boundary conditions:

$$u(0,t) = f_1(t) \text{ and } u(l,t) = f_2(t) \tag{3}$$

where c^2 and l are positive finite real constants, the functions:
$q(x,t)$, $f(x)$, $g(x)$, $f_1(t)$ and $f_2(t)$ are real continuous functions.
An example of hyperbolic partial differential equation is a one-dimensional wave equation for the amplitude function $u(x,t)$ with position x and time t. In order for this equation to be solvable the initial conditions Eq. (2) as well as the boundary conditions Eq. (3) should be provided [6]. To describe the scheme, we divide the interval $[0,l]$ and $[0,T]$ into N and M equal subintervals of mesh length h and k respectively.

Let $0 = x_0 < x_1 < x_2 < \ldots < x_{N-1} < x_N = l,$ and $0 = t_0 < t_1 < t_2 < \ldots < t_{N-1} < t_N = T$ be the mesh points with $x_i = x_0 + ih$ and $t_j = t_0 + jk,$ for $i = 1,2,\ldots,N$ and $j = 0,1,\ldots,M$. Assume that $u(x,t)$ has continuous higher order partial derivatives on the region $[0,l]\times[0,T]$. For the sake of simplicity, we use $u(x_i,t_j) = u(i,j), \frac{\partial^n u}{\partial x^n}(x_i,t_j) = \frac{\partial^n u}{\partial x^n}(i,j)$ and $\frac{\partial^n u}{\partial t^n}(x_i,t_j) = \frac{\partial^n u}{\partial t^n}(i,j)$, ($n \geq 1$ called n^{th} order derivatives). By using Taylor series expression, we have:

$$u(i+1,j) = u(i,j) + h\frac{\partial u}{\partial x}(i,j) + \frac{h^2}{2!}\frac{\partial^2 u}{\partial x^2}(i,j) + \frac{h^3}{3!}\frac{\partial^3 u}{\partial x^3}(i,j) + \frac{h^4}{4!}\frac{\partial^4 u}{\partial x^4}(i,j) + \frac{h^5}{5!}\frac{\partial^5 u}{\partial x^5}(i,j) + \ldots \tag{4}$$

$$u(i-1,j) = u(i,j) - h\frac{\partial u}{\partial x}(i,j) + \frac{h^2}{2!}\frac{\partial^2 u}{\partial x^2}(i,j) - \frac{h^3}{3!}\frac{\partial^3 u}{\partial x^3}(i,j) + \frac{h^4}{4!}\frac{\partial^4 u}{\partial x^4}(i,j) - \frac{h^5}{5!}\frac{\partial^5 u}{\partial x^5}(i,j) + \ldots \tag{5}$$

$$u(i,j+1) = u(i,j) + k\frac{\partial u}{\partial t}(i,j) + \frac{k^2}{2!}\frac{\partial^2 u}{\partial t^2}(i,j) + \frac{k^3}{3!}\frac{\partial^3 u}{\partial t^3}(i,j) + \frac{k^4}{4!}\frac{\partial^4 u}{\partial t^4}(i,j) + \frac{k^5}{5!}\frac{\partial^5 u}{\partial t^5}(i,j) + \ldots \tag{6}$$

$$u(i,j-1) = u(i,j) - k\frac{\partial u}{\partial t}(i,j) + \frac{k^2}{2!}\frac{\partial^2 u}{\partial t^2}(i,j) - \frac{k^3}{3!}\frac{\partial^3 u}{\partial t^3}(i,j) + \frac{k^4}{4!}\frac{\partial^4 u}{\partial t^4}(i,j) - \frac{k^5}{5!}\frac{\partial^5 u}{\partial t^5}(i,j) + \ldots \tag{7}$$

Subtracting Eq. (5) from Eq. (4) and Eq. (7) from Eq. (6), we obtain the second order finite difference ($\delta_c^1 u(i,j)$) for the first derivative of $u(i,j)$:

$$\delta_{cx}^1 u(i,j) = \frac{u(i+1,j)-u(i-1,j)}{2h} + \tau_1 \quad \text{and} \quad \delta_{ct}^1 u(i,j) = \frac{u(i,j+1)-u(i,j-1)}{2k} + \tau_2 \tag{8}$$

where $\tau_1 = -\frac{h^2}{6}\frac{\partial^3 u}{\partial x^3}(i,j)$ and $\tau_2 = -\frac{k^2}{6}\frac{\partial^3 u}{\partial t^3}(i,j)$

Similarly, adding Eq. (4) with Eq. (5) and Eq. (6) with Eq. (7), we obtain the second order finite difference ($\delta_c^2 u(i,j)$) for the first derivative of $u(i,j)$:

$$\delta_{cx}^2 u(i,j) = \frac{u(i+1,j)-2u(i,j)+u(i-1,j)}{h^2} + \tau_3$$

$$(9)\,\delta_{ct}^2 u(i,j) = \frac{u(i,j+1)-2u(i,j)+u(i,j-1)}{k^2} + \tau_4 \tag{10}$$

where $\tau_3 = -\frac{h^2}{12}\frac{\partial^4 u}{\partial x^4}(i,j)$ and $\tau_4 = -\frac{k^2}{12}\frac{\partial^4 u}{\partial t^4}(i,j)$

Substituting Eqs. (4) - (7) into Eqs. (9) and (10) yields:

$$\delta_{cx}^2 u(i,j) = \frac{\partial^2 u}{\partial x^2}(i,j) + \frac{h^2}{12}\frac{\partial^4 u}{\partial x^4}(i,j) + \tau_5 \quad \text{and}$$

$$(11)\,\delta_{ct}^2 u(i,j) = \frac{\partial^2 u}{\partial t^2}(i,j) + \frac{k^2}{12}\frac{\partial^4 u}{\partial t^4}(i,j) + \tau_6 \tag{12}$$

where $\tau_5 = \frac{h^4}{360}\frac{\partial^6 u}{\partial x^6}(i,j)$ and $\tau_6 = \frac{k^4}{360}\frac{\partial^6 u}{\partial t^6}(i,j)$

Using from Eq.(1) and successive differentiation, we have:

$$\frac{\partial^4 u}{\partial x^4}(i,j) = \frac{1}{c^2}\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u}{\partial t^2}(i,j)\right) - \frac{1}{c^2}\frac{\partial^2 q}{\partial x^2}(i,j) = \frac{1}{c^2}\delta_{ct}^2\delta_{cx}^2 u(i,j) - \frac{1}{c^2}\frac{\partial^2 q}{\partial t^2}(i,j) \tag{13}$$

$$\frac{\partial^4 u}{\partial t^4}(i,j) = c^2\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 u}{\partial x^2}(i,j)\right) + \frac{\partial^2 q}{\partial t^2}(i,j) = c^2\delta_{ct}^2\delta_{cx}^2 u(i,j) + \frac{\partial^2 q}{\partial t^2}(i,j) \tag{14}$$

Substituting Eqs. (13) and (14) into Eqs.(11) and (12) respectively, gives:

$$\delta_{cx}^2 u(i,j) = \frac{\partial^2 u}{\partial x^2}(i,j) + \frac{h^2}{12}\left(\frac{1}{c^2}\delta_{cx}^2\delta_{ct}^2 u(i,j) - \frac{1}{c^2}\frac{\partial^2 q}{\partial x^2}(i,j)\right) + \tau_5$$

$$\delta_{ct}^2 u(i,j) = \frac{\partial^2 u}{\partial t^2}(i,j) + \frac{k^2}{12}\left(c^2\delta_{ct}^2\delta_{cx}^2 u(i,j) + \frac{\partial^2 q}{\partial t^2}(i,j)\right) + \tau_6$$

$$\frac{\partial^2 u}{\partial x^2}(i,j)=\delta_{cx}^2 u(i,j)-\frac{h^2}{12}\frac{1}{c^2}\delta_{cx}^2\delta_{ct}^2 u(i,j)+\frac{h^2}{12}\frac{1}{c^2}\frac{\partial^2 q}{\partial x^2}(i,j)-\tau_5 \tag{15}$$

$$\frac{\partial^2 u}{\partial t^2}(i,j)=\delta_{ct}^2 u(i,j)-\frac{k^2}{12}c^2\delta_{ct}^2\delta_{cx}^2 u(i,j)-\frac{k^2}{12}\frac{\partial^2 q}{\partial t^2}(i,j)-\tau_6 \tag{16}$$

Again, substituting Eqs.(15) and (16) into Eq.(1), we obtain:

$$\delta_{ct}^2 u(i,j)-\frac{k^2}{12}c^2\delta_{ct}^2\delta_{cx}^2 u(i,j)-\frac{k^2}{12}\frac{\partial^2 q}{\partial t^2}(i,j)-\tau_6=c^2[\delta_{cx}^2 u(i,j)-\frac{h^2}{12}\frac{1}{c^2}\delta_{cx}^2\delta_{ct}^2 u(i,j)$$
$$+\frac{h^2}{12}\frac{1}{c^2}\frac{\partial^2 q}{\partial x^2}(i,j)-\tau_5]+q(i,j)$$
$$\delta_{ct}^2 u(i,j)-\frac{k^2}{12}c^2\delta_{ct}^2\delta_{cx}^2 u(i,j)-c^2\delta_{cx}^2 u(i,j)+\frac{h^2}{12}\delta_{cx}^2\delta_{ct}^2 u(i,j)=\frac{h^2}{12}\frac{\partial^2 q}{\partial x^2}(i,j)+\frac{k^2}{12}\frac{\partial^2 q}{\partial t^2}(i,j)+\tau_6-c^2\tau_5+q(i,j)$$
$$\frac{u(i,j+1)-2u(i,j)+u(i,j-1)}{k^2}+\tau_4-\frac{c^2k^2}{12h^2}\delta_{ct}^2\left(u(i-1,j)-2u(i,j)+u(i+1,j)\right)-\frac{c^2}{h^2}\{u(i-1,j)$$
$$-2u(i,j)+u(i+1,j)-c^2\tau_3\}+\frac{h^2}{12k^2}\delta_{cx}^2\left(u(i,j-1)-2u(i,j)+u(i,j+1)\right)=\frac{h^2}{12}\frac{\partial^2 q}{\partial x^2}(i,j)+$$
$$\frac{k^2}{12}\frac{\partial^2 q}{\partial t^2}(i,j)+\tau_6-c^2\tau_5+q(i,j)$$

After simplification, we obtain:

$$\begin{aligned}&\left(h^2-c^2k^2\right)u(i-1,j+1)+\left(10h^2+2c^2k^2\right)u(i,j+1)+\left(h^2-c^2k^2\right)u(i+1,j+1)=\\&\left(2h^2+10c^2k^2\right)u(i-1,j)+\left(20h^2+20c^2k^2\right)u(i,j)+\left(2h^2+10c^2k^2\right)u(i+1,j)\\&-\left(h^2-c^2k^2\right)u(i-1,j-1)-\left(10h^2-2c^2k^2\right)u(i,j-1)-\left(h^2-c^2k^2\right)u(i+1,j-1)+\\&12h^2k^2q(i,j)+h^4k^2\frac{\partial^2 q}{\partial x^2}(i,j)+h^2k^4\frac{\partial^2 q}{\partial t^2}(i,j)+\tau_7\end{aligned} \tag{17}$$

where $\tau_7=12h^2k^2(\tau_6-c^2\tau_5+c^2\tau_3-\tau_4)$ is a local truncation error.
Eq. (17), can be written as:

$$\begin{aligned}&\alpha u(i-1,j+1)+\beta u(i,j+1)+\alpha u(i+1,j+1)=\gamma u(i-1,j)+\eta u(i,j)+\\&\gamma u(i+1,j)-\alpha u(i-1,j-1)-\beta u(i,j-1)-\alpha u(i+1,j-1)+H(i,j)\end{aligned} \tag{18}$$

for $i=1,2,3,\ldots,N-1$ and $j=1,2,3,\ldots,M-1$
where

$$\alpha = h^2 - c^2k^2$$
$$\beta = 10h^2 + 2c^2k^2$$
$$\gamma = 2h^2 + 10c^2k^2$$
$$\eta = 20(h^2 - c^2k^2)$$
$$H(i,j) = 12h^2k^2q(i,j) + h^4k^2\frac{\partial^2 q}{\partial x^2}(i,j) + h^2k^4\frac{\partial^2 q}{\partial t^2}(i,j)$$

For $j = 0$, using the initial conditions Eq. (2) and central finite difference method, we obtain:

$$u(i,-1) = u(i,1) - 2k\frac{\partial u}{\partial t}(i,0) \tag{19}$$

Using Eq. (19) into Eq. (18) and putting i-1and $i+1$ in terms of i at $j = 0$, we get:

$$\begin{aligned} &2\alpha u(i-1,1) + 2\beta u(i,1) + 2\alpha u(i+1,1) = \gamma u(i-1,0) + \eta u(i,0) + \gamma u(i+1,0) \\ &+2k\alpha\frac{\partial u}{\partial t}(i-1,0) + 2k\beta\frac{\partial u}{\partial t}(i,0) + 2k\alpha\frac{\partial u}{\partial t}(i+1,0) + H(i,0) \end{aligned} \tag{20}$$

Thus, using the finite difference schemes given in Eqs. (18) and (20), which is a system of $N-1$ equations that gives an accurate numerical solution of one dimensional wave equation implicitly using the matrix inverse method.

3. Stability and Convergence Analysis

As cited in [1], [2] and [5], assume that the solution of Eq. (18) at the grid point (ih, jk) is:

$$u(i,j) = \lambda^j e^{ip\theta} \tag{21}$$

where $p = \sqrt{-1}$, θ is a real number and λ is a complex number.
Substituting Eq.(21) into Eq.(18) gives:

$$\begin{aligned} &\alpha\lambda^{j+1}e^{(i-1)p\theta} + \beta\lambda^{j+1}e^{ip\theta} + \alpha\lambda^{j+1}e^{(i+1)p\theta} = \gamma\lambda^j e^{(i-1)p\theta} + \eta\lambda^j e^{ip\theta} + \gamma\lambda^j e^{(i+1)p\theta} \\ &\qquad -\left(\alpha\lambda^{j-1}e^{(i-1)p\theta} + \beta\lambda^{j-1}e^{ip\theta} + \alpha\lambda^{j-1}e^{(i+1)p\theta}\right) + H(i,j) \\ &\Rightarrow \left(\lambda^{j+1} + \lambda^{j-1}\right)\left(\alpha e^{ip\theta}e^{-p\theta} + \beta e^{ip\theta} + \alpha e^{ip\theta}e^{p\theta}\right) = \lambda^j(\gamma e^{ip\theta}e^{-p\theta} + \eta e^{ip\theta} + \gamma e^{ip\theta}e^{p\theta}) + H(i,j) \\ &\Rightarrow (\alpha e^{2p\theta} + \beta e^{p\theta} + \alpha)\lambda^2 - (\gamma e^{2p\theta} + \eta e^{p\theta} + \gamma) + (\alpha e^{2p\theta} + \beta e^{p\theta} + \alpha) + H(i,j) = 0 \\ &\qquad a\lambda^2 + b\lambda + d = 0 \end{aligned} \tag{22}$$

where

$$a = \alpha e^{2p\theta} + \beta e^{p\theta} + \alpha,$$
$$b = -(\gamma e^{2p\theta} + \eta e^{p\theta} + \gamma) \text{ and}$$
$$d = \alpha e^{2p\theta} + \beta e^{p\theta} + \alpha + H(i,j)$$

By using Routh Hurtiz criteria and using the transformation [5], $\lambda = \dfrac{1+z}{1-z}$ in Eq.(22), we have:

$$(a-b+d)\lambda^2 + 2(a-d)\lambda + (a+b+d) = 0 \tag{23}$$

If $|\lambda| < 1$, then the difference scheme of Eq.(18) is stable. It is sufficient to show that:

$b < 0$ and $a + d > 0$

From Eq. (22),

$$\begin{aligned}
b &= -(\gamma e^{2p\theta} + \eta e^{p\theta} + \gamma) = -\left((2h^2 + 10c^2k^2)e^{2p\theta} + 20(h^2 - c^2k^2)e^{p\theta} + 2h^2 + 10c^2k^2\right) \\
&= -\left(2h^2(e^{2p\theta} + 10e^{p\theta} + 1) + 10c^2k^2e^{p\theta}(e^2 - 2) + 10c^2k^2\right) \\
a + d &= \alpha e^{2p\theta} + \beta e^{p\theta} + \alpha + \alpha e^{2p\theta} + \beta e^{p\theta} + \alpha + H(i,j) \\
&= 2\alpha e^{2p\theta} + 2\beta e^{p\theta} + 2\alpha + H(i,j) \\
&= 2h^2\left(e^{2p\theta} + 10e^{p\theta} + 1\right) + k^2\left(2c^2(2e^{p\theta} - e^{2p\theta} - 1) + 12h^2q(i,j) + h^4\frac{\partial^2 q}{\partial x^2}(i,j) + h^2k^2\frac{\partial^2 q}{\partial t^2}(i,j)\right)
\end{aligned}$$

Clearly, for sufficiently small k , $b < 0$ and $a + d > 0$.
Thus, the finite difference scheme given in Eq. (18) is absolutely stable for wave equation.
Now, expand Eq. (17) in Taylor series and replace the derivatives involving x and t for the relation,

$$\frac{\partial^2 u}{\partial t^2}(i,j) - c^2\frac{\partial^2 u}{\partial x^2}(i,j) = q(i,j) \tag{24}$$

and then we drive the local truncation error. The principal part of the local truncation error of the proposed method using Eqs. (10), (12) and (17) for the wave equation is:

$$\begin{aligned}
T(i,j) &= 12h^2k^2(\tau_6 - c^2\tau_5 + c^2\tau_3 - \tau_4) \\
&= 12h^2k^2\left\{\frac{k^4}{360}\frac{\partial^6 u}{\partial t^6}(i,j) - c^2\frac{h^4}{360}\frac{\partial^6 u}{\partial x^6}(i,j) - c^2\frac{h^2}{12}\frac{\partial^4 u}{\partial x^4}(i,j) + \frac{k^2}{12}\frac{\partial^4 u}{\partial t^4}(i,j)\right\} \\
&= -c^2h^4k^2\frac{\partial^4 u}{\partial x^4}(i,j) + h^2k^4\frac{\partial^4 u}{\partial t^4}(i,j) + O(h^6k^2 + h^2k^6)
\end{aligned}$$

Thus, $T \to 0$ as h and $k \to 0$.

So that, the scheme is consistent with the order of $O(h^4k^2 + h^2k^4)$. Hence the scheme is convergent.

4. Numerical Examples

To validate the applicability of the method, two wave equations have been considered. For each N, the point wise absolute errors are approximated by the formula, $|E(i,j)| = |u(x_i,t_j) - u(i,j)|$ for $i = 0,1,2,\ldots,N$ and $j = 0,1,2,\ldots,M$, where $u(x_i,t_j)$ and $u(i,j)$ are the exact and computed approximate solution of the given problem respectively, at the nodal point (i,j).

Example 1: Consider the one dimensional wave equation given as [5]

$$u_{tt} - c^2 u_{xx} = 0, \quad 0 \le x \le l,\ t \ge 0$$

$c = 1$, with initial conditions: $u(x,0) = \cos(\pi x)$ and $u_t(x,0) = 0$
and boundary conditions: $u(0,t) = \cos(\pi t)$ and $u(l,t) = -\cos(\pi t)$

The exact solution for this problem is $u(x,t) = \frac{1}{2}\cos(\pi(x+t)) + \frac{1}{2}\cos(\pi(x-t))$. The numerical solution in terms of point wise absolute errors by comparing with the previous method is given in Table 1.

Table 1: The comparison of absolute errors for Example 1 at different values of the step size in the x-direction h and time step size k = *0.0001*.

Rashidina and Mohsenyzadeha [5]				Our method			
x_i	t_i	Method I	Method II	*h = 0.05*	*h = 0.025*	*h = 0.01*	*h = 0.005*
0.05	0.0003	5e-10	1e-9	1.2279e-12	6.8723e-14	1.1102e-15	1.1102e-15
0.05	0.0005	1e-10	1e-9	3.4114e-12	1.9107e-13	4.1078e-15	3.9968e-15
0.1	0.0003	2.1e-9	7e-10	1.0612e-12	6.6391e-14	2.7756e-15	5.5511e-16
0.1	0.0005	9.7e-9	4e-10	2.9475e-12	1.8463e-13	7.7716e-15	9.9920e-16
0.2	0.0003	6.2e-9	1e-10	9.1205e-13	5.6843e-14	1.7764e-15	5.5511e-16
0.2	0.0005	1.3e-8	1e-10	2.5338e-12	1.5843e-13	4.8850e-15	1.3323e-15

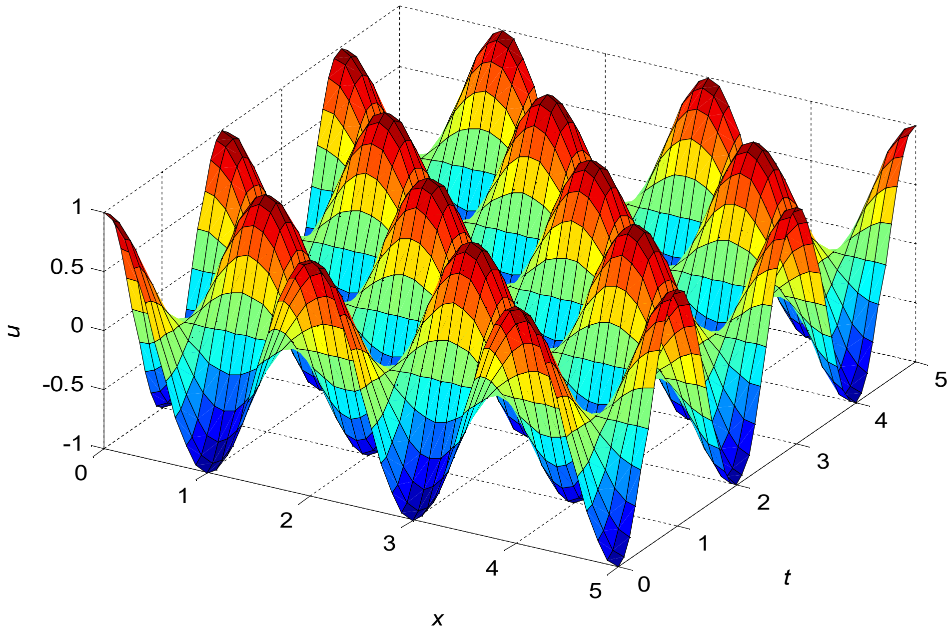

Fig. 1. Space-time graph of the solution for Example 1 when $0 \le t \le 5$, $k = 0.1 = h$.

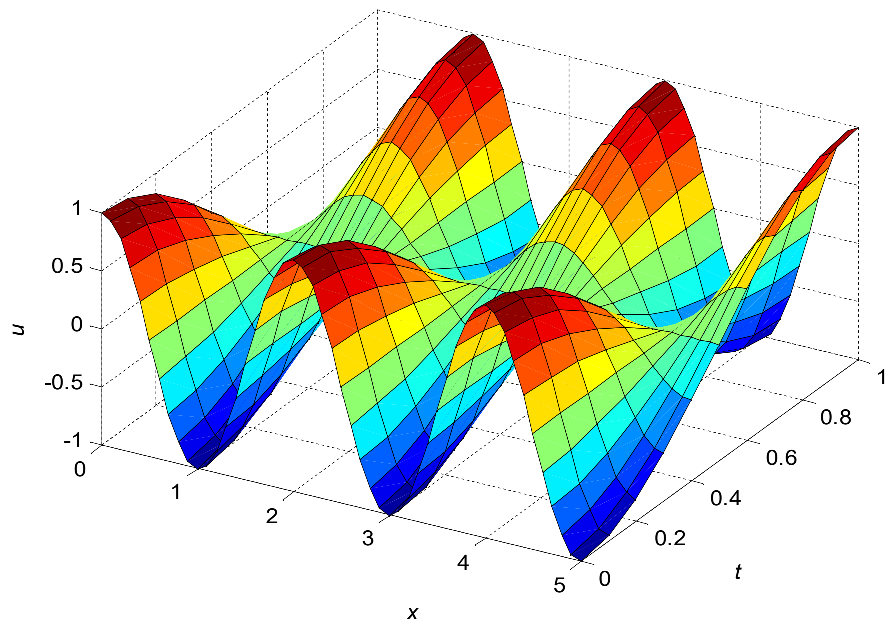

Fig. 2. Space-time graph of the solution for Example 1, when $0 \le t \le 1$ and $k = 0.1 = h$.

Example 2: Consider the non-homogeneous one dimensional wave equation given in [2]

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = \sin x, \quad 0 < x < \pi$$

with the boundary conditions: $u(0,t) = 0 = u(\pi,t); \quad t > 0$

and initial conditions: $u(x,0)=\sin x, \quad \dfrac{\partial u}{\partial t}(x,0)=\sin x$

Exact solution given as: $u(x,t)=\sin x(1+\sin t)$. The numerical solution in terms of absolute errors is given in Table 2.

Table 2: The comparison of absolute errors for Example 2 at $h=0.1\pi$ and $t=0.02$.

Zafar et al. [2]				Our Method	
x_i	Exact	FEM	Absolute error	Fourth order CFDM	Absolute error
0.000000000	0.000000000	0.000000000	0.000000000	0	0
0.314159265	0.315196922	0.314917808	0.000279114	0.3151973325794360	4.1033e-07
0.628318531	0.599954017	0.599009267	0.000530907	0.5995409541370464	7.8050e-07
0.942477796	0.825196256	0.824465508	0.000730748	0.8251973298562680	1.0743e-06
1.256637061	0.970076379	0.969217354	0.000859025	0.9700776414412834	1.2629e-06
1.57796327	1.019998667	1.019095436	0.000903231	1.019999994553664	1.3279e-06
1.884955592	0.970076379	0.969217354	0.000859025	0.9700776414412835	1.2629e-06
2.199114858	0.825196256	0.824465508	0.000730748	0.8251973298562682	1.0743e-06
2.513274123	0.599954017	0.599009267	0.000530907	0.5995409541370464	7.8050e-07
2.827433388	0.315196922	0.314917808	0.000279114	0.3151973325794360	4.1033e-07
3.141582654	0.000000000	0.000000000	0.000000000	0	0

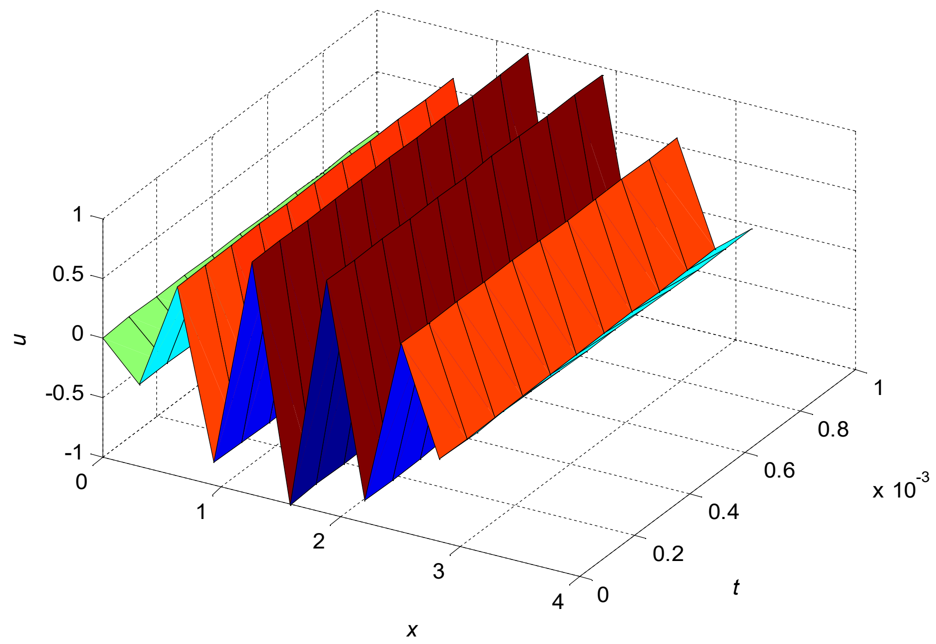

Fig. 3. Space-time graph of the solution for Example 2, $h=0.1\pi$ and $k=0.01$.

5. Discussion and Conclusion

In this paper, we presented fourth order compact finite difference method for solving quadratic one-dimensional wave equations. To further collaborate the applicability of the proposed method; tables of point wise absolute error and graphs have been plotted for Examples 1 and 2, for the exact solution versus the numerical solutions at different values of mesh size h and k. Table 1, shows the absolute errors obtained by fourth order compact finite difference method have been compared with absolute errors obtained by [5] and it show that the point wise absolute error decreases as the mesh size h decreases, which in turn shows the convergence of the computed solution. Table 2, also shows the absolute errors obtained by the present method have been compared with absolute errors obtained by [2]. Generally, the present method is computationally: stable, effective, simple to use, convergent and give accuracy solution than some previously existing methods.

References

[1] Jain M.K., Iyengar S.R.K, Jain R. K., computational methods for partial differential equations, New age international publishers, 2007.

[2] Zafar Z. U., Pervaiz A., Ahmed M.O. and Rafiq M., Finite Element Model for Linear Second Order One Dimensional Inhomogeneous Wave Equation, Pak. *J. Engg. & Appl. Sci. Vol. 17, Jul.,* 2015 (*p.* 58–63).

[3] Rajni Arora, Suruchi Singh, Swarn Singh, Exponential B-Spline Collocation Method For The Numerical Solution Of One-Space Dimensional Nonlinear Wave Equation With Strong Stability Preserving Time Integration, 2nd international conference on recent innovations in science, engineering and management, 22 November 2015, www.conferencewarld.in ISBN: 978-81-931039-9-9.

[4] Abbas Saadatmandi, Mehdi Dehghan, Numerical Solution of the One-dimensional Wave Equation with an Integral Condition, Published online 14 July 2006 in Wiley Inter Science (www.interscience.wiley.com). DOI 10.1002/num.20177.

[5] Rashidinia J. and Mohsenyzadeha, Numerical Solution of One-Dimensional Heat and Wave Equation by Non-Polynomial Quintic Spline, *International Journal of Mathematical Modelling* & *Computations* Vol. 05, No. 04, (2015), 291- 305.

[6] Won Y. Yang, Wenwu Cao, Tae-Sang Chung, John Morris, Applied Numerical methods using MATLAB, Wiley student Edition, 2013.