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On double summability methods $\left|\mathscr{A}_f\right|_k$ and $|C,0,0|_s$

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Double series, Factorable matrix, Absolute Cesàro summability

Abstract — Recently, for single series, the necessary and sufficient conditions for $|C,0| \Rightarrow \left|A_f\right|_k$ and vise versa, and $\left|A_f\right| \Rightarrow |C,0|_k$ and vise versa have been established, where $1 < k < \infty$ and A is a factorable matrix. The present study extends these results to double summability, and also provides some new results.

Subject Classification (2020): 40D25, 40F05.

1. Introduction

The summability theory has an important role in applied mathematics, engineering sciences, and analysis essentially in functional analysis, approximation theory, calculus, quantum mechanics, probability theory, Fourier analysis. The main purpose of the theory is to assign a limit value for divergent series or sequences by using a transformation which is given by the most general linear mappings of infinite matrices. The reason why matrices are used for a general linear operator is that a linear operator from a sequence space to another one can be given by an infinite matrix. In this regard, the literature in the field of summability theory continues to develop not only on the generation of sequence spaces through the matrix domain of a particular matrices such as Hölder, Euler, Cesàro, Hausdorff, Nörlund and weighted mean matrices and on the investigation of their topological, algebraic structures and matrix transformations but also on examinations about new series spaces derived by several absolute summability methods from a different perspective (see, [1–9]). Besides of all these, recently, a many of new article using by double series are also placed in literature. For instance, in [10], the characterizations of the equivalence $|C,0,0|_k \Leftrightarrow \left|R,p_n,q_n\right|_k$ for doubly sequences given by Sarıgöl and the necessary and sufficient conditions for the equivalence of absolute weighted mean summability methods of doubly infinite series are given in [11],(see also [12–16]). The main purpose of this paper is to extend certain theorems given by Hazar and Gökçe in [7] to double infinite series a different approach.

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2. Preliminary

Let $\sum x_v$ be an infinite series of complex numbers with partial sums s_n and σ_n^α denote the $n-$th term of the Cesàro $(C;\alpha)$-transform of $s=(s_n)$. If (see [1])

$$\sum_{n=1}^{\infty} n^{k-1}|x_n|^k<\infty$$

then, it is said that the series $\sum x_v$ is summable $|C,0|_k, k\geq 1$.

Let $A_f=(a_{nv})$ be a factorable matrix i.e., the lower triangular with entries

$$a_{nv}=\begin{cases}\widehat{a}_n a_v, & 0\leq v\leq n\\ 0, & v>n\end{cases} \tag{2.1}$$

where $(\widehat{a}_n)$ and (a_n) are any sequences of real numbers. The series $\sum x_v$ is said to be summable $\left|A_f\right|_k$, $k\geq 1$, if

$$\sum_{n=1}^{\infty} n^{k-1}\left|\widehat{a}_n\sum_{v=1}^{n} a_v x_v\right|^k<\infty \tag{2.2}$$

[9].

Let Λ and Γ be two methods of summability. Γ is said to include Λ if every series summable by the method Λ is also summable by the method Γ and it is written $\Lambda\Rightarrow\Gamma$. Also, Λ and Γ are said to be equivalent if each methods includes the other and it is written $\Lambda\Leftrightarrow\Gamma$.

Through the whole paper k^* denotes the conjugate index of k, i.e., $\frac{1}{k}+\frac{1}{k^*}=1$. The following theorems are given by Hazar and Gökçe [7] for single series:

Theorem 2.1. [7] Let $1<k<\infty$ and A be a factorable matrix given by (2.1) such that $\widehat{a}_v$, $a_v\neq 0$ for all v. Then, $\left|A_f\right|_k|\Rightarrow|C,0|$ if and only if

$$\sum_{v=1}^{\infty}\frac{1}{v}\left\{\frac{1}{|\widehat{a}_v|}\left(\frac{1}{|a_v|}+\frac{1}{|a_{v+1}|}\right)\right\}^{k^*}<\infty$$

Theorem 2.2. [7] Let $1<k<\infty$ and A be a factorable matrix. Then, $|C,0|_k\Rightarrow\left|A_f\right|$ if and only if

$$\sum_{v=1}^{\infty}\frac{1}{v}\left(|a_v|\sum_{n=v}^{\infty}|\widehat{a}_n|\right)^{k^*}<\infty$$

Theorem 2.3. [7] Let $1\leq k<\infty$ and A be a factorable matrix. Then, $|C,0|\Rightarrow\left|A_f\right|_k$ if and only if

$$\sum_{n=v}^{\infty} n^{k-1}|\widehat{a}_n a_v|^k=O(1) \text{ as } v\to\infty$$

Theorem 2.4. [7] Let $1\leq k<\infty$ and A be a factorable matrix such that $\widehat{a}_v$, $a_v\neq 0$ for all v. Then, $\left|A_f\right|\Rightarrow |C,0|_k$ if and only if

$$\frac{v^{k-1}}{|\widehat{a}_v|^k}\left(\frac{1}{|a_v|^k}+\frac{1}{|a_{v+1}|^k}\right)=O(1) \text{ as } v\to\infty$$

For any double sequence $\left(x_{ij}\right)$, define

$$\Delta_{11}x_{ij} = x_{ij} - x_{i-1,j} - x_{i,j-1} + x_{i-1,j-1};\ i,j \geq 1$$

Let $\sum\sum x_{ij}$ be a double infinite series with partial sums s_{nm}. By $t_{nm}^{\alpha\beta}$, we denote the double Cesàro means $\left(C,\alpha,\beta\right)$ of the double sequence (s_{nm}), that is

$$t_{nm}^{\alpha\beta} = \frac{1}{A_n^{\alpha}A_m^{\beta}}\sum_{i=0}^{n}\sum_{j=0}^{m}A_{n-i}^{\alpha-1}A_{m-j}^{\beta-1}s_{ij} \tag{2.3}$$

where

$$A_0^{\alpha} = 1,\ A_n^{\alpha} = \frac{(\alpha+1)(\alpha+2)\cdots(\alpha+n)}{n!},\ A_{-n}^{\alpha} = 0 \text{ for } n \geq 1$$

The series $\sum\sum x_{ij}$ is said to be summable $\left|C;\alpha;\beta\right|_k$, $k \geq 1$, if

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}(nm)^{k-1}\left|\Delta_{11}t_{nm}^{\alpha\beta}\right|^k < \infty \tag{2.4}$$

[12, 13]. In the special case for $\beta = 0$ and $\alpha = 0$ the summability method $\left|C;\alpha;\beta\right|_k$, reduces to $|C;0;0|_k$.

A double infinite matrix is called factorable if there exist sequences $a_n^{(1)}, \hat{a}_n^{(1)}, a_n^{(2)}, \hat{a}_n^{(2)}$ such that

$$a_{nmij} = \begin{cases} a_i^{(1)}\hat{a}_n^{(1)}a_j^{(2)}\hat{a}_m^{(2)}, & 0 \leq i \leq n, 0 \leq j \leq m \\ 0, & \text{otherwise} \end{cases}$$

where $a_n^{(1)}, \hat{a}_n^{(1)}, a_n^{(2)}, \hat{a}_n^{(2)}$ are any sequences of real numbers.

We say that the series $\sum\sum x_{ij}$ is summable $\left|\mathscr{A}_f\right|_k$, $k \geq 1$, if

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}(nm)^{k-1}\left|\hat{a}_n^{(1)}\hat{a}_m^{(2)}\sum_{i=1}^{n}a_i^{(1)}\sum_{j=1}^{m}a_j^{(2)}x_{ij}\right|^k < \infty \tag{2.5}$$

where $\mathscr{A}$ is factorable matrix.

Let us consider the space

$$\mathscr{L}_k = \left\{x = (x_{ij}) \in \Omega : \sum_{i,j=0}^{\infty}\left|x_{ij}\right|^k < \infty\right\}, 1 \leq k < \infty$$

which is the set of double sequences corresponding to the well known space ℓ_k of single sequences [16], where Ω is the set of all double sequences of complex numbers. Also, in the case $k = 1$ the space reduces to $\mathscr{L}$, studied by Zeltser [17]. On the other hand, $\mathscr{L}_k$ is the Banach space [16] according to its natural norm

$$\|x\|_{\mathscr{L}_k} = \left(\sum_{i,j=0}^{\infty}\left|x_{ij}\right|^k\right)^{1/k}, 1 \leq k < \infty$$

and, for $k = \infty$, $\mathscr{L}_\infty$ is the space of all bounded double sequences, which is a Banach space with the norm $\|x\|_{\mathscr{L}_\infty} = \sup_{i,j}\left|x_{ij}\right|$.

The following lemmas play significant role in our paper:

Lemma 2.5. [14] Let $k \geq 1$ and $A = (a_{mnrs})$ be a four dimensional infinite matrix of complex numbers. Then, $A \in (\mathscr{L}, \mathscr{L}_k)$ if and only if

$$\sum_{m,n=0}^{\infty} |a_{mnrs}|^k = O(1) \text{ as } r,s \to \infty \tag{2.6}$$

Lemma 2.6. [14] Let $1 \leq k < \infty$ and $A = \left(a_{mnij}\right)$ be an four dimensional infinite matrix of complex numbers. Define $W_k(A)$ and $w_k(A)$ by

$$W_k(A) = \sum_{r,s=0}^{\infty} \left(\sum_{m,n=0}^{\infty} |a_{mnrs}| \right)^k \tag{2.7}$$

and

$$w_k(A) = \sup_{MXN} \sum_{r,s=0}^{\infty} \left| \sum_{(m,n)\in MXN} a_{mnrs} \right|^k \tag{2.8}$$

where M and N are finite subsets of natural numbers. Then, the following statements are equivalent:

i. $W_{k^*}(A) < \infty$

ii. $A \in (\mathscr{L}_k, \mathscr{L})$

iii. $A^t \in (\mathscr{L}_\infty, \mathscr{L}_{k^*})$

iv. $w_{k^*}(A) < \infty$

Lemma 2.7. [11] Let $k > 0$, (p_n) and (q_n) are positive sequences such that $P_n = p_0 + p_1 + ... + p_n \to \infty, n \to \infty$, $(P_{-1} = p_{-1} = 0)$. Then, there exists two strictly positive constans M and N, depending only on k, such that

$$\frac{M}{P_{i-1}Q_{j-1}} \leq \sum_{n=i}^{\infty}\sum_{m=j}^{\infty} \frac{p_n q_m}{P_n P_{n-1}^k Q_m Q_{m-1}^k} \leq \frac{N}{P_{i-1}Q_{j-1}}$$

for all $i, j \leq 1$, where M, N are independent of $(p_m), (q_n)$.

3. Main Results

In this section, we prove the following theorems mentioned the relations between the summability methods $|C,0,0|_k$, $\left|\mathscr{A}_f\right|_k$, for several case of k.

Theorem 3.1. Let $1 < k < \infty$ and $\mathscr{A}$ be factorable matrix given by (2.1). Then, $|C,0,0|_k \Rightarrow \left|\mathscr{A}_f\right|$ if and only if

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \frac{1}{ij} \left(\left| a_i^{(1)} a_j^{(2)} \right| \sum_{n=i}^{\infty}\sum_{m=j}^{\infty} \left| \hat{a}_n^{(1)} \hat{a}_m^{(2)} \right| \right)^{k^*} < \infty \tag{3.1}$$

Proof.

Let $y_{nm} = (nm)^{1/k^*} t_{nm}^{00}$ where $\left(t_{nm}^{00}\right)$ is as in (2.3), i.e.,

$$t_{nm}^{00} = \frac{1}{A_n^0 A_m^0} \sum_{i=0}^{n}\sum_{j=0}^{m} A_{n-i}^{-1} A_{m-j}^{-1} s_{ij} = x_{nm}$$

and

$$T_{nm} = \hat{a}_n^{(1)} \hat{a}_m^{(2)} \sum_{i=1}^{n} a_i^{(1)} \sum_{j=1}^{m} a_j^{(2)} x_{ij}$$

Then, the series $\sum\sum x_{ij}$ is summable $\left|\mathscr{A}_f\right|$ and $|C,0,0|_k$ if and only if $(T_{nm}) \in \mathscr{L}$ and $(y_{nm}) \in \mathscr{L}_k$, respectively, and

$$T_{nm} = \hat{a}_n^{(1)} \hat{a}_m^{(2)} \sum_{i=1}^{n} a_i^{(1)} \sum_{j=1}^{m} a_j^{(2)} (ij)^{-1/k^*} y_{ij} = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} c_{nmij} y_{ij}$$

where $C = (c_{nmij})$ is defined by

$$c_{nmij} = \begin{cases} (ij)^{-1/k^*} a_i^{(1)} a_j^{(2)} \hat{a}_n^{(1)} \hat{a}_m^{(2)}, & 1 \le i \le n, 1 \le j \le m \\ 0, & \text{otherwise} \end{cases}$$

Therefore, $|C,0,0|_k \Rightarrow \left|\mathscr{A}_f\right|$ if and only if $C = (c_{nmij}) \in (\mathscr{L}_k, \mathscr{L})$, or equivalently, by Lemma 2.6,

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \frac{1}{ij} \left(\left| a_i^{(1)} a_j^{(2)} \right| \sum_{n=i}^{\infty}\sum_{m=j}^{\infty} \left| \hat{a}_n^{(1)} \hat{a}_m^{(2)} \right| \right)^{k^*} < \infty$$

which concludes the proof.

Theorem 3.2. Let $1 \le k < \infty$ and $\mathscr{A}$ be factorable matrix such that $\hat{a}_n^{(1)}, a_n^{(1)}, \hat{a}_n^{(2)}, a_n^{(2)} \ne 0$ for all n. Then, $\left|\mathscr{A}_f\right| \Rightarrow |C,0,0|_k$ if and only if

$$\frac{(nm)^{k-1}}{\left|\hat{a}_n^{(1)} \hat{a}_m^{(2)}\right|^k} \left(\frac{1}{\left|a_n^{(1)}\right|^k} + \frac{1}{\left|a_{n+1}^{(1)}\right|^k} \right) \left(\frac{1}{\left|a_m^{(2)}\right|^k} + \frac{1}{\left|a_{m+1}^{(2)}\right|^k} \right) = O(1) \tag{3.2}$$

as $n, m \to \infty$.

Proof.
Let $T_{nm} = \widehat{a}_n \widehat{b}_m \sum\limits_{i=1}^{n} a_i \sum\limits_{j=1}^{m} b_j x_{ij}$. A few basic calculations give

$$x_{nm} = \frac{1}{a_n^{(1)} a_m^{(2)}} \left(\frac{T_{nm}}{\hat{a}_n^{(1)} \hat{a}_m^{(2)}} - \frac{T_{n-1,m}}{\hat{a}_{n-1}^{(1)} \hat{a}_m^{(2)}} - \frac{T_{n,m-1}}{\hat{a}_n^{(1)} \hat{a}_{m-1}^{(2)}} + \frac{T_{n-1,m-1}}{\hat{a}_{n-1}^{(1)} \hat{a}_{m-1}^{(2)}} \right) \tag{3.3}$$

Moreover,

$$\begin{aligned} y_{nm} = (nm)^{1/k^*} t_{nm}^{00} &= (nm)^{1/k^*} \frac{1}{a_n^{(1)} a_m^{(2)}} \left(\frac{T_{nm}}{\hat{a}_n^{(1)} \hat{a}_m^{(2)}} - \frac{T_{n-1,m}}{\hat{a}_{n-1}^{(1)} \hat{a}_m^{(2)}} - \frac{T_{n,m-1}}{\hat{a}_n^{(1)} \hat{a}_{m-1}^{(2)}} + \frac{T_{n-1,m-1}}{\hat{a}_{n-1}^{(1)} \hat{a}_{m-1}^{(2)}} \right) \\ &= \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} d_{nmij} T_{ij} \end{aligned}$$

where $D = (d_{nmij})$

$$d_{nmij} = \begin{cases} (nm)^{1/k^*} \frac{1}{a_n^{(1)} a_m^{(2)} \hat{a}_n^{(1)} \hat{a}_m^{(2)}}, & i = n, j = m \\ -(nm)^{1/k^*} \frac{1}{a_n^{(1)} a_m^{(2)} \hat{a}_{n-1}^{(1)} \hat{a}_m^{(2)}}, & i = n-1, j = m \\ -(nm)^{1/k^*} \frac{1}{a_n^{(1)} a_m^{(2)} \hat{a}_n^{(1)} \hat{a}_{m-1}^{(2)}}, & i = n, j = m-1 \\ (nm)^{1/k^*} \frac{1}{a_n^{(1)} a_m^{(2)} \hat{a}_{n-1}^{(1)} \hat{a}_{m-1}^{(2)}}, & i = n-1, j = m-1 \end{cases}$$

Thus, we have $\left|\mathscr{A}_f\right| \Rightarrow |C,0,0|_k$ if and only if $D \in (\mathscr{L}, \mathscr{L}_k)$. It follows from Lemma 2.5 that Equation (3.2) holds.

Corollary 3.3. Let $1 < k < \infty$ and $\mathscr{A}$ be factorable matrix such that $\hat{a}_n^{(1)}, a_n^{(1)}, \hat{a}_n^{(2)}, a_n^{(2)} \ne 0$ for all n. Then, $\left|\mathscr{A}_f\right| \Leftrightarrow |C,0,0|_k$ if and only if Equations (3.1) and (3.2) hold.

Theorem 3.4. Let $1 < k < \infty$ and $\mathscr{A}$ be factorable matrix such that $\hat{a}_n^{(1)}, a_n^{(1)}, \hat{a}_n^{(2)}, a_n^{(2)} \neq 0$ for all n. Then, $\left|\mathscr{A}_f\right|_k \Rightarrow |C,0,0|$ if and only if

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\frac{1}{ij}\left\{\frac{1}{\left|\hat{a}_i^{(1)}\hat{a}_j^{(2)}\right|}\left(\frac{1}{\left|a_i^{(1)}a_j^{(2)}\right|}+\frac{1}{\left|a_{i+1}^{(1)}a_j^{(2)}\right|}+\frac{1}{\left|a_i^{(1)}a_{j+1}^{(2)}\right|}+\frac{1}{\left|a_{i+1}^{(1)}a_{j+1}^{(2)}\right|}\right)\right\}^{k^*}<\infty \tag{3.4}$$

Proof.
Let $T'_{nm} = (nm)^{1/k^*} T_{nm}$. Using (3.3), we have

$$\begin{aligned} t_{nm}^{00} &= \frac{1}{a_n^{(1)}a_m^{(2)}}\left(\frac{(nm)^{-1/k^*}T'_{nm}}{\hat{a}_n^{(1)}\hat{a}_m^{(2)}}-\frac{((n-1)m)^{-1/k^*}T'_{n-1,m}}{\hat{a}_{n-1}^{(1)}\hat{a}_m^{(2)}}-\frac{(n(m-1))^{-1/k^*}T'_{n,m-1}}{\hat{a}_n^{(1)}\hat{a}_{m-1}^{(2)}}+\frac{((n-1)(m-1))^{-1/k^*}T'_{n-1,m-1}}{\hat{a}_{n-1}^{(1)}\hat{a}_{m-1}^{(2)}}\right)\\ &= \sum_i\sum_j e_{nmij}T'_{nm} \end{aligned}$$

where

$$e_{nmij} = \begin{cases} \frac{(nm)^{-1/k^*}}{a_n^{(1)}a_m^{(2)}\hat{a}_n^{(1)}\hat{a}_m^{(2)}}, & n=i, m=j\\ \frac{((n-1)m)^{1/k^*}}{a_n^{(1)}a_m^{(2)}\hat{a}_{n-1}^{(1)}\hat{a}_m^{(2)}}, & n-1=i, m=j\\ \frac{(n(m-1))^{1/k^*}}{a_n^{(1)}a_m^{(2)}\hat{a}_n^{(1)}\hat{a}_{m-1}^{(2)}}, & n=i, m-1=j\\ \frac{((n-1)(m-1))^{1/k^*}}{a_n^{(1)}a_m^{(2)}\hat{a}_{n-1}^{(1)}\hat{a}_{m-1}^{(2)}}, & n-1=i, m-1=j \end{cases}$$

Then, we get that $\sum\sum x_{ij}$ is summable $|C,0,0|$ whenever $\sum\sum x_{ij}$ is summable $\left|\mathscr{A}_f\right|_k$ if and only if $E = (e_{nmij}) \in (\mathscr{L}_k, \mathscr{L})$. So, we imply Equation (3.4) with Lemma 2.6. This concludes the proof.

Theorem 3.5. Let $1 \le k < \infty$ and $\mathscr{A}$ be factorable matrix. Then, $|C,0,0| \Rightarrow \left|\mathscr{A}_f\right|_k$ if and only if

$$\sum_{n=i}^{\infty}\sum_{m=j}^{\infty}(nm)^{k-1}\left|\hat{a}_n^{(1)}\hat{a}_m^{(2)}a_i^{(1)}a_j^{(2)}\right|^k = O(1) \text{ as } i,j\to\infty \tag{3.5}$$

Since the theorem can be proved by the similar way with Theorem 3.2, it has been left to reader.

Corollary 3.6. Let $1 < k < \infty$ and $\mathscr{A}$ be factorable matrix such that $\hat{a}_n^{(1)}, a_n^{(1)}, \hat{a}_n^{(2)}, a_n^{(2)} \neq 0$ for all n. Then, $\left|\mathscr{A}_f\right|_k \Leftrightarrow |C,0,0|$ if and only if Equations (3.4) and (3.5) hold.

It may be noticed that if we take $\hat{a}_n^{(1)} = p_n \backslash P_nP_{n-1}$, $a_n^{(1)} = P_{n-1}$ and $\hat{a}_n^{(2)} = q_n\backslash Q_nQ_{n-1}$, $a_n^{(2)} = Q_{n-1}$, then the summability method $\left|\mathscr{A}_f\right|_k$ is reduced to the double absolute Riesz summability method $\left|R,p_n,q_n\right|_k$. Hence, we get the following results:

Corollary 3.7. Let $k \ge 1$. Then, $|C,0,0| \Rightarrow \left|R,p_n,q_n\right|_k$ if and only if

$$\sum_{n=i}^{\infty}\sum_{m=j}^{\infty}(nm)^{k-1}\left(\frac{p_nq_m}{P_nP_{n-1}Q_mQ_{m-1}}\right)^k = O\left(\frac{1}{\left(P_{i-1}Q_{j-1}\right)^k}\right)$$

Corollary 3.8. Let $k \ge 1$. Then, $\left|R,p_n,q_m\right| \Rightarrow |C,0,0|_k$ if and only if

$$(ij)^{1/s^*}P_iQ_j = O\left(p_iq_j\right) \text{ as } i,j\to\infty$$

Moreover, Equations (3.1) and (3.4) are equivalent to

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\frac{1}{ij}<\infty$$

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\frac{1}{ij}\left(\frac{P_i}{p_i}\right)^{k^*}\left(\frac{Q_j}{q_j}\right)^{k^*}<\infty$$

which are impossible. Thus, we have the following result.

Corollary 3.9. If $k>1$, then $\left|R,p_n,q_m\right|_k \not\Rightarrow |C,0,0|$ and also $|C,0,0|_k \not\Rightarrow \left|R,p_n,q_m\right|$.

4. Conclusion

This paper aimed to adapt the summability method $\left|\mathscr{A}_f\right|_k$ to double series and extend some theorems given for single series to double series. The relations between other summability methods and $\left|\mathscr{A}_f\right|_k$ are worth studying.

Author Contributions

The author read and approved the last version of the paper.

Conflicts of Interest

The author declares no conflict of interest.

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